Novel method for hit-position reconstruction using voltage signals in plastic scintillators and its application to Positron Emission Tomography

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Currently inorganic scintillator detectors are used in all commercial Time of Flight Positron Emission Tomograph (TOF-PET) devices. The J-PET collaboration investigates a possibility of construction of a PET scanner from plastic scintillators which would allow for single bed imaging of the whole human body.

This paper describes a novel method of hit-position reconstruction based on sampled signals and an example of an application of the method for a single module with a 30 cm long plastic strip, read out on both ends by Hamamatsu R4998 photomultipliers. The sampling scheme to generate a vector with samples of a PET event waveform with respect to four user-defined amplitudes is introduced. The experimental setup provides irradiation of a chosen position in the plastic scintillator strip with an annihilation gamma quanta of energy 511 keV. The statistical test for a multivariate normal (MVN) distribution of measured vectors at a given position is developed, and it is shown that signals sampled at four thresholds in a voltage domain are approximately normally distributed variables. With the presented method of a vector analysis made out of waveform samples acquired with four thresholds, we obtain a spatial resolution of about 1 cm and a timing resolution of about 80 ps (σ).

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1. Introduction

Positron Emission Tomography (PET) [1,2] represents one of the most prominent perspective techniques of non-invasive imaging in medicine. The first demonstration of recording data in positron detection was taken in early 1950s [3], only 2 years after the first medical application of the positron was reported [4]. In 1973 Robertson and his co-workers built the first ring PET tomograph, which consisted of 32 detectors [5]. This scanner has become the prototype of the current shape of PET.

Since the early detection of small lesions and monitoring of the functionality of organs are critical for prophylaxis and efficient treatments, notable efforts are nowadays devoted to improve the resolution of reconstructed images. It was realized that the measurement of the difference of arrival times, or times of flight (TOF), of two gamma rays provides substantial progress in this domain [6–10]. The new class of instruments, called TOF-PET, better localize the emission source along a straight line of gamma coincidence, called the Line of Response (LOR). The LORs are basic components of image reconstruction algorithms.

Currently all commercial PET devices use inorganic scintillator materials, usually LSO or LYSO crystals, as radiation detectors. These are characterized by relatively long rise- and decay times, of the order of tens of nanoseconds. Time resolution in PET examinations is crucial and one observes persistent strive for improvement [11–16].

In recent articles [17–20], an utterly new concept of TOF-PET scanner was introduced. It incorporates plastic scintillators with the good resolving time and the TOF method. Disadvantages due to the low detection efficiency and negligible photoelectric effect in organic scintillators can be compensated by large acceptance...
and good time resolution [19]. In addition, the method allows for
drastic reduction of the production cost of PET scanners and is
promising for the construction of the single bed whole body PET
scanner. A single detection unit of the newly proposed TOF-PET
detector [18] is built out of a long strip of scintillator, read out
on both sides by photomultipliers. Such a solution enables the
reconstruction of coordinates of the gamma quantum interaction
along the scintillator strip by measuring signals at its two ends. A
similar solution for crystal scintillators has been recently de-
veloped by the AX-PET collaboration [16]. The 10 cm long LYSO
crystals were coupled to digital Silicon Photomultipliers (dSiPM),
and a very good coincidence time resolution of about 200 ps
(FWHM) was achieved. In plastic scintillators, the 511 keV quanta
from electron–positron annihilation produces signals burdened
with large fluctuations of the number of photoelectrons. Therefore
a usage of the typical techniques for time measurement, based on
the application of a single-level leading-edge or constant-fraction
discriminators, is not sufficient. Therefore a multithreshold sam-
pling method to generate samples of a PET event waveform with
respect to four user-defined amplitudes was proposed. A similar
idea may be found in Ref. [15], where a coincidence timing
resolution of about 340 ps was obtained for LSO crystals. In Ref.
[15] the four sample points that comprise the rising edge were fit
to a line, and the intersection of the fitted line with the zero
voltage level defined the event time of the pulse.

