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On production of η' mesons in pp collisions close to threshold

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Abstract. The near threshold behaviour of the reaction cross section for $pp \to pp\eta'$, recently measured in experiments at COSY and SATURNE, is analyzed. The interaction in the pp as well as in the $\eta'p$ final states is taken into account. The suppression of the total cross section for this process at excess energies $Q < 3$ MeV observed in these experiments is interpreted as an evidence for a strong repulsive $\eta'p$ interaction.

PACS. 13.75.-n Hadron-induced low- and intermediate-energy reactions and scattering (energy $\leq$ GeV) – 25.10.+s Nuclear reactions involving few-nucleon systems – 25.40.-h Nucleon-induced reactions

1 Introduction

Production of mesons in nucleon-nucleon (NN) collisions has been a subject of interest for both experiment and theory for decades. For energies near threshold these reactions are characterized by a relatively large momentum transfer between the nucleons in the production process. Indeed the invariant 4-momentum transfer t exactly at threshold is $t = -m\mu$, where m is the nucleon mass and μ the mass of the produced meson. Because of the required large momentum transfer the size of the production region is small, and therefore the meson may be considered as being produced from a point-like source. Then the energy dependence of the cross section is mainly determined by the interaction between the (slowly moving) particles in the final state [1,2]. This specific feature of high-momentum-transfer reactions is already seen in the pion-production reactions near threshold. In fact, in this case the energy dependence of the cross section is given solely by the phase-space behaviour plus the pp final state interaction (FSI) [3,4]. Since the πN interaction is rather weak in the (s-wave) partial waves relevant for threshold production whereas the proton-proton (pp) interaction is very strong it is not surprising that the latter plays the leading role in the FSI effects.

For the production of heavier mesons such as the η- and η' mesons Watson's argument [1] should work even better. However, it is an open question whether the FSI effects will again be only due to the pp interaction like in case of pion production. Indeed, a series of recent measurements on η production in NN collisions indicates that the ηp FSI might have a non-negligible influence on the energy dependence of the total cross section leading to an enhancement close to threshold for $pp \to pp\eta$ [5] as well as for $pn \to d\eta$ [6]. This effect could be induced by an attractive ηp force resulting from the presence of the $S_{11}(1535)$ resonance.

Recently the total cross section for the reaction $pp \to pp\eta'$ close to threshold was measured for the first time [7,8]. It was found that the cross section for this reaction at excess energies $Q \approx 1.5$ MeV is noticeably suppressed as compared to the prediction that follows from the phase-space behaviour plus pp FSI [7]. In the present paper we want to investigate possible origins of the experimentally observed suppression of the η'-production cross section close to threshold. In analogy to the η-production case mentioned above we assume that it is the $\eta'p$ interaction which is responsible for the modification of the energy dependence of the total cross section. However, contrary to the η-production case where an enhancement of the cross section is observed, now a reduction of the cross section near threshold is required. Specifically a mechanism is needed which compensates the strong pp final state attraction because, as can be seen in paper of Moskal et al. [7], the data follow pretty much the trend of the pure phase space. In practice we see three different ways to explain the observed suppression of the η'-production cross section near threshold:

i) The $\eta'p$ amplitude at threshold is relatively large and is of opposite sign to that of the pp amplitude implying an effectively repulsive $\eta'p$ interaction [9]. Indeed the assumption of a rather strong and repulsive $\eta'p$ interaction is not unreasonable. It might be due to the presence of a resonance in the $\eta'p$ system as indicated by a recent study of η' photo production on protons [10].

ii) The $\eta' p$ amplitude at threshold is not large, but the range of the effective $\eta' p$ potential is extremely small. This could be the case if the η' meson has a large admixture from a two-gluon state so that the range of the $\eta' p$ interaction is determined essentially by the range of the two-gluon effective potential which is expected to be small, say of the order of $0.2 - 0.4$ fm.

iii) The $\eta' p$ amplitude is large and strongly absorptive due to the presence of many open channels. There is some experimental evidence for this, see, e.g., [10] in which resonances with rather large widths were required for the description of near-threshold data on photo production of η' mesons on protons.

