



A method for time calibration of PET systems using fixed beta-plus radioactive source

KAMIL DULSKI

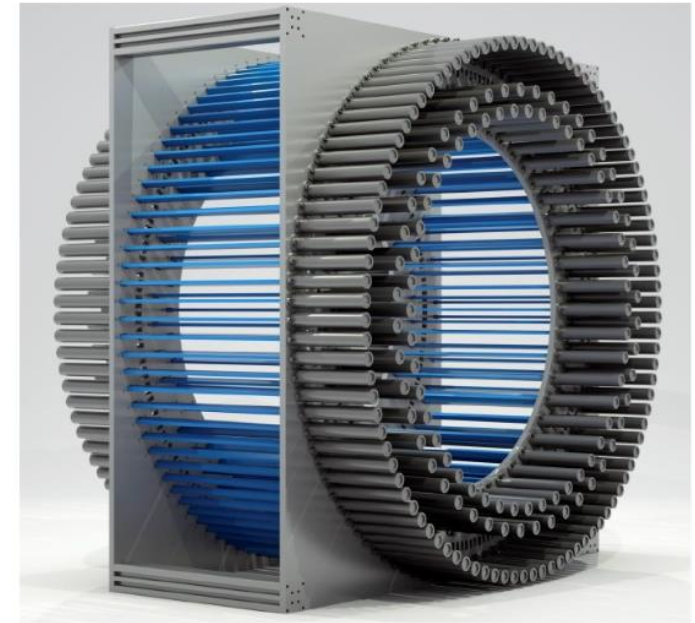
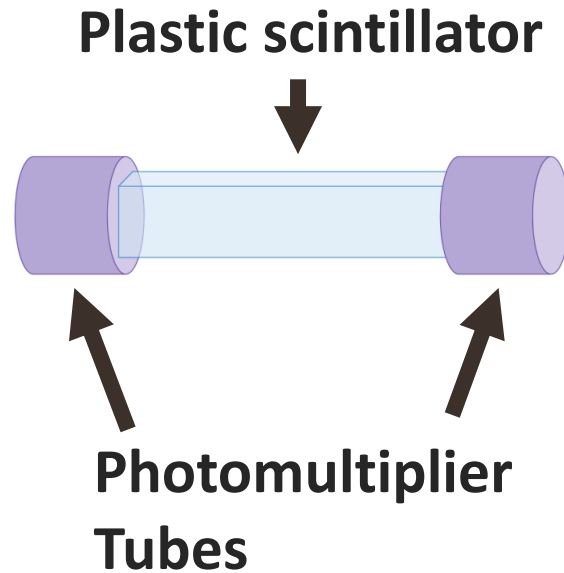
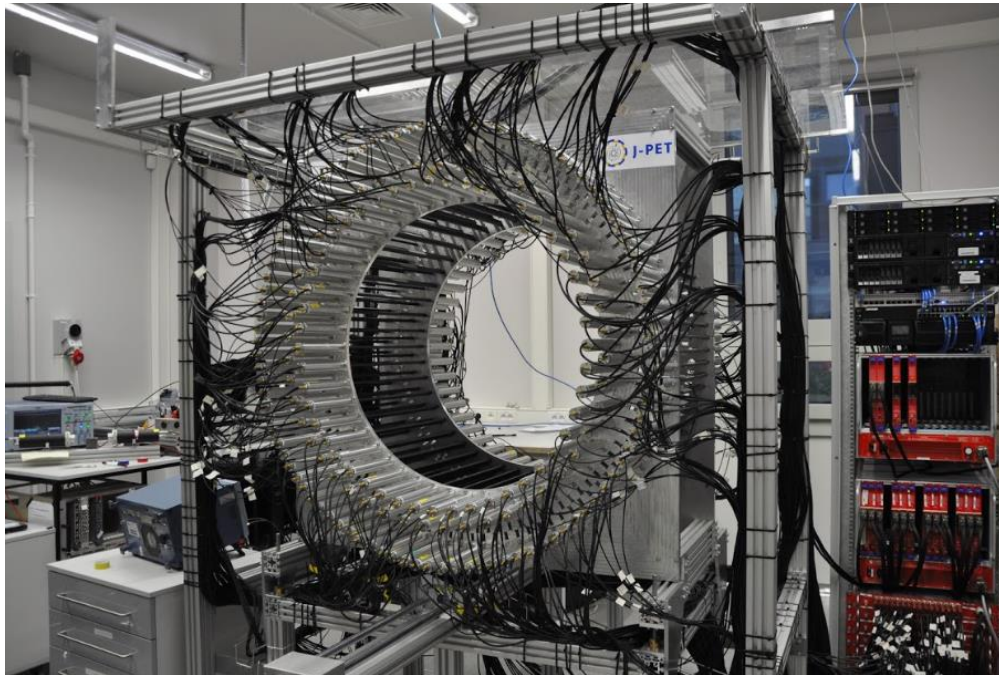
ON BEHALF OF THE J-PET COLLABORATION

3RD JAGIELLONIAN SYMPOSIUM ON FUNDAMENTAL AND APPLIED SUBATOMIC PHYSICS

Outline

1. J-PET – TOF-PET system
2. Calibration of TOF-PET systems – established methods
3. Calibration of a single module of J-PET
4. Calibration between modules

J-PET – (Time-Of-Flight) TOF-PET system

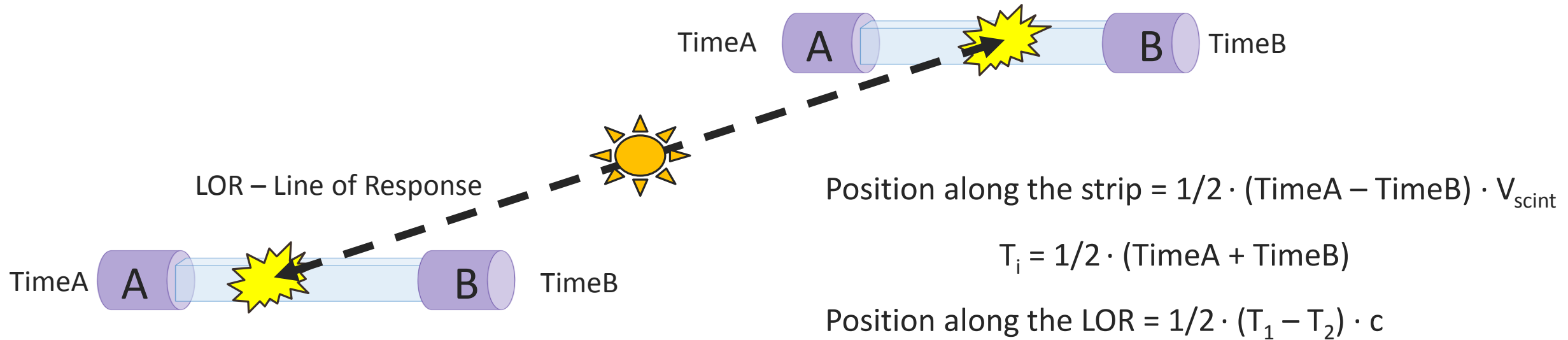


192 x Plastic
scintillator

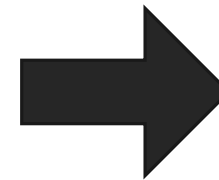


384 x Photomultiplier
Tubes

Principle of position reconstruction

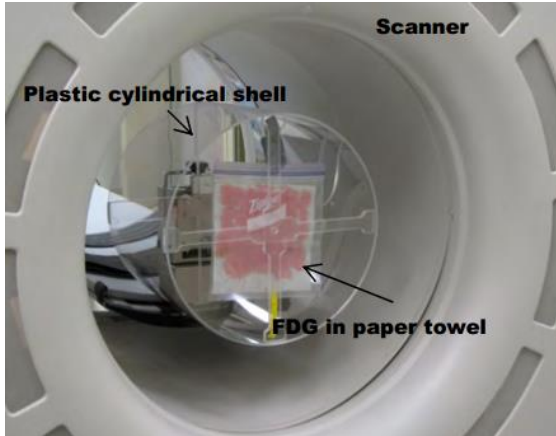


Reconstruction of position of the hit based on the time difference between signals

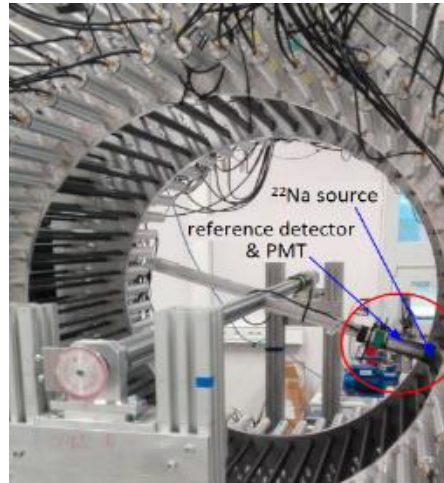


384 modules to calibrate with each other

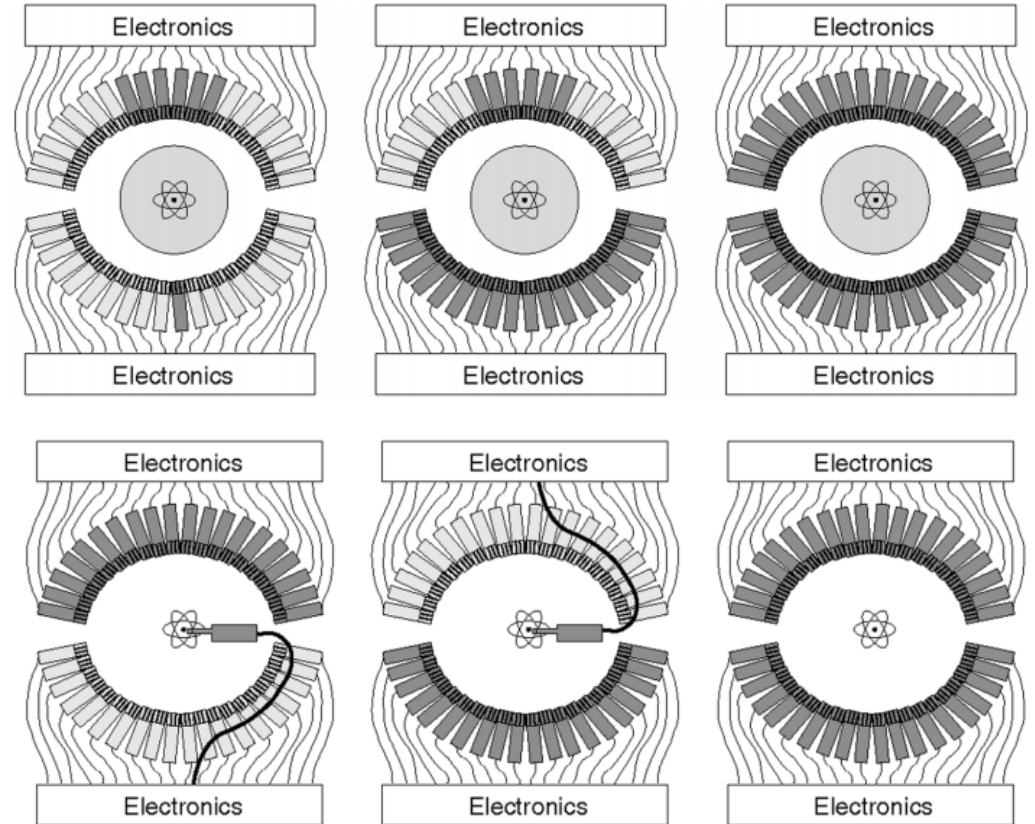
Calibration of TOF-PET systems - established methods



Large phantom:
Needs additional measurement,
with large phantom
to cover many LORs



Reference rotating system:
Needs additional measurement,
that scans over all detectors



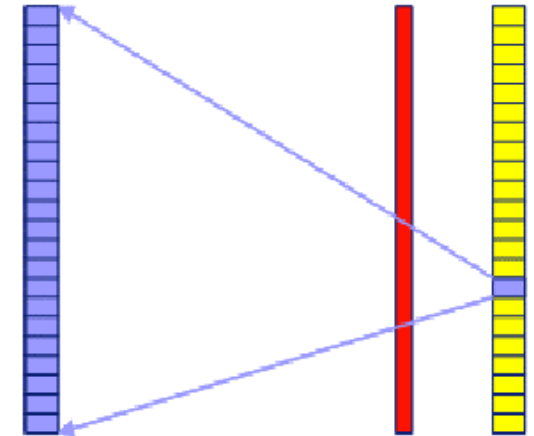
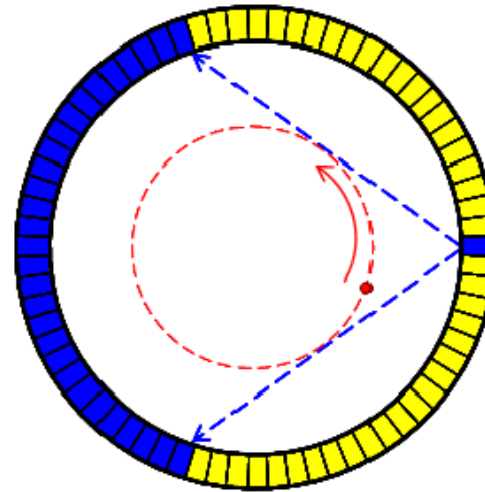
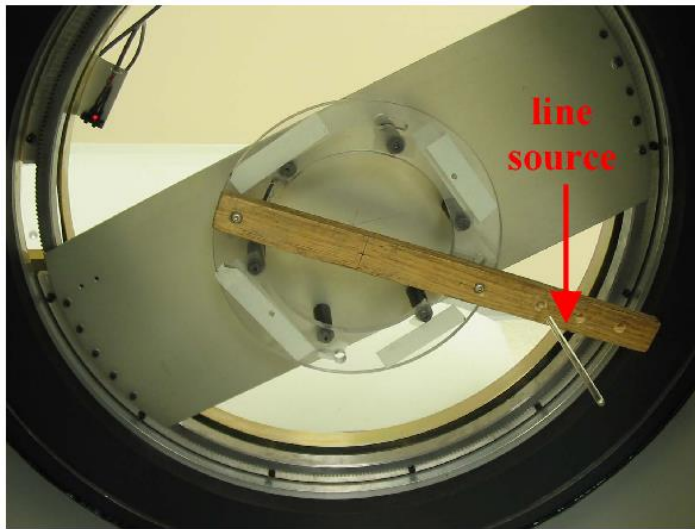
Xiaoli Li et al., IEEE Transactions on Nuclear Science 63, 3 (2016)

W. W. Moses and C. J. Thompson, IEEE Transactions on Nuclear Science 53, 5 (2006)

M. Skurzok et al., Acta Phys. Polon. A 132, 5 (2017)

Calibration of TOF-PET systems - established methods

**Rotating radioactive rod:
Needs additional measurement,
that scans over all LORs**



Performed measurement



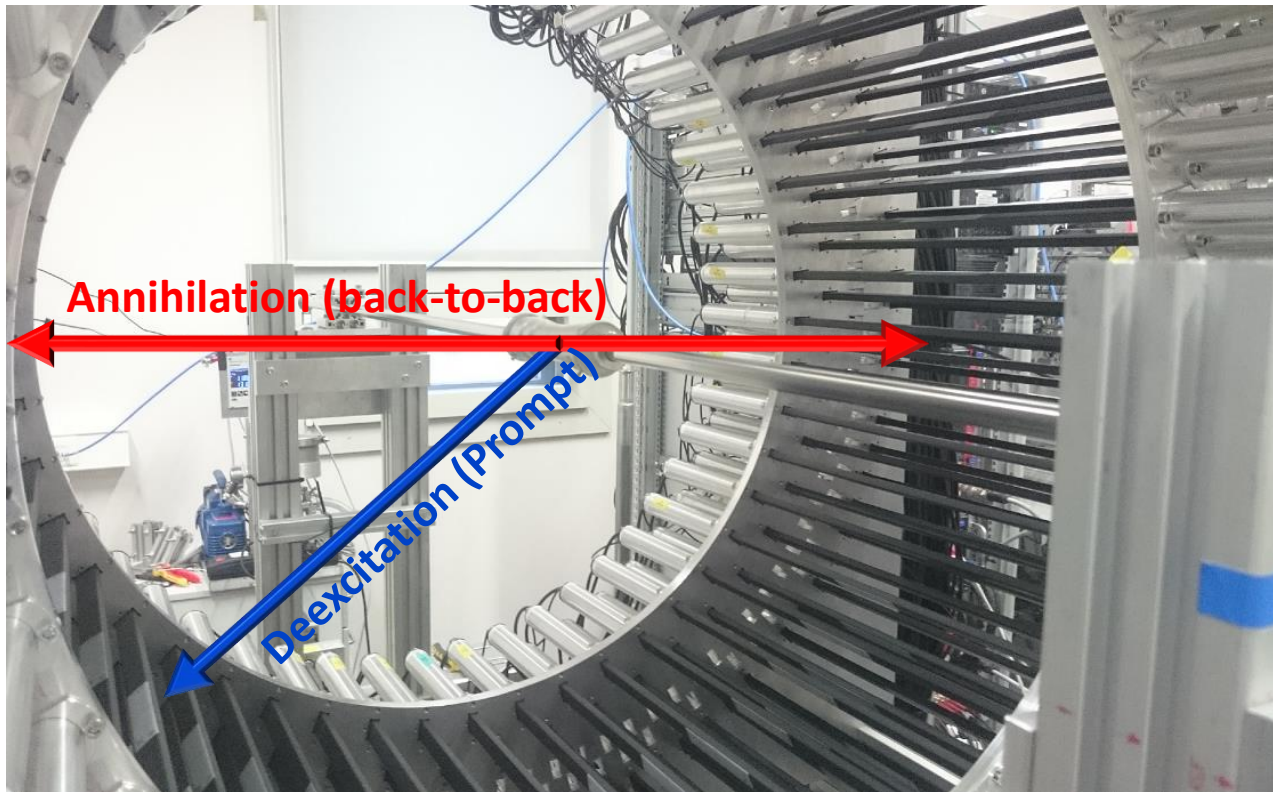
^{22}Na source (1 MBq activity) in Kapton foil was placed between two layers of XAD4¹ (porous polymer) and inserted inside aluminum chamber. Chamber was placed inside J-PET, in the center of the detector. Measurement done to study Positronium Annihilation in XAD4.

Calibration measurement details:

- Fixed source in the center
- Calibration based on prompt + annihilation quanta

¹ https://www.sigmaaldrich.com/content/dam/sigma-aldrich/docs/Sigma/Product_Information_Sheet/1/xad4pis.pdf

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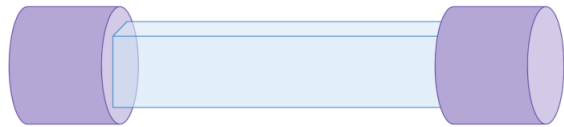
¹ https://www.sigmaaldrich.com/content/dam/sigma-aldrich/docs/Sigma/Product_Information_Sheet/1/xad4pis.pdf

Calibration procedure

1st step:
Calibrating Photomultiplier tubes, connected to one scintillator with each other



Modules calibrated

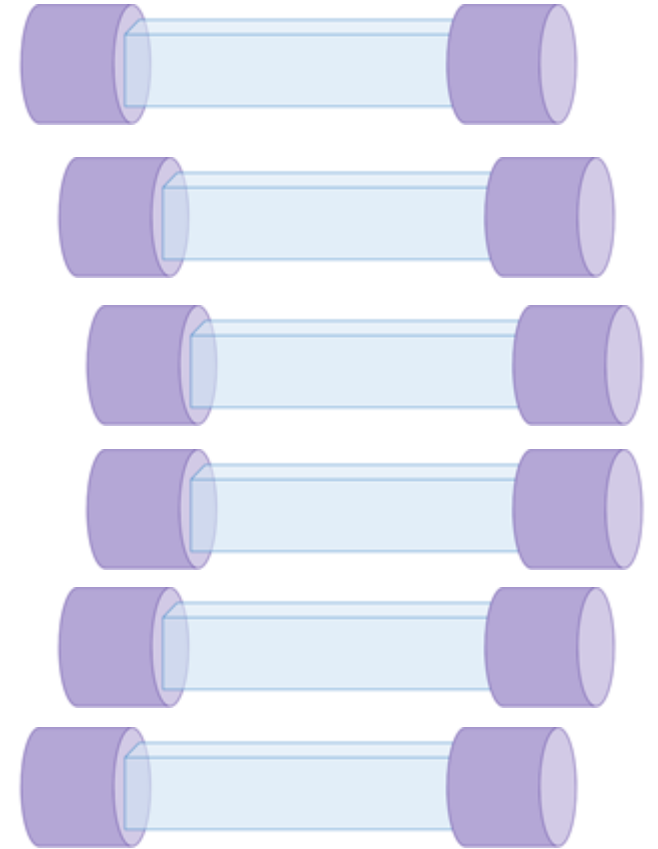


Photomultiplier Tubes

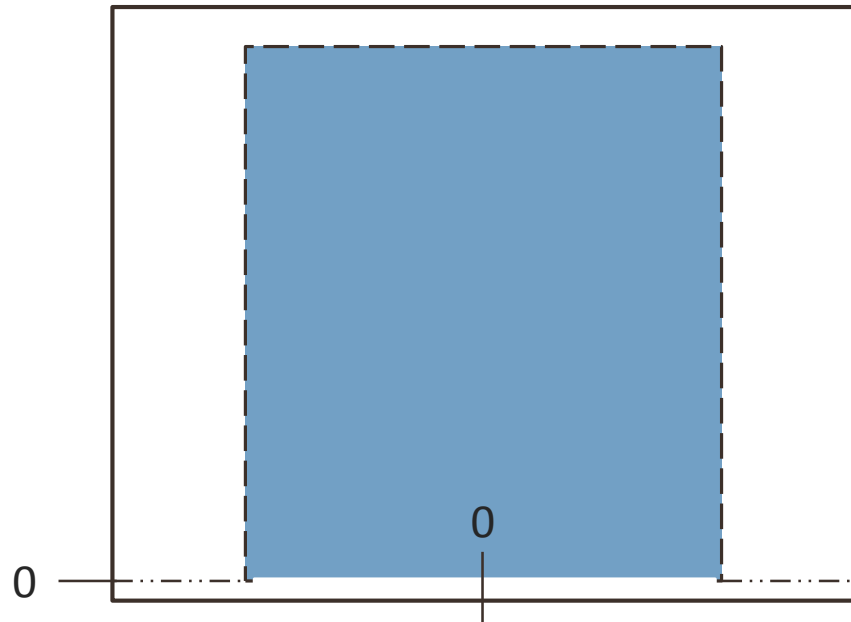
2nd step:
Calibrating all modules with each other at once