In this paper we propose a new method for reconstruction of the
gamma quantum hit position. The method is based on the statistical
model of signals probed in the voltage domain. An electronic
system for probing these signals is under development [21]. In
the following we describe a new concept of reconstruction of the
gamma quantum hit position. The description includes an explana-
tion of the methods used for the test of the normality of data,
determination of the effective number of degrees of freedom, as
well as explanation of the selection criteria applied to the experi-
mental sample. Then we describe an experimental setup used for
signal registration and present results of reconstruction of the hit
positions of gamma quanta in the 30 cm long plastic scintillator
strip, read out on both sides by the Hamamatsu photomultipliers
R4998. Signals from the photomultipliers were sampled in 50 ps
steps using the Lecroy Signal Data Analyzer 6000A.

2. Description of the reconstruction method

Light pulses produced in the strip propagate to its edges where
they are converted into electric signals. These are sampled in the
amplitude domain using a predefined number of voltage levels.
The measurement gives a vector of \(N\) values of times at which the
signal crosses the reference voltages. This vector carries informa-
tion about the shape of the signals and their times of arrivals to the
edges of the scintillator. These shapes depend on the hit position
and can be used for its reconstruction.

The method of hit-position reconstruction consists of two
steps. First, the scintillator’s volume is discretized and for each
bin a high statistics set of reference signals is created. In the
example discussed later, each set contains approximately 5000
signals generated by irradiation with gamma quanta at a fixed
scintillator position. The objective of the second part of the
procedure is to qualify the new measurement to one of the given
sets of signals and hence determine the hit position.

Consider \(L\) data sets \(S_i (i = 1, \ldots, L)\). Each \(S_i\) is an \(M \times N_i\) matrix of vectors representing signals gathered for the \(i\)th position; \(M\) is the
number of the collected signals and \(N_i\) stands for vector’s dimen-
sion equal to the number of samples per signal. In practice all
signals have the same dimension and \(N_i = N\) for all \(i\). The \(j\)th signal
in the \(i\)th data set corresponds to the \(j\)th row of the matrix \(S_i\) and is
denoted by the vector \(s_{ij}\). If the measured coordinates of vectors in
all \(L\) data sets are normally distributed then the mean value \(m_i\) and
covariance matrix \(C_i\) of the data set \(S_i\) describe it completely.

Assuming their normality, the proposed reconstruction proce-
dure qualifies a new measurement, represented by vector \(u\), to one
of the data sets \(S_i\) by using only information about \(m_i\) and \(C_i\). In
the first step of the reconstruction, the Mahalanobis distances \(d_{i}\)
between \(u\) and \(m_i\) are calculated:

\[
d_{ij} = (u - m_i) \cdot C_i^{-1} \cdot (u - m_i)^T, \quad i = 1, 2, \ldots, L.
\]

Next, the measured signal \(u\) is qualified to the data set \(S^*\) with
the smallest distance \(d_{ij}\):

\[
i^* = \arg(\min(d_{ij})).
\]

2.1. Test of the normality of data

There are number of existing procedures for testing whether multivariate vectors from a given data set have a multivariate
normal (MVN) distribution. For example Mardia [22] proposes
multivariate measures of skewness and kurtosis, which are special
cases of MVN moment restrictions. In addition, multivariate
generalization of the well known Kolmogorov–Smirnov [23] and
Shapiro–Wilks [24,25] tests has been established. There are also
available tests based on a \(x^2\) quantile–quantile (q–q) plot of the
observations squared Mahalanobis distances.

In this work, we propose an alternative procedure for testing a
MVN distribution as an extension of statistical test based on the q–
q approach [26]. In order to verify normality of the data set in
\(S_i\), the observations squared Mahalanobis distances for \(M_i\) vectors
from \(S_i\) data set are calculated:

\[
d_{ij}^2 = (s_{ij}^i - m_i) \cdot C_i^{-1} \cdot (s_{ij}^i - m_i)^T, \quad j = 1, 2, \ldots, M_i
\]

where \(m_i\) and \(C_i\) are estimated based on the data set \(S_i\). In Ref. [27]
authors assumed that the evaluated distances in Eq. (3) have a
\(x^2\) distribution with \(N\) degrees of freedom. In the following we will
show that this is not necessarily the case, and the number of
effective degrees of freedom may be smaller due to signal
correlation. The discussion about the effective number of degrees
of freedom (denoted hereafter as \(V\)) will be given in next section.