In the present paper we will study the consequences of these features of the $\eta' p$ amplitude. Specifically we are interested to see whether they indeed yield a suppression of the production cross section very close to threshold. Furthermore we would like to find out whether these features lead to any differences in the predicted observables which would allow to distinguish between them. We want to emphasize that the prime goal of our investigation is to provide possible explanations for the observed energy dependence of the η'-production cross section. Thus, it is complementary to the works of Sibirtsev et al. [11] and Bernard et al. [12] which aim at an understanding of the actual production mechanism.

In Sect. II we present our formalism for calculating the $pp \to pp\eta'$ production amplitude taking into account the pp- as well as the $\eta' p$ FSI. Coulomb corrections are also included. In Sect. III we make a comparison of our model calculation with the existing experimental data on the total production cross section. Furthermore, we present some predictions for the invariant mass distributions based on different assumptions about the $\eta' p$ scattering amplitude and the effective range of the $\eta' p$ force. Our results are summarized in Sect. IV.

2 Theoretical approach

We are interested only in studying the region of small excess energies $Q \leq 15$ MeV. Therefore we use nonrelativistic kinematics in the final state. Furthermore we consider only s-waves in the final state. Parity conservation and the Pauli principle then tell us that there is just one partial-wave amplitude that can contribute, namely the transition ${}^3P_0 \to {}^1S_0 s$ in the standard nomenclature. In the CM-frame the three-momenta $\boldsymbol{p}_i$ of each particle in the final state are related by the constraint $\sum_{i=1}^3 \boldsymbol{p}_i = 0$. It is also convenient to introduce the relative momenta $\boldsymbol{q}_{ij}$ between the two particles i and j, $\boldsymbol{q}_{ij} = (m_i \boldsymbol{p}_j - m_j \boldsymbol{p}_i)/(m_i + m_j)$. Notice that the phase volume can be expressed in terms of any chosen pair of momenta $\boldsymbol{q}_{ij}$ and $\boldsymbol{p}_k$. For example in terms of $\boldsymbol{q}_{12}$ and $\boldsymbol{p}_3$ the differential cross section is given by

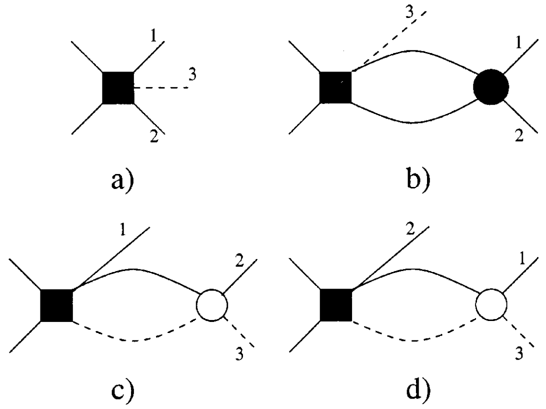

Fig. 1. Diagrammatic representation of the rescattering graphs included in our model calculation of the reaction $pp \to pp\eta'$. The filled square, filled circle and open circle represent the elementary η'-production amplitude and the pp- and $\eta' p$-scattering amplitudes, respectively

$$d\sigma = \frac{| M |^2}{4I\pi(4\pi)^3(2m + m_{\eta'})} \frac{1}{m_{12}} \times \sqrt{2\mu_3(Q - q_{12}^2/2/m_{12})} q_{12}^2 dq_{12} d\Omega_{\boldsymbol{p}_3 \boldsymbol{q}_{12}}, \quad (1)$$

where $m_{ij} = m_i m_j/(m_i + m_j)$, $\mu_i = m_i(m_j + m_k)/(m_i + m_j + m_k)$ are reduced masses. The excess energy Q is then given by, e.g., $Q = \frac{q_{12}^2}{2m_{12}} + \frac{p_3^2}{2\mu_3}$. $I = 2p_0\varepsilon_0$, with p_0 and ε_0 being the CM-momentum and energy of the incoming proton, respectively. The reaction amplitude M in (1) is related to the S-matrix by

$$S_{fi} = \delta_{fi} + i(2\pi)^4 \delta^4(P_f - P_i) \frac{M_{fi}}{2\varepsilon_0 \sqrt{2\varepsilon_{1f} 2\varepsilon_{2f} 2\varepsilon_{3f}}}, \quad (2)$$

where $\varepsilon_{\alpha f}$ are the energies of the particles in the final state.