Detector calibrated



Calibration of a single module of J-PET



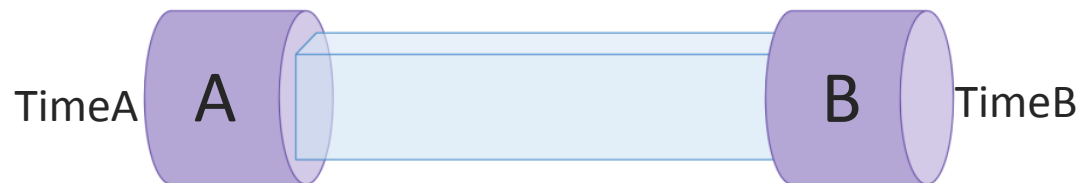
Perfect situation:

- Perfect time resolution
- Uniform irradiation of the strip

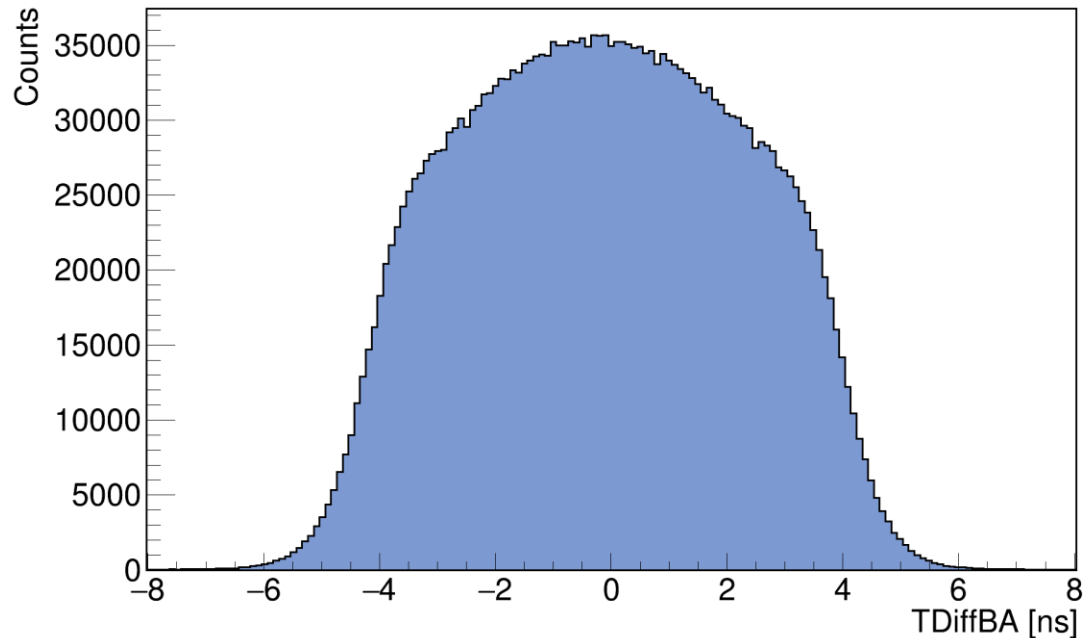


Rectangular distribution of Time differences

$$\text{TDiffBA} = \text{TimeA} - \text{TimeB}$$



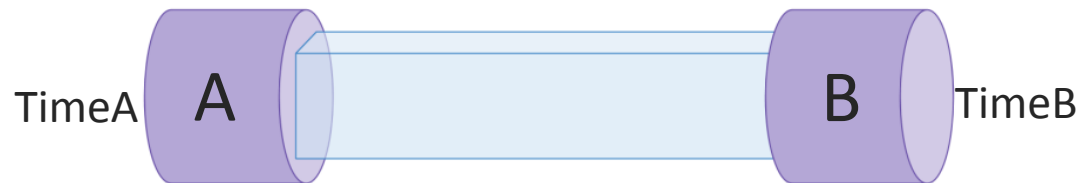
Calibration of a single module of J-PET



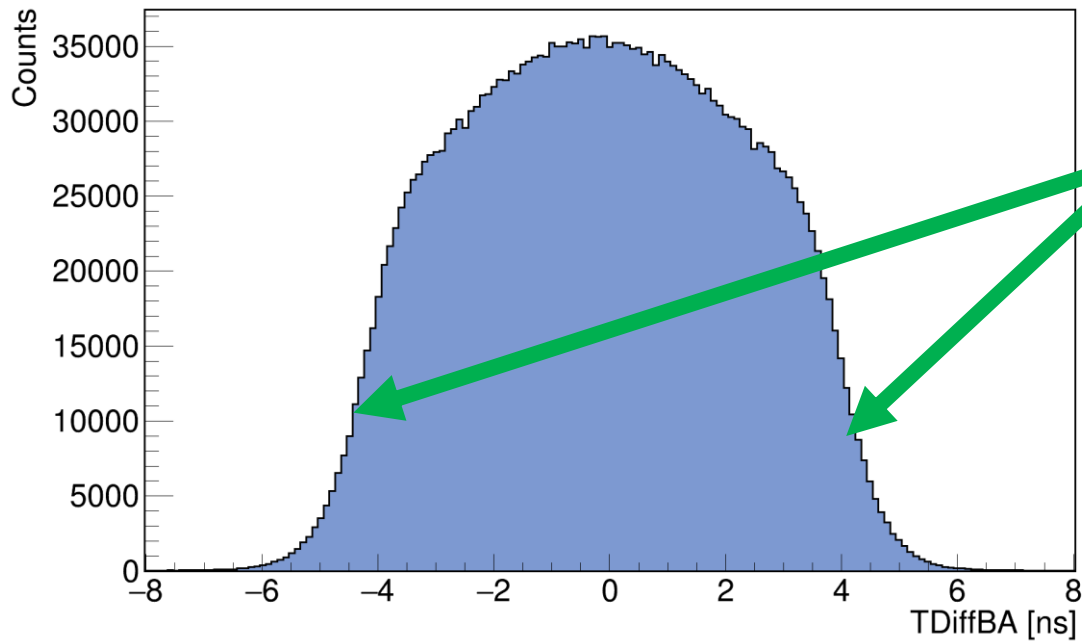
Real situation:

- Gauss-like smearing of photomultiplier
- Cosine squared distribution of irradiation for source in the center of the detector

$$\text{TDiffBA} = \text{TimeA} - \text{TimeB}$$



Calibration of a single module of J-PET



↑
MiddleEdgeLeft

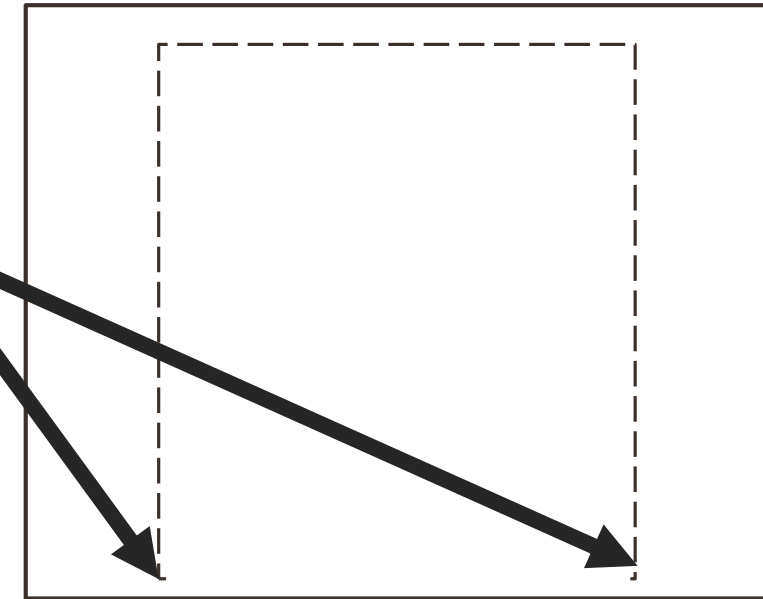
↑
MiddleEdgeRight

Taking the middles of these edges as the estimates of real edges

1 module (scintillator + 2 Photomultipliers) are treated as calibrated if

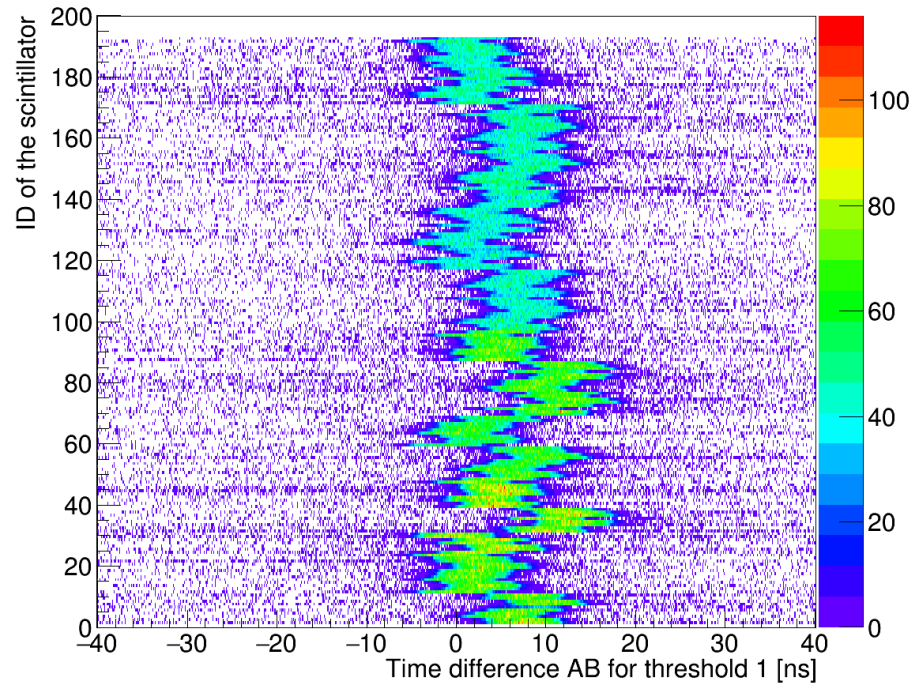
$$\text{MiddleEdgeRight} + \text{MiddleEdgeLeft} = 0$$

Middle calculated as a zero of a second derivative (point of inflection)

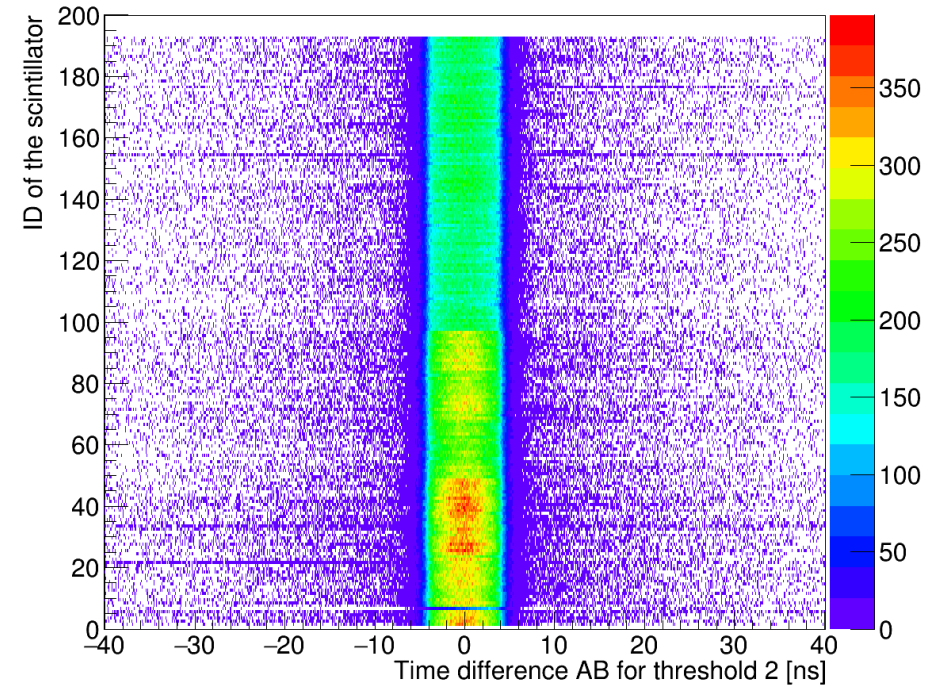


Calibration of a single module

No calibration



After calibration



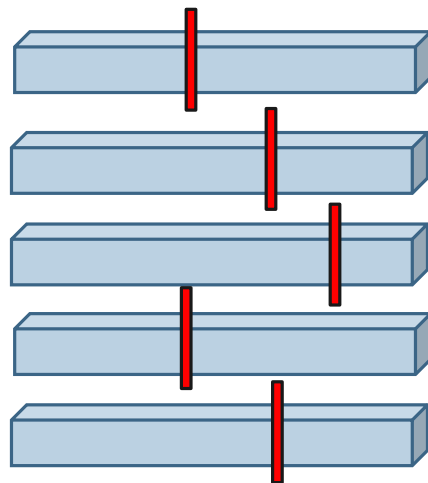
$$\text{TimeA}_{\text{new}} = \text{TimeA}_{\text{old}} - (\text{MiddleEdgeRight} - \text{MiddleEdgeLeft})/2$$

$$\text{TimeB}_{\text{new}} = \text{TimeB}_{\text{old}}$$

$$\text{TDiffAB} = \text{TimeA} - \text{TimeB}$$

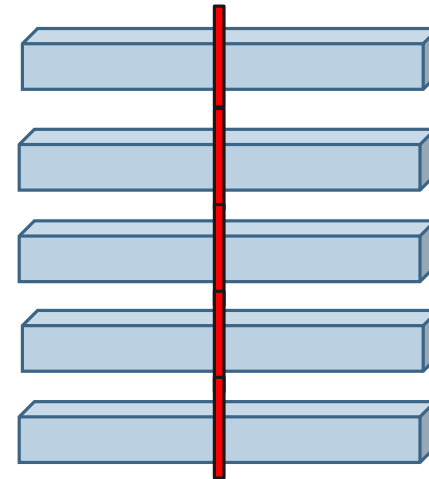
Intermediate status

Each module working in their own domain



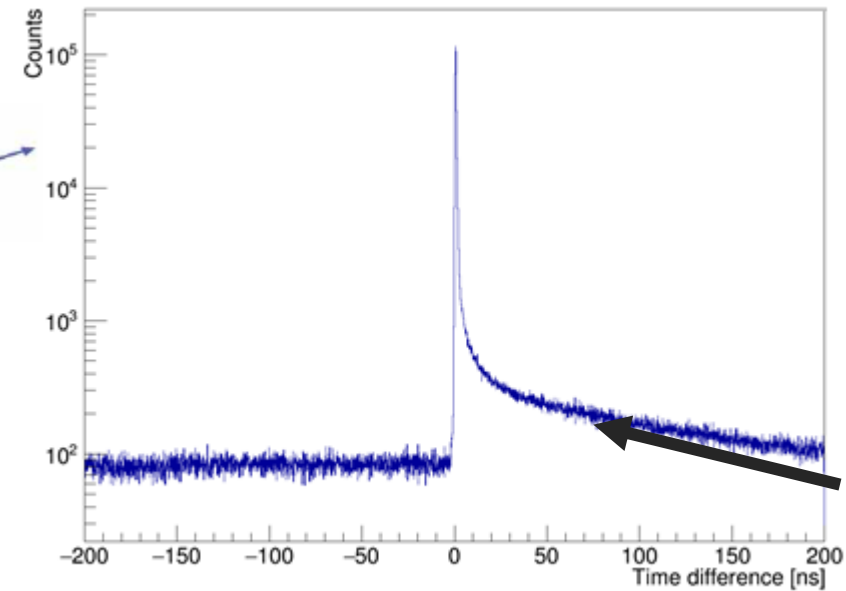
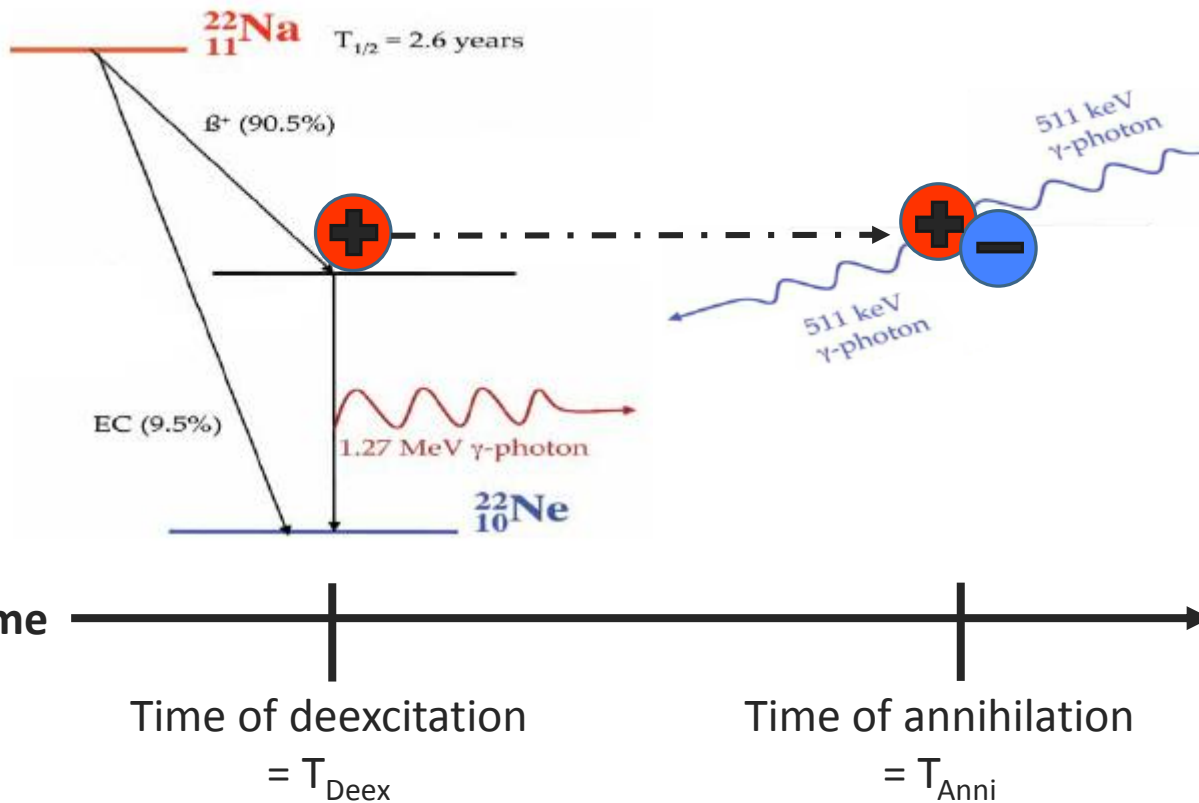
→ Global time

All modules working in one domain



→ Global time

^{22}Na decay + Positronium

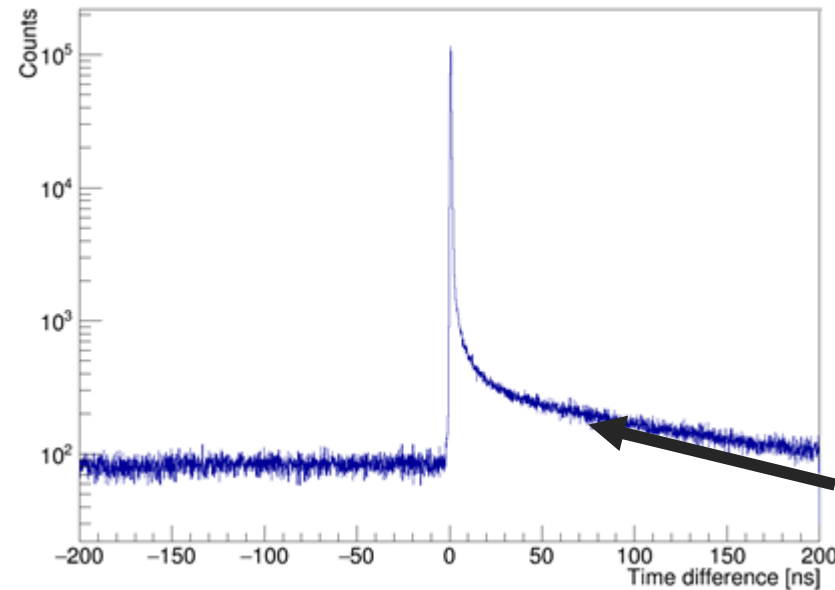
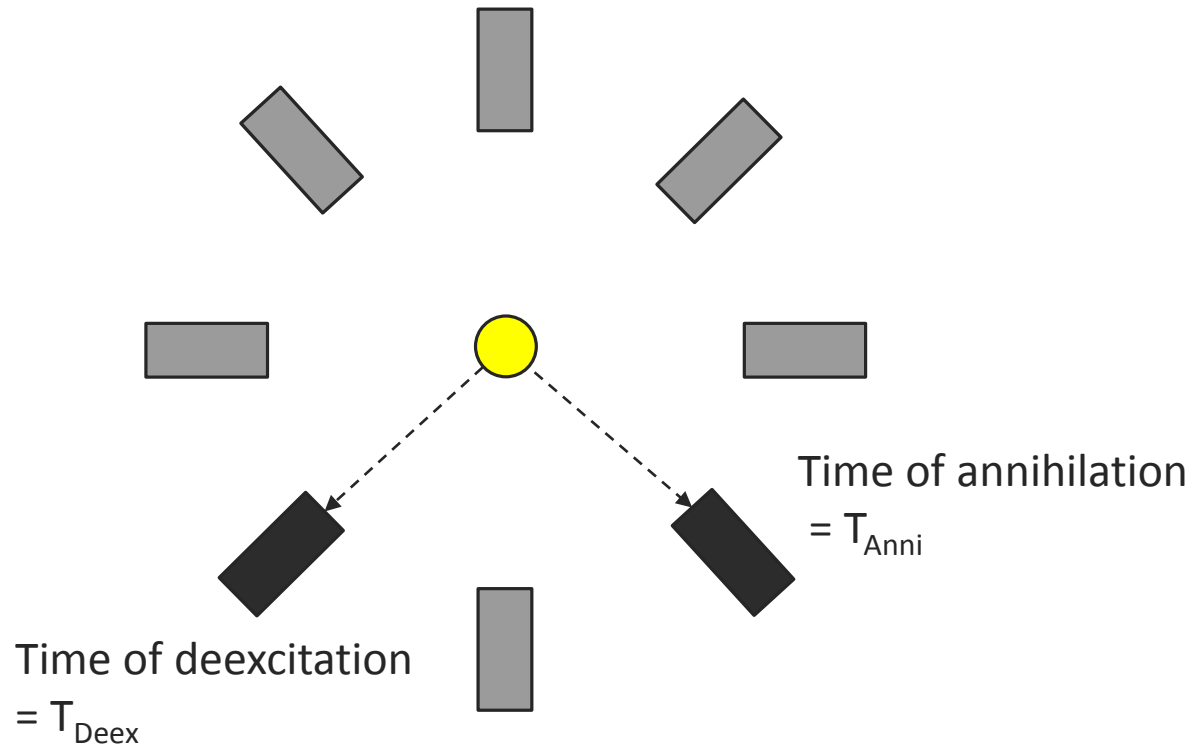


Calibration scan made for ^{22}Na source in Kapton foil with XAD4.