We provide a statistical test for data set \(S_i\) by comparing the
distribution of \(d_{ij}^2\) defined in Eq. (3) with the theoretical \(x^2\)
distribution with \(V\) degrees of freedom. The normalization of
theoretical histogram is provided to ensure that sum of counts in
both histograms is the same and equal to \(M_i\) (see Eq. (3)). We apply
uneven bin size, in order to store in each bin of the theoretical \(x^2\)
histogram a constant number of counts \(F_T\). This simplifies a control
of the assumption about the normal distribution of number of
counts in each bin. In the calculations, we have selected \(F_T = 30,\)
and therefore the Poisson distribution may be approximated
accurately by the normal distribution. Hence, we compare the
two histograms via statistical test \(R\) defined as follows:

\[
R(V) = \sum_{k=1}^{K} \frac{F_k^i - F_T}{F_T}
\]

where \(F_k^i\) value is the number of counts in the \(k\)th bin in
the experimental histogram from the \(i\)th data set, and \(K_i = M_i/F_T\) (the
number of bins in the histograms from the \(i\)th data set). The
bin sizes were calculated from the theoretical \(x^2\) with \(V\) degrees
of freedom. The test statistic \(R\) is a chi-squared random variable
with mean \(K_i\) and standard deviation \(\sqrt{2K_i}\); owing to well known
concentration inequalities, the probability that \(R\) exceeds its mean
plus three standard deviations is small. In the following we will
find the parameter \(\lambda\) that fulfills the equation:

\[
R_i(V) = K_i + \lambda \sqrt{2K_i}
\]
and we state that the null hypothesis that the experimental histogram has a $\chi^2$ distribution with $V$ degrees of freedom is true if $\lambda < 3$.

We wish to make one comment about the practical application of the test proposed in this section. The number of collected signals ($M_i$) and hence the number of bins ($K_i$) in Eq. (4) should be large enough to describe properly the smooth function $\chi^2$ with $V$ degrees of freedom. We will not provide an evaluation of a minimal number of bins, for arbitrary chosen degrees of freedom. We will not provide an evaluation of a minimal number of bins, for arbitrary chosen degrees of freedom.

2.2. Number of effective degrees of freedom

Components of the signal vector are mutually correlated in a complicated manner so the effective $V$ has to be determined empirically. Its upper bound $V_{\text{max}}$ is equal to the number of independent variables $N$. In order to determine the minimal $V_{\text{min}}$, the Principal Component Analysis (PCA) [28] of data set $S_i$ is performed. Before full PCA examinations, the column means of the data set $S_i$ are subtracted, in order to standardize distributions of the vectors' components. The data set with 0 mean value will be depicted with $S_i^0$. We define the orthonormal matrix $W_i = R^{N\times N}$ that maps the vectors from data set $S_i^0$ into a new space,

$$S_i = S_i^0 \cdot W_i$$

in such a way that the projection with successive basis vectors inherits the greatest possible variance in data set $S_i^0$. The covariance matrix of data set $S_i$ will be denoted with $\hat{C}_i$ and is given as $\hat{C}_i = E(S_i^2 - \bar{S}_i \bar{S}_i^\top)$. It is diagonal, with values sorted in non-increasing order. We define the Total Variance (TV) parameter as a normalized sum of $k$ variances on the diagonal of $\hat{C}_i$,

$$TV_k = O_k \cdot \hat{C}_i \cdot O_k^\top = (O_k \cdot \hat{C}_i \cdot O_k^\top)^{-1}$$

where $O_k$ is the $N$-dimensional row vector with ones at positions from 1 to $k$, and zeros from $k+1$ to $N$. According to this definition, $O_N$ is a vector with all $N$ values equal to one. The $TV$ is a non-decreasing function. We assume that at least $TV > 0.95$ is necessary to describe data set $S_i$ properly. The minimal number of variables $V_{\text{min}}$ is equal to the smallest $k$ for which $TV_k > 0.95$.