We assume that the amplitude M can be presented by the sum of diagrams shown in Fig. 1. Here the filled squares correspond to the primary production amplitude A and the filled and open circles to pp- and $\eta' p$ amplitudes, respectively. In other words, in addition to the born term (Fig. 1a) we take into account also the pp (Fig. 1b) and $\eta' p$ (Fig. 1c,d) FSI. Assuming that the energy dependence of the cross section is given solely by the FSI, as discussed in the Introduction, the production amplitude A can be taken out of the integrals corresponding to the diagrams Fig. 1b,c,d [1]. Thus we get

$$M = A \cdot F_{FSI}, \quad (3)$$

where the final-state-interaction factor F_{FSI} now incorporates all effects from the diagrams of Fig. 1. When the FSI is absent, i.e. when only the digram in Fig. 1a is taken into account then $F_{FSI} \to 1$. Let us now focus on the calculation of the factor F_{FSI} and neglect for the moment the Coulomb interaction between the protons. For the evaluation of the loop diagrams in Fig. 1 we must make assumptions about the off-shell behaviour of the pp- and $\eta' p$

amplitudes. We adopt the following simple expression for the half-off-shell pp amplitude:

$$f_{pp}^{off}(q_{12}, q) = \frac{q_{12}^2 + \beta^2}{q^2 + \beta^2} f_{pp}^{on}(q_{12}) \tag{4}$$

Here $q_{12}(q)$ is the relative momentum between the protons in the final (intermediate) state. The form (4) for the half-off-shell amplitude corresponds to the S-wave scattering amplitude resulting from a (separable) Yamaguchi potential. Since the scattering length in the $^1S_0$ pp partial wave is fairly large (a_{pp} = -7.8 fm) we can make use of an approximate relation between the parameter β and the effective-range parameter r_{pp} that follows for this potential in such a case, namely $\beta \approx 3/r_{pp}$ [13]. By taking the experimental value of the pp effective range, $r_{pp} \approx 2.8$ fm, we get $\beta \approx 0.2$ GeV. With regard to the $\eta' p$ interaction we assume that also its half-off-shell amplitude can be presented in a form analogous to (4), i. e.

$$f_{\eta' p}^{off}(q_{i3}, q') = \frac{q_{i3}^2 + C^2}{q'^2 + C^2} f_{\eta' p}^{on}(q_{i3}), \tag{5}$$

where the index i refers to proton 1 or 2. However, now (unlike β in the pp case) the value of the range parameter C cannot be extracted directly from experimental information. Clearly, if the range of the $\eta' p$ effective potential is large, then the parameter C will be comparable to β, i.e. $C \approx 1 - 1.5 m_\pi$, with m_π being the pion mass. But if the range of the $\eta' p$ potential is small, say of the order of only $0.2 - 0.4$ fm, then the parameter C should be much larger. In the subsequent investigation we will employ different values of C in order to study its influence on observables of the reaction $pp \to pp\eta'$. Taking the off-shell pp- and $\eta' p$ amplitudes in the form (4) and (5), we immediately get

$$\begin{aligned} F_{FSI} &= 1 + (\beta + iq_{12}) f_{pp}^{on}(q_{12}) \\ &+ (C + iq_{23}) f_{\eta' p}^{on}(q_{23}) + (C + iq_{13}) f_{\eta' p}^{on}(q_{13}). \end{aligned} \tag{6}$$

Let us now turn back to the Coulomb interaction between the protons and describe how it can be incorporated into our formalism. The procedure we follow was first introduced in [14,15] and we refer the reader to these papers for further details. According to [14] we have to substitute the expression $F_{FSI}^{pp} = 1 + (\beta + iq_{12}) f_{pp}^{on}(q_{12})$ in (6) by the function $\psi_{q_{12}}^{(-)}(0)$, where $\psi_{q_{12}}^{(-)}(0)$ is related to the (S-wave) Jost function $f(q_{12})$ via

$$\psi_{q_{12}}^{(-)}(0) = \frac{e^{i(\delta_C + \delta_{CS})}}{|f(q_{12})|}. \tag{7}$$