Mean Lifetime of Positronium in XAD4 ≈ 90 ns

Estimate of the lifetime of positronium
 Time difference = $T_{\text{Anni}} - T_{\text{Deex}}$

^{22}Na decay + Positronium



Calibration scan
made for ^{22}Na
source in Kapton
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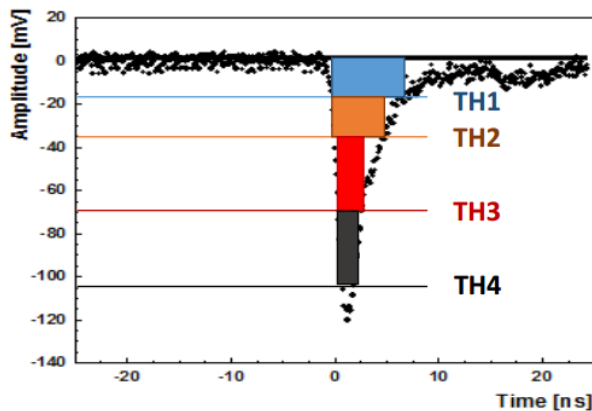
^{22}Na decay + Positronium

Condition to fulfill:

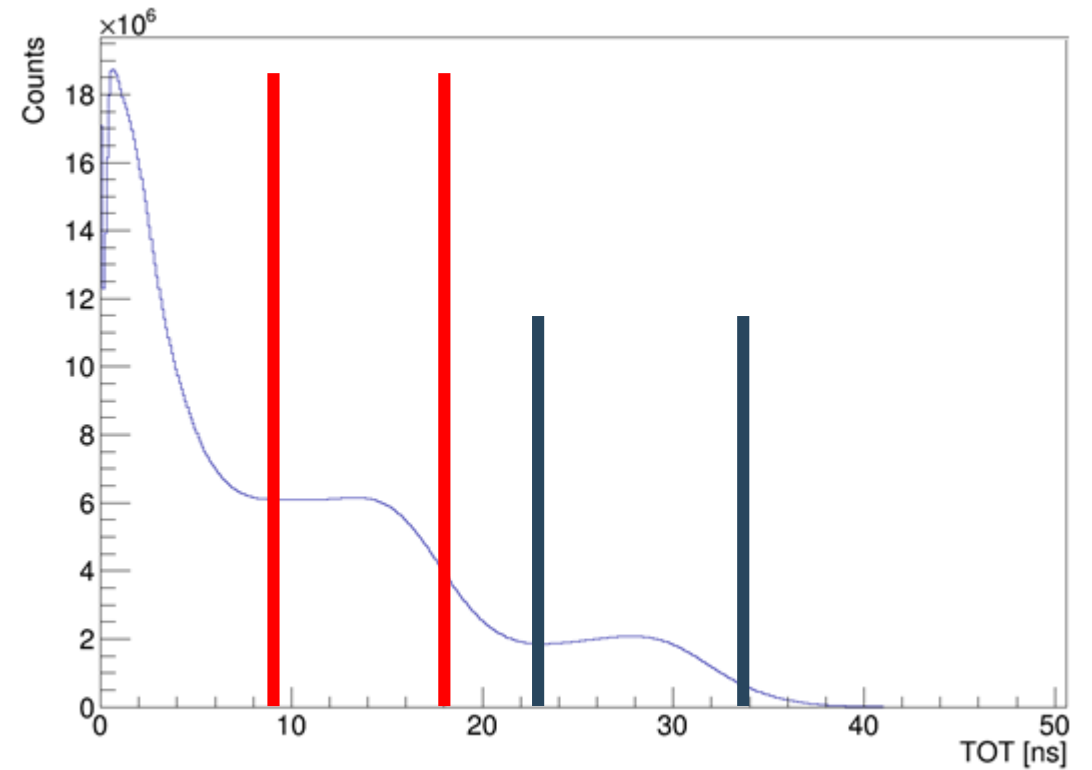
- 1 **Deexcitation** gamma quantum
- 1 **Annihilation** gamma quanta

Proceeding:

- Distinction of the origin of the registered gamma quantum



Area under signal depends on the deposited energy



TOT (Time over Threshold) – Sum of widths of signal on 4 thresholds

Correction on Time-of-Flight (TOF)

Condition to fulfill:

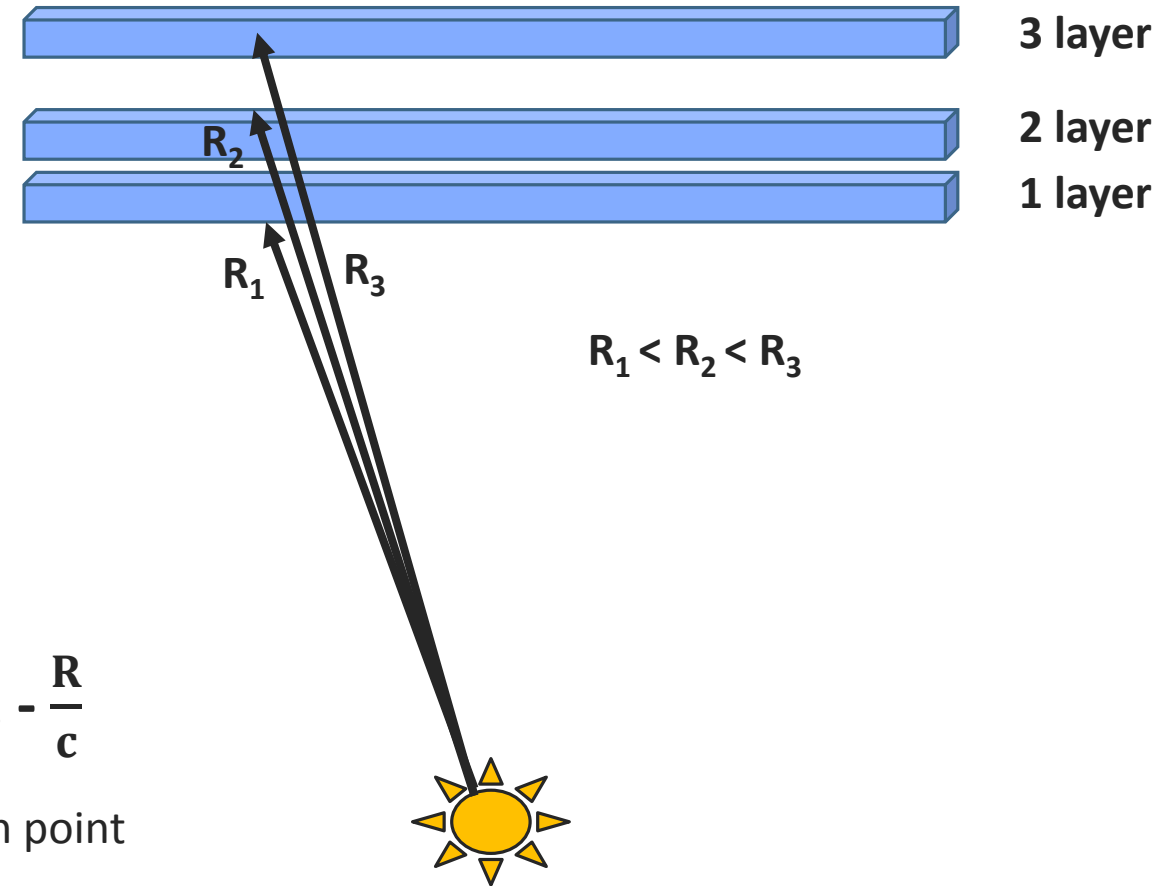
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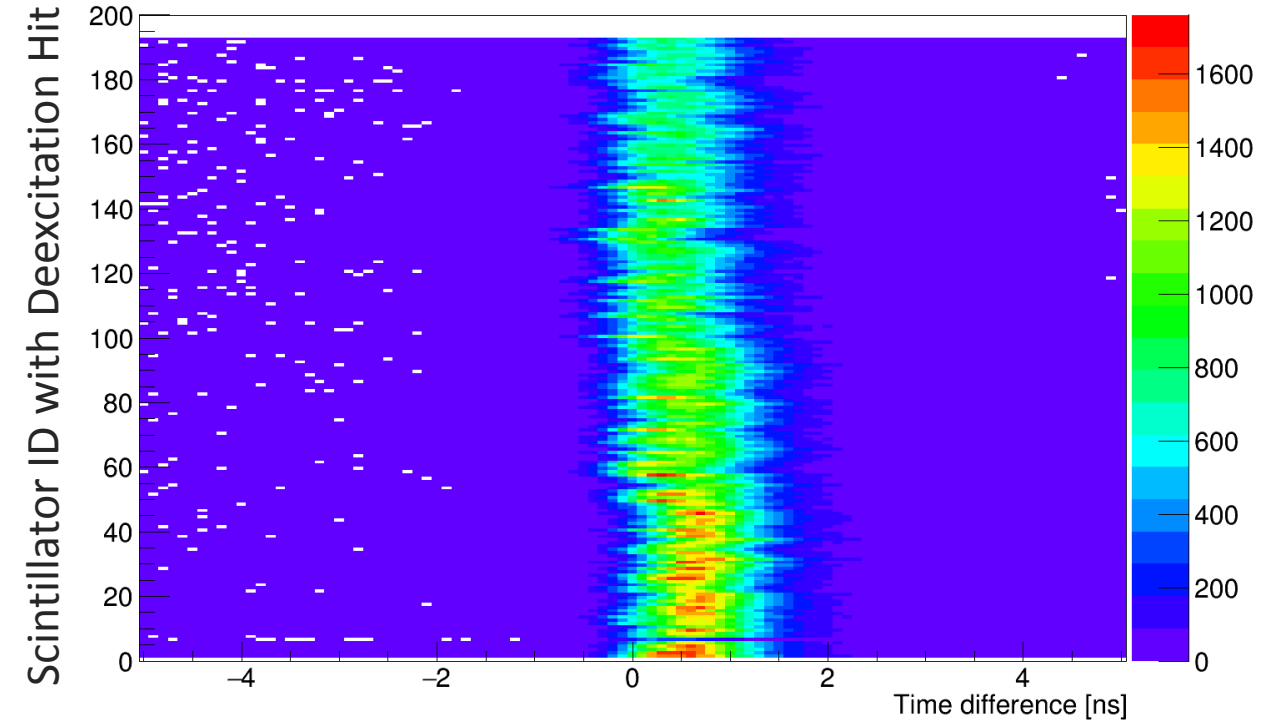
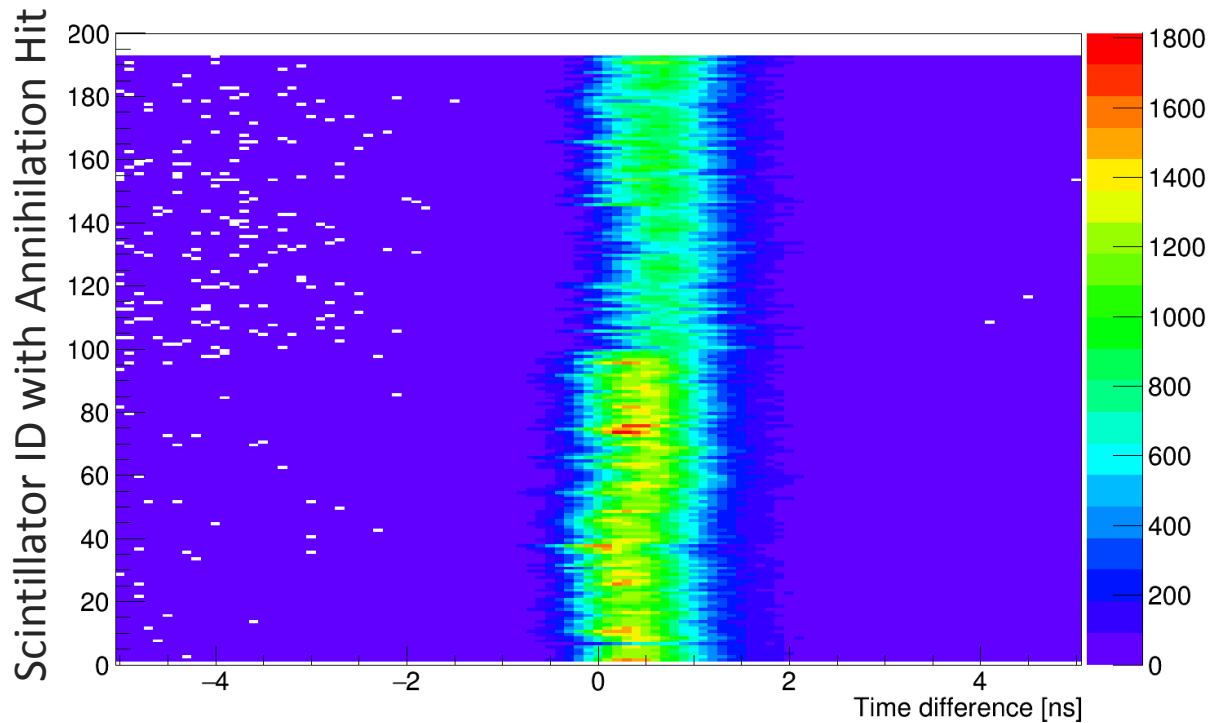
- Distinction of the origin of the registered gamma quantum
- 1 Annihilation + 1 Deexcitation
- Times of the Hits corrected on Time-Of-Flight

$$\text{Time}_{\text{new}} = (\text{Time}_{\text{sideA}} + \text{Time}_{\text{sideB}})/2 - \frac{R}{c}$$

R – Distance from the position of the source and the registration point
c – speed of light in vacuum



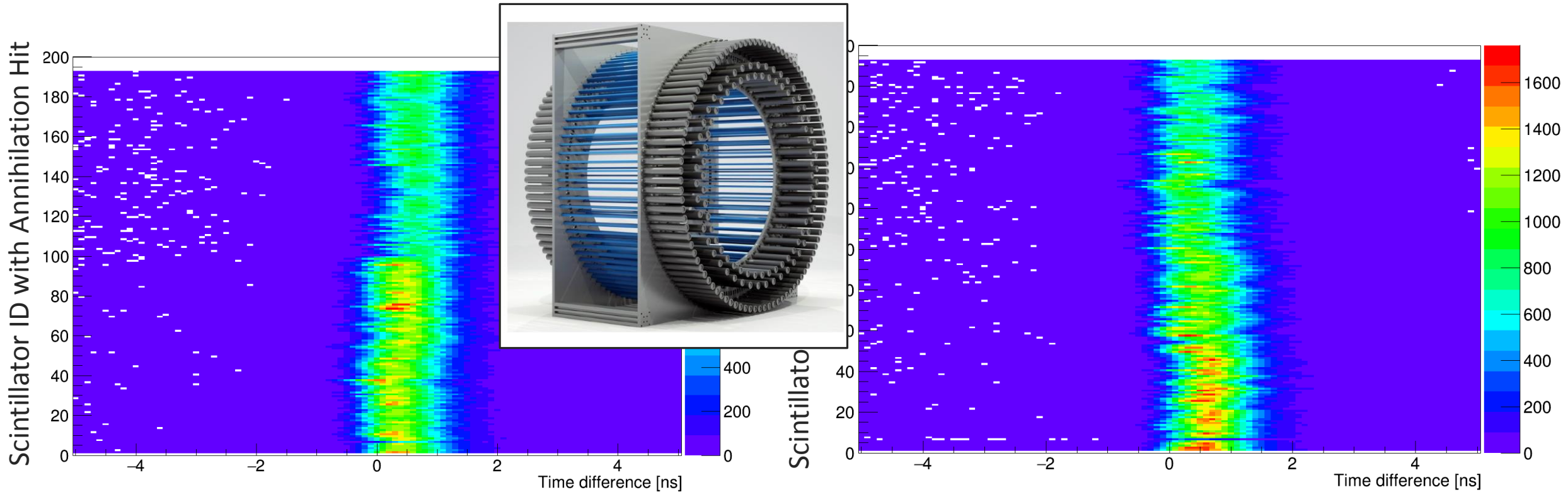
Calibration between modules



Positronium Lifetime spectrum should not depend on the pair of modules that were hit (if all are calibrated)

**And most important which scintillator
Annihilation/Deexcitation gamma quantum hit**

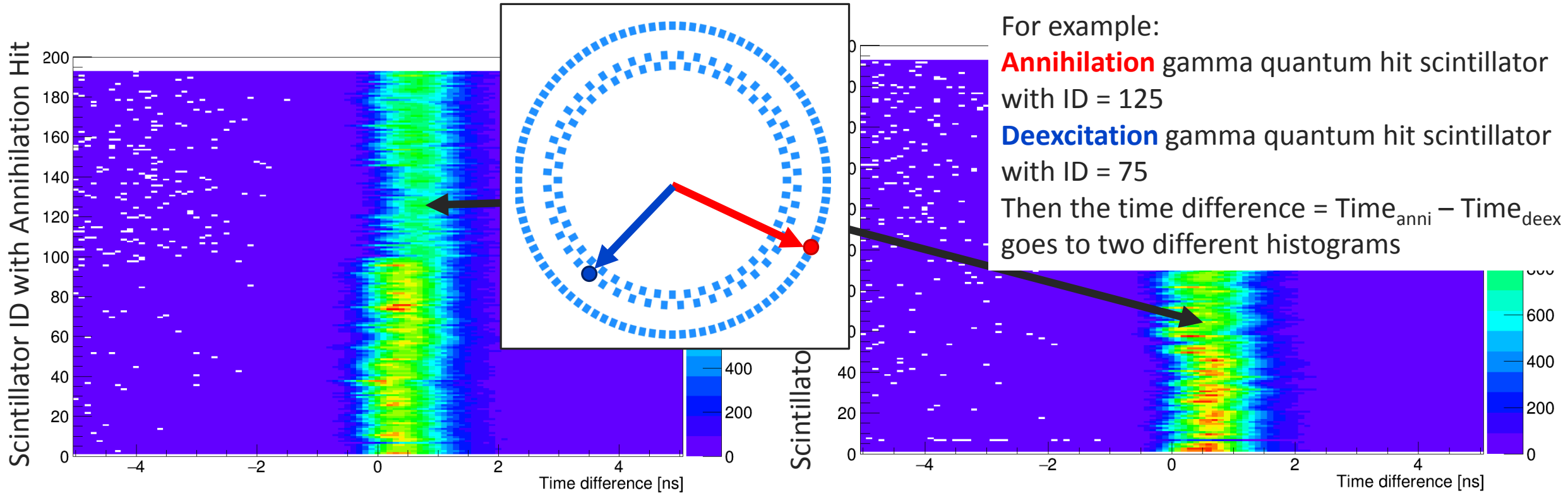
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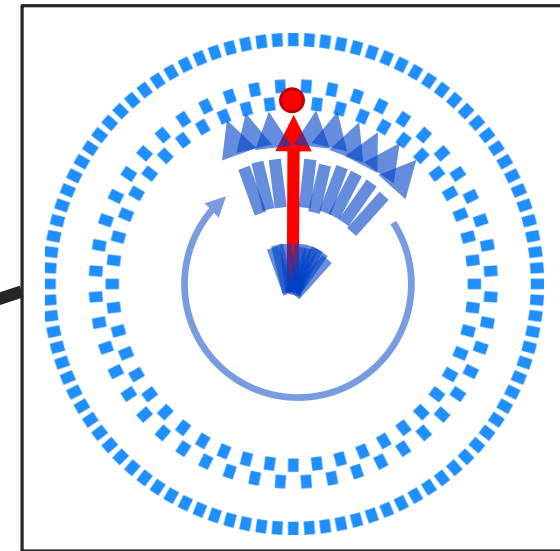
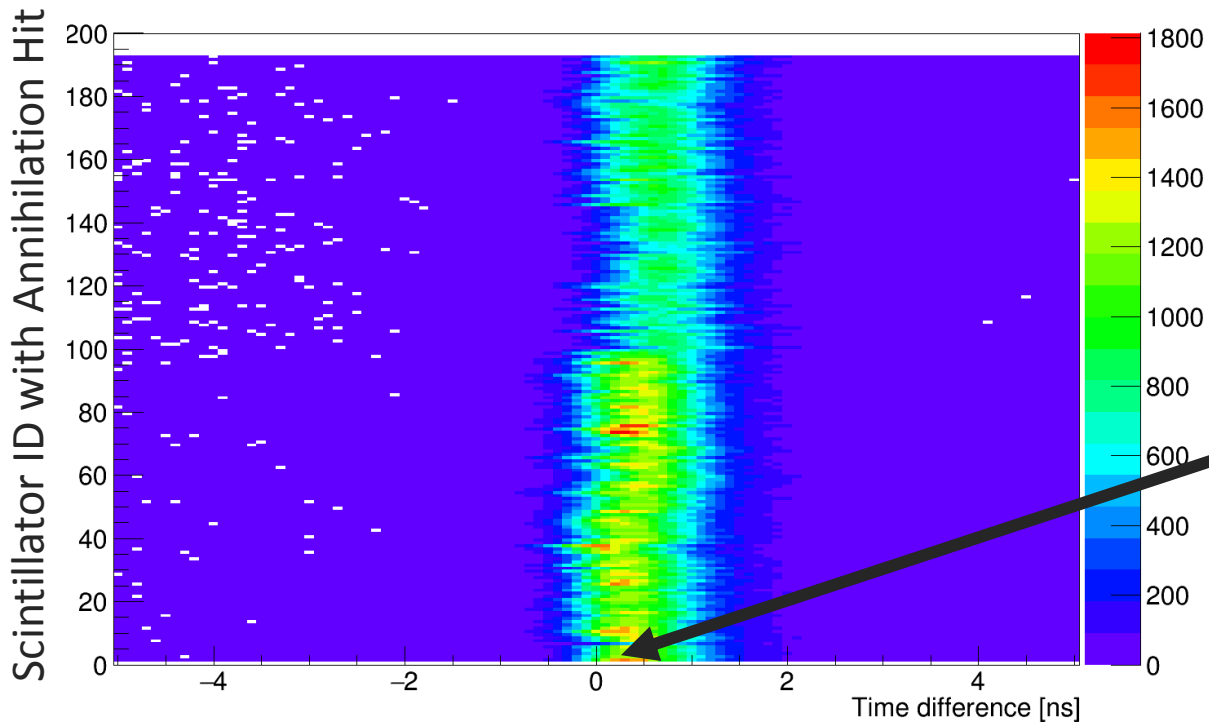
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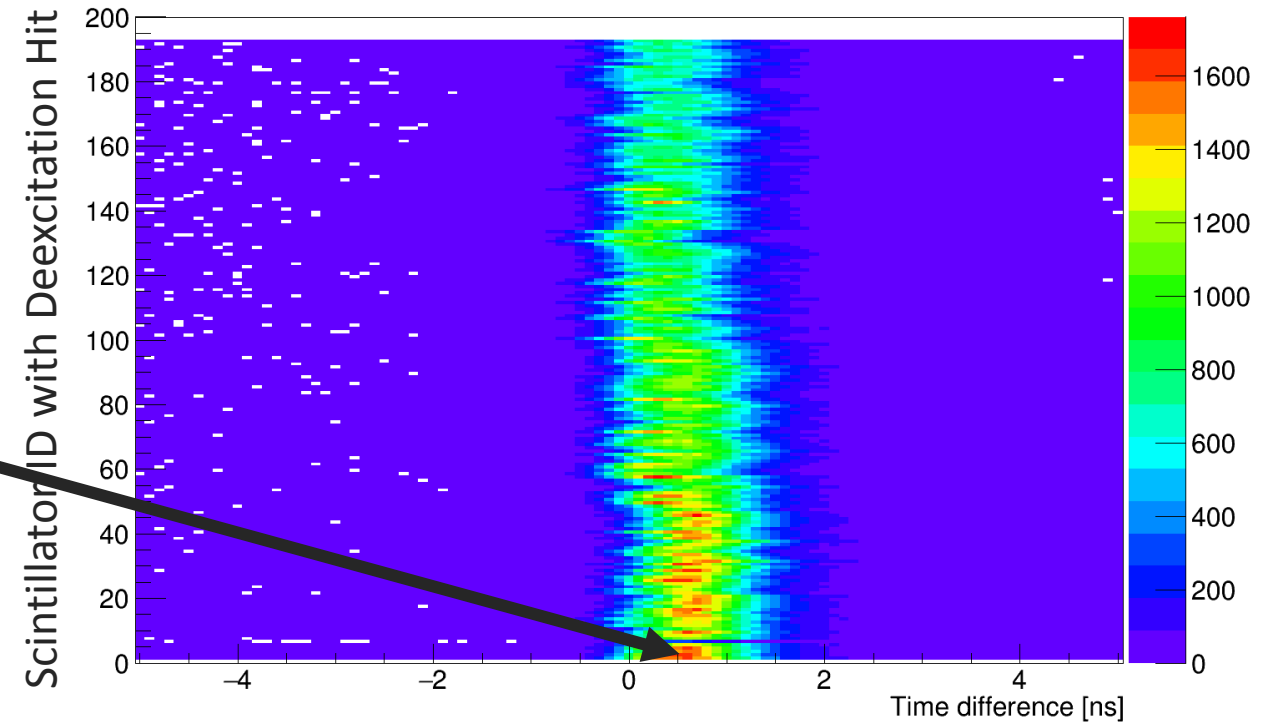
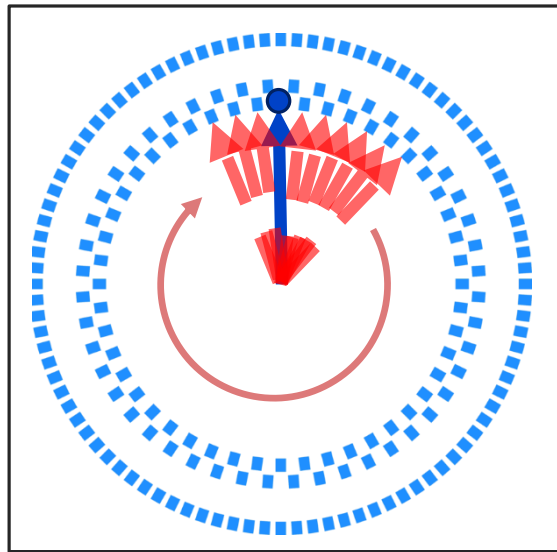
Calibration between modules



