Physics Letters B 677 (2009) 24–29

Contents lists available at ScienceDirect

Physics Letters B

www.elsevier.com/locate/physletb

Measurement of the $\eta \to 3\pi^0$ Dalitz plot distribution with the WASA detector at COSY

WASA-at-COSY Collaboration

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doi:10.1016/j.physletb.2009.03.063

ARTICLE INFO

Article history:
Received 17 November 2008
Received in revised form 23 February 2009
Accepted 29 March 2009
Available online 1 April 2009
Editor: V. Metag

PACS:
13.25.-k
13.75.Lb

Keywords:
η
$\eta \to 3\pi^0$
Dalitz plot

ABSTRACT

In the first production run of the WASA experiment at COSY, the eta decay into three neutral pions was measured in proton–proton interactions at a proton beam kinetic energy of 1.4 GeV. The Dalitz plot of the three pions was studied using 1.2×10^5 fully reconstructed events, and the quadratic slope parameter α was determined to be $-0.027 \pm 0.008(\text{stat}) \pm 0.005(\text{syst})$. The result is consistent with previous measurements and further corroborates the importance of pion–pion final state interactions.

1. Introduction

The η meson plays a special role in understanding low-energy Quantum Chromo Dynamics (QCD). Chiral symmetry, its realization in hadron physics at low energies and the role of explicit chiral symmetry breaking due to the masses of the light quarks (u, d, s) can be investigated using η decays.

The η meson decays into three pions ($\eta \to \pi^0\pi^0\pi^0$ and $\eta \to \pi^+\pi^-\pi^0$) are among the major decay modes (the branching ratios are 32.6% and 22.7%, respectively [1]) despite the fact that isospin is not conserved in these processes. It appears that these decays are driven by an isospin breaking term in the QCD Lagrangian which is proportional to the quark mass difference $m_d - m_u$ [2]. Any contribution from electromagnetic processes is suppressed [3–5]. The decay is closely related to pion–pion scattering, an elementary low-energy QCD process. The lowest order contribution to the decay mechanism is given by Current Algebra (CA) [6,7]. The predicted partial decay width is more than four times lower than the measured value. On the other hand, the distributions of the decay products are described within a few percent accuracy.

The decay amplitude of η into three pions can be described using the following two Dalitz variables:

$$x \equiv \frac{1}{\sqrt{3}} \frac{T_1 - T_2}{\langle T \rangle}, \qquad y \equiv \frac{T_3}{\langle T \rangle} - 1. \tag{1}$$

Here T_i are the kinetic energies of the pions in the rest frame of the η meson, $3\langle T \rangle \equiv T_1 + T_2 + T_3 = m_\eta - 3m_\pi$ where m_π is the pion mass and the π^0, π^+ mass difference is neglected. The boundaries of the Dalitz plot are shown in Fig. 1.

The decay amplitude predicted by CA is a linear function of y for the $\eta \to \pi^+\pi^-\pi^0$ decay what implies that the $\eta \to \pi^0\pi^0\pi^0$ decay amplitude ($\bar{A}$) is constant. Experimentally, a small deviation from a uniform Dalitz Plot density distribution is observed. The lowest term in the expansion about the center of the Dalitz plot is given by:

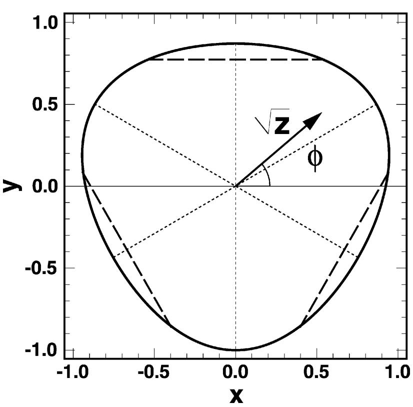

Fig. 1. Symmetrized Dalitz plot for the $\eta \to \pi^0\pi^0\pi^0$ decay. Dashed lines represent threshold for $\pi^0\pi^0 \to \pi^-\pi^+$ rescattering.

$$\left|\bar{A}(z, \phi)\right|^2 = c(1 + 2\alpha z) \tag{2}$$

where z and ϕ variables are related to x and y via:

$$x = \sqrt{z}\cos\phi, \qquad y = \sqrt{z}\sin\phi. \tag{3}$$

c is a constant factor and α is the quadratic slope parameter.