Here δ_C and δ_{CS} are the pure Coulomb and the Coulomb-modified strong (S-wave) phase shifts, respectively. The Jost function $f(q)$ for the pp interaction is known in analytical form for the case of Coulomb + separable Yamaguchi potential [16]. The final expression for the FSI factor, taking the Coulomb repulsion between the protons into account, is then given by

$$\begin{aligned} F_{FSI} &= \psi_{q_{12}}^{(-)}(0) + (C + iq_{23}) f_{p\eta'}^{on}(q_{23}) \\ &+ (C + iq_{13}) f_{p\eta'}^{on}(q_{13}) \end{aligned} \tag{8}$$

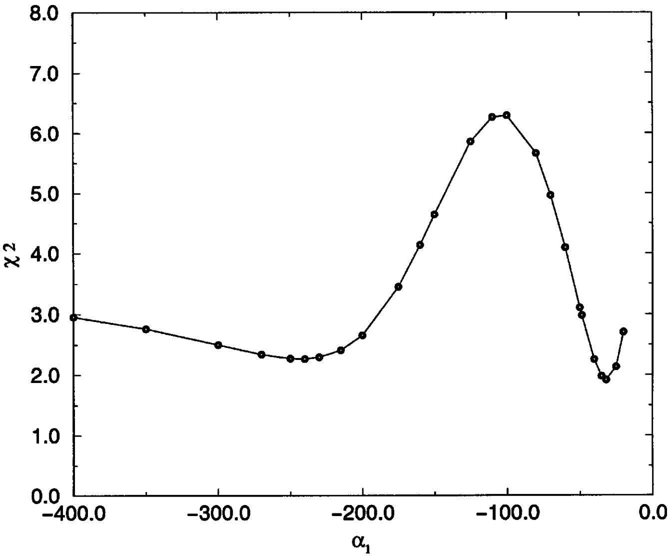

Fig. 2. The dependence of χ^2 on the parameter α_1 for $\alpha_2 = 0$, cf. (9)

This is the form of F_{FSI} employed in the present calculation. Note that there is, in principle, also a Coulomb interaction between the protons in the $\eta' p$ FSI loops (Fig. 1c and d). However, judging from test calculations based on an approximative treatment by means of multiplying the corresponding amplitudes with Coulomb (Gamov) factors we expect its effect to be of minor importance. Therefore we ignore it in the present study. Finally let us mention that we use the scattering length approximation for the $\eta' p$ amplitude $f_{\eta' p}^{on}(q)$, i. e.

$$f_{\eta' p}^{on}(q) = \frac{1}{-1/a_{\eta' p} - iq} = \frac{1}{\alpha - iq}, \tag{9}$$

where $\alpha = \alpha_1 - i\alpha_2$ is complex because of the presence of absorption ($\alpha_2 \geq 0$). In our model study we take α as free parameter that will be determined by a fit to the η' production data.

3 Fitting to the data and main results

In fitting to the data we have four parameters to be determined: α_1, α_2, C and a normalization constant. We proceed in the following way: First a concrete parameter set $\{\alpha_1, \alpha_2, C\}$ is fixed and then the normalization constant is determined by a best (i.e. χ^2) fit to the data. Note that the χ^2 fit is done for two cases, namely (a) utilizing all experimental data available in the threshold region and (b) excluding the data points of the SATURNE measurement. We make this distinction because the two data sets suggest somewhat different trends in the energy dependence of the production cross section and we want to study their implications.