After the determination of $V_{\text{min}}$, calculations of statistics $R$ are repeated for different $V$ in the range from $V_{\text{min}}$ to $V_{\text{max}}$. The theoretical $\chi^2$ distribution with $V$ degrees of freedom for which the smallest statistics $R$ (Eq. (4)) and hence smaller parameter $\lambda$ (from Eq. (5)) was calculated, is selected. The experimental distribution is said to be a MVN distribution with $V$ degrees of freedom, if $\lambda$ is smaller than 3.

2.3. Method for data cleaning

If the data are normally distributed, the statistical significance of assignment of the measurement $u$ to data set $S_i$ can be provided. The Mahalanobis distance $d$ in Eq. (3), from the $\chi^2$ distribution with $V$ degrees of freedom, can be interpreted using p-values [29]. The hypothesis that $u$ may be assigned to $S_i$ is rejected when the $p$-value is below the predetermined significance level (e.g. 0.01), indicating that the signal $u$ is very unlikely under this hypothesis. Equivalently, a threshold on the Mahalanobis distance $d$, depicted with $d_{\text{max}}$, ensuring a minimal expected $p$-value, can be provided. Finally, the measurement $u$ is qualified to the data set $i$ if, and only if, the distance $d$ to $S_i$ is smaller than the predefined threshold $d_{\text{max}}$. In practice, the $d$- or $p$-value criterion may be used for the rejection of background events due to e.g. signal distortion by gamma quantum rescattering [30].

3. Experimental setup

The method described in the previous sections was tested on the example of reconstruction of hit-position in a single module of the J-PET detector [18]. The measurement was performed with a single module consisting of the 30 cm plastic scintillator strip EJ-230 [31] with the rectangular profile $1.9 \text{ cm} \times 0.5 \text{ cm}$. The strip was connected on two sides, via optical gel, to the Hamamatsu photomultipliers R4998, denoted as PM1(2) in Fig. 1. A series of measurements was performed using collimated gamma quanta from the $^{22}$Na source placed between the scintillator strip and the reference detector. A collimator was located on a dedicated mechanical platform allowing one to shift it along the line parallel to the scintillator strip with a submillimeter precision. The $^{22}$Na source was moved from the first to the second end in steps of 6 mm. At each position, about 5000 pairs of signals from PM1 and PM2 were registered in coincidence. These signals were sampled using the Signal Data Analyzer 6000A with a probing interval of 50 ps. As a trigger, a coincidence between signals from PM1 and a reference detector was required. Such trigger conditions, together with the 1.5 mm slit in a 20 cm long lead collimator, enabled us to select annihilation quanta. The background of gamma quanta from deexcitation of $^{22}$Ne is less than 0.1%. The size of the spatial profile of such triggered annihilation quanta was determined to be about 2 mm (FWHM). Examples of two signals registered at PM1 and PM2 are shown in Fig. 2. The upper (lower) panel of Fig. 2 shows a signal registered in PM1(2) for the case when the scintillator was irradiated at 7 and 23 cm to PM1 and PM2, respectively.

In the first step of the analysis the distributions of signal amplitudes were investigated. Experimental results based on the
We use the following ones: linear combinations of time values are equivalent to each other. The effective number of registered variables is thus for two signals, where only differences of times are physically meaningful. The effective number of registered variables is thus smaller by one and equal to $N=15$. Hence, each data set contains signals registered at a specified position, where single measurement is represented by a 15-dimensional real number vector.