The slope α was measured to be negative and small, which leads to a decrease of the Dalitz plot density at the border by a few percent. The explanation of this effect poses a challenge for Chiral Perturbation Theory (ChPT), the effective field theory of QCD in the low-energy region. The leading order calculations in ChPT coincide with CA whereas the next-to-leading order (NLO) calculations [8] significantly improve the agreement for the partial decay width but predict a small positive value for α. The large uncertainty of the recently carried out next-to-next-to-leading order (NNLO) calculations [9] does not allow to decide the sign of the slope. The origin of the non-uniform density distribution in the $3\pi^0$ Dalitz plot are in part pion–pion interactions in the final state. In contrast to the perturbative calculations, as soon as the rescattering effects are considered to infinite orders via unitarization of the decay amplitude using dispersion relations [10,11] or iteration of the Bethe–Salpeter equation [12] the sign of α turns out to be in accordance with experiment. The calculations indeed predict negative values for α in agreement with the experimental results which are summarized in Table 1 together with the different theoretical predictions.

Furthermore, $\eta \to 3\pi$ decays are considered to be a source of precise constraints for the light quark mass ratios [13]. In a recent

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approach the constraints are derived entirely from the experimental partial decay width and from the data on $\eta \to \pi^0\pi^0\pi^0$ Dalitz plot slope [14].

The first precise experimental determination of the α parameter was carried out by Crystal Ball at AGS [15] in 2001, using a pion beam impinging on a liquid hydrogen target. In 2005, KLOE released their first result of a high statistics measurement of α (using $e^+e^- \to \Phi \to \eta\gamma$) [16] which differed from the Crystal Ball result by three standard deviations. The CELSIUS/WASA [17] experiment has confirmed the negative sign of the slope parameter α. However, the achieved accuracy did not allow to resolve the discrepancy between the Crystal Ball and the early KLOE results. Very recently, the KLOE data were reanalyzed and a new α value was presented [18] that is consistent with Crystal Ball. However, the final results are not published yet. The Crystal Ball Collaboration has collected additional statistics at the MAMI facility and the results were recently submitted for publication [19,20].

The phase space of the $\eta \to 3\pi^0$ decay covers the threshold for the $\pi^0\pi^0 \to \pi^+\pi^-$ rescattering process as a consequence of the fact that the neutral pion mass is slightly lower than the charged pion mass. The boundaries correspond to the $\pi^0\pi^0$ invariant mass being equal to $2m_{\pi^\pm}$. In 2004 a cusp like structure in the $\pi^0\pi^0$ invariant mass for the $K^+ \to \pi^0\pi^0\pi^+$ decay was observed by the NA48/2 Collaboration [23]. The effect was predicted already in 1997 by Meißner et al. [24] and the interpretation of the NA48/2 results was given by Cabibbo [25]. The process provides a precise determination of a combination of the $I = 0$ and $I = 2$ pion–pion s-wave scattering lengths, a_0 and a_2. A similar effect was observed in the $K_L \to 3\pi^0$ decay [26] and it should be present also in the $\eta \to 3\pi^0$ and $\eta' \to \pi^0\pi^0\eta$ decays. There are important consequences of this phenomenon for the analysis of the $\eta \to 3\pi^0$ decay [17,27–30]. The amplitude is no longer a function only of the z variable but has to explicitly depend on the Mandelstam variables. The description of the amplitude as a polynomial function will fail in the cusp region. For example the dependence of the ϕ averaged amplitude squared ($|\bar{\mathcal{A}}|^2$):

$$|\bar{\mathcal{A}}|^2 \equiv \frac{1}{2\pi}\int_0^{2\pi} \left|\bar{\mathcal{A}}(z,\phi)\right|^2 d\phi \tag{4}$$

on the z variable cannot be linear for $0.6 < z < 0.9$. However the most sensitive variables to search for the cusp effect are the invariant masses of the pion pairs (or kinetic energies of the pions). They correspond to the projections onto the dotted lines in Fig. 1.

