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Reconstruction of hit position of gamma quanta in scintillators based on sampling of signals in voltage and fraction domains.

Praca magisterska

na kierunku informatyka

Praca wykonana pod kierunkiem:
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Abstract

Faculty of Physics, Astronomy and Applied Computer Science

Reconstruction of hit position of gamma quanta in scintillators based on sampling of signals in voltage and fraction domains.

by Natalia ZOŃ

With the ongoing development of novel Positron Emission Tomography solutions, there exist a demand for researching methods of processing data allowing for the reconstruction of 3D human body images from signals gathered by the device's detectors.
This thesis describes a method of reconstructing the position of gamma quanta hit along a single polymer scintillator based on the calculation of similarity of signals incoming from a PET device with respect to signals in a previously created database.
The similarity of two signals is computed using either a set of times corresponding to a set of voltage thresholds common for the two compared signals (Chi-square method) or two sets of points designating two curves representing the signals (Frechet distance method). The theoretical basis of the concept and its general idea as well as individual steps of the proposed algorithm are explained in detail in the next chapters of this thesis.
The realization of the method in form of a computer program was implemented in Python (version 2.7), a high-level, general purpose programming language which allows for employing multiple programing paradigms including object-oriented, functional, procedural or imperative programming.
This thesis is supplemented with Appendix containing the source code of the computer program.

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Streszczenie

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Rekonstrukcja miejsca uderzenia kwantów gamma w scyntylatorach w oparciu o próbkowanie sygnałów w dziedzinach napięć i frakcji.

autor: Natalia ZOŃ

Wraz z rozwojem nowych rozwiązań w dziedzinie Pozytronowej Tomografii Emisyjnej pojawia się potrzeba tworzenia i badania metod pozwalających na rekonstrukcję trójwymiarowego obrazu organów ciała pacjenta z sygnałów zebranych przy pomocy detektorów w tomografie.

Niniejsza praca magisterska zawiera opis jednej z możliwych metod rekonstrukcji miejsca uderzenia kwantów gamma w pojedynczym polimerowym scyntylatorze, opartej na wyznaczaniu podobieństwa pomiędzy sygnałami pochodzącymi z tomografu a sygnałami znajdującymi się we wcześniej utworzonej bazie danych.

Podobieństwo pomiędzy dwoma sygnałami jest obliczane przy użyciu jednej z dwóch reprezentacji syngałów: jako zbiorów czasów odpowiadających zbiorowi progów napięcia, wspólnego dla obydwu porównywanych sygnałów (chi-kwadrat), lub jako zbiorów punktów wyznaczających dwie krzywe będące reprezentacją sygnałów (odległość Frecheta).

W dalszej części pracy zostały opisane podstawy teoretyczne, ogólna koncepcja oraz poszczególne kroki proponowanego algorytmu.

Przedstawiona została realizacja omawianej metody w formie programu komputerowego napisanego w języku Python (wersja 2.7), który jest wysokopoziomowym językiem ogólnego zastosowania, pozwalającym na wykorzystywanie licznych paradygmatów programowania, w szczególności: programowania obiektowego, funkcjonalnego, proceduralnego oraz imperatywnego.

Do niniejszej pracy dołączony jest kod źródłowy opisanego programu (Załącznik A).

Acknowledgements

I would like to express my deep gratitude toward my supervisor, prof. Paweł Moskal for all of his encouragement, persistent help and guidance throughout the making of this thesis, and also for spending time to proofread it and provide numerous useful suggestions. Thank you for giving me the opportunity to be a part of the J-PET research group.

I am grateful to all colleagues from the group for participating in vigorous discussions each time I presented my research advancements during our meetings. This work could not have been completed without all of your constructive criticism and invaluable advice.

Last but not least, I thank my parents, for all of their love, support and endless patience.

Contents

Introduction

Positron Emission Tomography is a non-invasive nuclear medicine method used for obtaining three dimensional (3D) images of metabolic processes in the body of living organisms. The image is produced by reconstructing the density distribution of a radioactive tracer in the patient's organs. To allow for such reconstruction, a PET device detects gamma quanta emitted from the radiopharmaceutical substance incorporated into molecules of body tissues. Detectors constituting the PET system are arranged around the examined body in form of a ring. In the currently used modalities, the detector ring consists of inorganic scintillator crystals, divided into sub-crystals, with attached photomultipliers. The determination of the position of the gamma quanta emission point requires the time of flight of the gamma quanta (for reconstruction of the radial coordinate) as well as the knowledge of the position of sub-crystals in which the gamma quanta have been registered (for reconstruction of the longitudinal and angular coordinate). The resolution of the obtained image depends solely on the number and size of scintillator sub-crystals constituting the detector ring. The high cost of producing crystal scintillators limits the number of detectors used in a single device. The currently standard width of a detector ring is equal to about 20 cm [1], allowing for only such length of a patient's body to be examined at one time.

Jagiellonian PET (J-PET) is based on a novel method [1, 2] which substitutes crystal scintillators with polymer scintillators. J-PET group aims at construction of a detector composed of scintillator strips having up to 1 m length. The large field of view of such device would allow for simultaneous imaging of a larger part of the body than currently used devices can visualize at a given time. The geometry of J-PET devices allows to join PET and CT or NMR scanners into one working unit which could produce images obtained from both methods, captured at the same time [2–4]. In addition, polymer scintillators have the advantage of price over the more expensive crystal scintillators. In contrast to the current solutions, in the J-PET detector the hit position of a gamma quantum cannot be determined based on the position of the detector and instead it will be reconstructed based on the sampling of signals in the domain of voltage. Therefore,

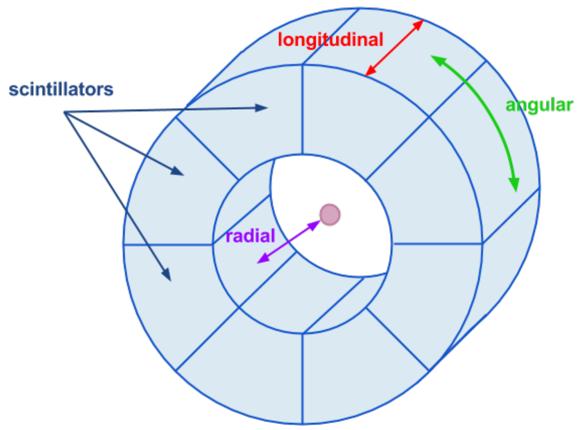


FIGURE 1: A schematic representation of a detector ring used in Positron Emission Tomography.

there arises a need for the elaboration of a method of reconstructing the longitudinal coordinate of gamma quanta hit point in a single polymer scintillator strip.
A gamma quantum interacting in the scintillator creates a light signal which propagates to both sides of the scintillator, and on each side is converted into electric signal by means of photomultipliers. In the J-PET detector a dedicated multi-threshold electronics is designed [5] to sample electric signals in the domain of voltage.

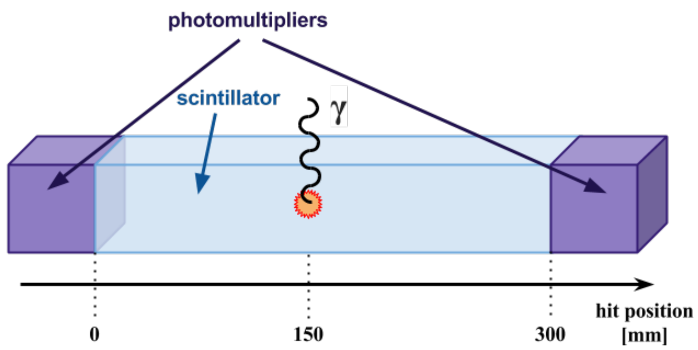


FIGURE 2: A single scintillator strip module. Scintillator and photomultipliers as indicated by arrows.

In this thesis I describe a method for reconstructing the hit position of gamma quanta in polymer scintillators based on the comparison of measured signals with a database of previously saved signals for which their corresponding gamma quanta hit positions are known [6].

In a working J-PET Tomography Unit an implementation of the described method will be integrated as one of the modules in the J-PET framework [7].
'Chapter One: Theory' introduces the general idea of the presented method and contains the detailed explanation of the mathematical basis of the proposed algorithm.
'Chapter Two: Implementation' contains a description of an application of the presented method in form of a computer program. The design of the program's structure as well as the technical tools used in implementation are discussed.
'Chapter Three: Results' specifies the data and the types of tests used to evaluate the proposed method's efficiency and presents results obtained from conducting these tests.

Chapter 1

Theory

During positron emission tomography imaging, gamma quanta emitted from the patient are registered by the detectors. The position and time of interaction of two gamma quanta registered within a few nanosecond interval allows to reconstruct a line including a place of the decay of radiopharmaceutical tracer. Determination of high statistics sample of such lines permits to reconstruct a density distribution of radiopharmaceutical inside the patient's body.

This chapter contains the description of a basic idea of the reconstruction of the position and time of the interaction of gamma quantum in a long scintillator strips (used in the J-PET detector).

1.1 A general concept of database-search reconstruction method

For the clarity of the explanation I will first define basic notions used hereafter in this thesis.

A 'signal' is a digital representation of analog output generated by a photomultiplier. An example of two signals is shown in Fig. 1.1.

An 'event' is a set of two signals, each incoming from a different photomultiplier, measured at the same time, both being results of the interaction of one particular gamma quantum in the scintillator strip.

The shape of signals incoming from photomultipliers at each scintillator's end changes with the position of the interaction along the strip. Signals produced by the photomultiplier closer to the gamma quanta hit point are larger and have more sharp edges than signals measured at the other end. When the interaction occurs at the center of the strip, the signals generated at both photomultipliers have similar shapes. The upper panel of

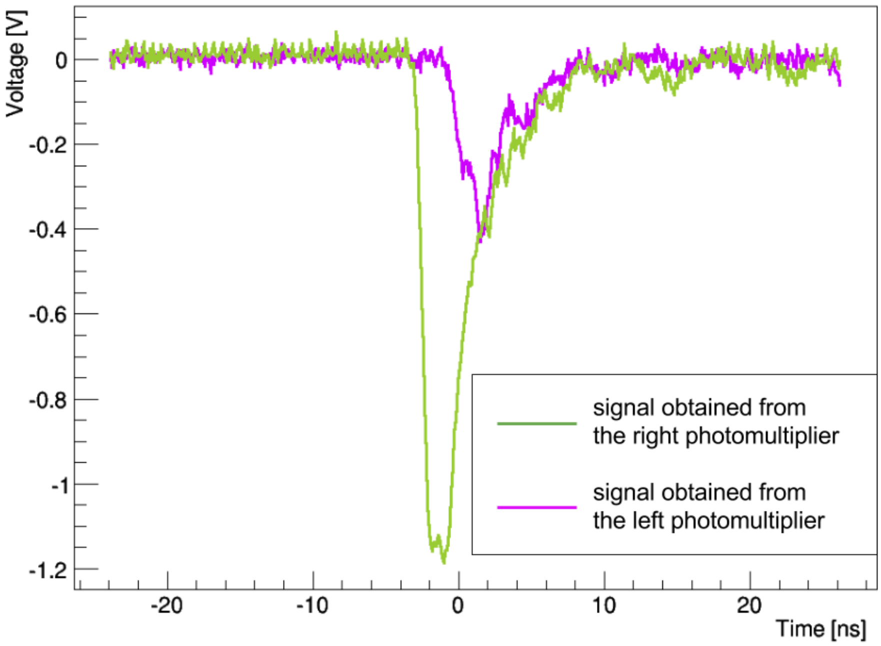


FIGURE 1.1: An example of signals resulting from the interaction of gamma quantum close to the right photomultiplier.

Fig. 1.2 is a pictorial illustration of variations of signals as a function of the position of interaction of a gamma quantum.

The general concept of database-search based gamma quanta hit position reconstruction relies on the premise that signals' shapes are largely dependent on the hit position. Employing such an assumption, it follows that the measure of similarity between the shapes of signals constituting two events is correlative with the measure of distance between the hit positions of gamma quanta corresponding to each of the compared events. One possible way of utilizing this correlation in retrieving gamma quanta hit position is to create a database of events measured at known positions of irradiation. Any incoming event whose corresponding hit position needs to be reconstructed should be compared (using one of possible distance metrics, see Chapter 1.3) with a number of events from the database. Based on the result of such comparisons, a best matching position can be defined for each incoming event.