From the point of view of further reconstruction procedure, all linear combinations of time values are equivalent to each other. We use the following ones:

$$x_i - y_i, \quad i = 1, \ldots, 4 \tag{8}$$

$$x_i - x_{i+1}, \quad i = 1, \ldots, 3 \tag{9}$$

$$x_i - w_i, \quad i = 1, \ldots, 4 \tag{10}$$

$$y_i - z_i, \quad i = 1, \ldots, 4. \tag{11}$$

They correspond to time difference of signals from two PMTs at a given voltage levels (Eq. (8)), time differences of a signal from PM1 at adjacent levels (Eq. (9)), and width of the signal on PM1 and PM2 at given levels (Eqs. (10) and (11), respectively). According to this choice, the $j$th measurement in the $i$th data set may be represented as $S_j^{(i)} = [x_1 - y_1, x_2 - y_2, x_3 - y_3, x_4 - y_4, x_1 - x_2, x_2 - x_3, x_3 - x_4, x_1 - w_1, x_2 - w_2, x_3 - w_3, x_4 - w_4, y_1 - z_1, y_2 - z_2, y_3 - z_3, y_4 - z_4]$.

The signals were registered in 50 ps steps (blue curves in Fig. 2). In order to evaluate the time value at given thresholds, interpolation must be applied. Due to the very high sampling rate, results obtained with different interpolation methods (linear, spline [32]) were found to be very close to each other. We used the linear interpolation in order to minimize computational cost.

4. Application of the hit-position reconstruction method to the experimental data

Though in principle all points from the sampled signal could be used for the reconstruction, in practice, in the case of hundreds of detection modules, the front-end electronics are able to perform at only a few samples per signal. Therefore in the following, the signals are probed at four fixed-voltage levels, providing eight time values for each signal from PM1 or PM2 – enough to estimate the resolution of hit positions.

4.1. Choice of variables

Based on the measurements of fully sampled signals, we simulate a four-level measurement with sampling in the voltage domain at 60, 120, 180 and 240 mV (see Fig. 2). Sampling times from PM1(2) at a given level are denoted with $x$ ($y$) and $w$ ($z$), for the rising and falling slope, respectively (Fig. 2) in total 16 values for two signals, where only differences of times are physically meaningful. The effective number of registered variables is thus smaller by one and equal to $N=15$. Hence, each data set contains signals registered at a specified position, where single measurement is represented by a 15-dimensional real number vector.

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$$y_i - z_i, \quad i = 1, \ldots, 4. \tag{11}$$

They correspond to time difference of signals from two PMTs at a given voltage levels (Eq. (8)), time differences of a signal from PM1 at adjacent levels (Eq. (9)), and width of the signal on PM1 and PM2 at given levels (Eqs. (10) and (11), respectively). According to this choice, the $j$th measurement in the $i$th data set may be represented as $S_j^{(i)} = [x_1 - y_1, x_2 - y_2, x_3 - y_3, x_4 - y_4, x_1 - x_2, x_2 - x_3, x_3 - x_4, x_1 - w_1, x_2 - w_2, x_3 - w_3, x_4 - w_4, y_1 - z_1, y_2 - z_2, y_3 - z_3, y_4 - z_4]$.

The signals were registered in 50 ps steps (blue curves in Fig. 2). In order to evaluate the time value at given thresholds, interpolation must be applied. Due to the very high sampling rate, results obtained with different interpolation methods (linear, spline [32]) were found to be very close to each other. We used the linear interpolation in order to minimize computational cost.

4.2. Number of effective degrees of freedom

According to the procedure described in Section 2, the PCA is performed and subsequently the TV is determined as a function of the number of variables. The result is shown in Fig. 4. Two curves representing signals registered in two most peripheral places are very similar. The minimal $V$ of the $\chi^2$ statistics...
4.3. Validation of the normality of the collected data sample

The hypothesis of normality was tested for numbers of degrees of freedom ranging from $V_{\text{min}} = 8$ to 15. The comparison of the experimental distribution with the theoretical one was performed based on the statistical test $R$ defined in Eq. (4).