Let us first look at the case were all data points are used. We consider two different values for the parameter

Table 1. A selection of parameter values obtained in the χ^2 fit together with the resulting scattering length for the $\eta' p$ interaction in the $^1S_0$ partial wave. Entries under (a) refer to fits utilizing all η' production data whereas in (b) only the data of [7] are considered. Solutions for which results are shown in Figs. 3 and 4 are marked with an asterisk *). Predictions for both solutions in (b) are shown in Fig. 5

		$\alpha_1 - i\alpha_2$ [MeV/c]	$a_{\eta' p}$ [fm]	χ^2, best fit	C [m_π]
		-240−i0	0.82−i0	2.266	2.5
		-470−i0$^{*)}$	0.42−i0	2.267	5
a)	I	-240−i50$^{*)}$	0.79−i0.16	2.163	2.5
		-490−i50	0.40−i0.04	2.215	5
		-140−i300$^{*)}$	0.25−i0.54	2.283	2.5
		-450−i300	0.30−i0.20	2.175	5
		-32−i0	6.16−i0	1.914	2.5
		-40−i0	4.93−i0	2.140	5
	II	-32−i50	1.79−i2.79	2.110	2.5
		-30−i50	1.74−i2.90	2.297	5
b)	I	-110−i50	1.48−i0.67	0.539	2.5
		-180−i50	1.01−i0.28	0.116	5

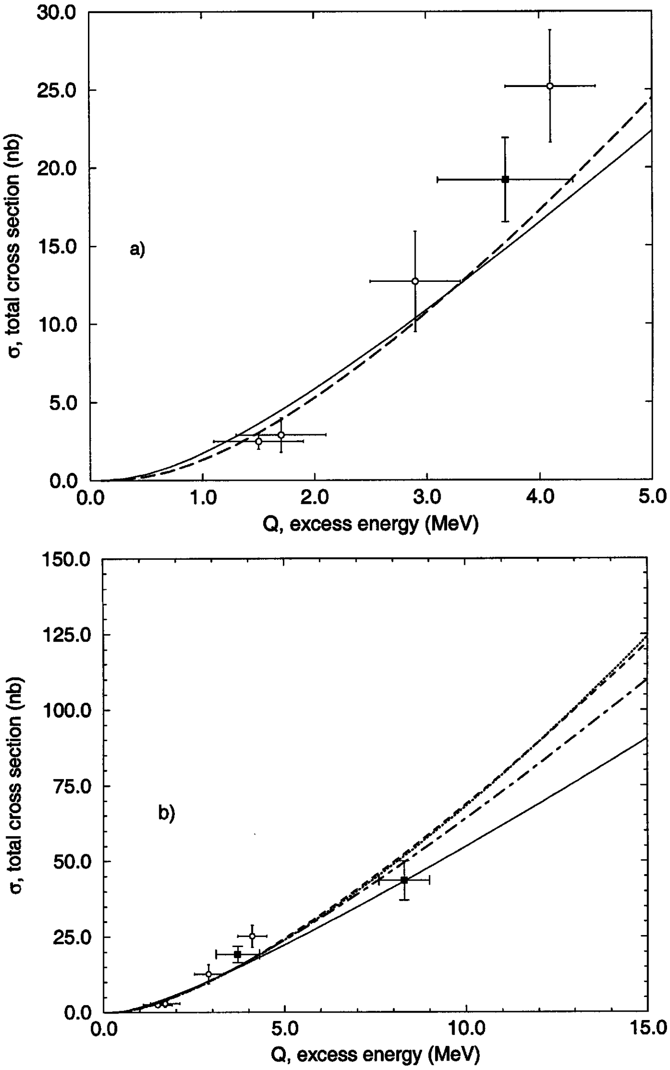

Fig. 3. Total cross section for the reaction $pp \to pp\eta'$: (a) near threshold and (b) over a larger energy range. The solid line is the result that follows from just phase space plus pp FSI. The other curves correspond to calculations where the $\eta' p$ FSI is included as well. The dashed line is based on solution I with $a_{\eta' p} = (0.79 - 0.16i) fm$ and $C = 2.5 m_\pi$. The dotted and dash-dotted curves are obtained for $a_{\eta' p} = (0.42 - i0) fm$, $C = 5 m_\pi$ and $a_{\eta' p} = (0.25 - 0.54i) fm$, $C = 2.5 m_\pi$, respectively. The experimental data are from [7] (open circles) and [8] (filled squares). Note that all results with the $\eta' p$ interaction included basically coincide below Q=5 MeV and therefore only one curve is shown in this case