The results presented in this Letter are based on a measurement using proton–proton interactions with the WASA detection system recently installed at the Cooler Synchrotron COSY at the Forschungszentrum Jülich GmbH.

2. The experiment

The WASA detection system [31] was transported to COSY (COoler SYnchrotron) in the Summer 2005 [32]. The COSY facility at Jülich offers polarized and phase space cooled proton and deuteron beams with momenta up to 3.7 GeV/c [33,34]. The available beam momentum range allows the production of π, K, η, ω, and $\eta\prime$ mesons well above the production thresholds in proton–proton and proton–deuteron interactions. Most of the final states in pN, pd, and dd reactions can be detected due to the nearly 4π acceptance of the WASA detector for charged and neutral particles.

The WASA detector [31] was designed and optimized for studies of production and decays of light mesons in hadronic interactions. It consists of three main components: the Forward Detector—

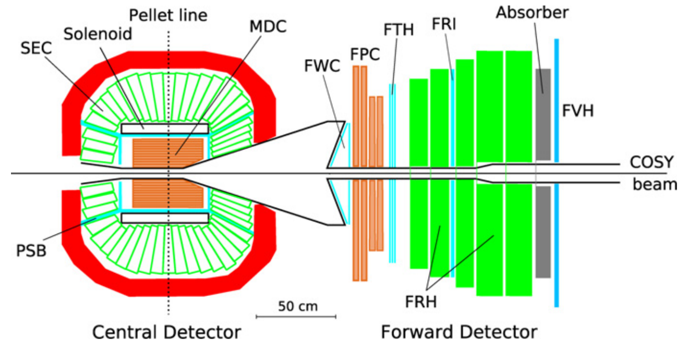

Fig. 2. Schematic side view of the WASA detector setup at COSY.

used for tagging and triggering on meson production, the Central Detector—used for measuring neutral and charged meson decay products, and the unique pellet target system. The target beam consists of 30 μm diameter pellets of hydrogen or deuterium, providing a high target density in the order of 10^{15} atoms/cm^2. (See Fig. 2.)

The Central Detector surrounds the interaction region and is designed for detection and identification of photons, electrons, and charged pions. It consists of an inner drift chamber (MDC), a superconducting solenoid providing the magnetic field for momentum measurements, a barrel of thin plastic scintillators for particle identification and triggering, and an electromagnetic calorimeter. The amount of structural material is kept to a minimum to reduce disturbance of the particles. The beryllium beam pipe wall is 1.2 mm thick and the material of the superconducting solenoid corresponds to 0.18 radiation lengths.

The calorimeter (SEC) is used for detection and reconstruction of particles in the polar angle range from 20° to 169°, which is about 96% of the geometrical acceptance. The calorimeter consists of 1012 sodium doped CsI crystals. The trapezoidally shaped crystals with lengths from 20 to 30 cm correspond to ∼16 radiation lengths. The energy resolution of the calorimeter is $\sigma(E)/E \approx 5\%/\sqrt{E[\mathrm{GeV}]}$ [31].

The forward detector is designed for detection of particles emitted in the polar angles from 3° to 18°. It allows for identification and reconstruction of protons from the $pp \to pp\eta$ reaction close to threshold. The precise track coordinates are given by four sets of straw proportional chambers (FPC). Kinetic energies are reconstructed using the ΔE information in the layers of plastic scintillators of different thickness. In addition, the signals are used for triggering. The kinetic energy of the protons can be reconstructed with a resolution of $\sigma(T)/T \sim 1.5$–3% for kinetic energies below 400 MeV.