Fig. 5 presents results for the data collected by the irradiation of the edge of the scintillator strip. The $\lambda$ parameter is shown as a function of $V$ degrees of freedom of the theoretical $\chi^2$. The minimum value of $\lambda$ is obtained for 13 degrees of freedom and is ca. 8. Large values of $\lambda (> 3)$ indicate that the data are not exactly normally distributed. The $\lambda$ parameter was evaluated for the theoretical $\chi^2$ with 13 degrees of freedom for all positions. The $\lambda$ values vary between 7 and 12. From this analysis one infers that the data in each set may be approximated with 13 independent and normally distributed variables. We will follow this assumption and investigate a simple hit-position reconstruction method based on a MVN distribution of signals. However, in the future different methods not necessarily fulfilling the normality assumption should be considered, and compared with the presented one. In the following the information about the means and covariance matrices, for all data sets, and the estimated $V$ of $\chi^2$ statistics will be used to calculate the significance of assignments ($p$-values).

4.4. An example of the hit-position reconstruction

In the previous sub-sections it was shown that the collected data samples approximately fulfill the assumption of normality so it is worth trying to apply the method for the hit-position reconstruction introduced in Section 2. Reconstruction is equivalent to the qualification of the signal to one of the predefined data sets established for the various positions along the scintillator. Fig. 6 shows an example of the position reconstruction for the signal created by the gamma hitting at known position, referred to as true.

The distances $d$ to all data sets were calculated according to Eq. (1), and are marked in Fig. 6 as circles. The hit-position is defined as the one for which distance $d$ acquires a minimum (full blue circle). In this example the reconstruction procedure yielded a hit-position different by 1.8 cm from the true position marked with red square. Knowing that the distance $d$ is derived from $\chi^2$ distribution with 13 degrees of freedom, the $p$-value of the assignment of the signal to the best-matching data set may be evaluated. The red dotted line in Fig. 6 indicates the maximum acceptable value of $d_{\text{max}} = 27.7$ corresponding to the $p$-value threshold of 0.01. The statistical significance of the assignment of a given measurement to the best-matching data set allows for the filtering of the data sample. Because of the multiple interactions of gamma quanta, the analyzed signal may be produced as a random coincidence of two signals in different positions. Hence, the $p$-value, or equivalently distance $d$, analysis helps to reject such distorted signals and improve the reconstruction process.

4.5. Spatial resolution of the hit-position reconstruction

The reconstruction method was verified using signals from all data sets from all the irradiation positions. Due to the fact that each data set $S_i$ consists of about 5000 measurements, the influence of the single measurement on parameters ($m_i, C_i$) of set $S_i$ is negligible. Hence, all signals in each data set $S_i$ were used to evaluate the parameters of a MVN distribution defined by ($m_i, C_i$). The hit position was reconstructed for each signal using the method presented in Section 2. Knowing the true hit-position for each measurement, differences between the true and the reconstructed positions ($r$) were evaluated (cf. Fig. 7).

In our method only the index of the best-matching data set is found, hence the $r$ acquires discrete values. The standard deviation ($\sigma$) of $r$ is equal to 1.05 cm. Results presented in Fig. 7 were obtained after filtering data provided the $p$-value is larger than 0.01 and the amplitude of the signal is larger than 0.6 $A_{\text{max}}$, where $A_{\text{max}}$ corresponds to the amplitude at the Compton edge observed at given position of irradiation (see Fig. 3). The last criterion is used in order to reject signals with the small number of photoelectrons which spoils the resolution and anyhow is discarded in the image reconstruction in order to filter out the scattering of annihilation quanta inside the diagnosed patient [18,19]. In comparison, the proposed method using the lowest threshold (60 mV) alone, under the same filtering conditions, gives 1.08 cm ($\sigma$) spatial resolution. In the case of using the highest threshold level (240 mV) alone the spatial resolution of 1.25 cm ($\sigma$) is obtained.
4.6. Time resolution of the event time reconstruction