C, namely $C = 2.5\ m_\pi$ and $5\ m_\pi$, respectively. This allows us to investigate the influence of the range of the $\eta' p$ interaction. Furthermore we consider separately regions of small and large α_2 values in order to check the sensitivity of the calculated η'-production cross section to the amount of $\eta' p$ absorption. A typical example of the resulting $\chi^2(\alpha)$ curve is depicted in Fig. 2 for the particular case of $\alpha_2 = 0$. One can see that the curve exhibits two distinct minima. This structure is basicially independent of the value of α_2 and the positions of the minima of $\chi^2(\alpha)$ move only slightly when α_2 is increased. The first minimum corresponds to relatively small values for the scattering lengths, Re $a_{\eta' p} < 1$, as can be seen from Table 1 where some typical values for α_1 and α_2 are compiled (cf. the section labelled with I). We call them "normal" solutions.

The second minimum (solutions II in Table 1) corresponds to fairly small values of α_1 and hence, in general, to rather large values of the $\eta' p$ scattering length. We call these solutions abnormal. Unfortunately, these solutions are outside of the range where the present rescattering model can be reliably applied. Namely, if the $\eta' p$ as well as the pp scattering length are large, we cannot limit ourselves to the diagrams of Fig. 1 only but must sum up the FSI-diagrams to all orders. We are planning to consider this situation in a future investigation. In the present paper, however, we will restrict our study to the normal solutions I.

The behaviour of the total cross section for some selected values of α_1, α_2, and C (representing the features i)-iii) of the $\eta' p$ amplitude) is presented in Fig. 3. One can see that the introduction of an $\eta' p$ interaction leads indeed to an improvement in the description of the energy dependence of the η' production cross section near threshold (cf. Fig. 3a). In particular, all considered features yield a suppression of the cross section very close to threshold as shown by the data. As a matter of facts, the resulting cross sections are almost identical for excess energies below 5 MeV. For energies above Q = 5 MeV, however, there are noticeable differences in the predicted cross sections as is demonstrated in Fig. 3b. Specifically we observe that all results including an $\eta' p$ interaction are significantly enhanced in comparison to what follows from a model calculation without an $\eta' p$ FSI (cf. the solid line). Future experiments [17] will certainly allow to discern between these predictions. But one should be aware