The detector setup is nearly the same as used in the previous CELSIUS/WASA experiment [17]. The main change is a completely new readout system with charge-to-digital converters based on a flash ADC [35,36]. In addition, the fourth layer of the forward range hodoscope (FRH) was replaced by two new thicker layers (15 cm instead of 11 cm).

The η mesons have been produced in proton–proton interactions at a beam proton energy of 1.4 GeV (momentum 2.141 GeV/c). The beam energy corresponds to a center of mass excess energy of 56 MeV for the $pp \to pp\eta$ reaction and the cross section is 10 μb. The results presented here are based on about 2.4 pb^{-1} of data collected during the first WASA-at-COSY production run and correspond to 94 hours of data taking. At the trigger level, events with two tracks in the forward detector and at least two hit groups in the calorimeter with energy deposits of more than 50 MeV were accepted. In addition, a veto on signals in the plastic barrel (PSB) was required, aiming to select only neutral particles.

Table 1
Overview of experimental and theoretical results for the slope parameter α.

Slope parameter $\alpha \pm$ stat $\pm$ syst	Comment	Year	Ref.
-0.022 ± 0.023	GAMS-2000	(1984)	[21]
$-0.052 \pm 0.017 \pm 0.010$	Crystal Barrel	(1998)	[22]
$-0.031 \pm 0.004 \pm 0.004$	Crystal Ball/BNL	(2001)	[15]
$-0.014 \pm 0.004 \pm 0.005$	KLOE, preliminary	(2005)	[16]
$-0.027 \pm 0.004^{+0.004}_{-0.006}$	KLOE, reanalysis	(2007)	[18]
$-0.026 \pm 0.010 \pm 0.010$	CELSIUS/WASA	(2007)	[17]
$-0.027 \pm 0.008 \pm 0.005$	WASA-at-COSY	(2008)	this result
$-0.0322 \pm 0.0012 \pm 0.0022$	Crystal Ball/Mami-C	(2008)	[19]
$-0.032 \pm 0.002 \pm 0.002$	Crystal Ball/Mami-B	(2008)	[20]
0	CA	(1967)	[7]
$+0.015$	ChPT NLO	(1984)	[8]
$-(0.007$–$0.014)$	ChPT NLO + dispersive	(1995)	[4]
-0.007	UChPT	(2003)	[11]
-0.031 ± 0.003	UChPT fit	(2005)	[12]
$+0.013 \pm 0.032$	ChPT NNLO	(2007)	[9]

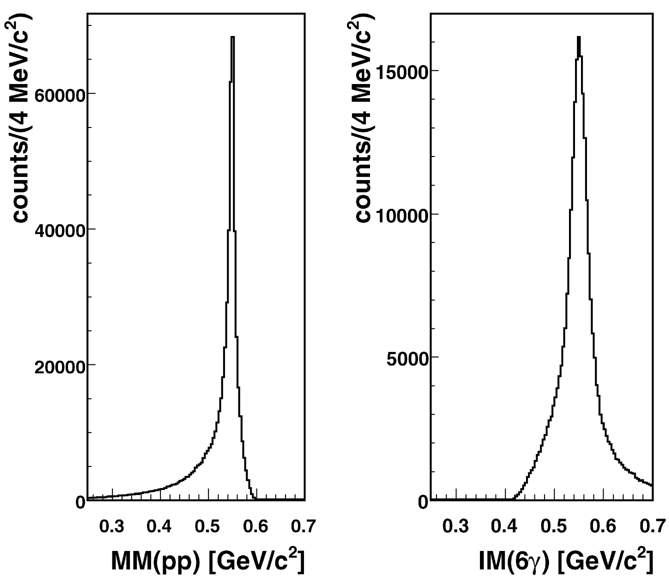

Fig. 3. Missing mass of two protons, $MM(pp)$ after selection of the $pp6\gamma$ final state (left). Invariant mass of six photons, $IM(6\gamma)$ after applying cut $0.535\ \text{GeV}/c^2 < MM(pp) < 0.565\ \text{GeV}/c^2$ and after fitting the individual pions (right).