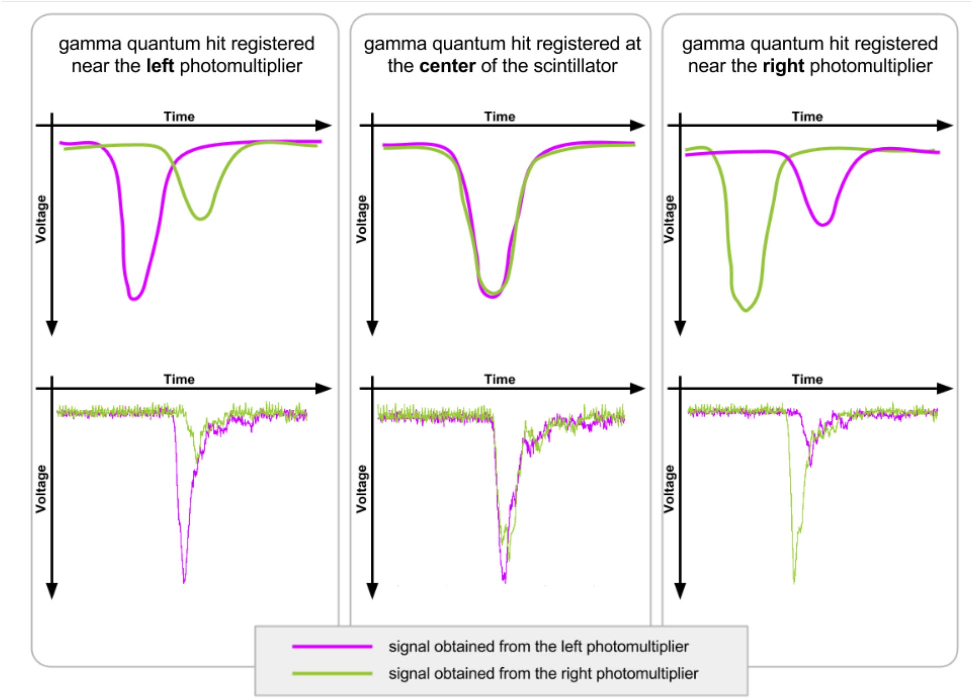


FIGURE 1.2: A pictorial (above) and real data (below) representation of how the shape of signals changes with respect to collimator position.

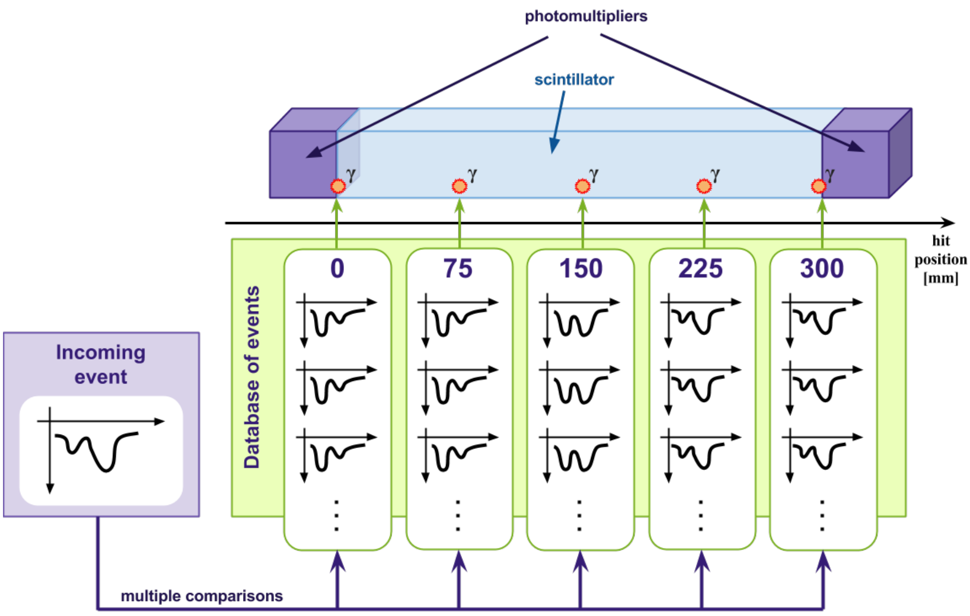


FIGURE 1.3: A schematic representation of the general concept behind the presented reconstruction method.

1.2 Sampling

Events obtained from photomultipliers are represented as sets of points in a 2-dimensional metric space. To produce this discrete representation of an analog signal, one of two approaches can be taken. One approach is to measure the voltage present on a given channel, at a number of points in time. This method will be referred to as "time domain sampling" (see Fig 1.4). The other possibility is to select a number of voltage threshold levels and detect the times at which a given signal's voltage was equal to each of those thresholds, i.e. "voltage domain sampling" (see Fig.1.5).

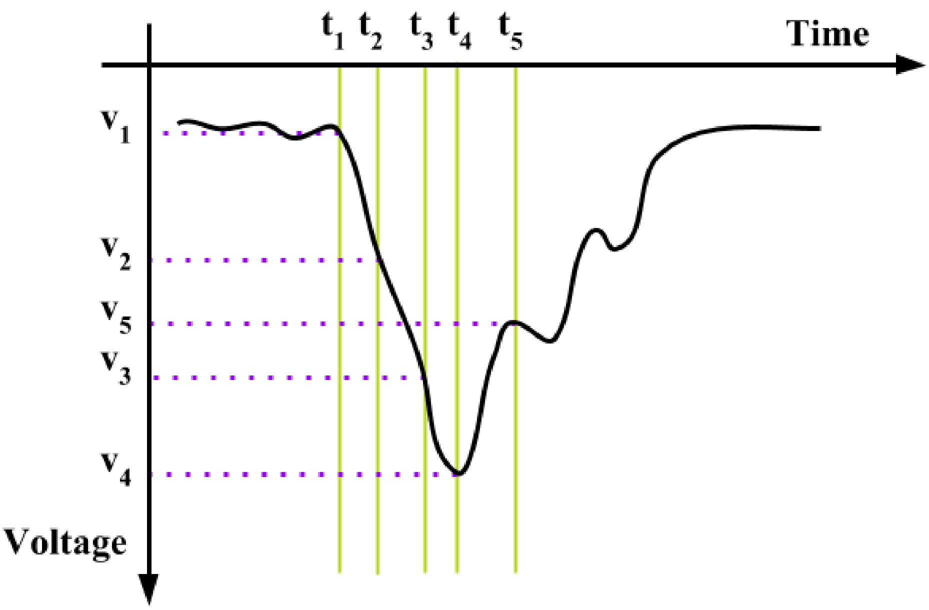


FIGURE 1.4: A pictorial representation of time domain sampling

Having obtained a set of discrete points carrying all available information about a given signal, a continuous representation can be acquired by interpolating the values of measurement at points located in between of sampling time values (time domain sampling) or voltage thresholds (voltage domain sampling). In such a case, a signal is represented by the original set of points along with a function (i.e. an interpolator) defined on a continuous interval from the first sampling value to the last one, relating each input point to an interpolated voltage value (time domain sampling) or time (voltage domain sampling).

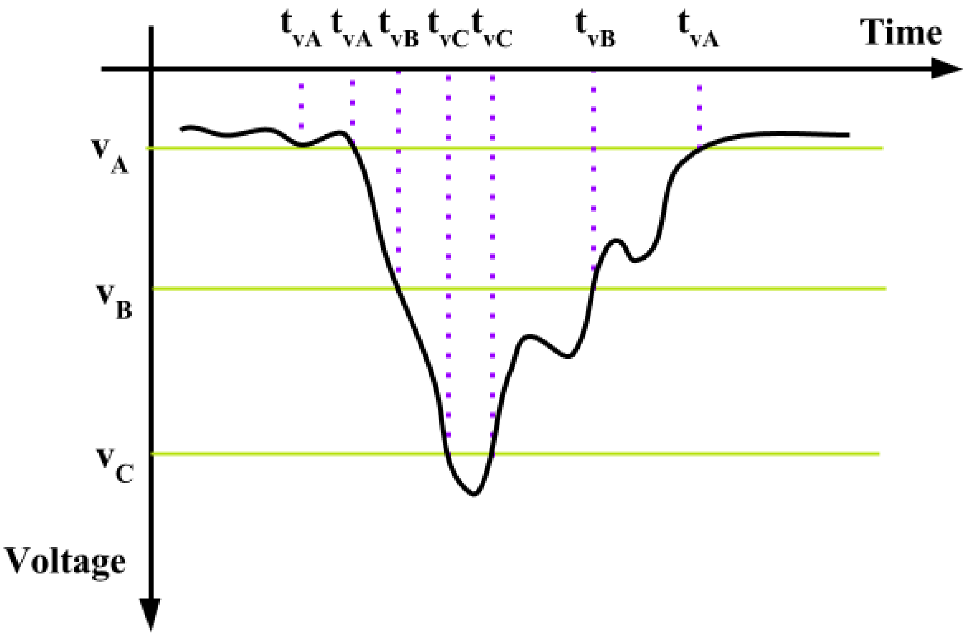


FIGURE 1.5: A pictorial representation of voltage domain sampling

1.2.1 Interpolation in time domain sampling

There are a number of interpolation methods which can be applied to signals. The simplest among them is linear interpolation.

With decreasing number of points in the original data set representing a given signal, the error of linear interpolation grows significantly. To reduce this error, different interpolation methods can be applied, including but not limited to spline interpolation [8] and akima interpolation [9].

1.2.2 Interpolation in voltage domain sampling

When considering the sampling of time at voltage thresholds, the same methods as in the case of time domain sampling would be applicable, if a function that maps voltage values to points in time could be unambiguously defined. Such function would be the inverse of a given signal's interpolator used in time domain sampling. However, the interpolator is not an injective function, and hence non-invertible. For this reason, interpolation of time at a given voltage threshold level requires following a more complex procedure than in the case of interpolating voltage at a given time.

A given signal can be split at the time at which it reaches maximum amplitude into two separate parts, each representing either the rising or the falling edge of the original signal. When ignoring noise fluctuations, both of these edges can be interpreted as injective relations from time domain into voltage codomain, which allows for finding their corresponding inverse functions.

To neglect noise fluctuations present in signals, a simple method of approximating the time at which a given signal crosses a voltage threshold was proposed. First, a chosen edge of the signal is divided into two subsets, one containing points which have voltage values larger than the threshold, and the other containing points whose voltage values are smaller than the threshold (if there is a point having voltage exactly equal to the threshold level, it is immediately returned as the threshold's corresponding time). Next, a point closest to the threshold level is chosen from each of these two subsets. Having obtained the two points, the time can be interpolated as the time at which a line containing these points is crossing a horizontal line designated by the threshold level, as it is illustrated in Fig. 1.6.

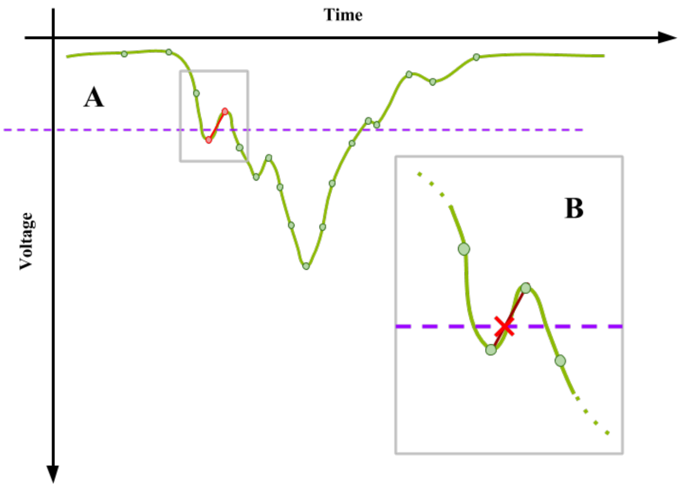


FIGURE 1.6: The simplified method for interpolation in voltage domain sampling (A) and a magnification of the region where the interpolated time was found (B).

The voltage thresholds used in sampling can be defined as either constant voltage values or as fractions of a given signal's amplitude.

1.3 Distance metrics

A distance metric is the mapping $D : A \times A \rightarrow \mathbb{R}$ satisfying the following metric space axioms:
$(M1) : \forall x \in A : d(x, x) = 0$
$(M2) : \forall x, y, z \in A : d(x, y) + d(y, z) \geq d(x, z)$
$(M3) : \forall x, y \in A : d(x, y) = d(y, x)$
$(M4) : \forall x, y \in A : x \neq y \Rightarrow d(x, y) > 0$

In the case of defining a distance function in the events domain, events are considered as points in a 2N-dimensional space where N denotes a number of samples of a signal. 2N originates from the fact that signals from both ends of scintillator constitute one event. The difference in time between two signals belonging to one event is correlated with the hit position of gamma quantum along a scintillator strip [2]. For this reason, both signals that constitute a given event are collectively considered as one dataset. The shape, which is the subject of comparison consists of two curves on a 2-dimensional plane placed in fixed locations with respect to each other.
In contrast, the shift in time between two compared events is not correlated with their corresponding hit position and, if taken into account, can be a source of bias, e.g. two events measured with similar conditions (time of measurement, oscilloscope settings) might be interpreted as more similar than they actually are. Therefore, before computing the value of distance measure for a pair of events, one of them is shifted so that they are maximally aligned in time domain. The maximal alignment of two events can be defined in a number of ways. One possibility is to find such a translation in time domain of one event with respect to the other, that the chosen distance measure for the pair of events is minimal. A simplified and less computationally complex solution (without the need for the minimization of a function) is to find the value of a distance measure corresponding to time domain only, and use it as the time by which one of the events should be shifted. A simple method of defining such a distance measure is to sample signals belonging to both events at a chosen number of voltage thresholds, and, for each pair of signals corresponding to the same photomultiplier, find the differences in time at any given threshold as illustrated in Fig. 1.7. The arithmetic mean of all time differences found in such way is the simplified distance measure value. Using this approximate measure, one of the compared events can be shifted with respect to the other to allow for the accurate computation of their respective similarity, using more advanced methods (see Fig. 1.8).