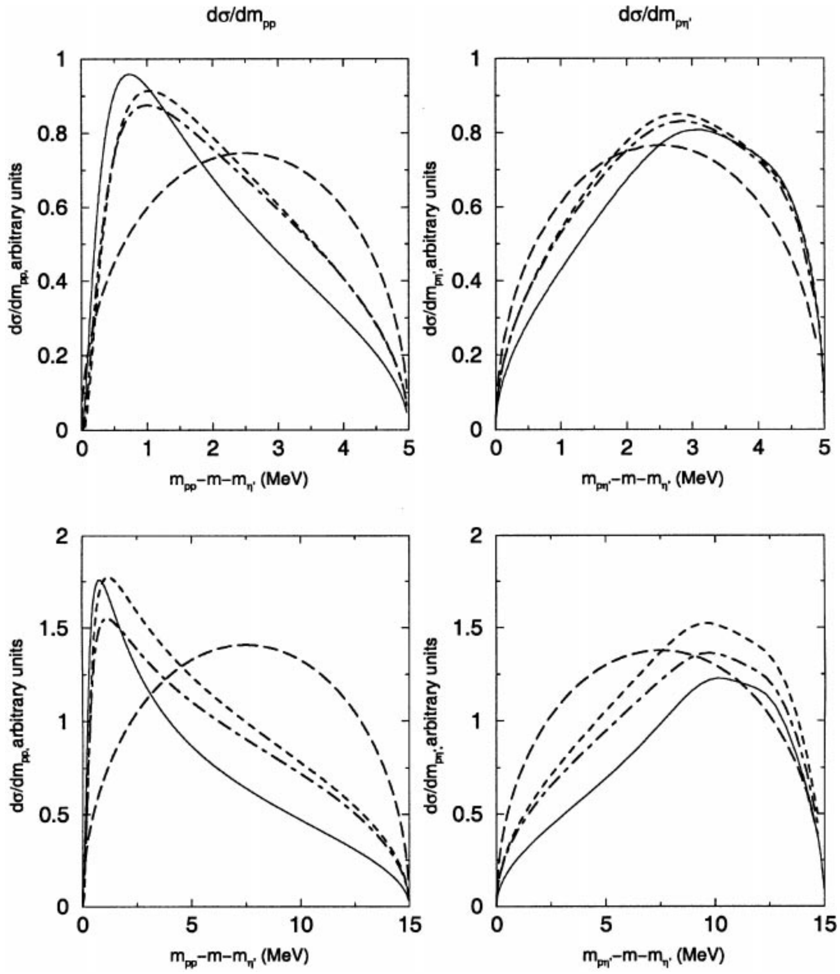

Fig. 4. Spectra of pp and $p\eta'$ mass. The long dashed curve is the results which follows from the pure phase space, normalized arbitrary. The solid curve is a calculation with phase space plus pp FSI. The dashed and dash-dotted curves include in addition an $\eta' p$ FSI; they meaning is the same as Fig. 3. The normalization for all curves, except for the pure phase space results, is the same as in Fig. 3

that, in general, the energy dependence does not provide a unique signal for the underlying properties of the $\eta' p$ interaction. E.g., the dotted and dashed curves in Fig. 3b are almost identical up to $Q = 15$ MeV despite the fact that the range parameters (C) of the employed $\eta' p$ interactions are rather different (cf. Table 1). Thus, it is difficult to relate differences in the predicted energy dependence to particular properties of the $\eta' p$ interaction.

Please note that all model solutions compiled in Table 1 yield positive values for the real part of the $\eta' p$ scattering length. This expresses the fact that the $\eta' p$ interaction has to be effectively repulsive in order to produce a noticeable suppression of the cross section at small Q.

In Fig. 4 we present invariant mass distributions of the pp and $\eta' p$ systems for some excess energies. It is evident that the $\eta' p$ FSI resulting from our fits leads to a significant modification of the mass distribution. Thus a measurement of those mass distributions can provide further evidence for a possible effect of the $\eta' p$ FSI. Not unexpectedly however, also those mass distributions are not very sensitive to the details of the $\eta' p$ force. I. e. solutions that give similar results for the total production cross section lead also to basically the same predictions for the invariant mass distributions.

Results for the cross section obtained in the χ^2-fit excluding the SATURNE data are shown in Fig. 5. In this case the description of the experimental data is significantly better up to Q=5 MeV, cf. the corresponding χ^2 values in Table 1. On the other hand, the predicted cross sections rise rather rapidly with increasing Q. Additional experimental information about the η'-production cross section at somewhat larger energies is required for judging about the validity of such a model solution.

4 Summary

In this paper we have presented a model analysis of the recent data [7,8] on the reaction $pp \to pp\eta'$ close to threshold. Specifically we tried to investigate the origin of the experimentally observed suppression of the production cross section for excess energies $Q < 3$ MeV. For that purpose we carried out a DWBA calculation of the reaction $pp \to pp\eta'$ taking into account the usual pp FSI but in addition also the interaction in the $\eta' p$ system. It turned out that the introduction of the $\eta' p$ interaction leads indeed to a modification of the energy dependence of the η' production cross section. Specifically, with an effectively