3. Data analysis

In the offline analysis, the $pp6\gamma$ final state is selected by requiring six hit clusters with energy deposit at least 20 MeV in the calorimeter consistent with photons and two proton tracks in the forward detector. The conditions provide a clean data sample of about 8×10^5 events. In order to select π^0 candidates from the six reconstructed photons, all fifteen possible combinations of the three photon pairs were considered. For each combination, the quantity χ_j^2 is calculated:

$$\chi_j^2 \equiv \sum_i^3 \frac{(IM(j,i) - m_{\pi^0})^2}{\sigma^2}, \quad j = 1, 2, 3, \ldots, 15 \tag{5}$$

where $IM(j,i)$ is the invariant mass of the ith $\gamma\gamma$ pair for the jth combination, $\sigma = 14\ \text{MeV}/c^2$ is the resolution of the $\gamma\gamma$ invariant mass. The combination with minimum value of χ_j^2 is selected. Only events with $\chi_j^2 < 15$ and a missing mass of two protons between $0.535\ \text{GeV}/c^2$ and $0.565\ \text{GeV}/c^2$ are kept. Fig. 3 shows the missing mass of two protons (left panel) and the invariant mass of six photons (right panel) after applying the cut on the missing mass and after fitting the individual pions.

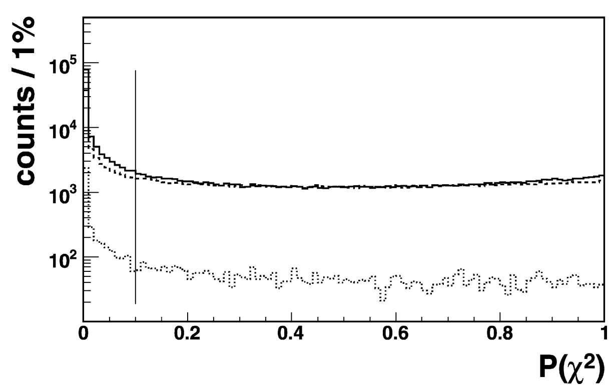

Fig. 4. The distribution of the kinematic fit probability with the $\eta \to 3\pi^0$ decay hypothesis ($P(\chi^2)$) for the experimental data (solid line), Monte Carlo simulation of the signal (dashed line), and background (dotted line). The vertical line represents the selection for the final Dalitz plot analysis.

In order to reconstruct the $3\pi^0$ Dalitz plot, a kinematic fit is applied. In Ref. [17], the full final state including the two protons was considered in the fitting procedure. Here, only the $\eta \to 3\pi^0$ decay system is fitted, based on the reconstructed photon angles and momenta. This approach reduces systematic uncertainty due to proton reconstruction. Different reconstruction uncertainties (as a function of energy and angle) were obtained from a GEANT Monte Carlo detector simulation, with resolution parameters matched to reproduce experimental distributions. The experimental distribution of the kinematic fit probability ($P(\chi^2)$) is compared in Fig. 4 to the Monte Carlo simulation. The simulation includes background from direct three pion production, $pp \to pp\pi^0\pi^0\pi^0$, the cross section was obtained by interpolation of the results from the CELSIUS/WASA experiment [37]. The relative amount of background is less than 4% in the final event sample after $P(\chi^2) > 0.1$ cut (vertical line in Fig. 4).

4. Extraction of the slope parameter

Based on the analysis procedure, the efficiency corrected radial density distribution $|\bar{\mathcal{A}}|^2$ is shown in Fig. 5. The efficiency correction is obtained by dividing the measured z distribution by the result of the Monte Carlo simulation, assuming $\alpha = 0$. A linear fit to the data points is applied to extract the slope parameter α, yielding a statistical uncertainty of $\sigma(\alpha) = 8 \times 10^{-3}$. The achieved resolution in z is $\sigma(z) = 0.055$.