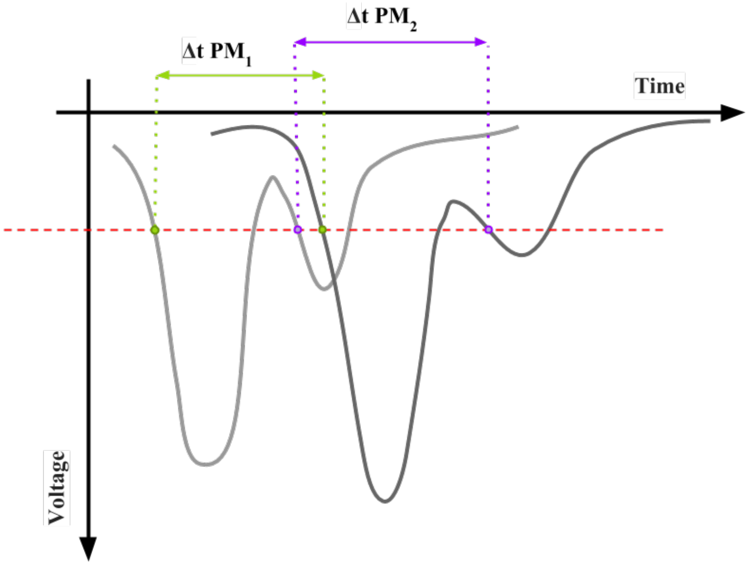


FIGURE 1.7: A pictorial representation of the simplified method of finding the maximal alignment of two events.

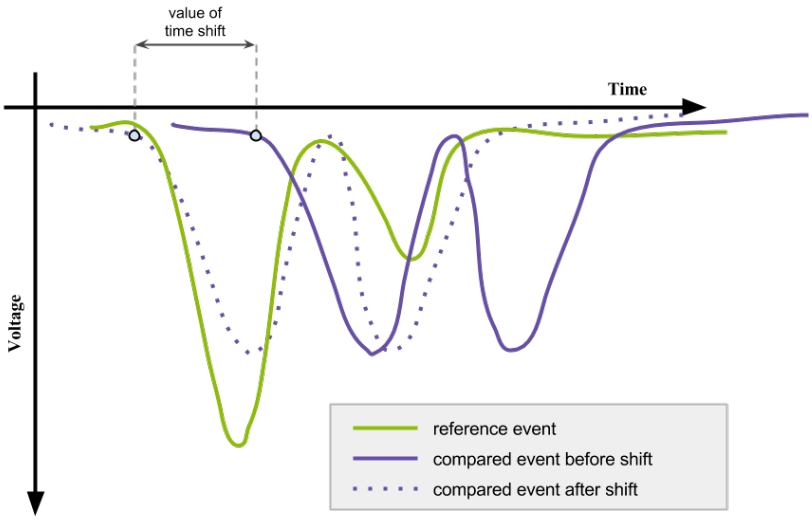


FIGURE 1.8: A pictorial representation of two events before and after applying the maximal alignment condition.

1.3.1 Chi-square test measure

The chi-square test distance measure is a representation of the collective distance between all corresponding points of two given events. The partial distances are defined in time domain, while the corresponding pairs of points are designated by voltage thresholds, as shown in eq. 1.1

$$\chi^2(event_1, event_2) = \frac{\sum_{i=0}^{n-1}(t_{1(Left),i} - t_{2(Left),i})^2}{n} + \frac{\sum_{i=0}^{m-1}(t_{1(Right),i} - t_{2(Right),i})^2}{m}, \tag{1.1}$$

where i values denote voltage thresholds whose corresponding sampled times are represented by $t_{1,i}$ (for $event_1$) and $t_{2,i}$ (for $event_2$). Left and Right denote the side of the photomultiplier at which the signals were measured.

1.3.2 Frechet distance

The Frechet distance is a measure of distance between two curves, which takes into account the ordering and location of points along the curves [10]. In its original version it is defined on polygonal curves constituting of continuous line segments, however for the purpose of gamma quanta hit position reconstruction a discrete variant of the method and its recursive implementation proposed by Eiter and Manilla [11] was used.
The mathematical formula for Frechet distance is given in eq. 1.2

$$F(A, B) = \inf_{\alpha,\beta} \max_{t\in[0,1]} \left\{ d\Big(A(\alpha(t)),\, B(\beta(t))\Big) \right\}, \tag{1.2}$$

where d is the distance function of a metric space S, A, B are curves in the metric space S, α and β are continuous, non-decreasing, surjective reparametrizations of $[0, 1]$ into itself.

1.3.3 Applying similarity measures on signals

The similarity of two signals is computed using either a set of times corresponding to a set of voltage thresholds common for the two compared signals (Chi-square method) or two sets of points designating two curves representing the signals (Frechet distance method). In the Frechet method, the points can be arbitrarily defined along signals' curves, and for this reason the original discrete representations of signals (obtained from digital measurement) were used for the comparison. In the chi-square method, the partial differences are computed on times corresponding to given voltage thresholds, and so the method requires each pair of points to have the same voltage value. To satisfy this requirement, the value of chi-sqare was computed using the original points on one of the signals while the second signal's times corresponding to their thresholds were interpolated. In both Frechet distance and chi-square methods, only the rising edges of signals were used for the comparison. A given signal's rising edge was restricted in the voltage domain to the smallest amplitude among compared signals, to ensure that the interpolation does not go out of range.

1.3.4 Evaluation

So far I have defined the possible ways of determining the distance between two given events. In this section I describe the methods of utilizing this knowledge in the search for the hit position of gamma quanta in scintillator strip, assuming that a database of events corresponding to various hit positions along the strip is available.
An input event, whose gamma quanta hit position needs to be reconstructed should be compared with a number of database events corresponding to each measured position. After computing a chosen distance measure values for each of those database events, they need to be collectively interpreted in such a way that the resulting reconstructed gamma quantum hit position of the input event can be defined.

1.3.4.1 Global minimum method

One possible method of determining the resulting reconstructed position of the input event is to define it as the position of the database event which returned the minimum distance measure value among all conducted comparisons.

1.3.4.2 Minimum of arithmetic means method

Assuming that for every position in the database more than one event was compared with the input event, the average distance measure value can be defined for each position by finding the arithmetic mean of all distances returned from comparisons of events corresponding to that position. The position whose average measure is minimal is returned as the reconstructed hit position.

1.3.4.3 Minimum of standard deviation method

When conducting multiple comparisons of the input event with events belonging to each database position, the resulting sets of distance measure values can have different statistical dispersions. The reconstructed position is determined under the presupposition that the best matching database position is the one whose result shows the smallest statistical spread, i.e. has minimal standard deviation as defined by the formula shown in Eq. 1.3.

$$s = \sqrt{\frac{\sum(x-\bar{x})^2}{n-1}}, \tag{1.3}$$

where x denotes the observed values and $\bar{x}$ is the mean value of those observations and $n-1$ is the sample size.

1.3.4.4 Minimum of fitted quadratic function method

The reconstructed position can also be defined as the minimum of a quadratic function fitted to the dataset of results obtained from multiple comparisons. In this case the reconstructed position is not represented as a discrete number corresponding to one of the positions defined in the database but can be any real value.

1.4 Data filtering

After collecting events measured by photomultipliers, an initial filtering of the data must be performed. Only signals from the greater half of the charge spectra of all collected signals should be used in the process of hit position reconstruction. Such signals should be taken into account for the image reconstruction exclusively, in order to decrease blurring of the image due to the gamma quanta scattered inside a patient's body [1, 2].

The shapes of charge spectra depend on the hit position. Because of that, the point designating the middle of a given spectrum (i.e. the point of cut) must be found separately for each set of data. In theory, the point of cut for a given spectrum could be determined simply as the middle point in between the start (in this case - the zero value) and the end of the histogram. However, values composing such histograms are subject to measurement uncertainties, and so the end of the histogram cannot be accurately defined. For this reason, a method for determining the cut point must not be sensitive to experimental fluctuations, but rather should rely on the analysis of the general shape of a spectrum.

It is important to note that the charge of a measured signal is proportional to the kinetic energy of electrons hit by the gamma quanta. In the case of the ideal detector the shape of the charge spectrum correspond to the spectrum of the kinetic energy of electron hit by the gamma quantum which is described by formula 1.4.

To obtain a simulated histogram a simple Monte Carlo method can be implemented. The first step is to generate a number of uniformly distributed pseudo-random points on a 2-dimensional plane. Next, each of these points is tested to determine whether at its x-coordinate, the value of the function in Eq. 1.4 is greater or smaller than the y-coordinate of the point. If the point happens to be positioned under the function curve, it passes on to next steps of the algorithm, otherwise it is rejected.

$$f(t) = const \cdot ((\frac{m^2 + (m-t)^2}{m(m-t)}) + (\frac{2t-m}{t-m})^2 - 1), \tag{1.4}$$

where m and t denote mass and kinetic energy of the scattered electron, respectively. Energy and mass is expressed in the units of MeV.
Equation 1.4 was derived [12] from the Klein-Nishina formula [13], and describes kinetic energy distribution of electrons scattered by annihilation gamma quanta.
For the simulated histogram to accurately represent the shape of experimental data histograms it is also necessary to take into account the experimental resolution causing the smearing of the spectrum (see Fig. 1.9 (left panel)), as well as varying scale of histograms (they can be scaled independently in either x or y axis, or in both). Scaling in x accounts for the relation between kinetic energy of the scattered electron and the charge of electric signals generated by photomultipliers, and scaling in y accounts for the ratio between number of measured and number of simulated events.
This adjustment of the simulated histogram's shape to fit the experimental one is done in a number of steps. The first modifiable parameter (i.e. nf) is responsible for experimental resolution. Experimental smearing is approximated by the gaussian distribution whose standard deviation changes as a function of energy [14]. Thus, each generated point, after being accepted, is smeared in the x axis domain by a pseudo-random value generated from a gaussian distribution having the standard deviation (σ) as defined in eq. 1.5.

$$\sigma = nf \cdot \sqrt{a_x}; \qquad b_x = rand(\sigma, a_x), \tag{1.5}$$

where a_x denotes the x-coordinate of the generated point, $rand$ denotes the pseudo-random number generator of gaussian distrubution with mean equal to a_x and standard deviation equal to σ, b_x is the resulting point of simulated histogram, which includes experimental smearing.
Two additional parameters, β and norm, are associated with the amounts by which the simulated histogram is scaled in the x and y axes, respectively [14]. The parameter β is applied to the set of generated and smeared points, using the formula in eq. 1.6 for

every point.

$$c_x = \beta \cdot b_x \tag{1.6}$$

In the next step of the algorithm, the points are converted into a histogram, by assigning each point's x-coordinate to one of the histogram 'bins', i.e. segments on the x-axis. The height of a given bin is defined as the number of entries which were assigned to it.

The third parameter, 'norm', is applied after converting the generated points into a histogram. Each bin's height is multiplied by the norm, which squeezes or stretches the histogram in the y-axis domain.

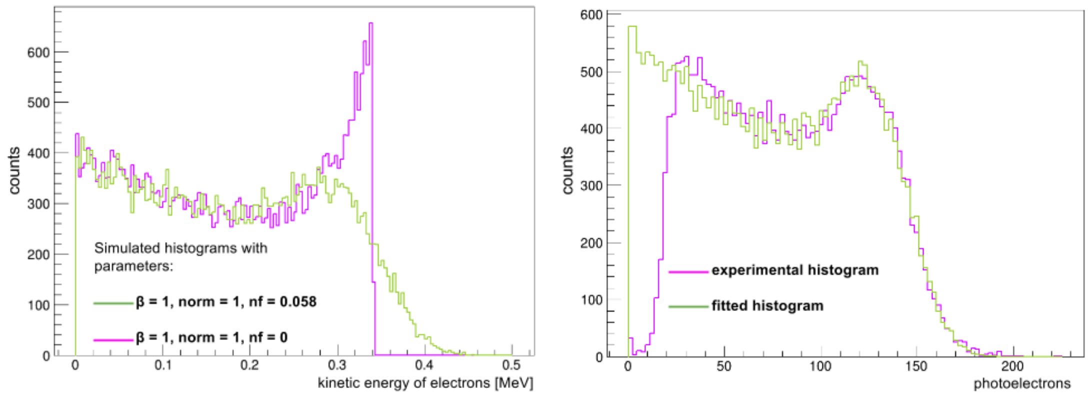


FIGURE 1.9: An example of two generated hitograms with and without simulating the experimental smearing (left). An experimental histogram with the corresponding fitted simulated histogram (right).