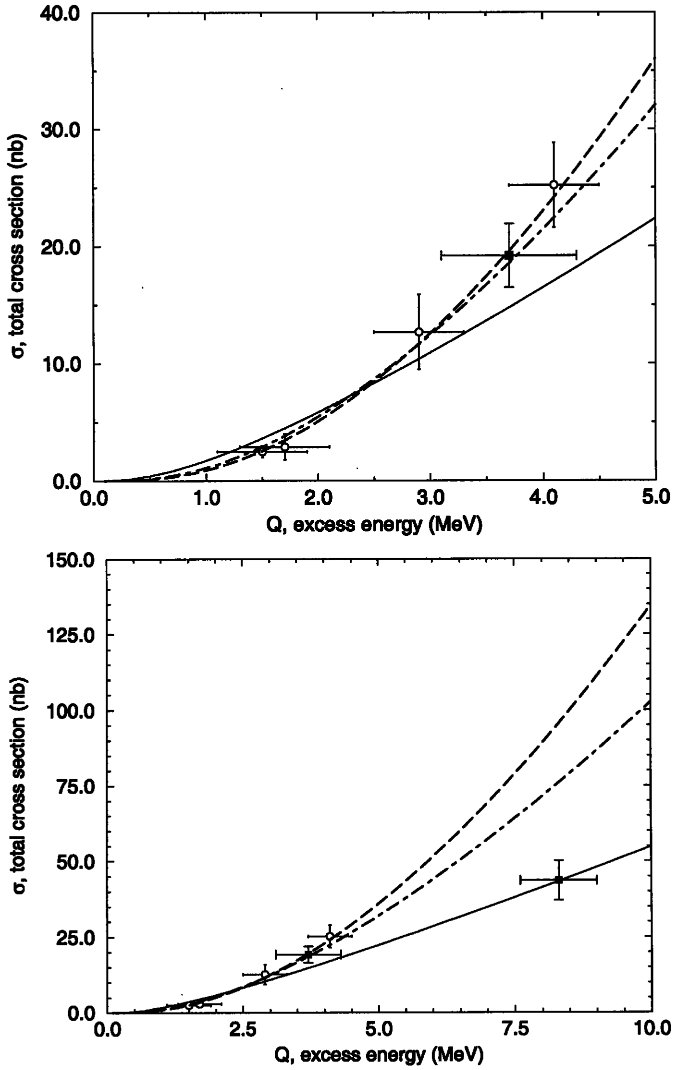

Fig. 5. Results for the reaction $pp \to pp\eta'$ where the data point from SATURNE were excluded in the fitting procedure. The solid curve is the results with just phase space plus pp FSI. The dashed and dot-dashed curves include also the $\eta'p$ interaction with the parameters $a_{\eta'p} = (1.01 - 0.28i)fm,\ C = 5m_\pi$ and $a_{\eta'p} = (1.48 - 0.67i)fm,\ C = 2.5m_\pi$, respectively. The data are the same as in Fig. 3

repulsive $\eta'p$ interaction a suppression of the cross section close to threshold can be achieved leading to a visible improvement in the description of the data.

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References

1. K. Watson, Phys.Rev, **88**, 1163 (1952)
2. M. Goldberger and K. Watson, Collision Theory (Wiley, New York 1964)
3. H.O. Meyer et al., Phys. Rev. Lett. **65**, 2846 (1990)
4. G. Miller and P.U. Sauer, Phys. Rev. C **44**, 1725 (1991)
5. A.M. Bergdolt et al., Phys. Rev. C **48**, R2969 (1993); E. Chiavassa et al., Phys. Lett. B **322**, 270 (1994); H. Calén et al., Phys. Lett. B **366**, 39 (1996)
6. H. Calén et al., Phys. Rev. Lett. **79**, 2642 (1997); H. Calén et al., Phys. Rev. Lett. **80**, 2069 (1998)
7. P. Moskal et al., Phys. Rev. Lett. **80**, 3202 (1998)
8. F. Hibou et al., Phys. Lett. B **438**, 41 (1998)
9. P. Moskal, Acta Phys. Pol. B **29**, 3091 (1998)
10. R. Plötzke et al., Phys. Lett. B **444**, 555 (1998)
11. A. Sibirtsev and W. Cassing, Eur. Phys. J. A **2**, 333 (1998)
12. V. Bernard, N. Kaiser, and Ulf-G. Meißner, Eur. Phys. J. A **4**, 259 (1999)
13. Y. Yamaguchi, Phys. Rev. **95**, 1628 (1954)
14. A.E. Kudryavtsev, B.L. Druzhinin, V.E. Tarasov, JETP Lett. **63**, 235 (1996)
15. B.L. Druzhinin, A.E. Kudryavtsev, V.E. Tarasov, Z. Phys. A **359**, 205 (1997)
16. H. van Haeringen, Nucl. Phys. A **253**, 355 (1975)
17. P. Moskal et al., in preparation