The systematic uncertainty of the result is estimated by varying one by one all parameters that are important in the analysis. Table 2 shows the main contributions. The dominant contribution

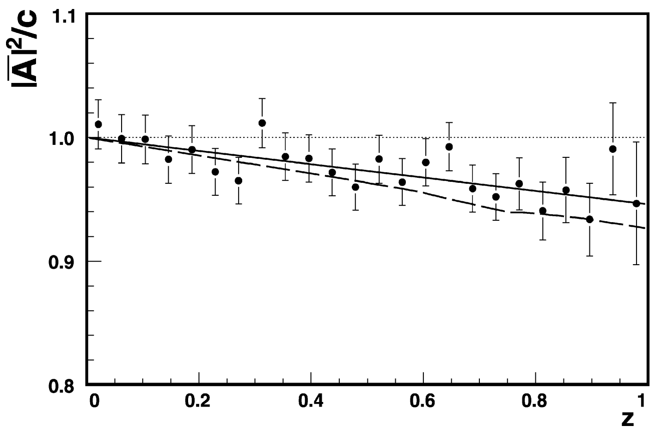

Fig. 5. Extracted dependence of $|\bar{A}|^2$ on the z variable. The solid line is a $c(1+2\alpha z)$ fit. The dashed line is a prediction of the cusp effect [38].

Table 2
The main contributions to the systematic uncertainty as discussed in the text. The overall systematic uncertainty was calculated as the square root of the summed squares of all contributions.

Source of systematic uncertainty	RMS
Vertex position	0.004
Combinatoric background	0.001
Missing mass cut	0.002
Error parametrization for the kinematical fit	0.002
Confidence level cut	0.001
Overall systematic uncertainty	0.005

comes from an uncertain fraction (of the order of few per cent) of the interactions with gas stemming from pellet evaporation or with pellets bouncing in the beam pipe. This causes that the vertex position for some events is located upstream or downstream the position of the pellet target stream. In the kinematic fit the nominal vertex position was assumed and therefore part of such events were rejected by the cut on the confidence level of the fit. The combinatoric purity of the selected event sample is 96% and can change with the cut on χ_j^2 probability of the best combination and on the confidence level of the fit. A variation of these parameters has only a minor effect on the obtained slope result. Another source of systematic errors is imposed by the background mainly originating from the $pp \to pp3\pi^0$ reaction. This effect is under control by changing the width of the proton–proton missing mass cut and, hence, varying the relative amount of background in the range from 4% to 12%.

The kinematic fit performance relies on the precise understanding of the errors of reconstructed quantities. This effect was studied by replacing the polar angle dependent parametrization of reconstruction uncertainties with a much simpler polar angle independent description, again showing only a small effect on the result (see Table 2). Finally the accepted confidence region was changed to 30–80% leading change of the α slope by 0.001.

Taking into account the systematic studies, the final result for the extracted slope parameter α is

$$\alpha = -0.027 \pm 0.008(\text{stat}) \pm 0.005(\text{syst})$$

based on 120 000 events. The result agrees within one standard deviation with the high statistics measurements of the slope parameter performed by the KLOE Collaboration [18] and the Crystal Ball Collaboration [15]. It is also consistent with the previous measurement performed by the CELSIUS/WASA Collaboration [17]. The shape of the z distribution including the cusp that emerges from a virtual $\pi^+\pi^-$ intermediate state followed by a $\pi^+\pi^- \to \pi^0\pi^0$ transition, can be calculated from the $\eta \to \pi^+\pi^-\pi^0$ amplitude and the π–π scattering lengths. The cusp effect leads to a broad local minimum in the radial density of the Dalitz plot for $0.6 < z < 0.9$ due to the contribution of the regions where the invariant mass of the two pions less than $2m_{\pi^\pm}$ (see Fig. 1). The dashed line in Fig. 5 was obtained using a parameterization of the $\eta \to \pi^+\pi^-\pi^0$ decay amplitude from the KLOE Collaboration [39] and a nonrelativistic effective field theory [30,38]. It is seen that the statistical precision of the data is insufficient to investigate the cusp effect.