Focusing attention on one ID = 1.

Every event in which **Annihilation** gamma quantum hit scintillator with ID = 1

Calibration between modules

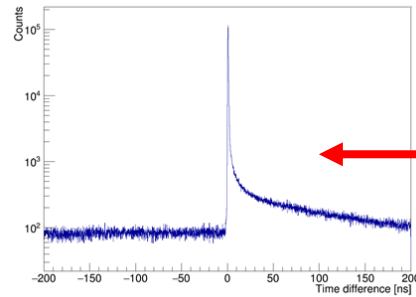
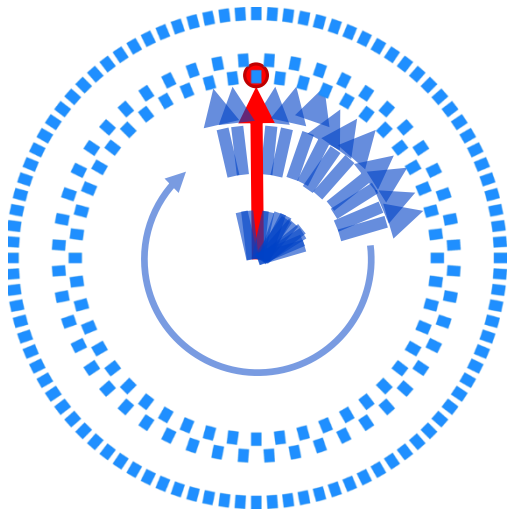


Focusing attention on one ID = 1.

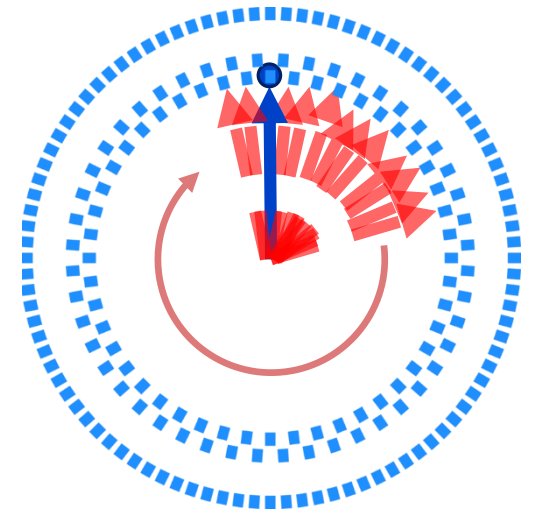
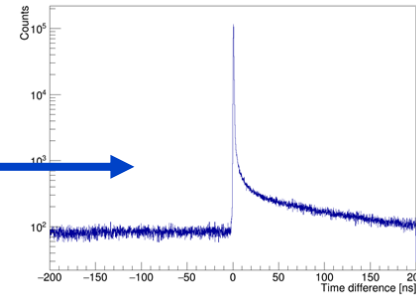
Every event in which **Deexcitation** gamma quantum hit scintillator with ID = 1

Calibration between modules

For one given ID of the scintillator we will have two histograms for consideration



In perfect situation
these will be the same



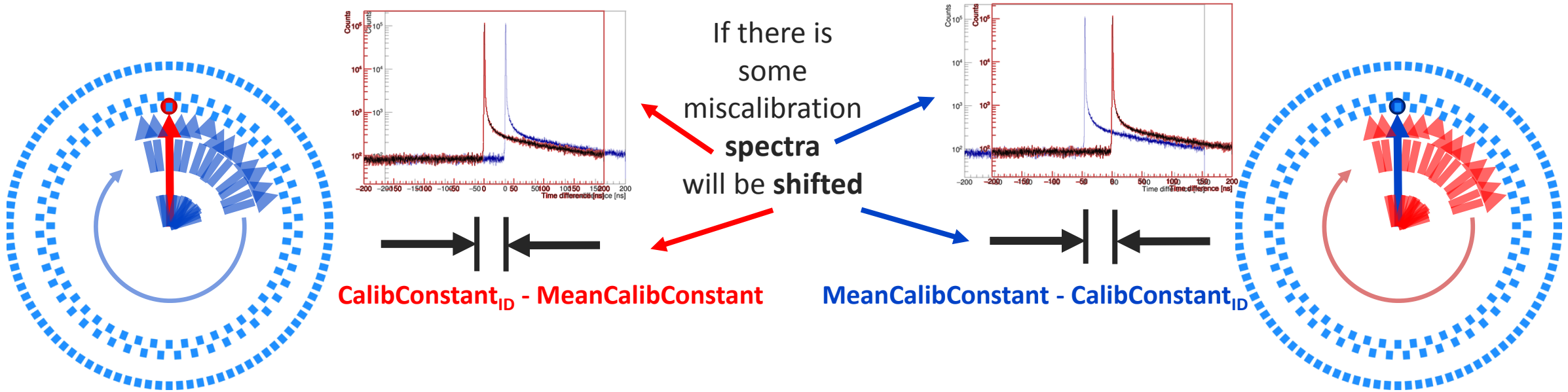
For given Annihilation ID:

For given Deexcitation ID:

$$(\text{Time}_{\text{Anni}} + \text{CalibConstant}_{\text{ID}}) - (\text{Time}_{\text{Deex}} + \text{MeanCalibConstant}) = \text{Time difference} = (\text{Time}_{\text{Anni}} + \text{MeanCalibConstant}) - (\text{Time}_{\text{Deex}} + \text{CalibConstant}_{\text{ID}})$$

Calibration between modules

For one given ID of the scintillator we will have two histograms for consideration



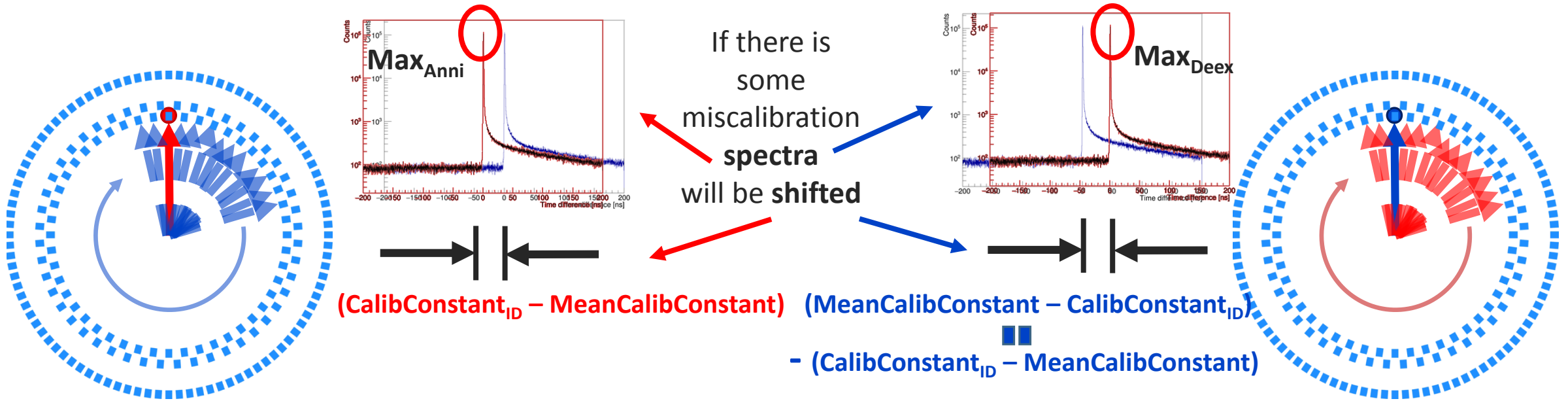
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Calibration between modules

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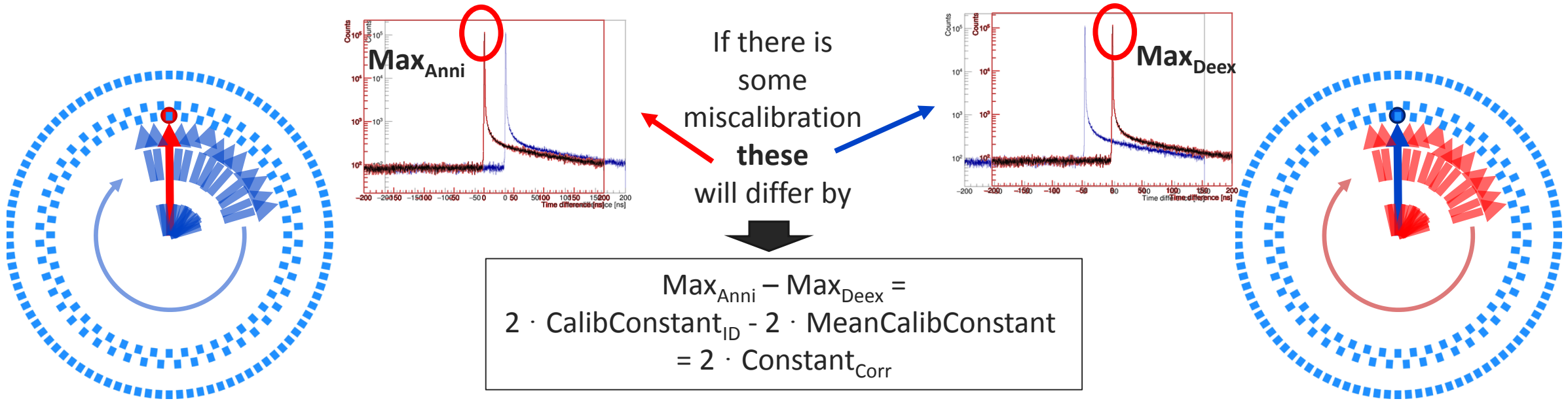
For given Annihilation ID:

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$$(\text{Time}_{\text{Anni}} + \text{CalibConstant}_{ID}) - (\text{Time}_{\text{Deex}} + \text{MeanCalibConstant}) = \text{Time difference} = (\text{Time}_{\text{Anni}} + \text{MeanCalibConstant}) - (\text{Time}_{\text{Deex}} + \text{CalibConstant}_{ID})$$

Calibration between modules

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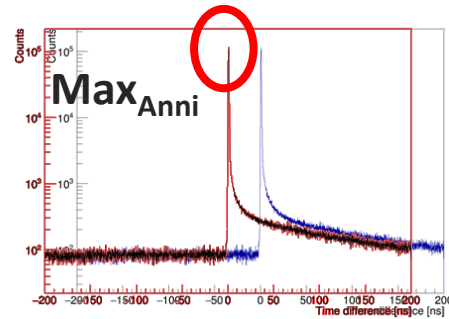
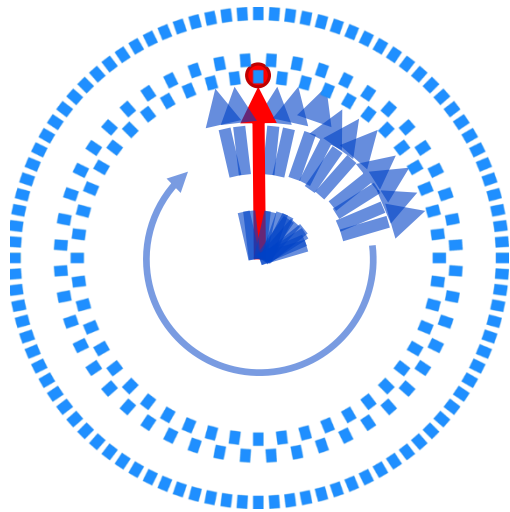
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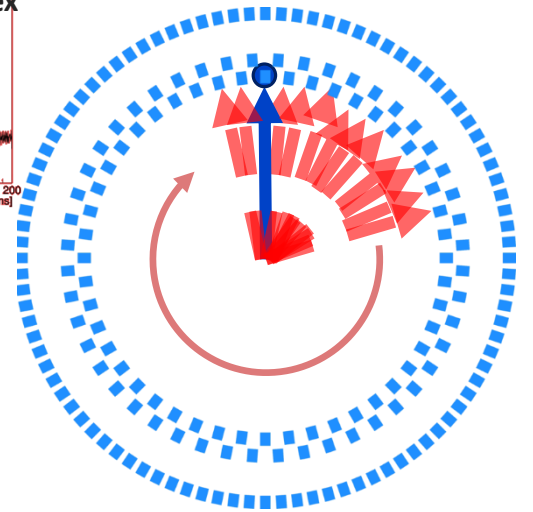
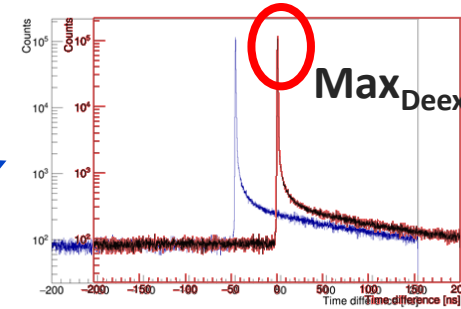
$$(\text{Time}_{\text{Anni}} + \text{CalibConstant}_{\text{ID}}) - (\text{Time}_{\text{Deex}} + \text{MeanCalibConstant}) = \text{Time difference} = (\text{Time}_{\text{Anni}} + \text{MeanCalibConstant}) - (\text{Time}_{\text{Deex}} + \text{CalibConstant}_{\text{ID}})$$

Calibration between modules

For one given ID of the scintillator we will have two histograms for consideration



If there is some miscalibration these will differ by



$$\begin{aligned} \text{Max}_{\text{Anni}} - \text{Max}_{\text{Deex}} &= \\ 2 \cdot \text{CalibConstant}_{\text{ID}} - 2 \cdot \text{MeanCalibConstant} &= \\ = 2 \cdot \text{Constant}_{\text{Corr}} \end{aligned}$$

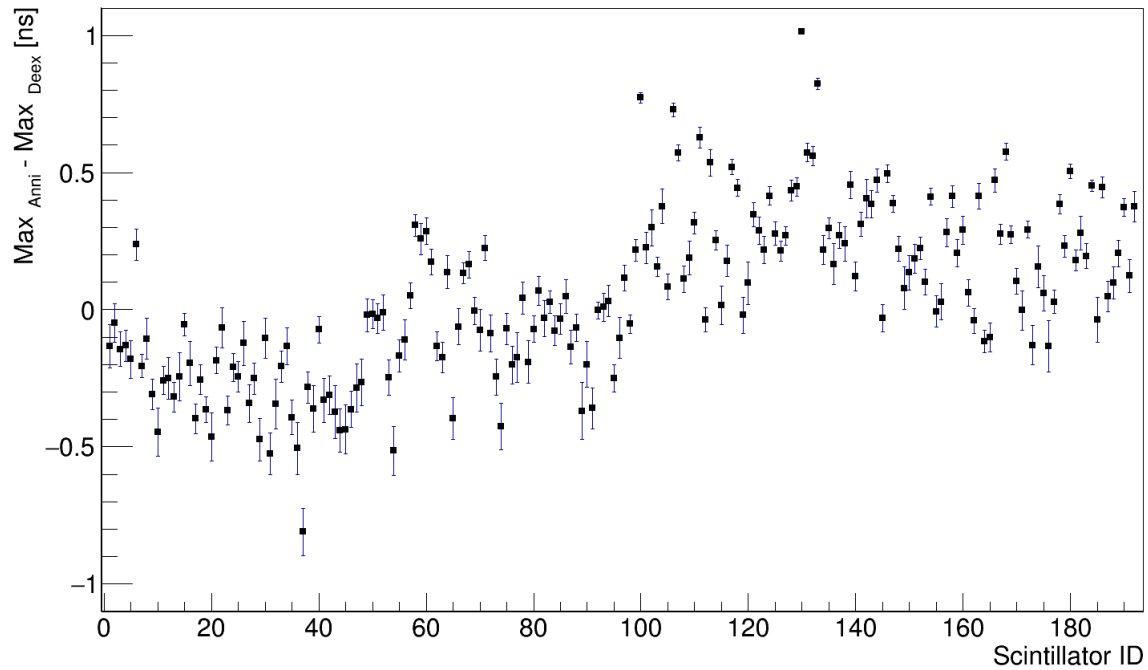
$$\text{Correction for Scintillator: } (\text{Max}_{\text{Anni}} - \text{Max}_{\text{Deex}})/2$$

Maximum of distribution calculated as zero of 1st derivative

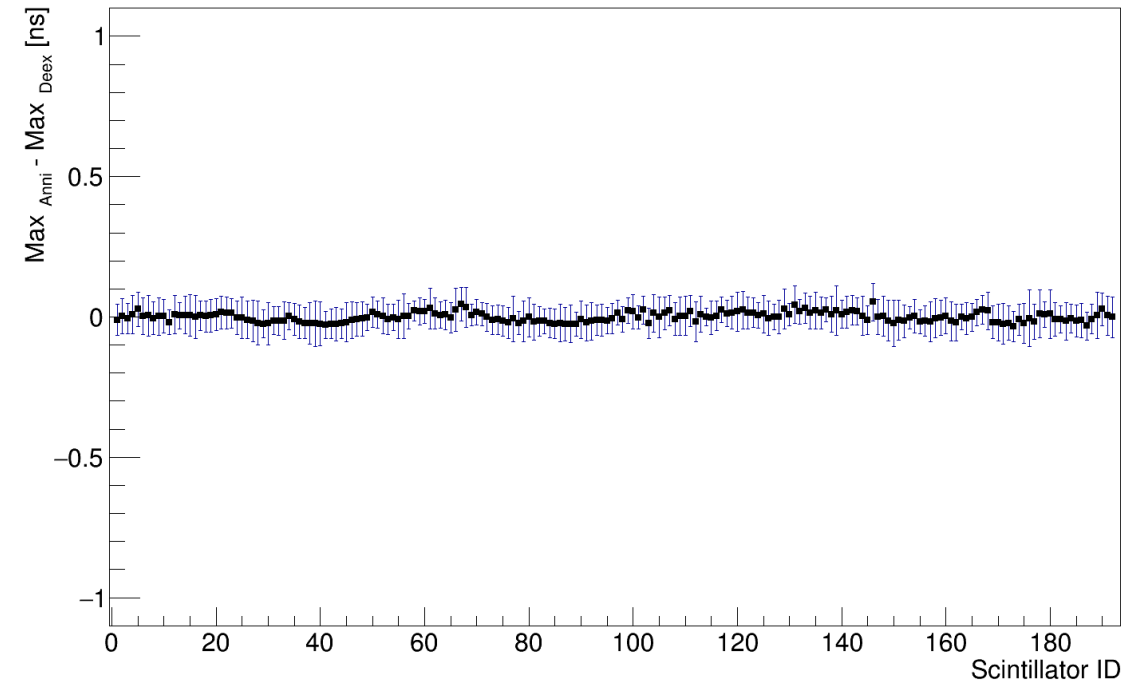
Correction for each side of module:
Correction for scintillator

Calibration between modules

Before Calibration (0 iteration)



After Calibration (3rd iteration)



Conclusions

New Iterative Time Calibration procedure was developed for the J-PET. As a first calibration method of TOF-PET systems, it uses point-like source that emits prompt gamma quantum for calibration. Calibration was performed for ^{22}Na source in Kapton foil with XAD4 (porous polymer) around it.