For every experimental histogram whose cut point needs to be found, a simulated histogram can be generated having such values of the parameters nf, β and norm that its shape is maximally similar to the original experimental one (as illustrated in Fig. 1.9 (right panel)). The resulting cut point is dependent on the β parameter of the obtained histogram, which is described in eq. 1.7.

$$cut = (\beta \cdot 0.34) \cdot fraction, \tag{1.7}$$

where 0.34 denotes the maximum energy of the scattered electron in units of MeV, β denotes the obtained value of the parameter β, and *fraction* is the desired fraction of spectrum that should be disregarded.

Chapter 2

Implementation

2.1 Tools used

The choice of tools used for the first implementation of the presented method was made under the main premise that the created software should be easily scalable, modifiable and well controlled in terms of any possible errors and malfunctioning. Another important aspect was to make the software relatively easy to use so that even people who are not experts in programming would be able to run it and modify at least some of the modules to fit their particular requirements. At present the optimization of running speed was not among the main concerns, nonetheless it will be an important factor during the production of the final version which can be installed on a working tomography unit.

2.1.1 Programming language

The software was implemented in Python (version 2.7), a high-level, general purpose programming language which allows for employing multiple programing paradigms including object-oriented, functional, procedural or imperative programming (https://www.python.org/). It provides an extensive standard library, which, when used together with the variety of third-party modules available for Python, can create a suitable environment for developing algorithmic problems of various nature.
The IDE used for the development of the source code was JetBrains Pycharm (http://www.jetbrains.com/pycharm/).

2.1.2 Libraries

In addition to using the standard library of Python, a number of third-party libraries were used in the implementation.

2.1.2.1 SciPy

SciPy is a system of Python-based open-source modules for mathematics, science and engineering (http://scipy.org/). It offers tools including numerical method implementations, signal processing, statistics, optimization, data structures. In the implementation of the hit position reconstruction method, SciPy was used mainly for interpolation and minimization purposes.

2.1.2.2 CodernityDB

CodernityDB is a Python-based NoSQL, schema-less database designed for providing storage and fast access (up to 100 000 insert and more than 100 000 get operations per second) to large amounts of simple-structured data (http://labs.codernity.com/codernitydb/). In this project, it is used as an alternative to ASCII files for storing signal data.

2.1.2.3 PyROOT

PyROOT (http://root.cern.ch/drupal/content/pyroot) is a Python interface providing access to C++ based scientific computation library, ROOT. Although ROOT is mainly a numerical computation library, in the implemented software it was used only for plotting, while SciPy modules were used for the calculations.

2.2 General structure of the system

The software presented in this thesis (the source code is supplemented in Appendix A) was designed to be a platform for testing new advancements in the hit position reconstruction methods, which allows for the careful examination and fine-tuning of the tested ideas. The architectural design of the system (see Fig. 2.1) was created to satisfy the need for modularity and modifiability of the application.

2.3 System modules in detail

2.3.1 Photoelectron Filter

The software responsible for designating the cut value on photoelectron spectra is a standalone Python program, completely separable from the reconstruction modules. It is also contained in its own Pycharm IDE project. This is due to the fact that this code was developed to be used for the purpose of filtering the input data for a number of various applications, one of which is the hit position reconstruction program.

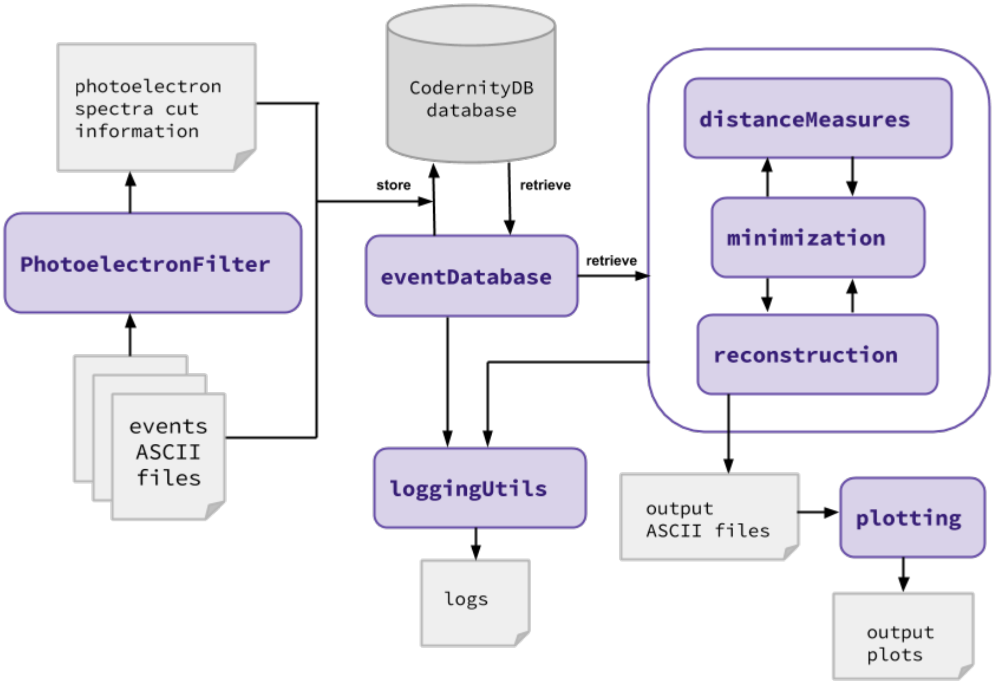


FIGURE 2.1: A schematic representation of the architecture of the system.

The program operates on ASCII files containing the photoelectron numbers specified for each position (catalog)/channel. It allows the user to designate which positions and channels should be processed, and performs batch processing of the chosen datasets.
The output is in the form of a file containing information about the obtained parameter values, *.png and *.root format plots showing the experimental and fitted histograms, and a single file enlisting the obtained cut value for positions/channels.
An additional package used to obtain photoelectron numbers from ASCII event files (oscilloscope output) is also included in the project.
The fitting of a simulated histogram to an experimental one is done using minimization of the chi-square distance measure for these two histograms, by varying parameters β, *norm* and nf[1] (See section 1.4 Data Filtering). The minimization is realized with the package optimize.minimize from SciPy library, using the Nelder-Mead minimization algorithm [15].

[1]The parameters nf is referred to as *const* in the source code supplemented in Appendix A.

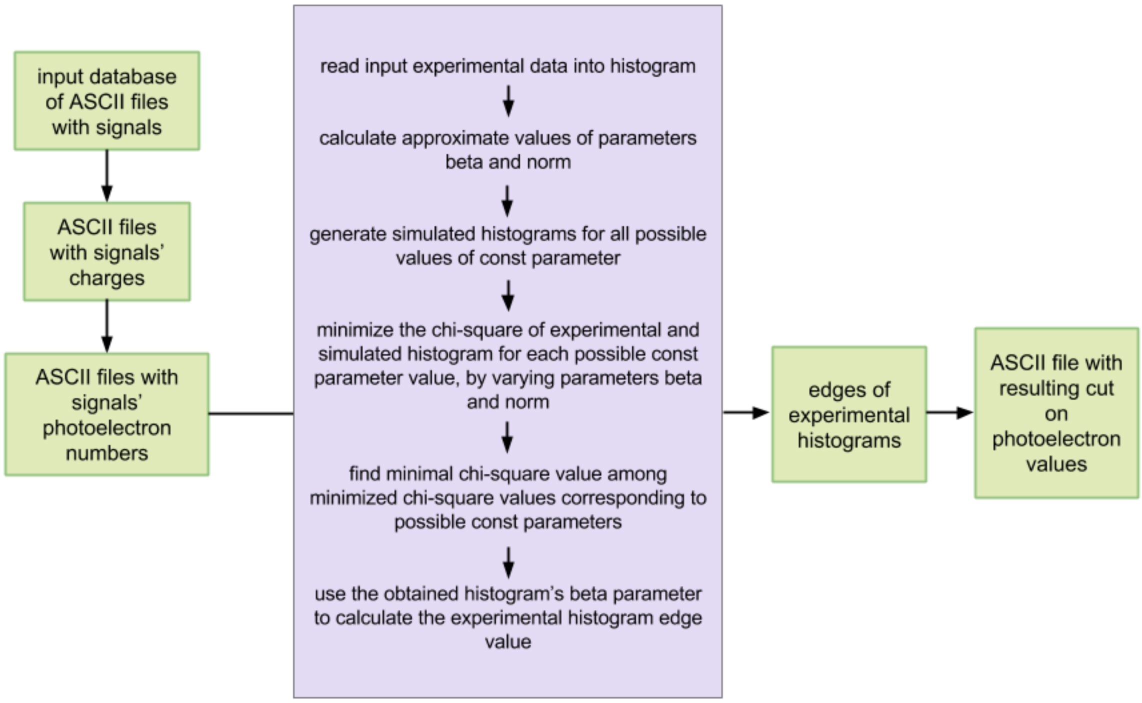


FIGURE 2.2: A schematic representation of the workflow of the Photoelectron Filter program.

2.3.2 Hit Position Reconstruction

2.3.2.1 Package: eventDatabase

This module encapsulates an interface to the database as well as the definitions of data structures. Before performing any tests of hit position reconstruction, events must be loaded into the database. During the loading a number of characteristics is computed for each event and stored along with it, including: charges, times of the signals' beginnings (designated by constant voltage thresholds), amplitudes, times when signals reach their amplitudes.

Due to oscilloscope malfunctioning, some events can be shifted in the y axis by a small value. This shift is also corrected during the process of loading signals into the database. The value of shift is obtained as the arithmetic mean of the y-coordinate values of points from a given signal's file up to the time of the signal's beginning.

Only the events which correspond to the electrons scattered with the energy larger that the fraction of the maximum energy specified (which can be obtained from PhotoelectronFilter) are loaded into the database.

During loading, the events are marshalled into string and float values and stored inside the database as Python dictionary instances. When events are retrieved from the database, they are unmarshalled into objects of the class Event.

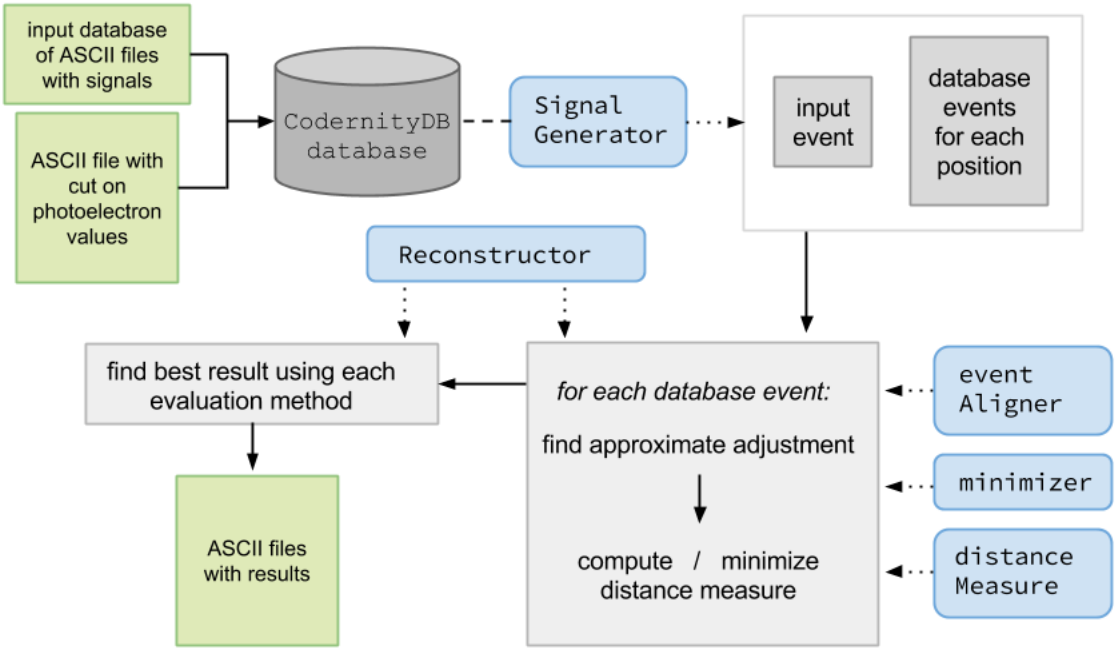


Figure 2.3: A schematic representation of the workflow of the Hit Position Reconstruction program.

2.3.2.2 Package: distanceMeasure

The algorithms for calculating the distance between two compared events are defined in the distanceMeasure package. Each distance measure should be defined in its own file. Any algorithm can be used, as long as the file contains a method 'measure(event1, event2)', which executes the calculations and returns the distance. Two distance measures, chi-square and Frechet distance, are already implemented.
This package also contains a sub-package called 'timeInterpolation' which is an implementation of the simple method for voltage domain sampling.