5. Outlook

One motivation for the presented study was the striking discrepancy between KLOE and Crystal Ball results for the $\eta \to 3\pi^0$ Dalitz plot slope parameter α. Now, after the recent reanalysis of the KLOE data, the three major experiments focusing on the measurement of η decays, KLOE at DAPHNE in Frascati, Crystal Ball now installed at MAMI in Mainz, and WASA-at-COSY in Jülich, obtain consistent results. The non-zero value of the slope parameter is clear indication for the importance of final state pion–pion interactions. The theoretical understanding of these processes has advanced significantly, also triggered by the firm experimental situation. The Dalitz plot slope parameter provides a very sensitive test of the ChPT predictions. The ChPT NLO and NNLO calculations that assume $m_{\pi^0} = m_{\pi^\pm}$ in the loops, indicate a positive sign of the slope parameter. The uncertainty is however large and the negative sign is not excluded [9].

Among the experiments studying η decays, WASA is presently the only experiment focusing on hadronic production of eta mesons in pp or pd scattering. The experiment combines the high production cross section especially in the pp reaction with the capability to run at high luminosities up to 10^{32} cm^{-2} s^{-1}. This will allow to collect high statistics data samples of η mesons (10^7 decays and more) with WASA-at-COSY. High statistics data samples provide further increased accuracy and sensitivity in the study of small effects, like the cusp structure in $\eta \to 3\pi^0$. Recently WASA-at-COSY has collected a large data sample of $pd \to {}^3\text{He}\eta$ events with low background and an unbiased trigger requirement, enabling studies *e.g.* of $\eta \to \pi^0\pi^0\pi^0$ and $\eta \to \pi^+\pi^-\pi^0$ decays simultaneously.

Acknowledgements

This work was in part supported by: the Forschungszentrum Jülich including the COSY-FFE program, the European Community under the FP6 program (Hadron Physics, RII3-CT-2004-506078), the German BMBF, the German–Indian DAAD-DST exchange program, VIQCD (VH-VI-231) and the German Research Foundation (DFG).

We gratefully acknowledge the financial support given by the Knut and Alice Wallenberg Foundation, the Swedish Research Council, the Göran Gustafsson Foundation, the Polish Ministry of Science and Higher Education under grants PBS 7P-P6-2/07, 3240/H03/2006/31, 1202/DFG/2007/03.

We also want to thank the technical and administration staff at the Forschungszentrum Jülich and at the participating institutes.

This work is part of the PhD Thesis of P. Vlasov.

References

[1] Particle Data Group, C. Amsler, et al., Phys. Lett. B 667 (2008) 1.
[2] J. Bijnens, J. Gasser, Phys. Scr. T 99 (2002) 34, arXiv:hep-ph/0202242.
[3] D.G. Sutherland, Nucl. Phys. B 2 (1967) 433.
[4] R. Baur, J. Kambor, D. Wyler, Nucl. Phys. B 460 (1996) 127, arXiv:hep-ph/9510396.