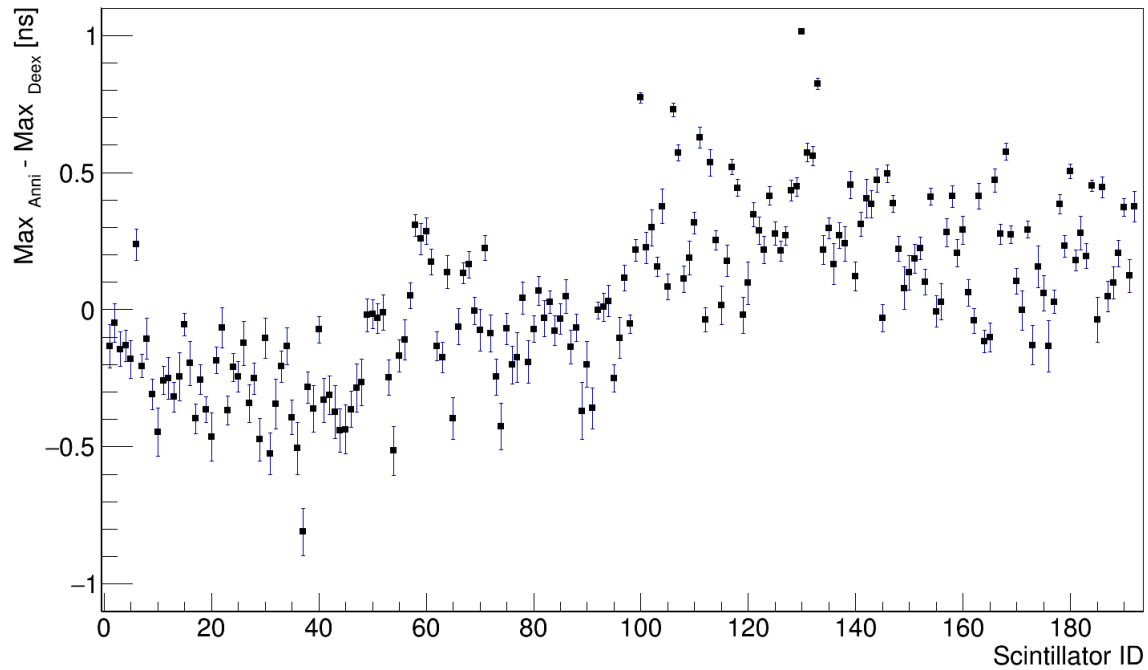
Compared to other methods, there is no need to mount complicated setup for calibration in TOF-PET detector. Every possible pair of detector is calibrated at once.

Additionally, because the measure of the miscalibration is the difference in the positions of the maxima, calibration does not depend much on the material that is around radioactive source.

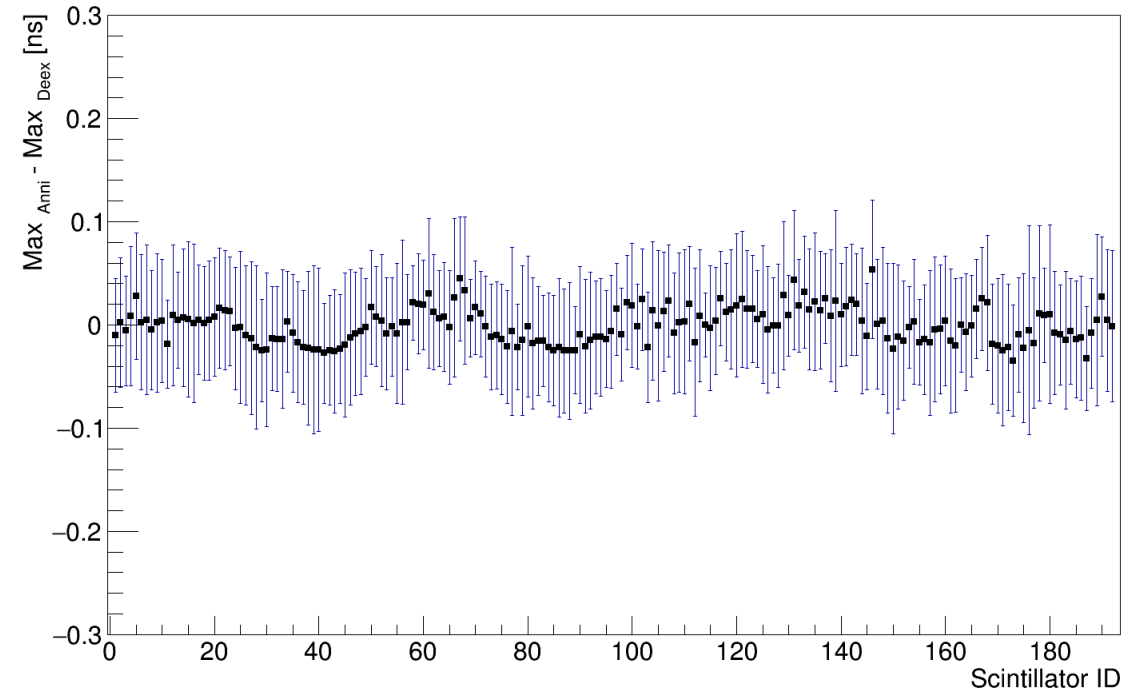
Thank You for Your attention

Calibration between modules

Before Calibration (0 iteration)



After Calibration (3rd iteration)



Calibration between modules

Uncertainty calculation:

Maximum calculated from the linear fit: $y = A \cdot x + B$

So $x_0 = -B/A$

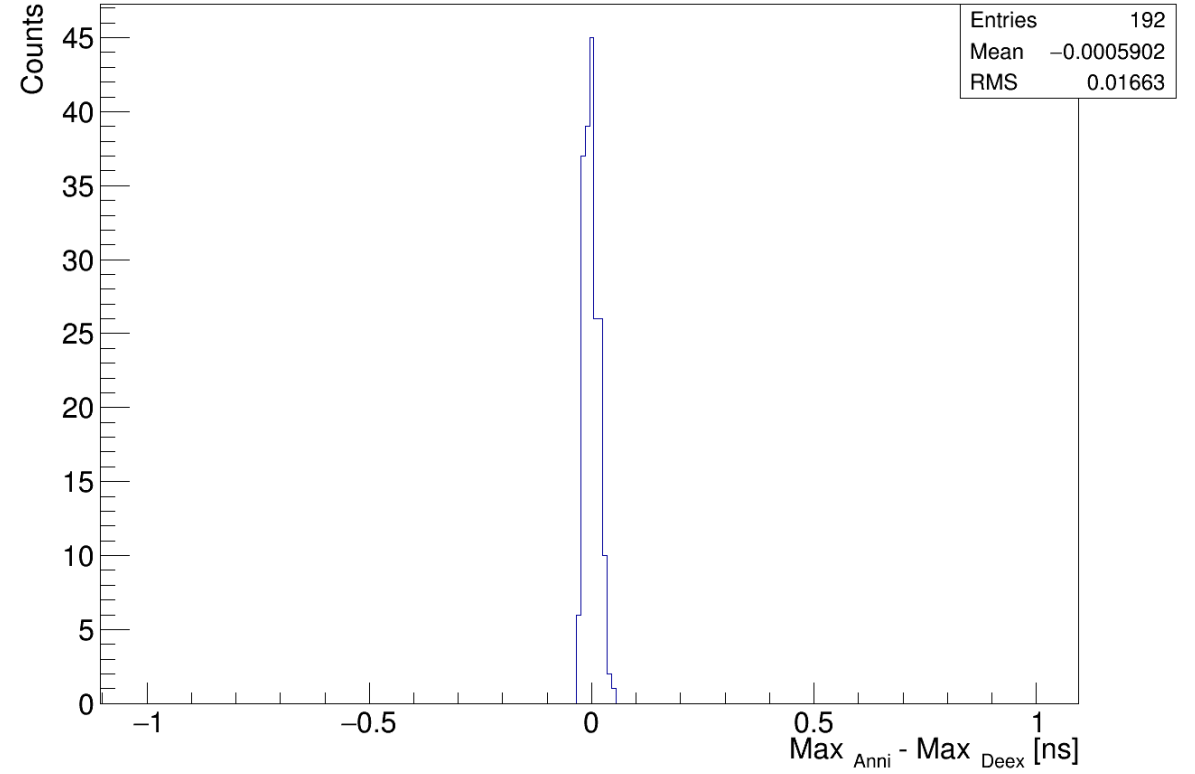
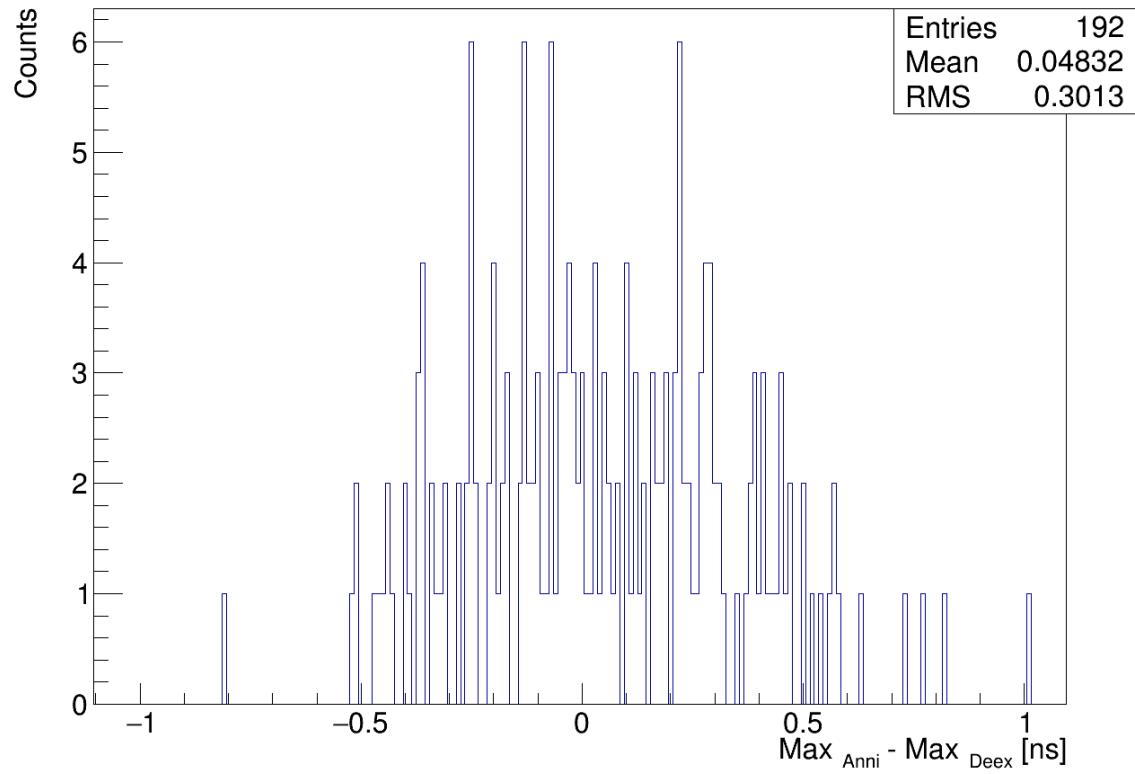
x_0 Error: $dx_0 = \sqrt{\left(\frac{dB}{A}\right)^2 + \left(\frac{B \cdot dA}{A^2}\right)^2}$

dA , dB from the formulas of uncertainty of linear fit

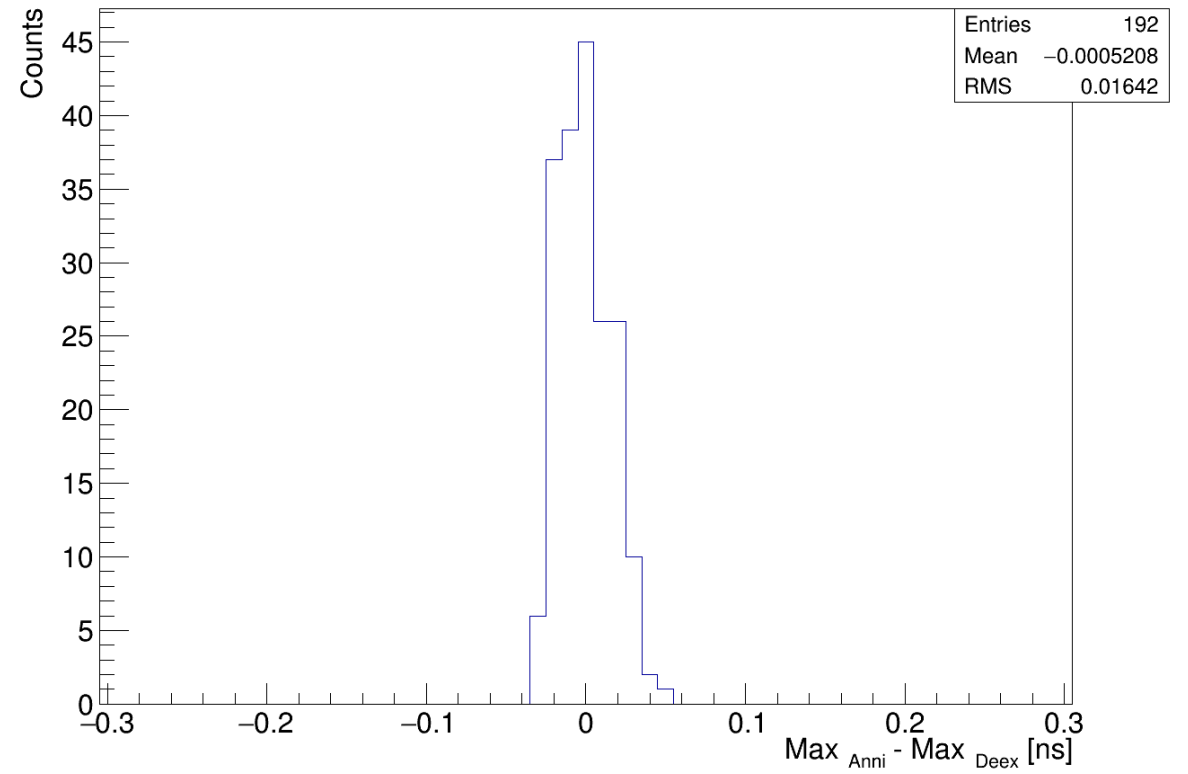
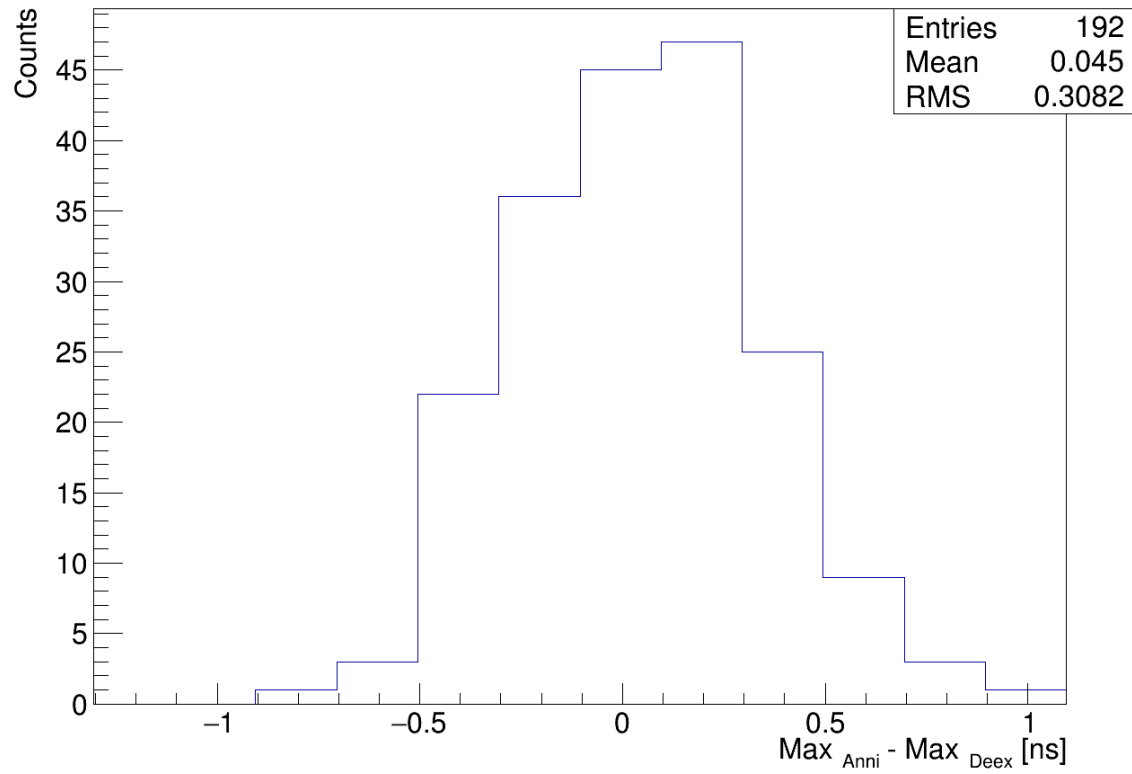
$$\hat{\epsilon}_i = y_i - B - A x_i.$$

$$s_A = \sqrt{\frac{\frac{1}{n-2} \sum_{i=1}^n \hat{\epsilon}_i^2}{\sum_{i=1}^n (x_i - \bar{x})^2}} \quad s_B = s_A \sqrt{\frac{1}{n} \sum_{i=1}^n x_i^2} = \sqrt{\frac{1}{n(n-2)} \left(\sum_{i=1}^n \hat{\epsilon}_j^2 \right) \frac{\sum_{i=1}^n x_i^2}{\sum_{i=1}^n (x_i - \bar{x})^2}}$$