The resolution of the time difference ($\Delta t$) between the signal arrivals to the scintillator ends may be derived directly from the previously calculated value of the spatial resolution,

$$\sigma(\Delta t) = \sigma(r) \frac{2}{v_{\text{eff}}}$$

(12)

where $v_{\text{eff}}$ denotes the effective speed of light signal in the used scintillator strip. In the recent work [33], the speed of the light in the scintillator was estimated to be 12.6 cm/ns. Hence, the resolution of $(\Delta t)$ may be estimated to be about 167 ps ($\sigma$). This corresponds to a twice better resolution of about 83 ps ($\sigma$) for the determination of the interaction moment of the gamma quanta hit the scintillator ($t_{\text{hit}}$). The interaction moment is given by

$$t_{\text{hit}} = \frac{t_{L} + t_{R}}{2} - \frac{D}{v_{\text{eff}}}$$

(13)

where $t_L$ and $t_R$ are the arrival times to the left and right photomultipliers and $D$ is the length of whole strip. We assume for the sake of simplicity that $v_{\text{eff}}$ in Eq. (13) is known exactly. Since the time difference $\Delta t = t_{R} - t_{L}$, we have

$$\sigma^2(\Delta t) = \sigma^2(t_{L}) + \sigma^2(t_{R})$$

(14)

and the resolution of $t_{\text{hit}}$ based on Eq. (13) may be expressed as

$$\sigma^2(t_{\text{hit}}) = \frac{\sigma^2(t_{L}) + \sigma^2(t_{R})}{4} = \frac{\sigma^2(\Delta t)}{4}$$

(15)

which implies that $\sigma(t_{\text{hit}}) = \frac{\sigma(\Delta t)}{2}$.

5. Conclusions

In this paper a novel method for hit-position reconstruction in plastic scintillator detectors was introduced. It was validated for the application in a new Positron Emission Tomography based on plastic scintillator strips [17–20]. The method can be applied to detectors enabling sampling of signals in the voltage domain. The distinctive feature of the proposed reconstruction technique is the determination of the Mahalanobis distances of the multivariate vector, representing the measured signal, from the vectors corresponding to the mean of the signals in data sets determined for known positions. The covariance matrix and the mean vector are calculated separately for each position. The reconstruction algorithm identifies the data set for which the Mahalanobis distance acquires a minimum and reconstructs the hit position as that where the identified data set was generated.

The procedure was tested using a large statistics sample of data registered by a dedicated setup in 50 ps intervals. Experimental setup provided irradiation of a chosen position in a plastic scintillator strip with 511 keV gamma quanta. Sampling in the voltage domain at four thresholds was simulated and each measurement was represented by a 15-dimensional vector holding information about the relative time values of signal’s arrival to both scintillator ends.

Using the introduced reconstruction procedure, the spatial and time resolutions of the hit-position and event time for annihilation quanta measured with a 30 cm plastic scintillator strip with sampling of signals at two edges were determined to be about 1 cm and 80 ps ($\sigma$), respectively. It should be noted that these resolutions can be slightly improved by decreasing smearing due to the finite size of the beam (0.2 cm) and due to the 0.6 cm step used for the determination of data sets.

The performance of the method has been validated on a single scintillator strip. However, an independent test for about 20 strip has been made and the results were stable in the sense of obtained resolution ($\sigma$). Anyhow, in order to avoid inhomogeneity of response in a full scale detector, separate calibrations of each module (scintillator, photomultipliers and electronics channels) will be provided. Moreover, the worsening of electronic performance for a full scale detector will be limited since the readout system is based on well recognized and tested components used in the particle physics experiments e.g. HADES [34].

It was shown that the measured signals may be approximated with a MVN distribution with 13 degrees of freedom. It is worth noting that the developed statistical test is general and may be incorporated in any other investigation where confirmation of multidimensional normality is needed. Since the $\lambda$ parameter (see Eq. (5)) is greater than 3, the further improvements in spatial and time resolutions can be achieved by applying different reconstruction methods where the assumption about normality is not obligatory, e.g. artificial neural networks. Furthermore, the resolution can be still improved by the optimization of threshold levels, an increase of their number, and enhancing light collection efficiency by optimizing the shape of scintillators and usage of silicon photomultipliers.

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