[5] C. Ditsche, B. Kubis, U.-G. Meißner, Eur. Phys. J. C 60 (2009) 83, arXiv:0812.0344 [hep-ph].
[6] W.A. Bardeen, L.S. Brown, B.W. Lee, H.T. Nieh, Phys. Rev. Lett. 18 (25) (1967) 1170.
[7] H. Osborn, D.J. Wallace, Nucl. Phys. B 20 (1970) 23.
[8] J. Gasser, H. Leutwyler, Nucl. Phys. B 250 (1985) 539.
[9] J. Bijnens, K. Ghorbani, JHEP 0711 (2007) 030, arXiv:0709.0230 [hep-ph].
[10] J. Kambor, C. Wiesendanger, D. Wyler, Nucl. Phys. B 465 (1996) 215, arXiv:hep-ph/9509374.
[11] N. Beisert, B. Borasoy, Nucl. Phys. A 716 (2003) 186, arXiv:hep-ph/0301058.
[12] B. Borasoy, R. Nißler, Eur. Phys. J. A 26 (2005) 383, arXiv:hep-ph/0510384.
[13] H. Leutwyler, Phys. Lett. B 378 (1996) 313, arXiv:hep-ph/9602366.
[14] A. Deandrea, A. Nehme, P. Talavera, Phys. Rev. D 78 (2008) 034032, arXiv:0803.2956 [hep-ph].
[15] Crystal Ball Collaboration, W.B. Tippens, et al., Phys. Rev. Lett. 87 (2001) 192001.
[16] KLOE Collaboration, S. Giovannella, et al., in: Proceedings of La Thuile 2005, Results and Perspectives in Particle Physics, Rencontres de Moriond, 2005, p. 241, arXiv:hep-ex/0505074.
[17] CELSIUS/WASA Collaboration, M. Bashkanov, et al., Phys. Rev. C 76 (2007) 048201, arXiv:0708.2014 [nucl-ex].
[18] KLOE Collaboration, F. Ambrosino, et al., in: Proceedings of LP07 Conference, Kyungpook National Univ. Press, 2007, pp. S8–356, arXiv:0707.4137 [hep-ex].
[19] S. Prakhov, et al., arXiv:0812.1999 [hep-ex].
[20] M. Unverzagt, et al., arXiv:0812.3324 [hep-ex].
[21] Serpukhov-Brussels-Annecy (LAPP) Collaboration, D. Alde, et al., Z. Phys. C 25 (1984) 225.
[22] Crystal Barrel Collaboration, A. Abele, et al., Phys. Lett. B 417 (1998) 193.
[23] NA48/2 Collaboration, J.R. Batley, et al., Phys. Lett. B 633 (2006) 173, hep-ex/0511056.
[24] U.-G. Meißner, G. Müller, S. Steininger, Phys. Lett. B 406 (1997) 154, arXiv:hep-ph/9704377;
U.-G. Meißner, G. Müller, S. Steininger, Phys. Lett. B 407 (1997) 454, Erratum.
[25] N. Cabibbo, Phys. Rev. Lett. 93 (2004) 121801, hep-ph/0405001.
[26] KTeV Collaboration, E. Abouzaid, et al., Phys. Rev. D 78 (2008) 032009, arXiv:0806.3535 [hep-ex].
[27] N. Cabibbo, G. Isidori, JHEP 0503 (2005) 021, arXiv:hep-ph/0502130.
[28] E. Gamiz, J. Prades, I. Scimemi, Eur. Phys. J. C 50 (2007) 405, arXiv:hep-ph/0602023.
[29] G. Colangelo, J. Gasser, B. Kubis, A. Rusetsky, Phys. Lett. B 638 (2006) 187, arXiv:hep-ph/0604084.
[30] M. Bissegger, A. Fuhrer, J. Gasser, B. Kubis, A. Rusetsky, Phys. Lett. B 659 (2008) 576, arXiv:0710.4456 [hep-ph].
[31] CELSIUS/WASA Collaboration, C. Bargholtz, et al., Nucl. Instrum. Methods A 594 (2008) 339, arXiv:0803.2657 [nucl-ex].
[32] WASA-at-COSY Collaboration, H.H. Adam, et al., arXiv:nucl-ex/0411038.
[33] D. Prasuhn, et al., Nucl. Instrum. Methods A 362 (1995) 16.
[34] R. Maier, Nucl. Instrum. Methods A 390 (1997) 1.
[35] H. Kleines, et al., IEEE Trans. Nucl. Sci. 53 (2006) 893.
[36] H. Kleines, et al., IEEE Trans. Nucl. Sci. 55 (2008) 261.
[37] CELSIUS/WASA Collaboration, C. Pauly, et al., Phys. Lett. B 649 (2007) 122, arXiv:nucl-ex/0602006.
[38] C.-O. Gullström, A. Kupść, A. Rusetsky, Phys. Rev. C 79 (2009) 028201, arXiv:0812.2371 [hep-ph].
[39] KLOE Collaboration, A. Antonelli, et al., JHEP 0805 (2008) 006, arXiv:0801.2642 [hep-ex].