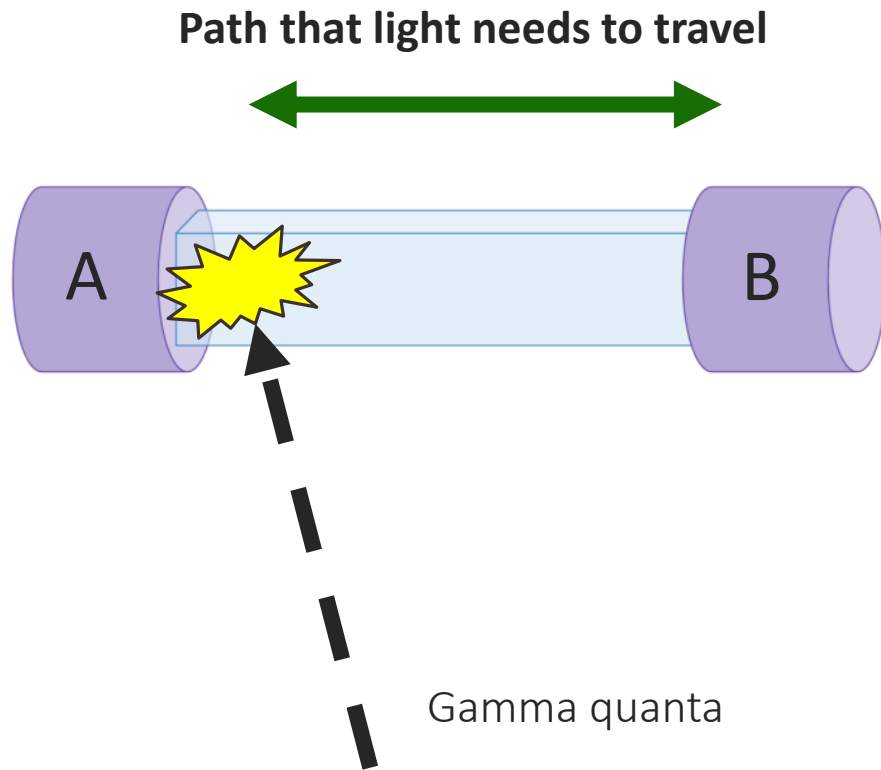
Calibration between modules



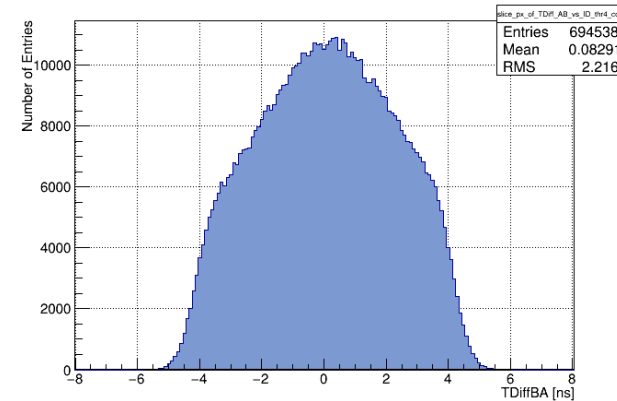
Calibration between modules



Velocity calibration

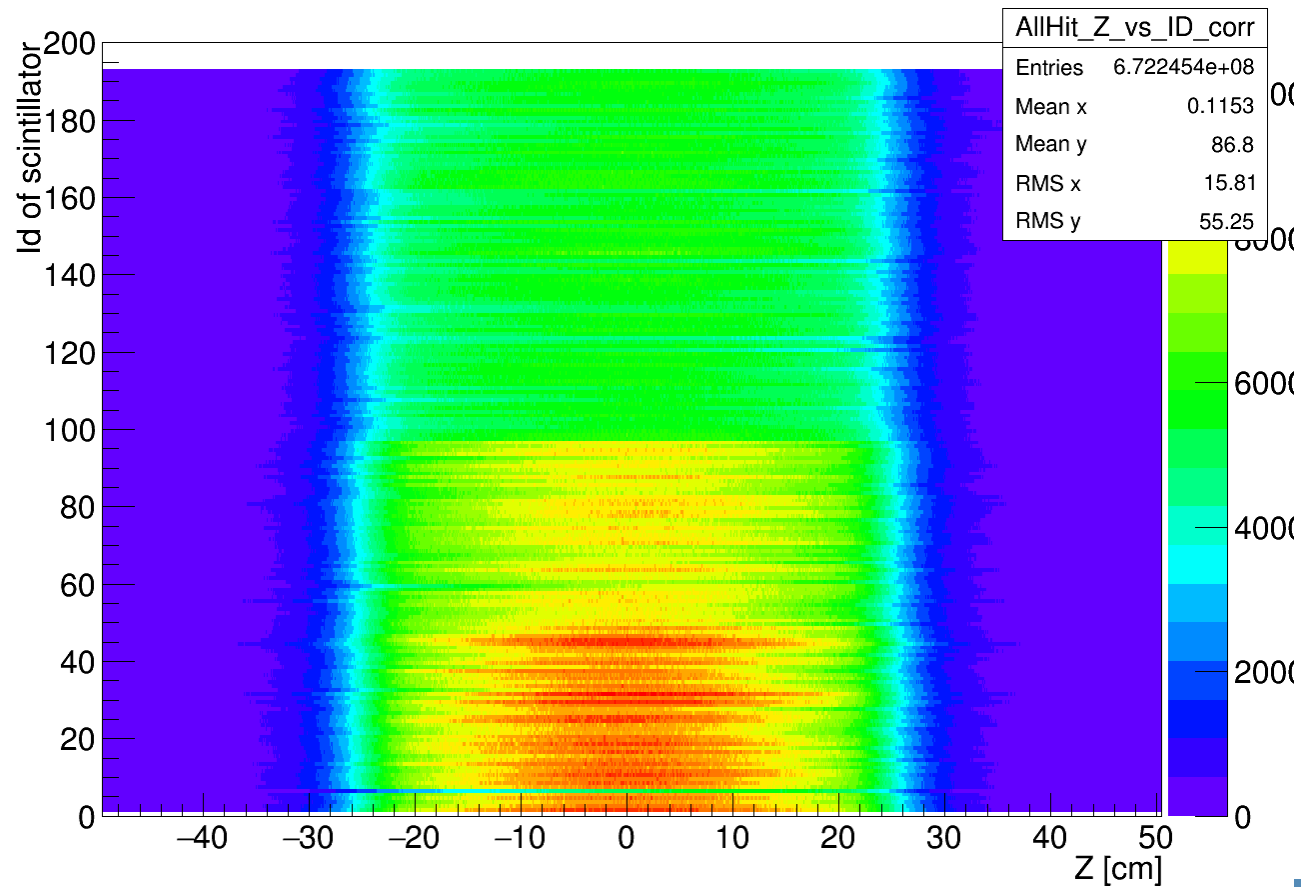


$$\text{Time difference BA} = \frac{\text{Path that light needs to travel}}{\text{Velocity of light in scintillator}}$$

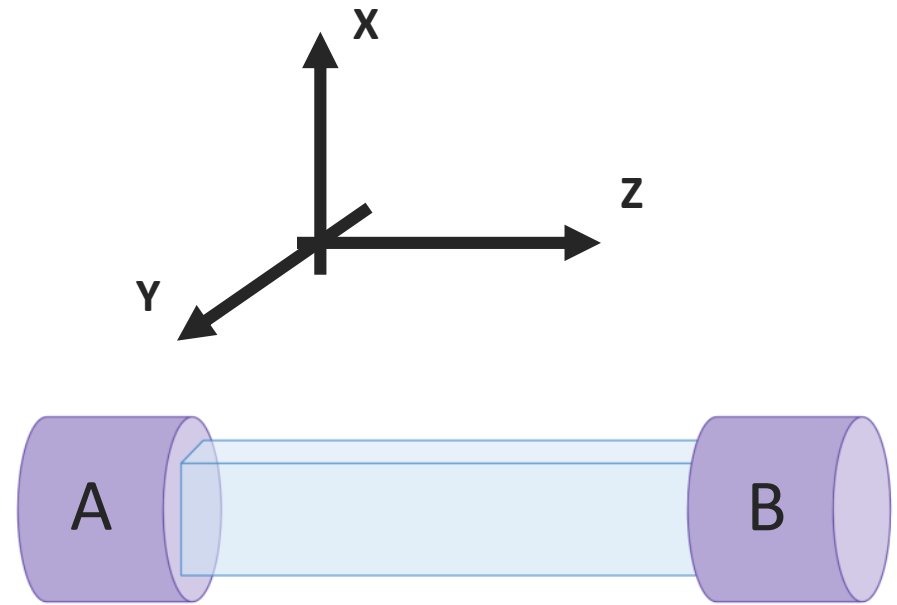


$$\text{Right Edge} - \text{Left Edge} = \frac{\text{Effective Length}}{\text{Velocity of light in scintillator}}$$

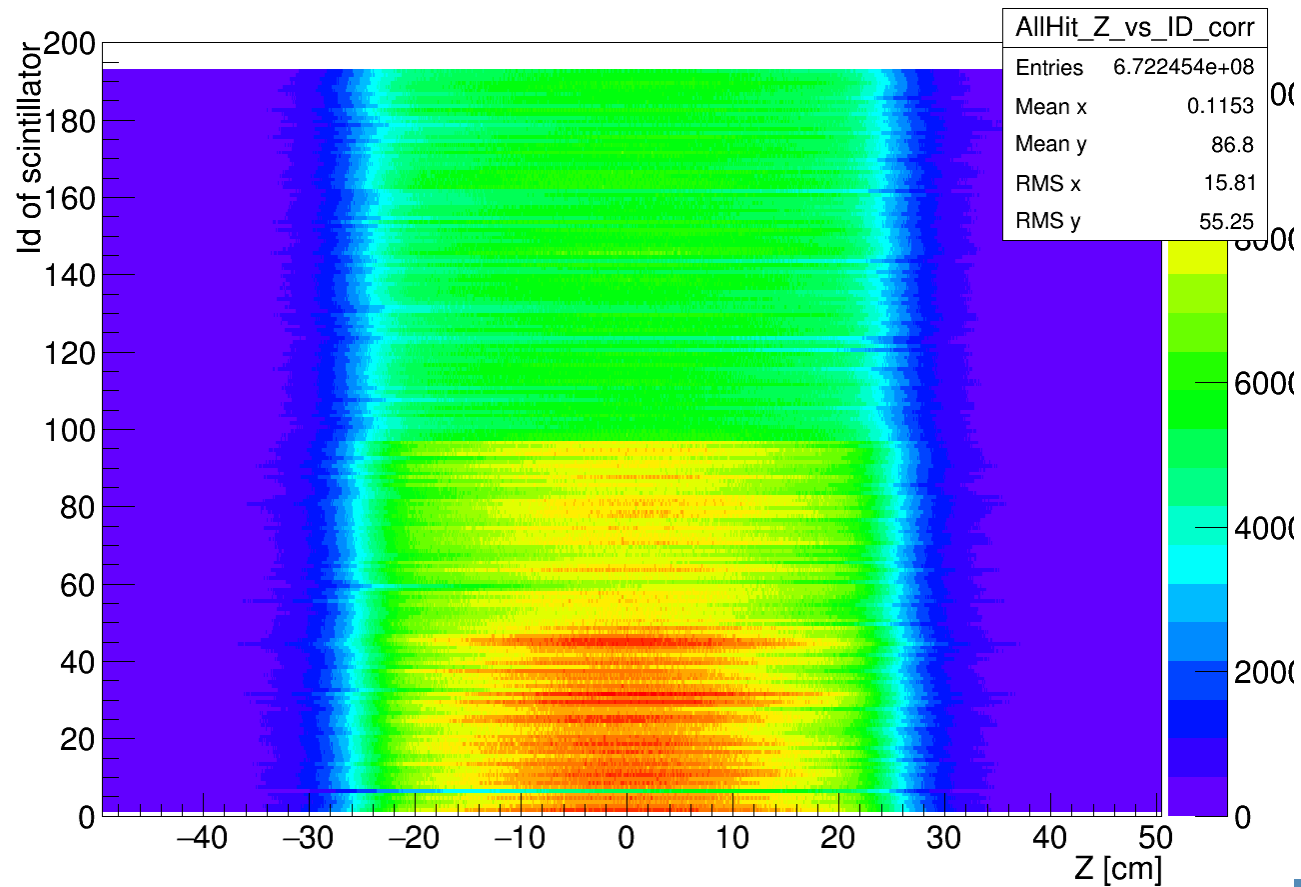
Velocity calibration



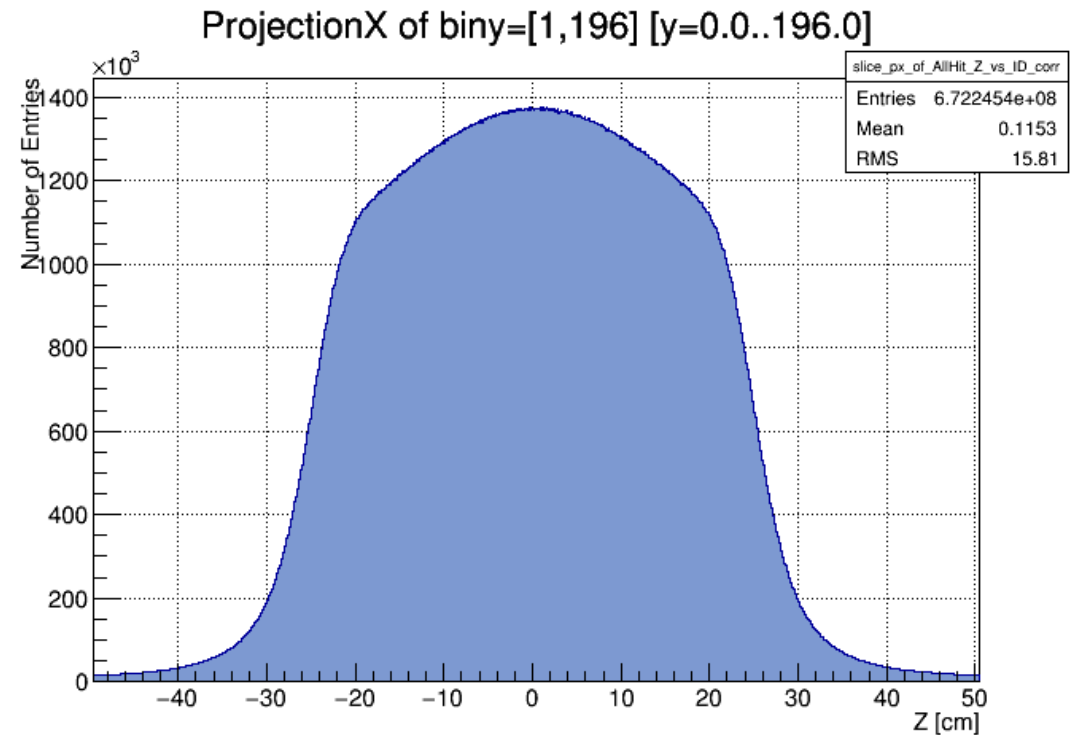
Assuming that Effective Length (later explanation) = 50 cm
Velocity = $50 / (\text{MiddleEdgeRight} - \text{MiddleEdgeLeft})$
Z position = $\text{Velocity} * (\text{TimeB} - \text{TimeA})/2$



Velocity calibration

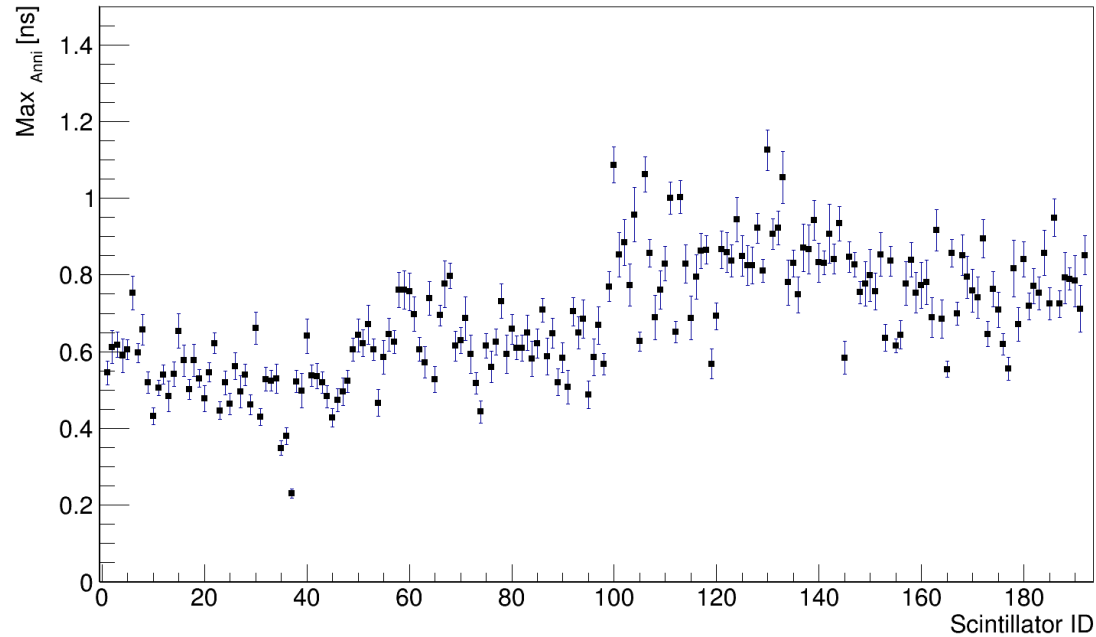


Assuming that Effective Length (later explanation) = 50 cm
Velocity = $50 / (\text{MiddleEdgeRight} - \text{MiddleEdgeLeft})$
Z position = $\text{Velocity} * (\text{TimeB} - \text{TimeA}) / 2$

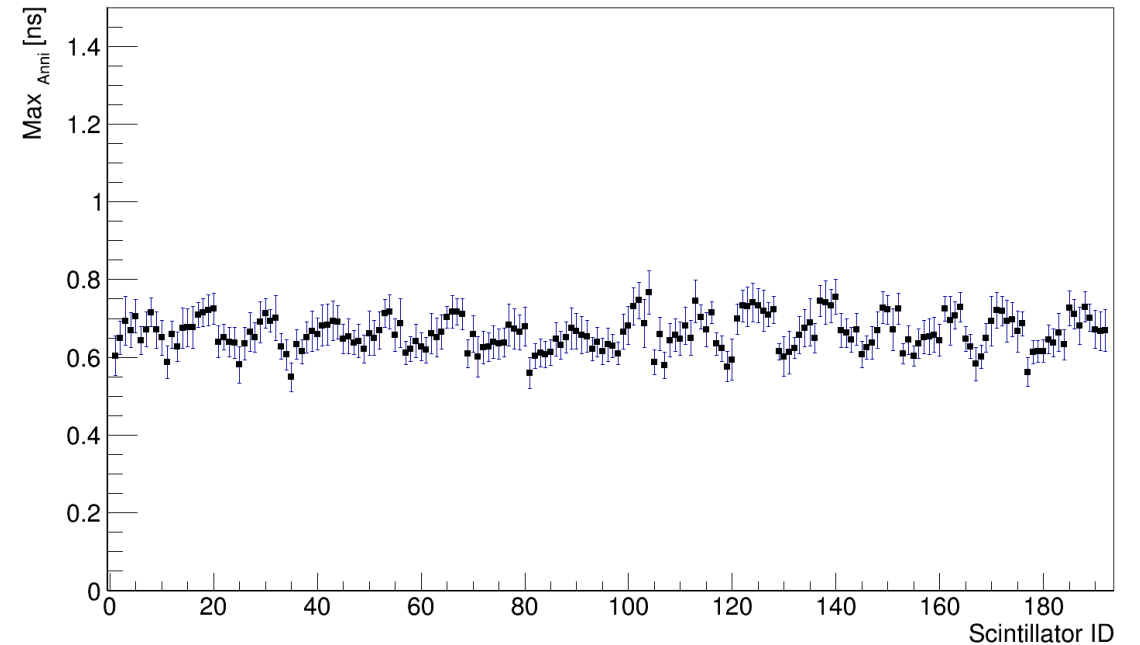


Calibration between modules

Before Calibration (0 iteration)

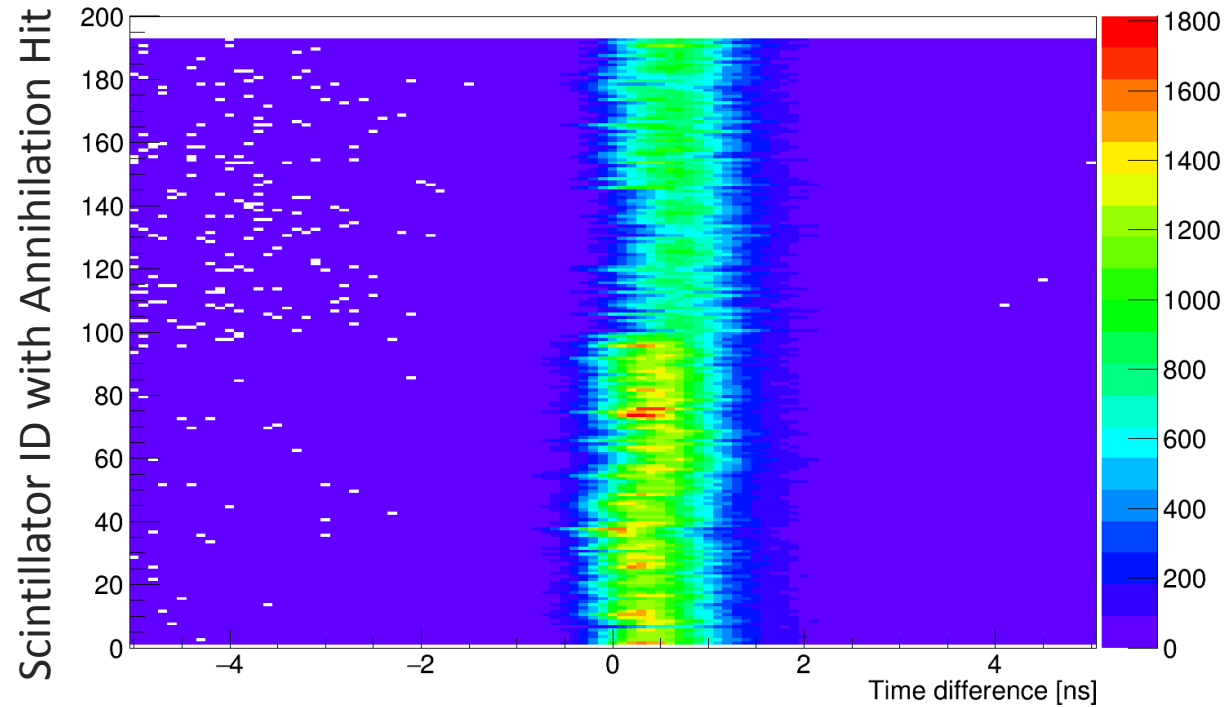


After Calibration (3rd iteration)

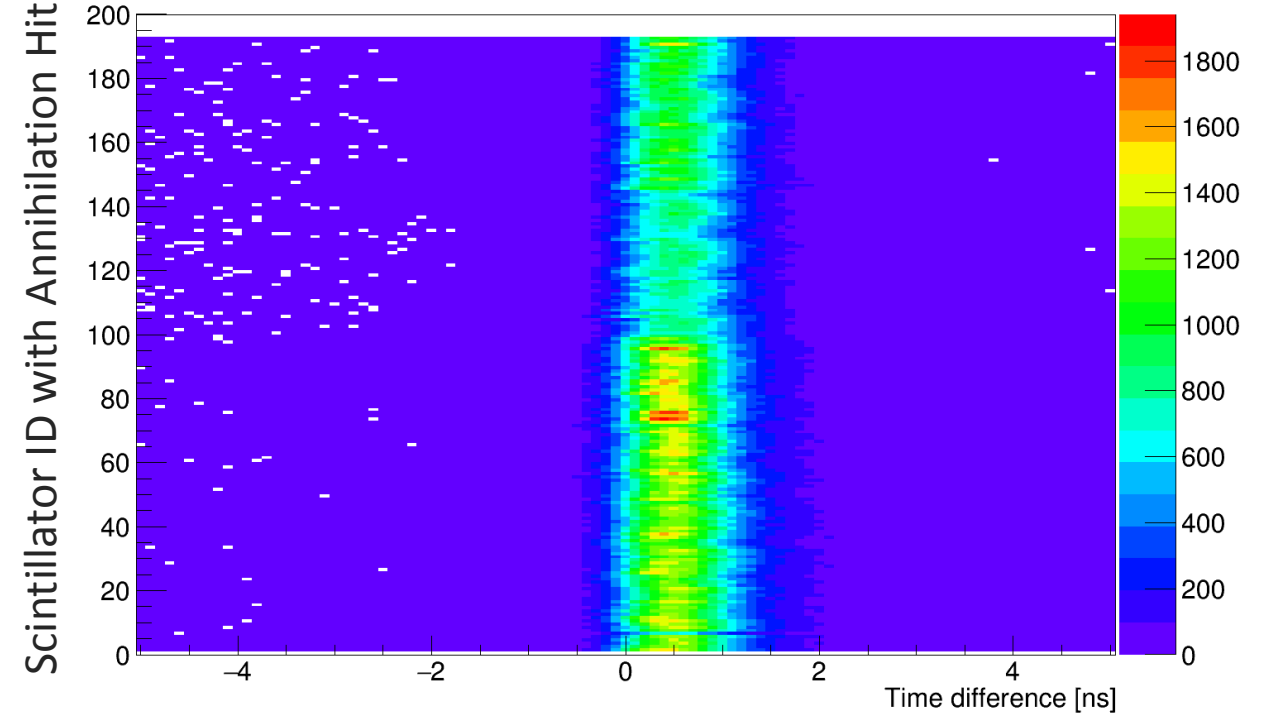


Calibration between modules

Before Calibration (0 iteration)