2.3.2.3 Package: minimization

The package 'minimization' implements the two methods of obtaining the distance measure between two events: approximate (simplified) method, and method using algorithmic minimization. The used algorithm is the 'Nelder-Mead' method [15] as implemented in the scipy.optimize.minimize module of the library SciPy. For the better accuracy of minimization, only the rising edges of signals are used to compute the minimized distance between two events.

2.3.2.4 Package: reconstruction

The package reconstruction encapsulates the main file of the project. It implements the highest-level of hit position reconstruction, making the usage of all the neccessary modules. The module is responsible for retrieving events from the database (randomly or sequentially, making sure no duplicate tests will be present in the result), applying distance measure algorithms, evaluating results and writing output files.

2.3.2.5 Package: plotting

This package contains code for generating plots presenting the reconstruction method results, using PyROOT. They can be run as a standalone python script.

Chapter 3

Results

3.1 Data used for determination of the results

3.1.1 Experimental Setup

Data needed to test the performance of the described method was gathered using the experimental setup shown in Fig. 3.1. During the measurement, a collimator encapsulating a radioactive source was placed in a number of positions along the Z axis. Starting from the position 3 mm away from the beginning of scintillator strips, the collimator was repeatedly shifted with the step of 3 mm, until reaching the last measured position: 297 mm. For each possible position of the collimator 5000 events were collected.

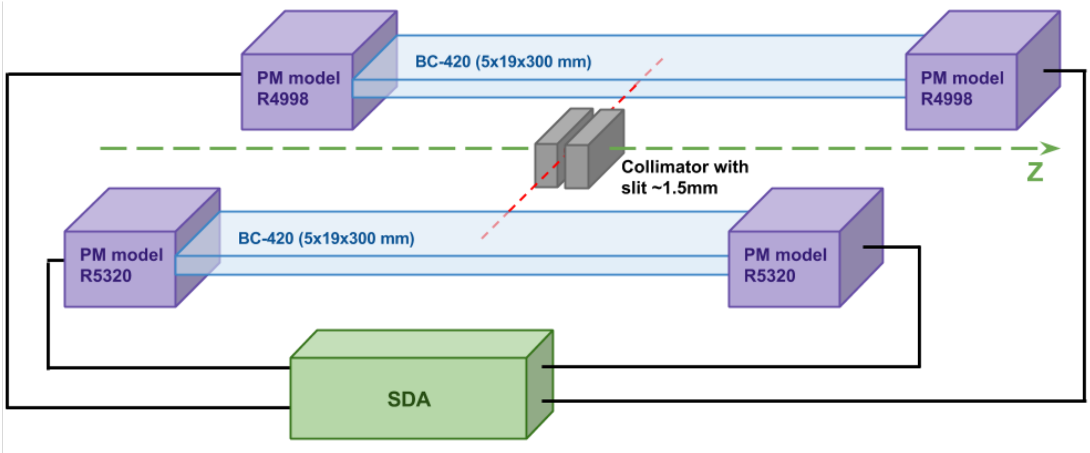


FIGURE 3.1: A schematic view of the experimental setup.

3.1.2 Format of the collected data

Events collected during the conducted measurement were obtained by and saved using the Serial Data Analyzer (Lecroy SDA6000A). The analog signals incoming from four photomultipliers, each connected to a SDA channel, were sampled in time domain with the step of 100 ps. Every collected event was saved in form of four ASCII files corresponding to four SDA channels.
A signal is represented by two columns of floating point numbers; first column denotes times of sampling and second column their respective values of measured voltage.

3.2 Conducted tests and results

3.2.1 Single reconstruction test

A single reconstruction test consists of comparing a single input event with a desired number of database events to reconstruct the input event's position. The plots presented below show distance measure values of each individual comparison as well as their average and minimum values per database signals positions.

3.2.1.1 Chi-square measure

The results shown in fig 3.2 were calculated using the chi-square similarity measure (see section 1.3.1), conducting 50 comparisons per position in the database.

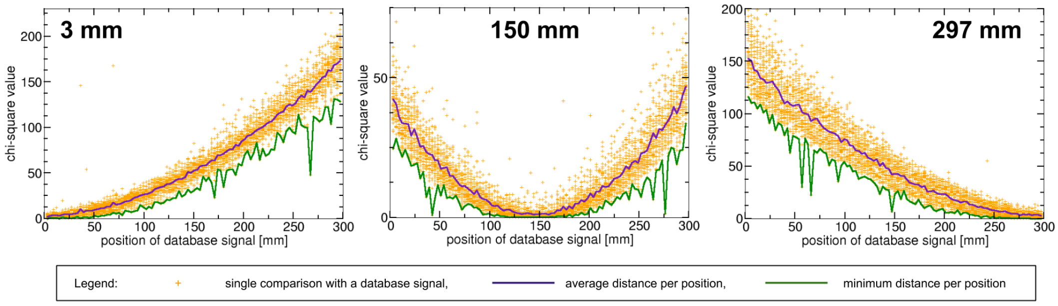


FIGURE 3.2: Chi-square distance measure values obtained for input signals measured at position 3 mm (left), 150 mm (center) and 297 mm (right).

3.2.1.2 Frechet measure

The results shown in fig 3.3 were calculated using the Frechet distance similarity measure (see section 1.3.2), conducting 50 comparisons per position in the database.

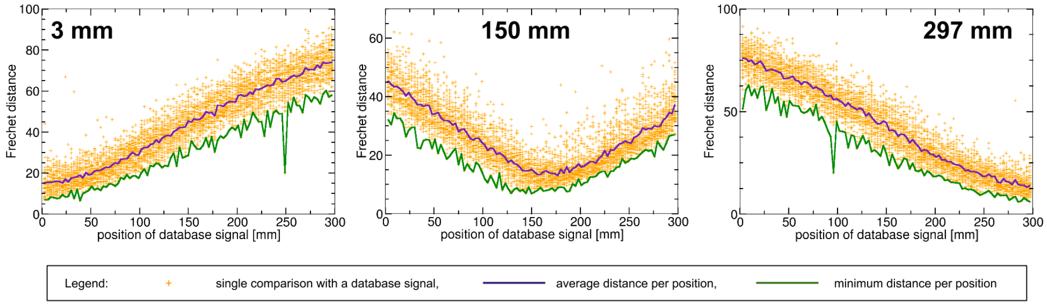


FIGURE 3.3: Frechet distance measure values obtained for input signals measured at position 3 mm (left), 150 mm (center) and 297 mm (right).

3.2.2 Resolution of position reconstruction as a function of number of comparisons per position

The test was conducted using chi-squared distance measure. The input events were randomly chosen from all possible positions. The maximal alignment of two events was computed using the simplified (approximate) method.

3.2.2.1 Evaluation method: global minimum

The resolution of reconstruction as a function of number of comparisons per position for evaluation method: global minimum, using the minimum distance point as the reconstructed value and using the minimum of quadratic function fitted to the comparison results as the reconstructed value is illustrated in figures 3.4 and 3.6, respectively. For both of these evaluation methods, example histograms used to obtain sigma and RMS of reconstruction are presended in Fig. 3.5 (evaluation using the minimum distance) and Fig. 3.7 (evaluation using quadratic fitting).

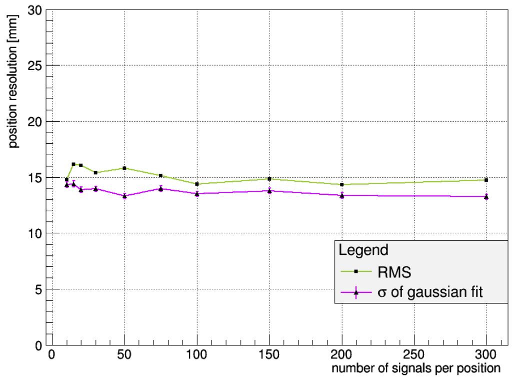


FIGURE 3.4: Sigma and RMS of reconstructed position as a function of number of comparisons per position for the evaluation method: global minimum.

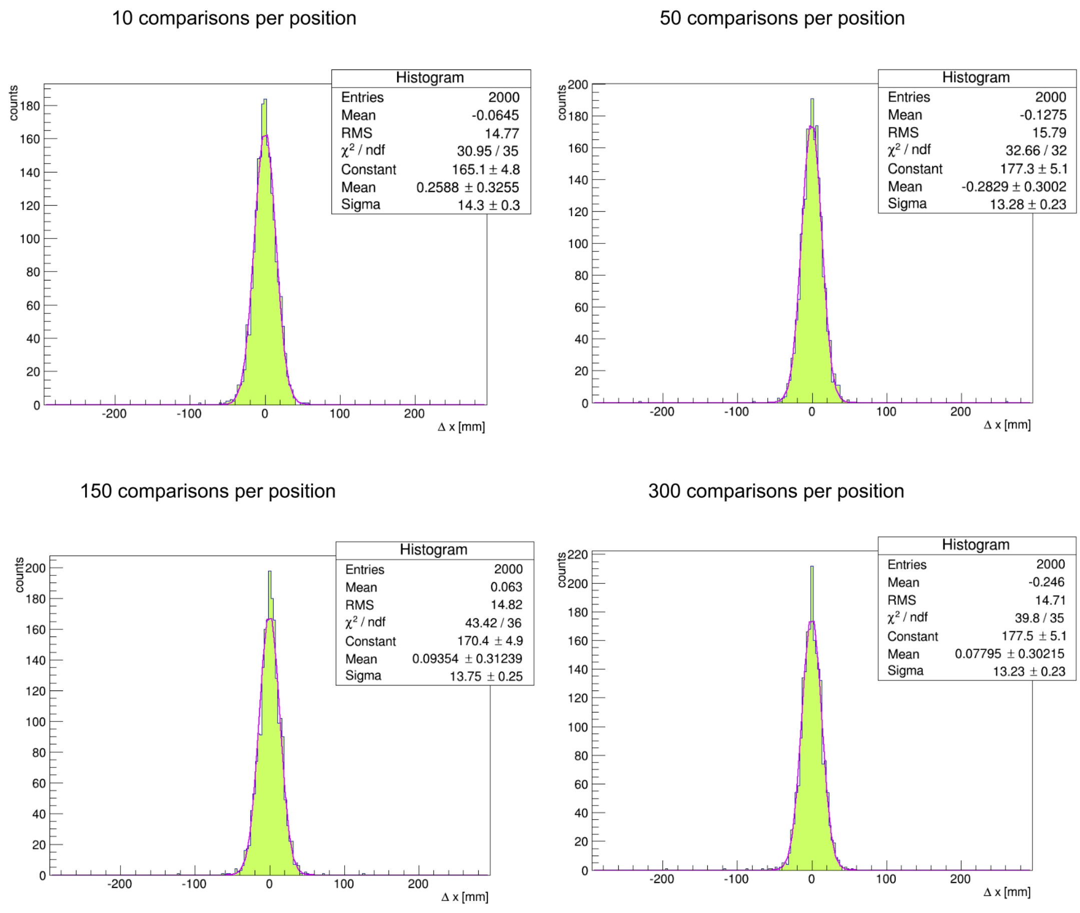


FIGURE 3.5: Example histograms used to obtain the data points for Fig. 3.4, for the numbers of comparisons per position: 10 (upper left), 50 (upper right), 150 (bottom left), 300 (bottom right).

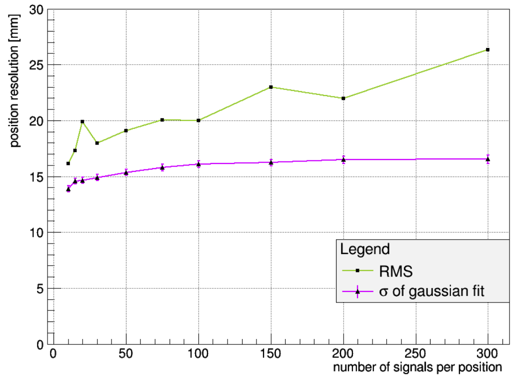


FIGURE 3.6: Sigma and RMS of reconstructed position as a function of number of comparisons per position for the evaluation method: minimum of quadratic function fitted to minimal distance per position.