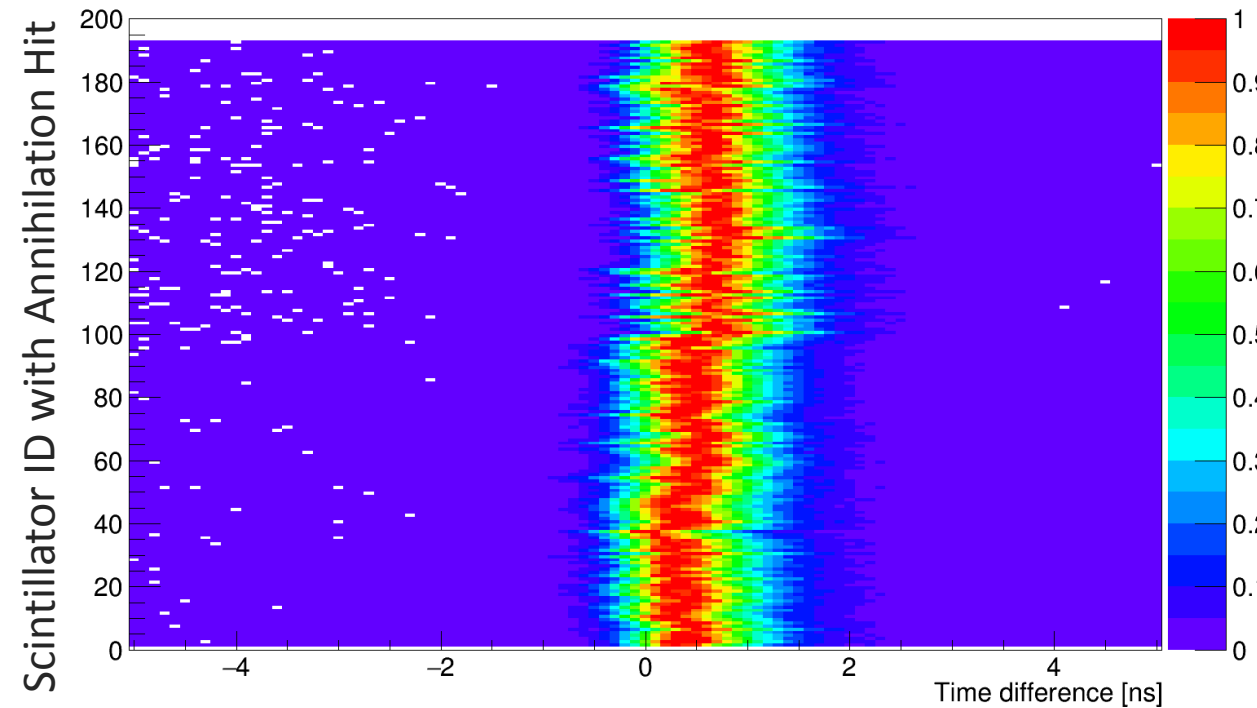


After Calibration (3rd iteration)

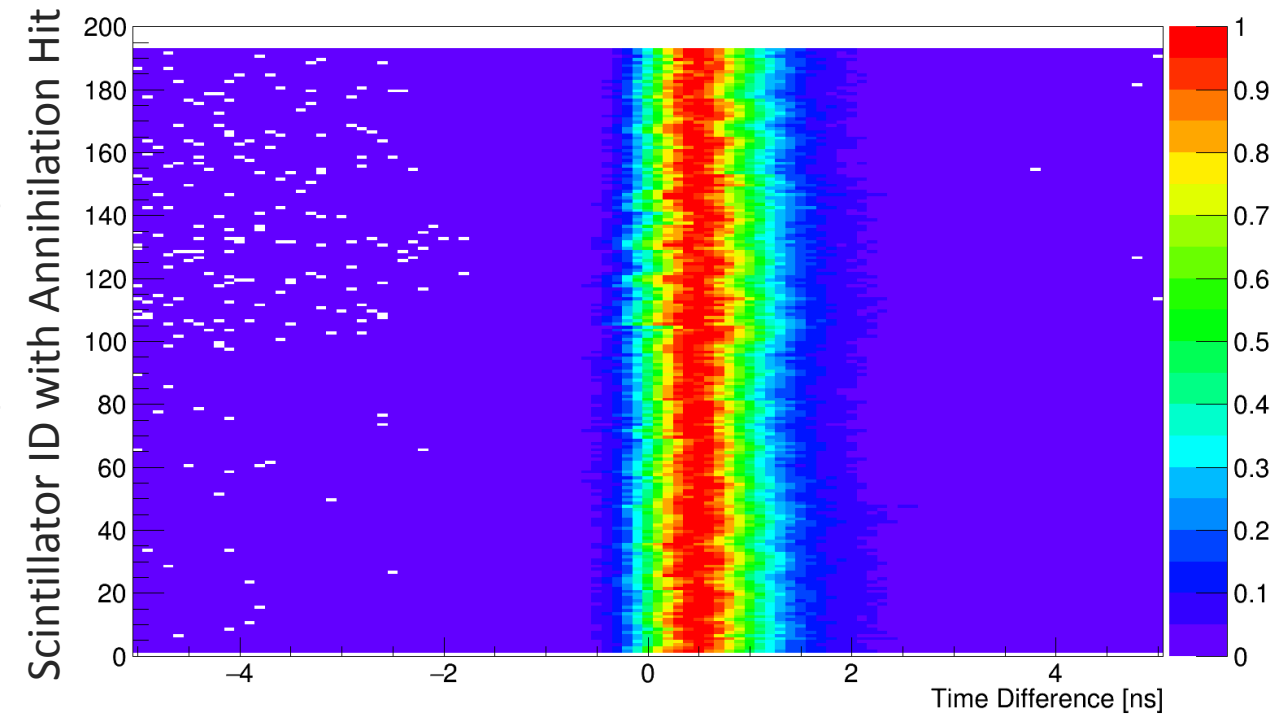


Calibration between modules

Before Calibration (0 iteration)

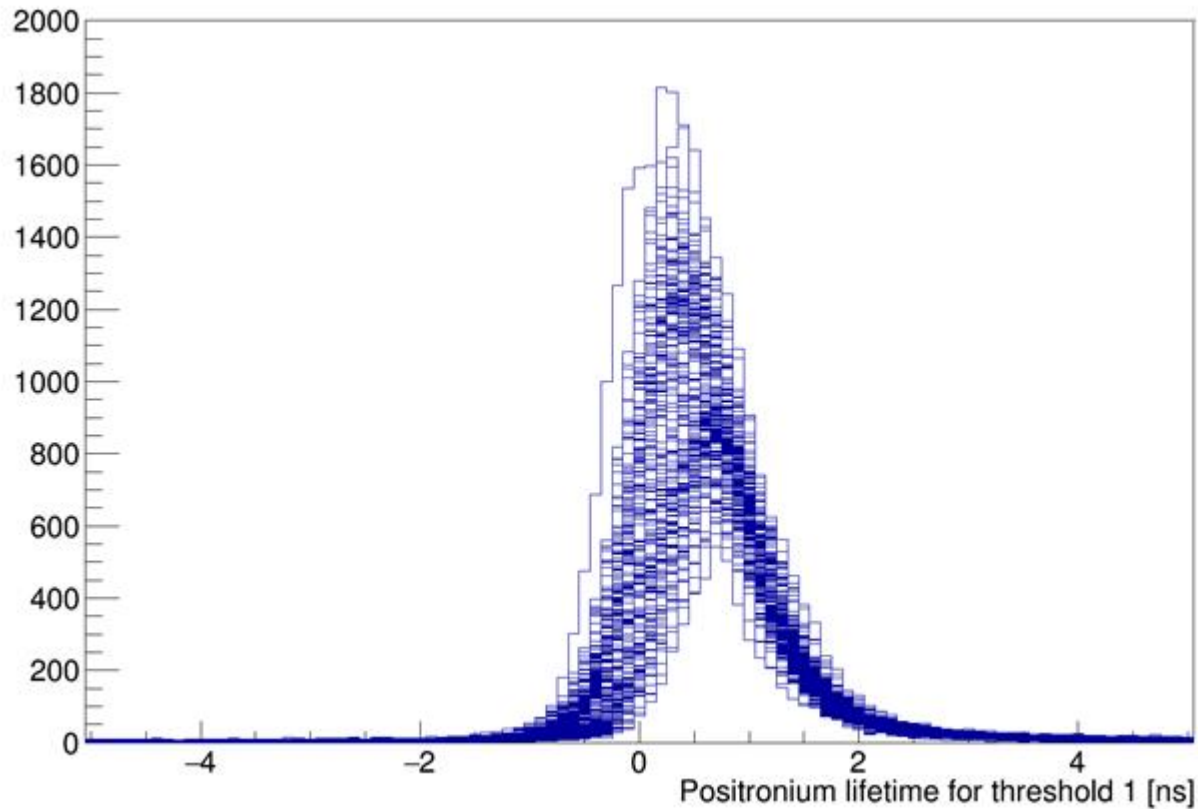


After Calibration (3rd iteration)

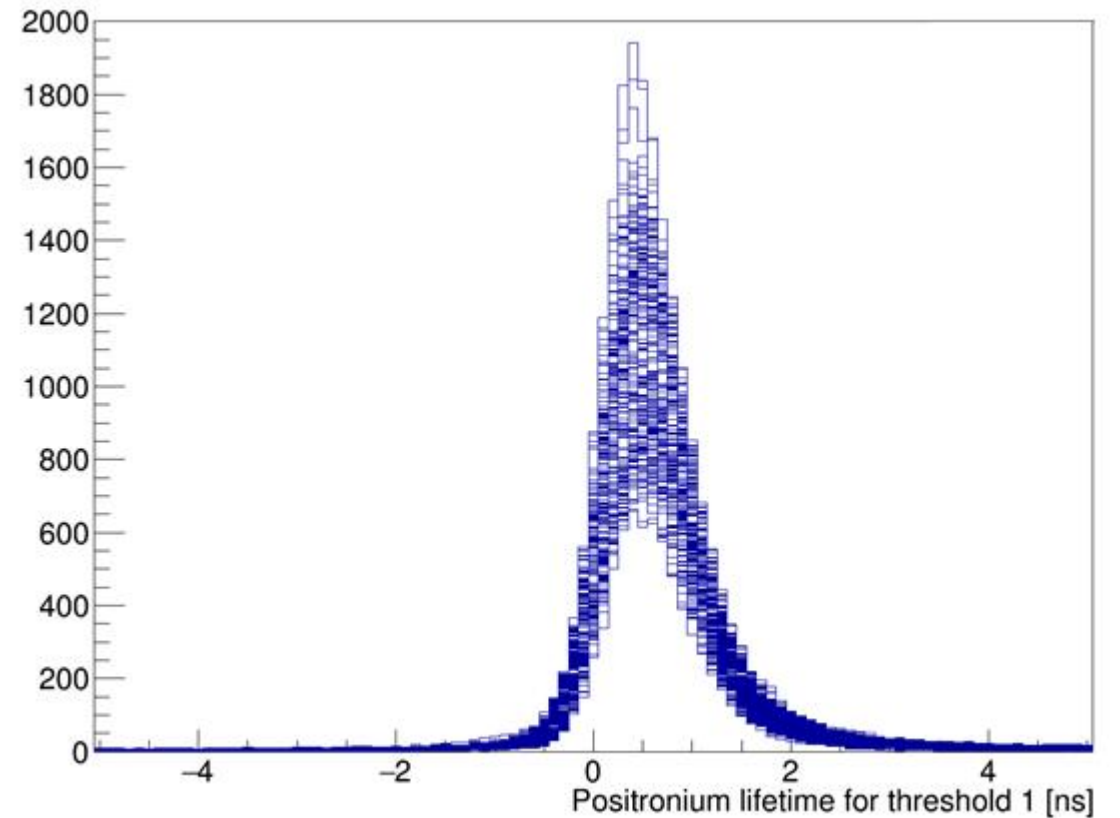


Calibration between modules

Before Calibration (0 iteration)



After Calibration (3rd iteration)



Effective Length studies

Calibration constant depends on chosen Effective Length, because:

- Effective Length used for calculation of the Z position of registered gamma quantum
- Time of the Hits used for Calibration, are corrected on TOF -> subtracting the path before deposition of Energy (path depends on the Z)

For different assumed Effective Length the same calibration procedure was applied, to check how calibration constants changes with the change of Effective Length.

Calibration constants for Effective Length = 50 cm chosen as reference

Velocity of light in scintillator = Effective Length

*** (Right Edge – Left Edge)**

Z position of Hit = Velocity of light in scintillator * (TimeB – TimeA)/2

Effective Length studies

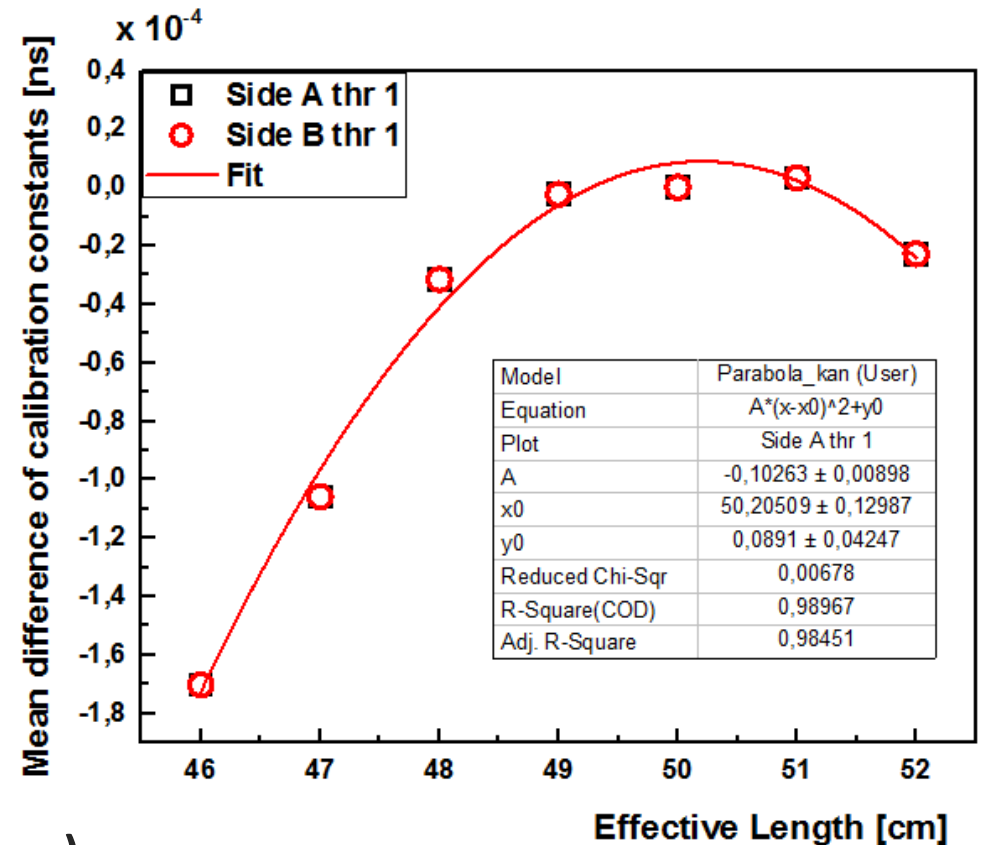
For different assumed Effective Length the same calibration procedure was applied, to check how calibration constants changes with the change of Effective Length.

Calibration constants for Effective Length = 50 cm chosen as reference.

Mean difference of calibration constant:

Mean over all IDs of:

$$\left(\text{Correction for 50 cm} \right)_{ID} - \left(\text{Correction for [Effective Length] cm} \right)_{ID}$$



Effective Length studies

Calibration constant for 50 cm

y_1
 y_2
 y_3
 ...

Calibration constant for x cm

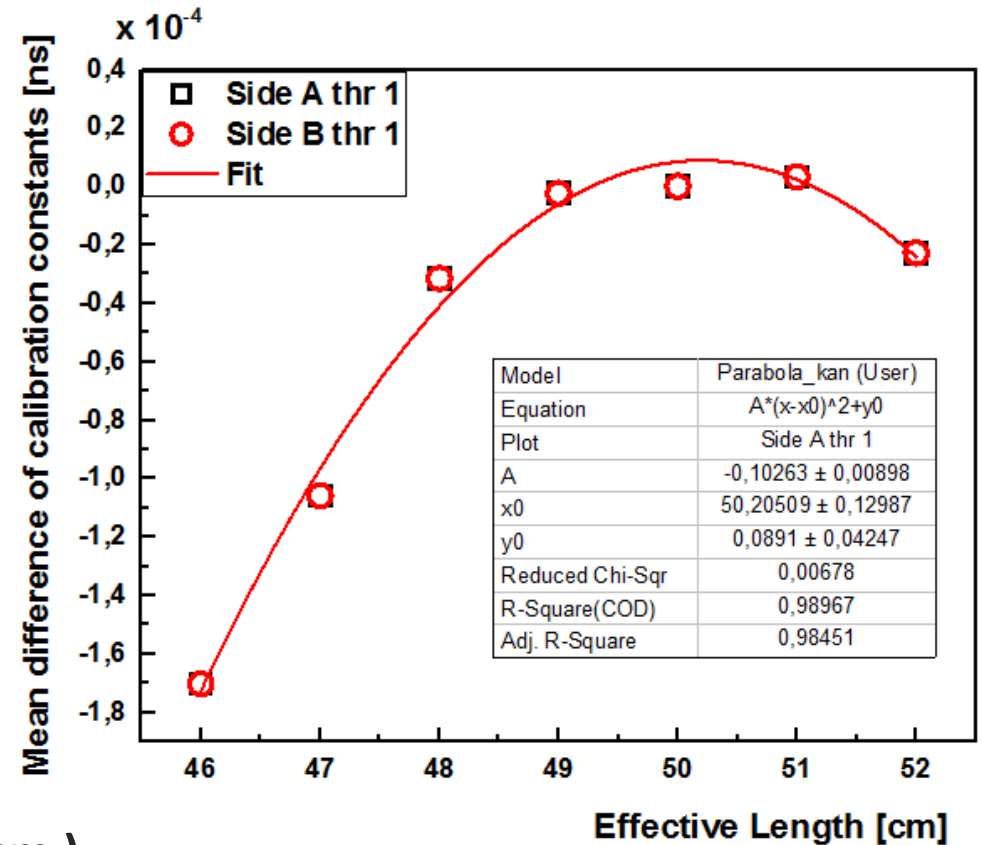
y_1'
 y_2'
 y_3'

Mean of set of $(y_i - y_i')$ for $i=1$ to $i=192$



Mean over all IDs of:

$$(\text{Correction for 50 cm})_{ID} - (\text{Correction for [Effective Length] cm})_{ID}$$



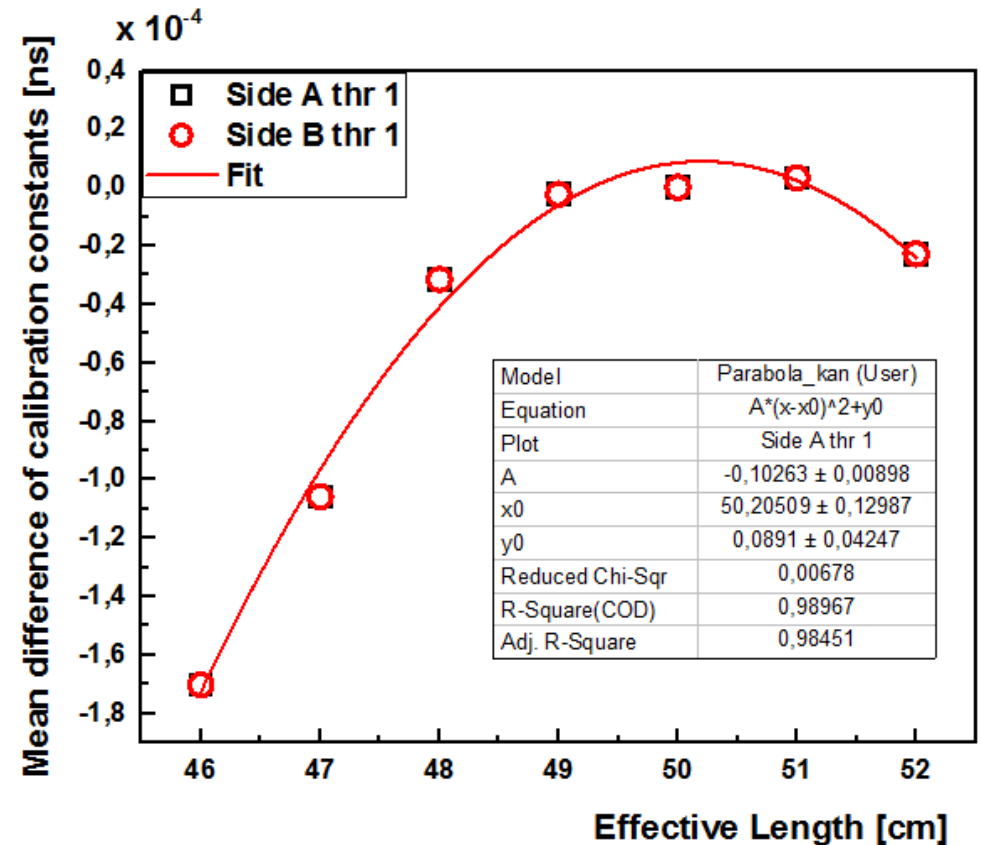
Effective Length studies

Nevertheless, the choice of Effective Length does not depend much on the Calibration Constant:

Effective length change by 1 cm causes the mean change of Calibration constant in the order of 0.1 ps
(Maximal change in the order of 1 ps)