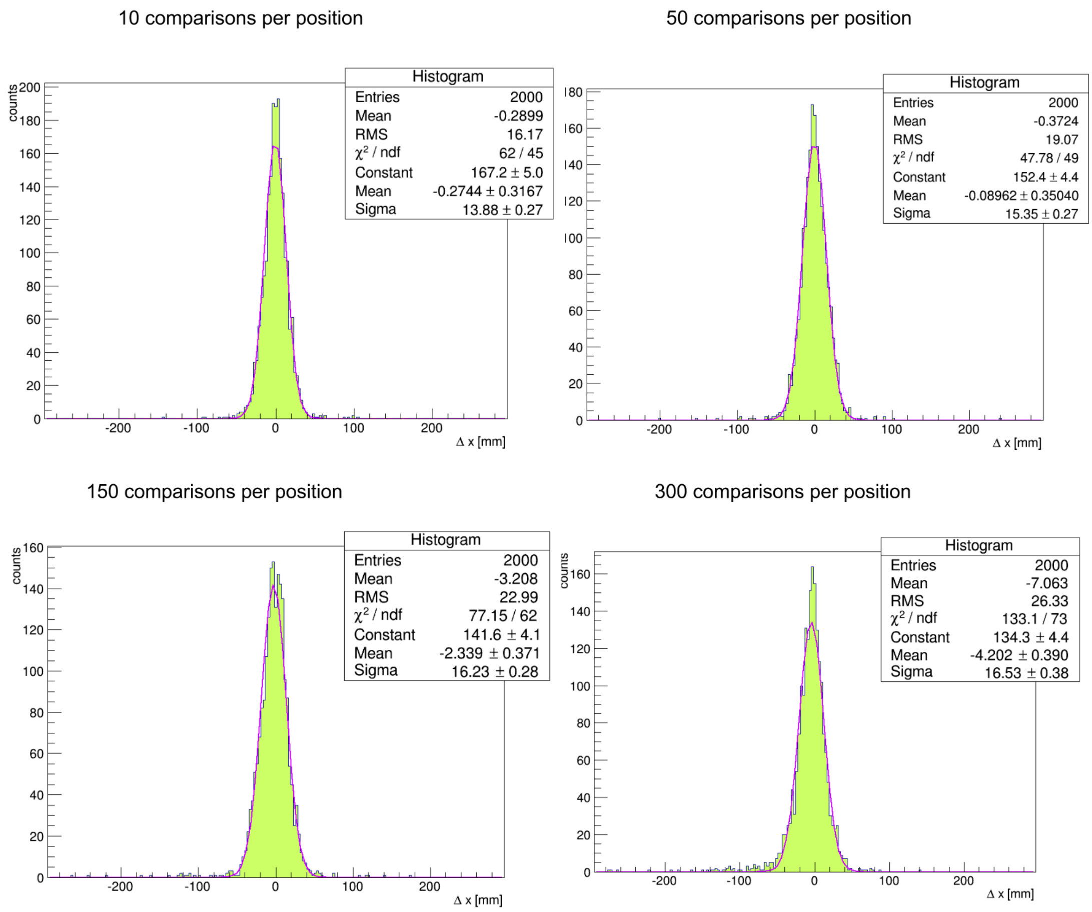


FIGURE 3.7: Example histograms used to obtain the data points for Fig. 3.6, for the numbers of comparisons per position: 10 (upper left), 50 (upper right), 150 (bottom left), 300 (bottom right).

3.2.2.2 Evaluation method: minimum of artithmetic means

The resolution of reconstruction as a function of number of comparisons per position for evaluation method: minimum of artithmetic means, using the minimum average distance point as the reconstructed value and using the minimum of quadratic function fitted to the comparison results as the reconstructed value is illustrated in figures 3.8 and 3.10, respectively. For both of these evaluation methods, example histograms used to obtain sigma and RMS of reconstruction are presended in Fig. 3.9 (evaluation using the minimum distance) and Fig. 3.11 (evaluation using quadratic fitting).

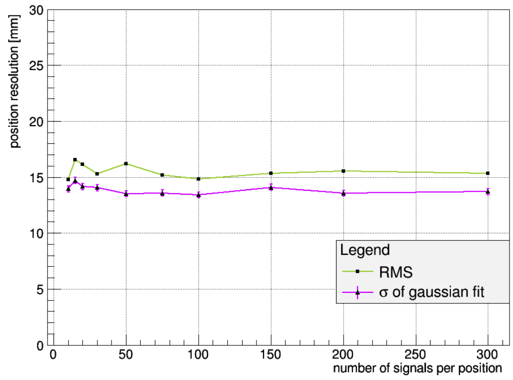


FIGURE 3.8: Sigma and RMS of reconstructed position as a function of number of comparisons per position for the evaluation method: minimum of arithmetic mean of distance measure per position.

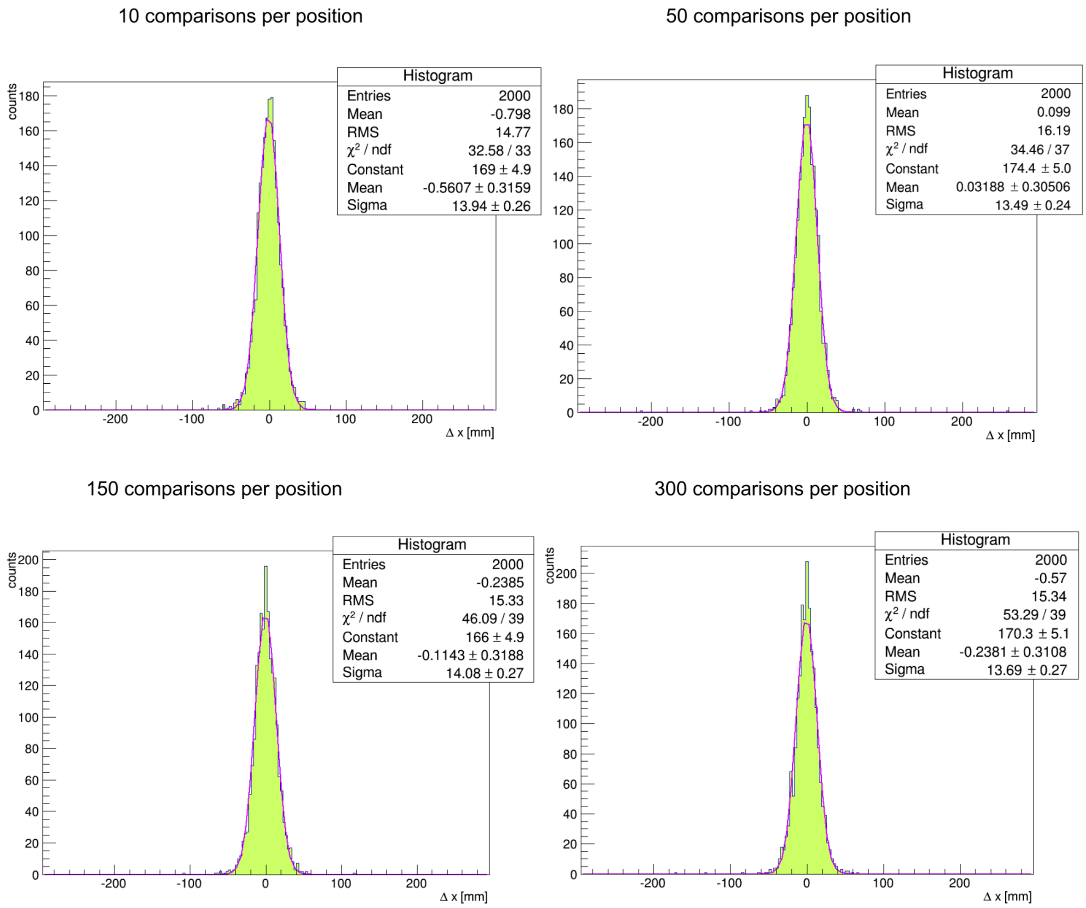


FIGURE 3.9: Example histograms used to obtain the data points for Fig. 3.8, for the numbers of comparisons per position: 10 (upper left), 50 (upper right), 150 (bottom left), 300 (bottom right).

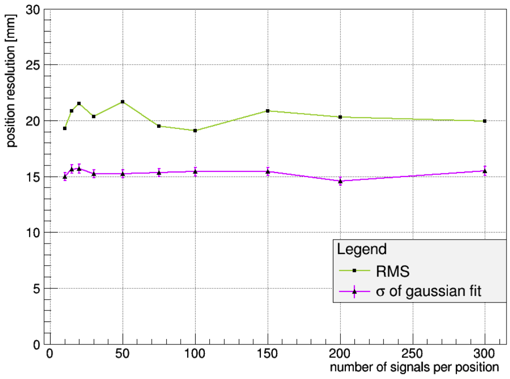


FIGURE 3.10: Sigma and RMS of reconstructed position as a function of number of comparisons per position for the evaluation method: minimum of quadratic function fitted to arithmetic mean of distance measure per position.

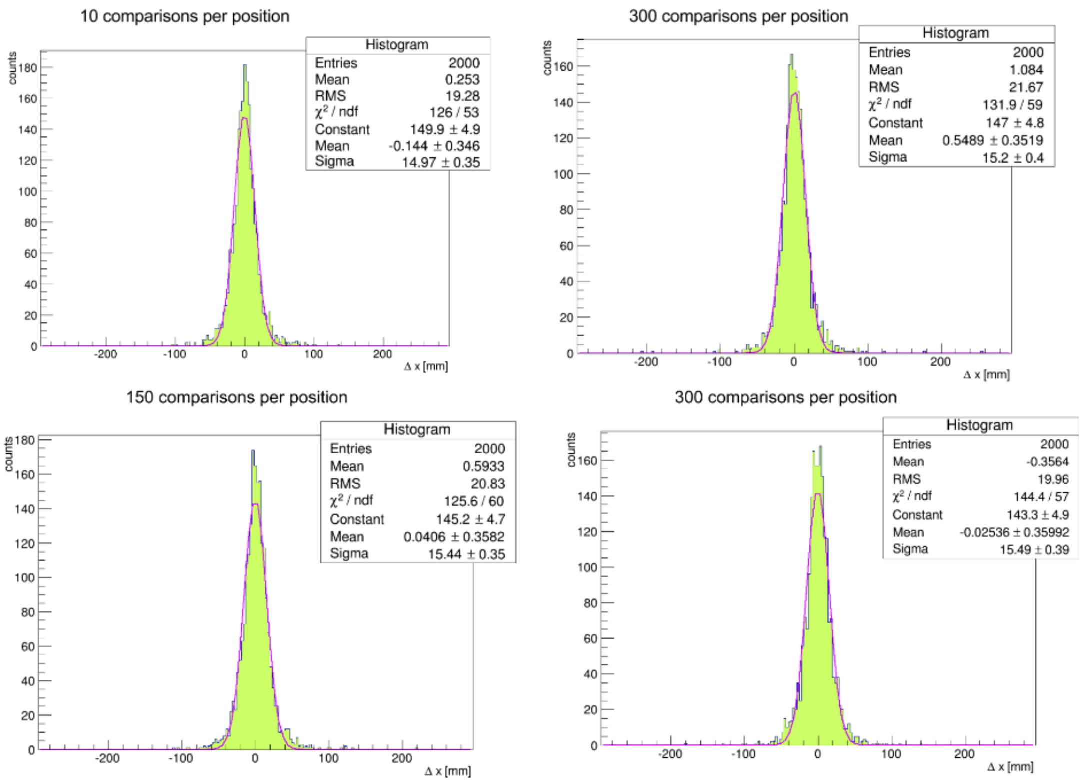


FIGURE 3.11: Example histograms used to obtain the data points for Fig. 3.10, for the numbers of comparisons per position: 10 (upper left), 50 (upper right), 150 (bottom left), 300 (bottom right).

3.2.2.3 Evaluation method: minimum of standard deviation

The resolution of reconstruction as a function of number of comparisons per position for evaluation method:minimum of standard deviation, using the minimum standard deviation point as the reconstructed value and using the minimum of quadratic function fitted to the comparison results as the reconstructed value is illustrated in figures 3.12 and 3.14, respectively. For both of these evaluation methods, example histograms used to obtain sigma and RMS of reconstruction are presended in Fig. 3.13 (evaluation using the minimum distance) and Fig. 3.15 (evaluation using quadratic fitting).

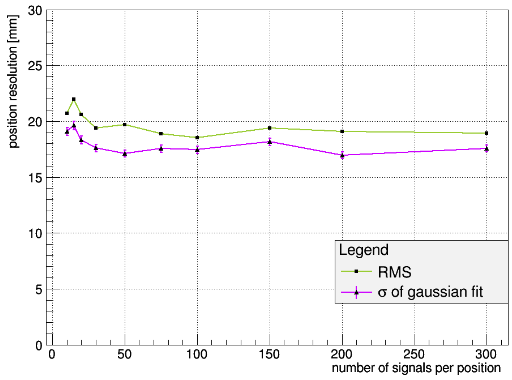


FIGURE 3.12: Sigma and RMS of reconstructed position as a function of number of comparisons per position for the evaluation method: minimum of standard deviation of distance measure per position.

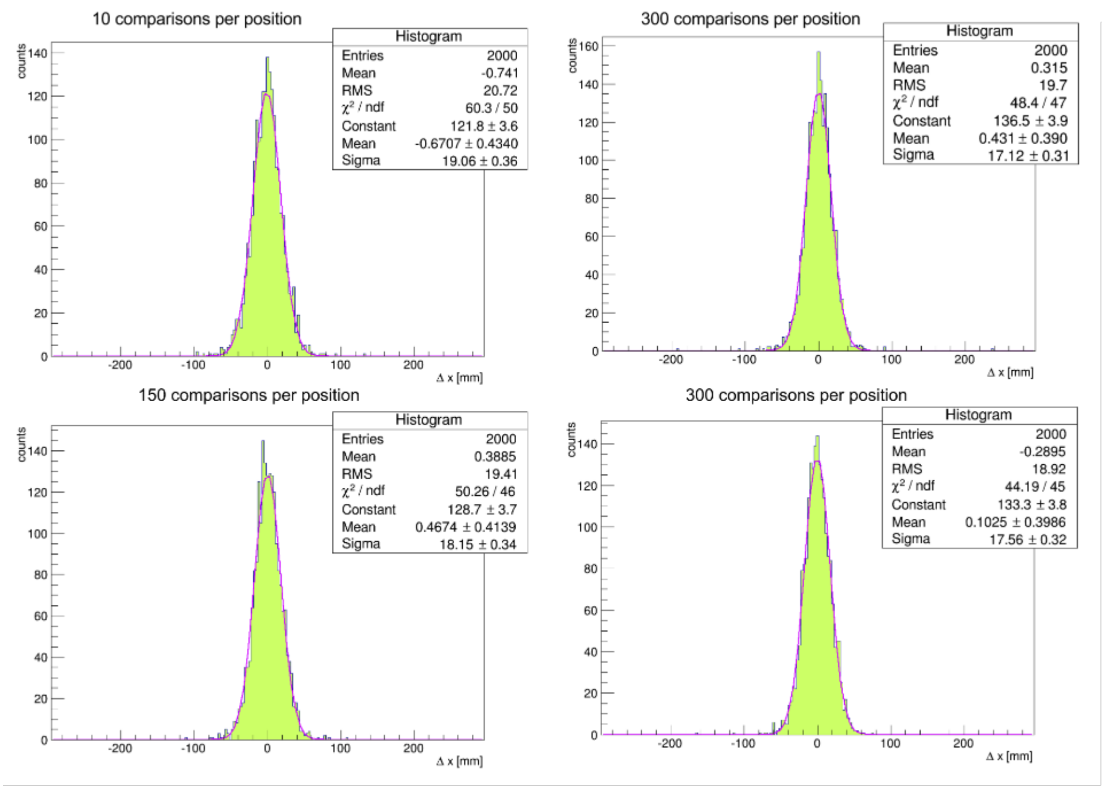


FIGURE 3.13: Example histograms used to obtain the data points for Fig. 3.12, for the numbers of comparisons per position: 10 (upper left), 50 (upper right), 150 (bottom left), 300 (bottom right).

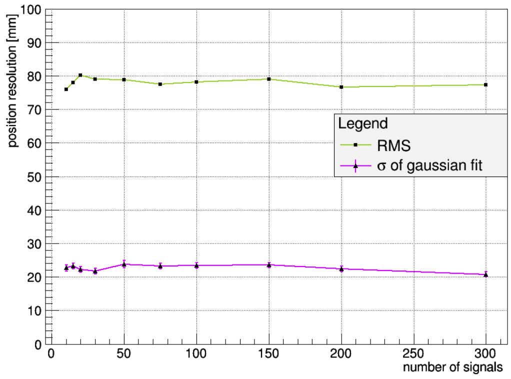


FIGURE 3.14: Sigma and RMS of reconstructed position as a function of number of comparisons per position for the evaluation method: minimum of quadratic function fitted to standard deviation of distance measure per position.

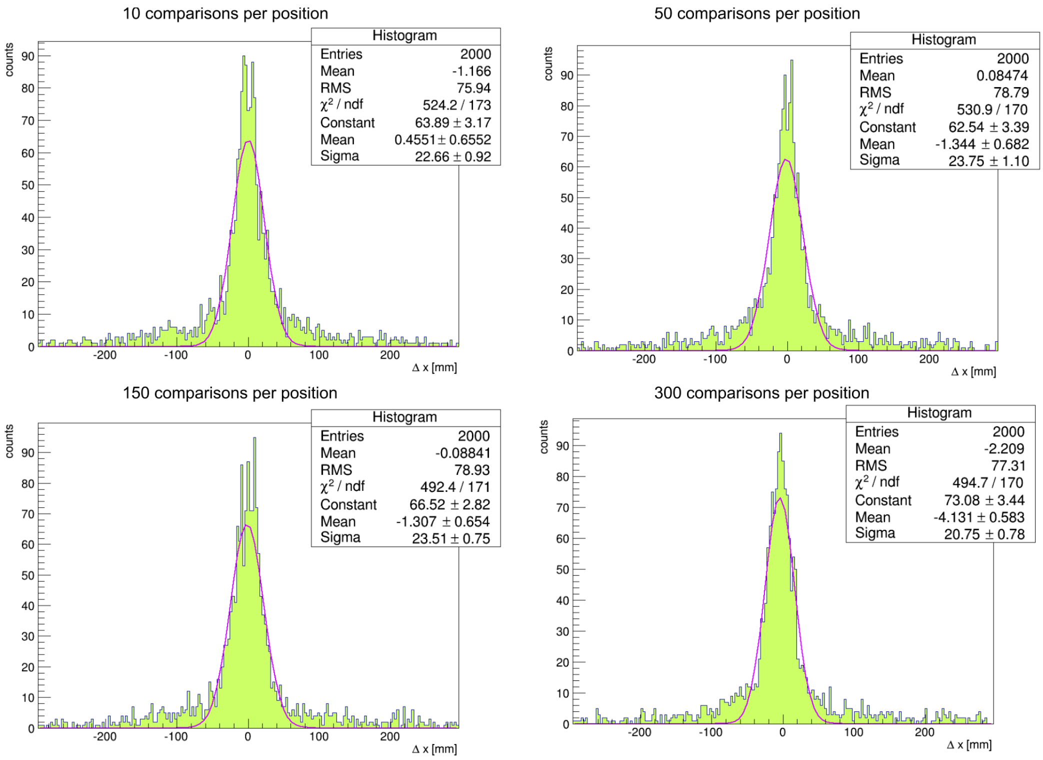


FIGURE 3.15: Example histograms used to obtain the data points for Fig. 3.14, for the numbers of comparisons per position: 10 (upper left), 50 (upper right), 150 (bottom left), 300 (bottom right).

3.2.3 Resolution of position reconstruction as a function of position of input signals

The test was conducted using chi-square distance measure and 50 comparisons per position in the database. The maximal alignment of two events was computed using the simplified (approximate) method (See section 1.3 Distance Metrics).

3.2.3.1 Evaluation method: global minimum

The resolution of reconstruction as a function of position of input signals for evaluation method: global minimum, using the minimum distance point as the reconstructed value and using the minimum of quadratic function fitted to the comparison results as the reconstructed value is illustrated in figures 3.16 and 3.18, respectively. For both of these evaluation methods, example histograms used to obtain sigma and RMS of reconstruction are presended in Fig. 3.17 (evaluation using the minimum distance) and Fig. 3.19 (evaluation using quadratic fitting).

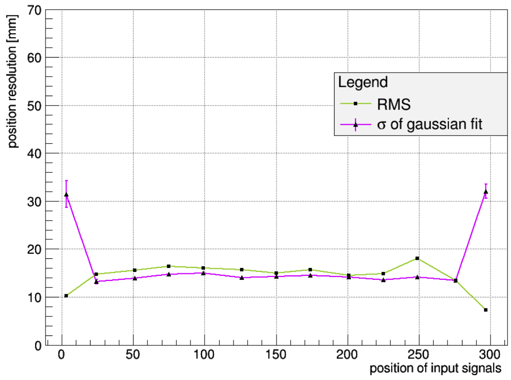


FIGURE 3.16: Sigma and RMS of reconstructed position as a function of position of input signals for the evaluation method: global minimum.

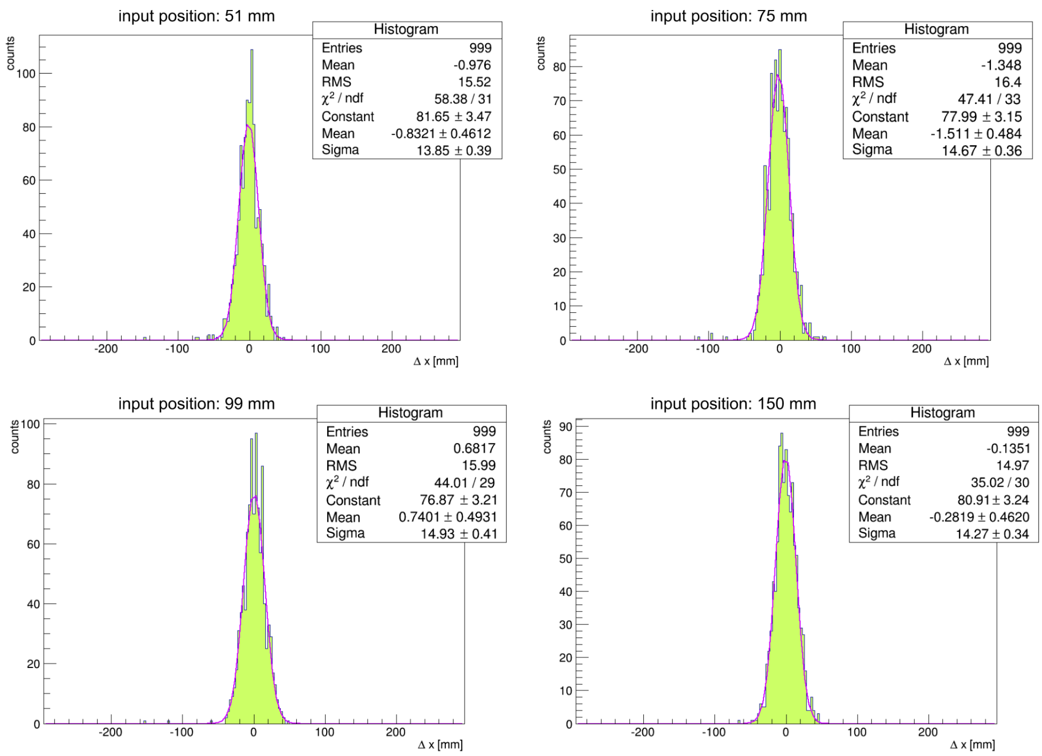


FIGURE 3.17: Example histograms used to obtain the data points for Fig. 3.16, for the positions of input signals: 51 mm (upper left), 75 mm (upper right), 99 mm (bottom left), 150 mm (bottom right).

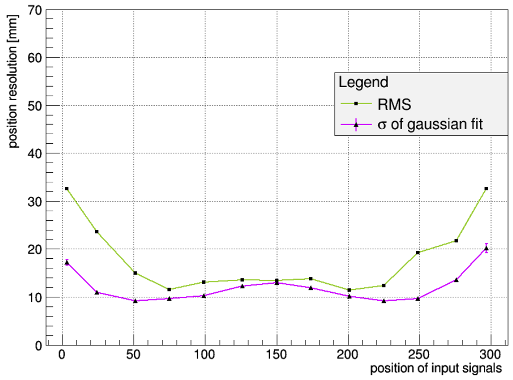


FIGURE 3.18: Sigma and RMS of reconstructed position as a function of position of input signals for the evaluation method: minimum of quadratic function fitted to minimal distance per position.

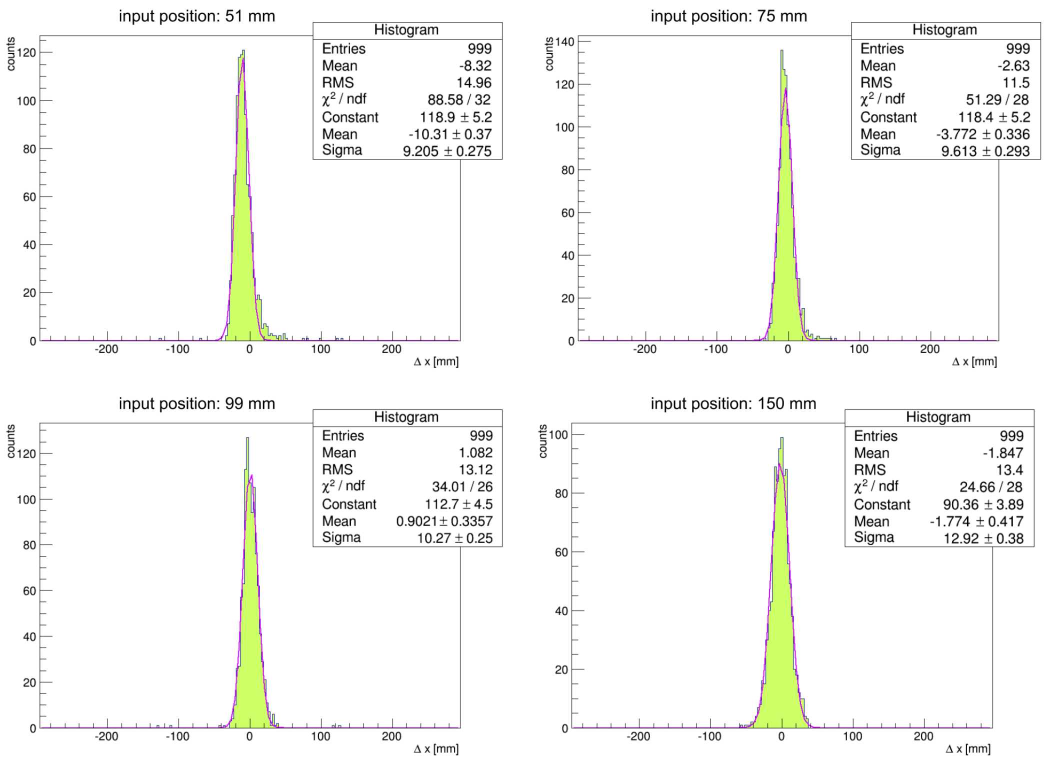


FIGURE 3.19: Example histograms used to obtain the data points for Fig. 3.18, for the positions of input signals: 51 mm (upper left), 75 mm (upper right), 99 mm (bottom left), 150 mm (bottom right).