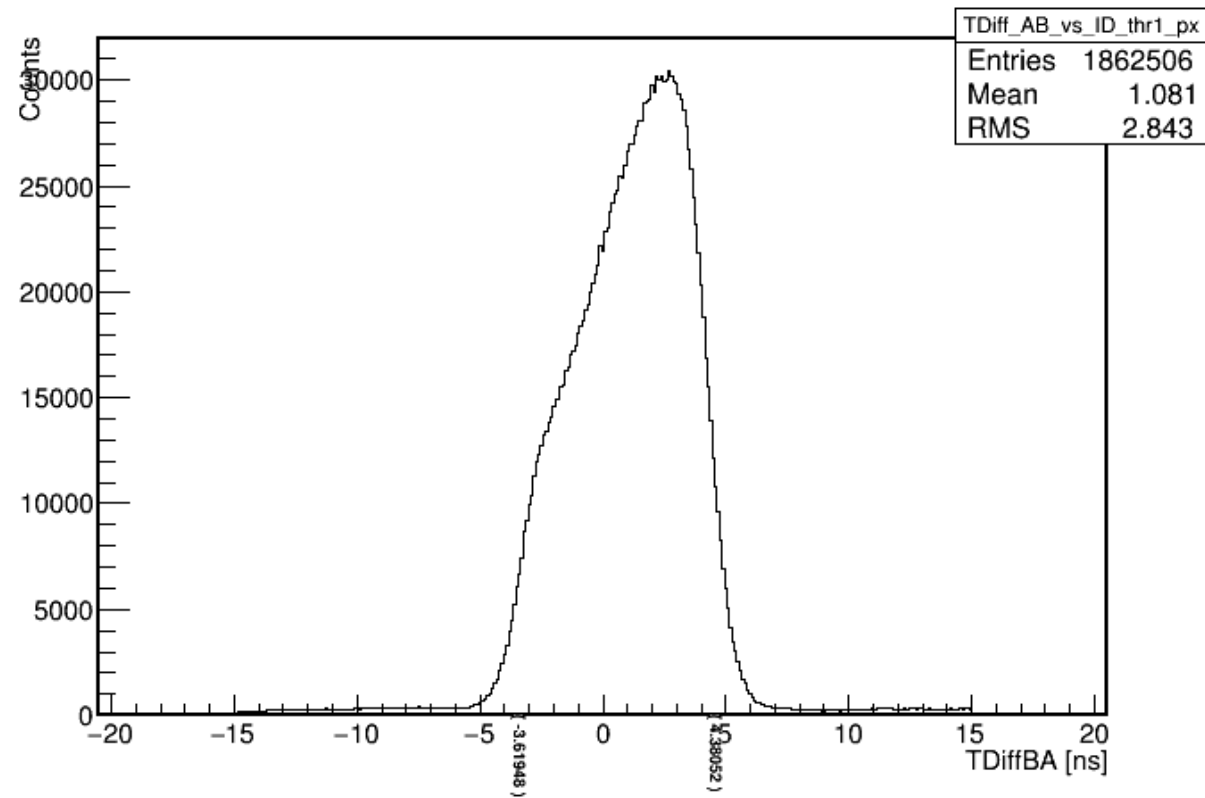
$$\text{Z position of Hit} = \text{Velocity} * (\text{TimeB} - \text{TimeA})/2$$

$$\text{Velocity} = \text{Effective Length} * (\text{Right Edge} - \text{Left Edge})$$

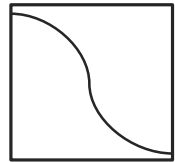


Supplementary slides

Example for problematic scintillator



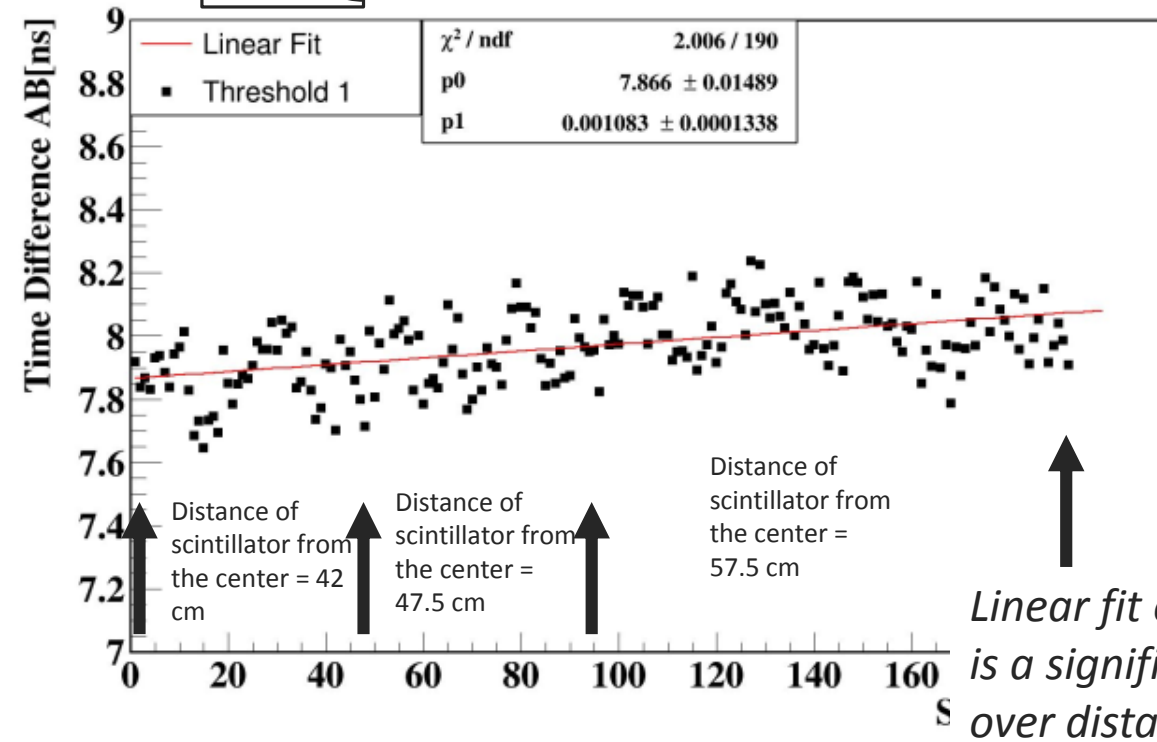
Different methods of determining middle of the edge



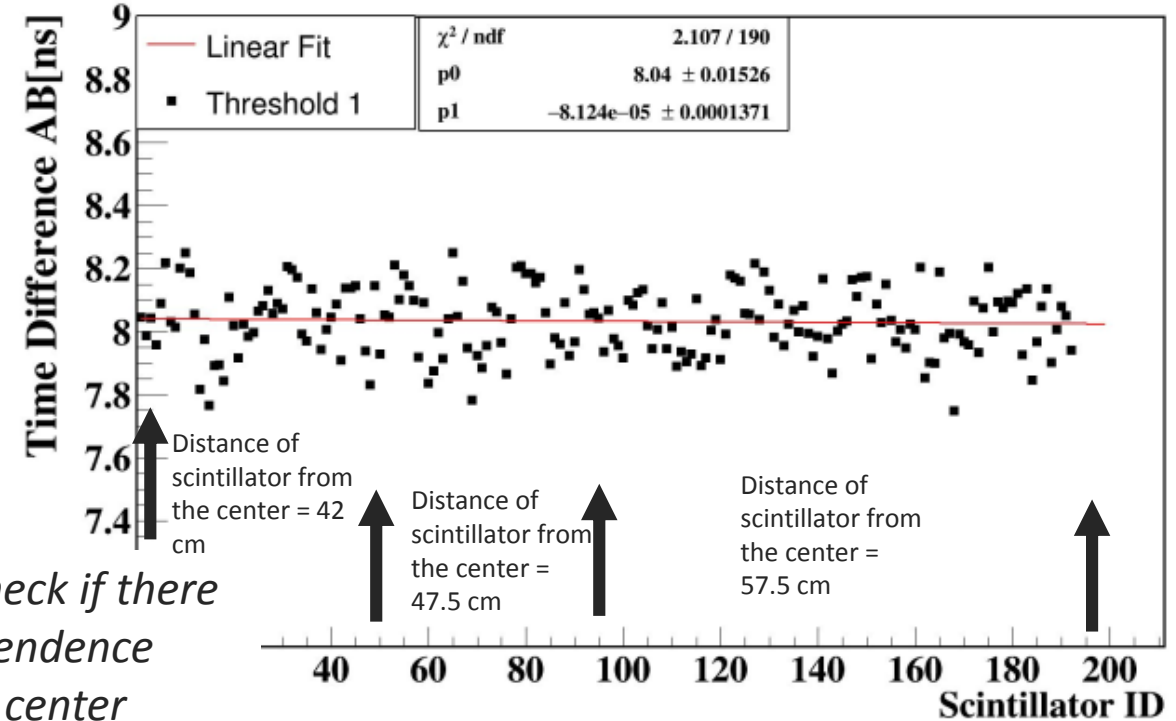
← Fermi Fit

Method should not depend on the layer of scintillators

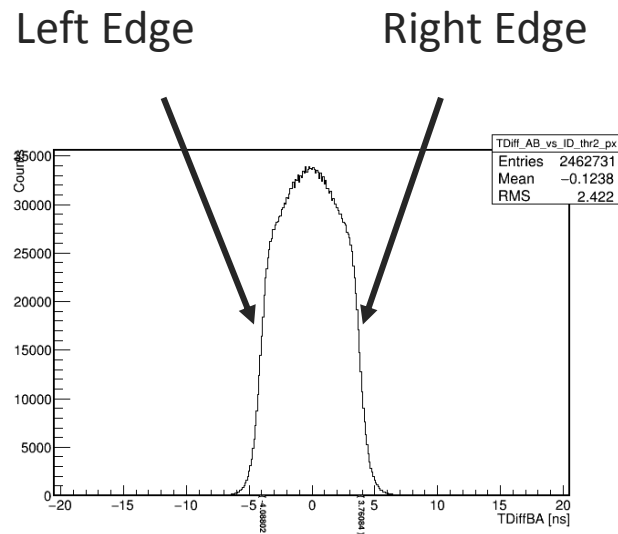
Zero of the second derivative



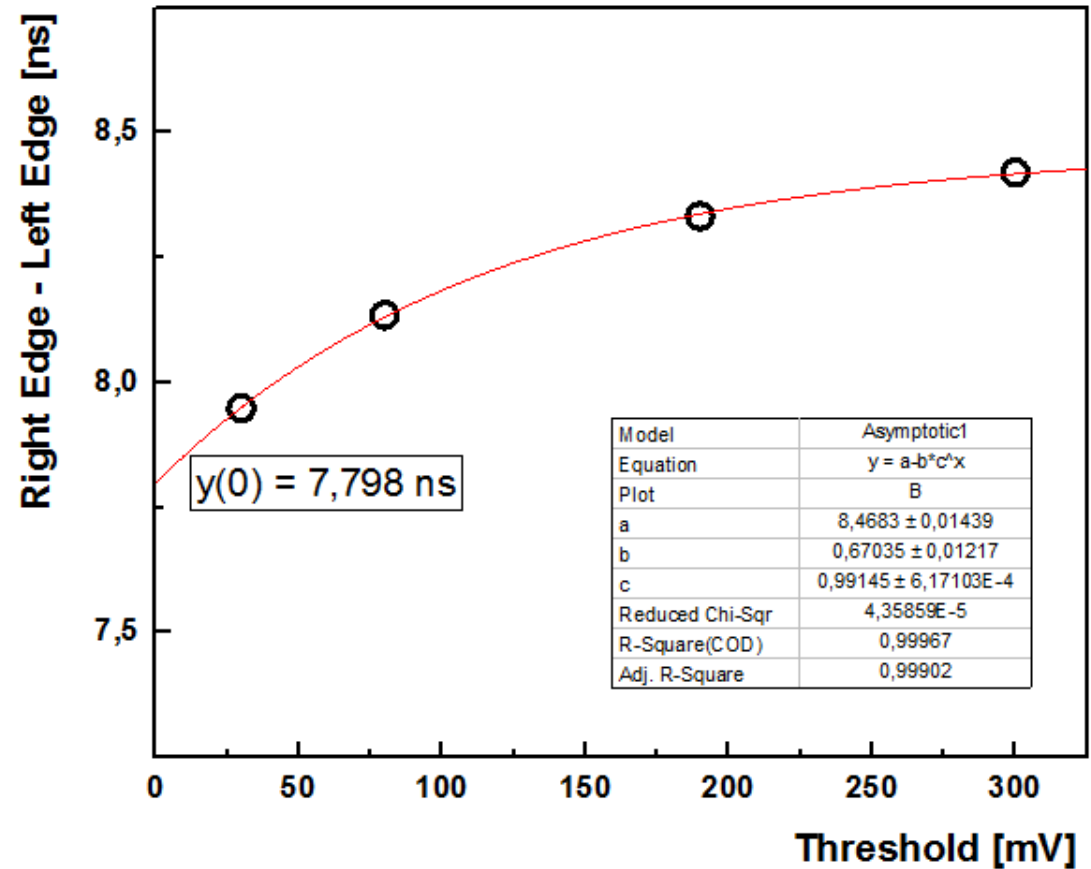
Linear fit only to check if there is a significant dependence over distance from center



Comparison of mean width of TDiffAB distribution

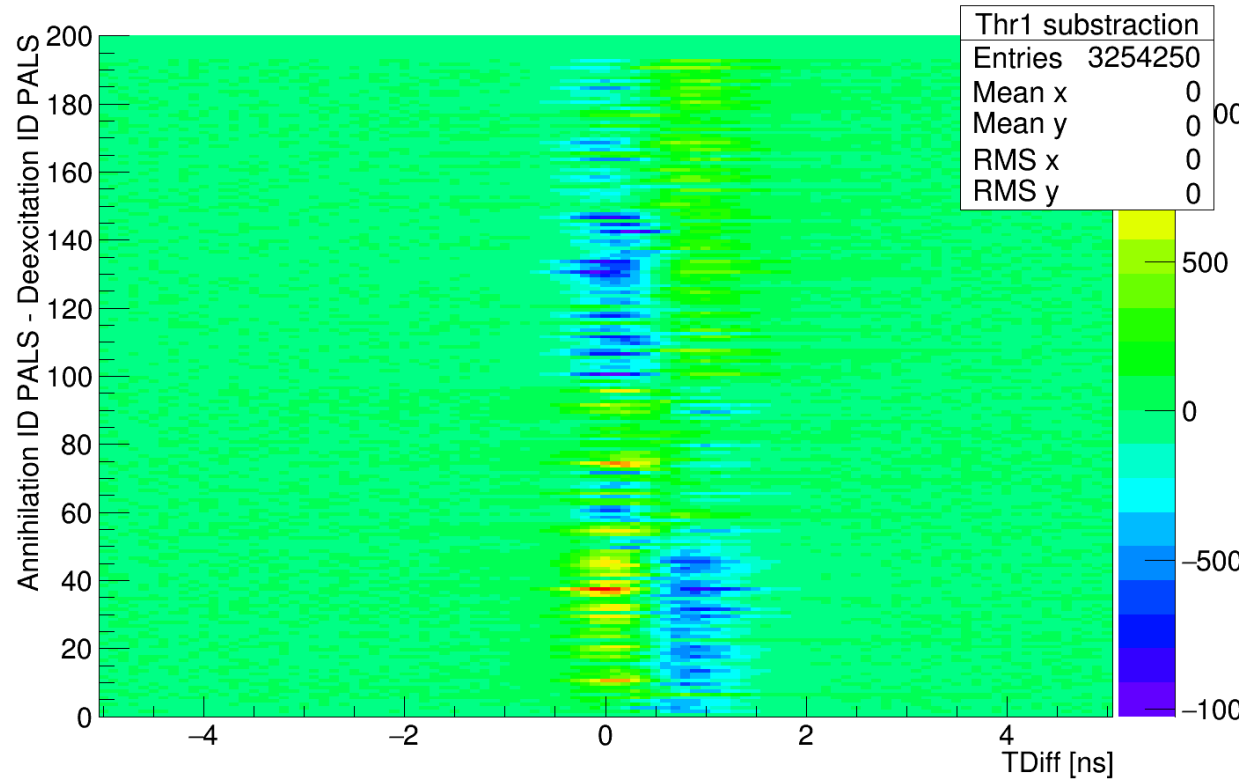


Higher Threshold – Greater WALK effect influence

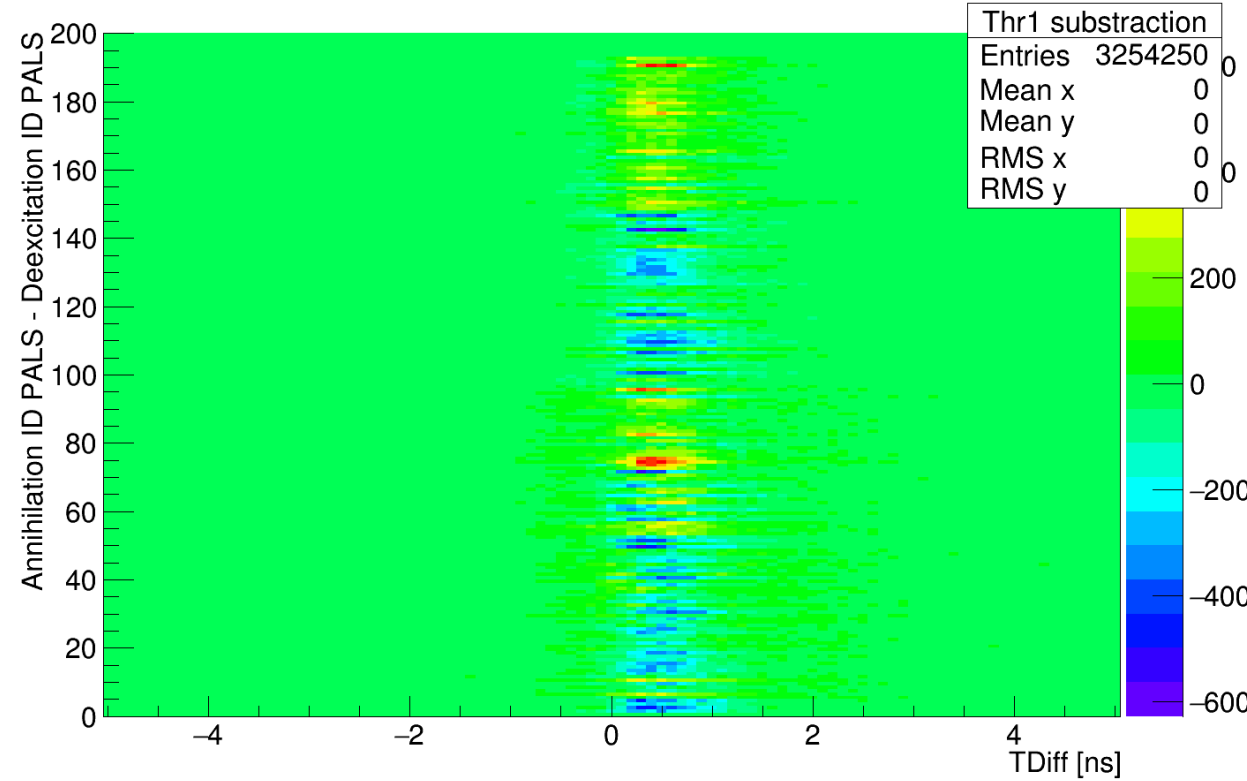


Calibration between modules

Before Calibration (0 iteration)

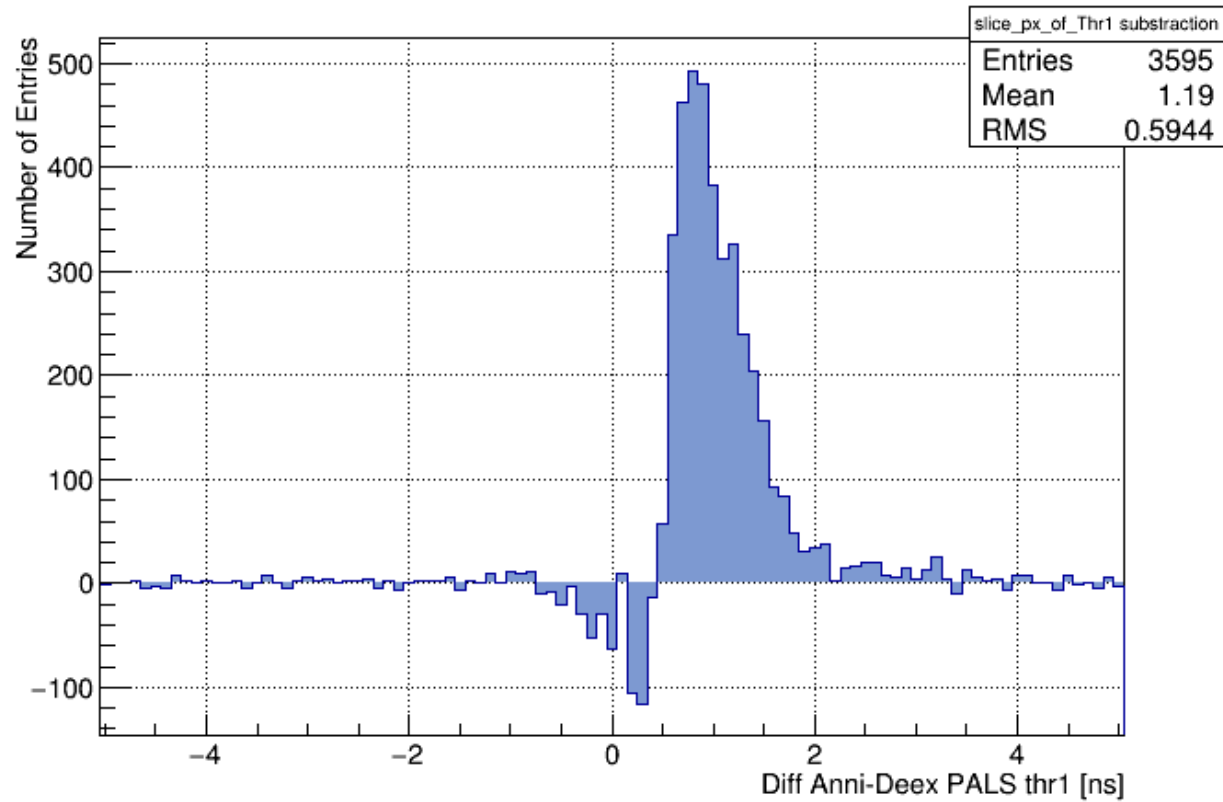


After Calibration (3rd iteration)



Calibration between modules

Before Calibration (0 iteration)



After Calibration (3rd iteration)