3.2.3.2 Evaluation method: minimum of artithmetic means

The resolution of reconstruction as a function of position of input signals for evaluation method: minimum of artithmetic means, using the minimum average distance point as the reconstructed value and using the minimum of quadratic function fitted to the comparison results as the reconstructed value is illustrated in figures 3.20 and 3.22, respectively. For both of these evaluation methods, example histograms used to obtain sigma and RMS of reconstruction are presended in Fig. 3.21 (evaluation using the minimum distance) and Fig. 3.23 (evaluation using quadratic fitting).

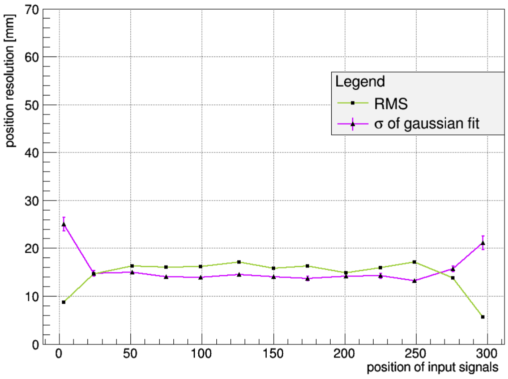


FIGURE 3.20: Sigma and RMS of reconstructed position as a function of position of input signals for the evaluation method: minimum of arithmetic mean of distance measure per position.

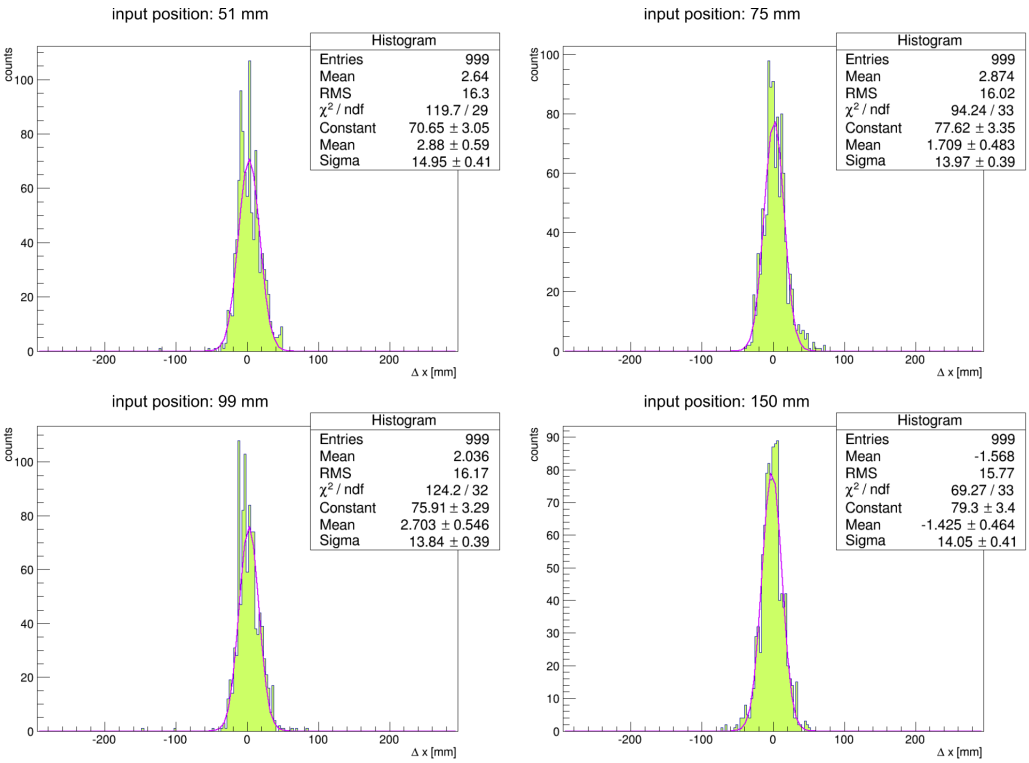


FIGURE 3.21: Example histograms used to obtain the data points for Fig. 3.20, for the positions of input signals: 51 mm (upper left), 75 mm (upper right), 99 mm (bottom left), 150 mm (bottom right).

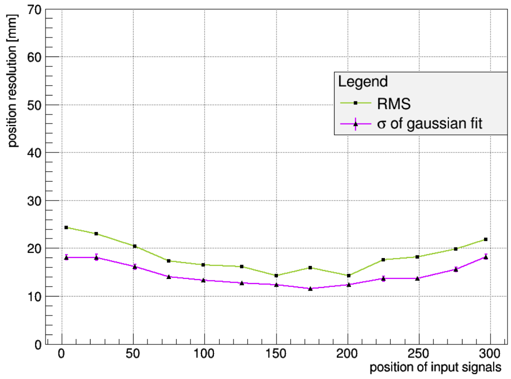


FIGURE 3.22: Sigma and RMS of reconstructed position as a function of position of input signals for the evaluation method: minimum of quadratic function fitted to arithmetic mean of distance measure per position.

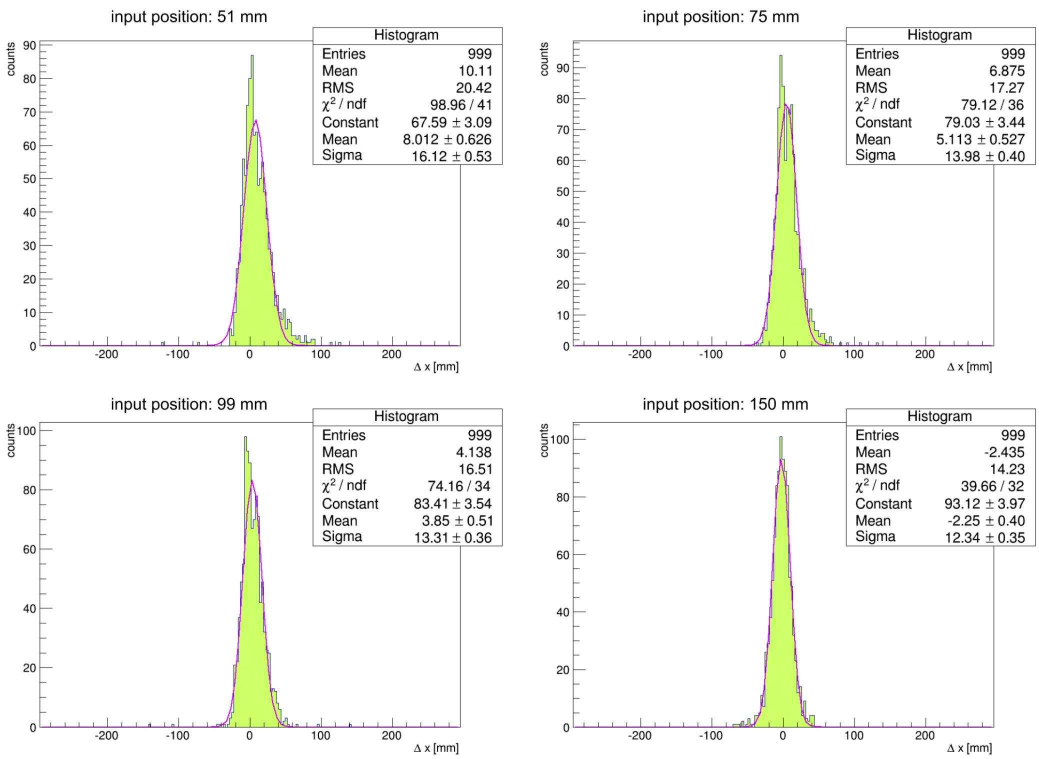


FIGURE 3.23: Example histograms used to obtain the data points for Fig. 3.22, for the positions of input signals: 51 mm (upper left), 75 mm (upper right), 99 mm (bottom left), 150 mm (bottom right).

3.2.3.3 Evaluation method: minimum of standard deviation

The resolution of reconstruction as a function of position of input signals for evaluation method: minimum of standard deviation, using the minimum standard deviation point as the reconstructed value and using the minimum of quadratic function fitted to the comparison results as the reconstructed value is illustrated in figures 3.24 and 3.26, respectively. For both of these evaluation methods, example histograms used to obtain sigma and RMS of reconstruction are presended in Fig. 3.25 (evaluation using the minimum distance) and Fig. 3.27 (evaluation using quadratic fitting).

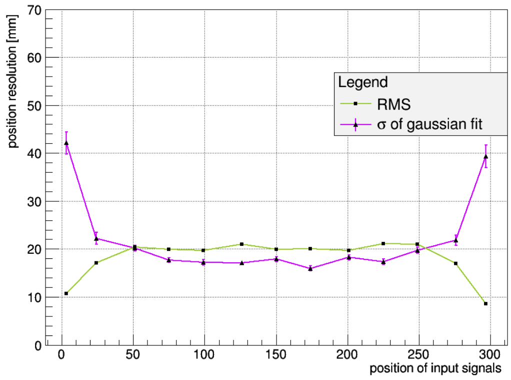


FIGURE 3.24: Sigma and RMS of reconstructed position as a function of position of input signals for the evaluation method: minimum of standard deviation of distance measure per position.

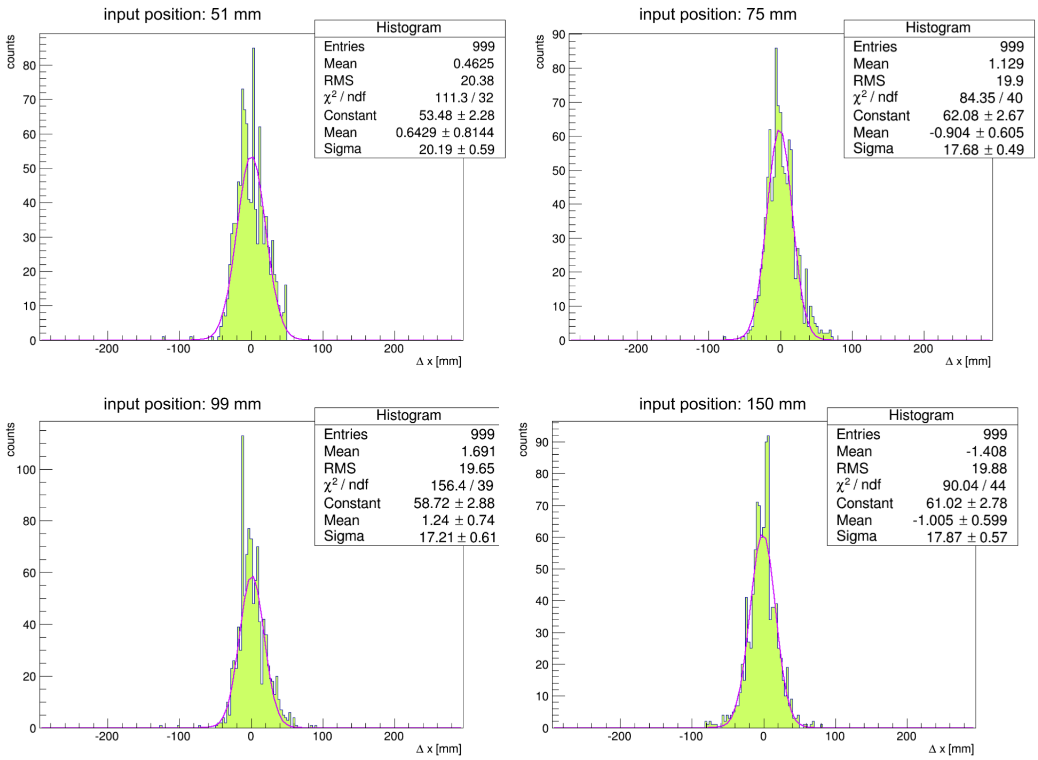


FIGURE 3.25: Example histograms used to obtain the data points for Fig. 3.24, for the positions of input signals: 51 mm (upper left), 75 mm (upper right), 99 mm (bottom left), 150 mm (bottom right).

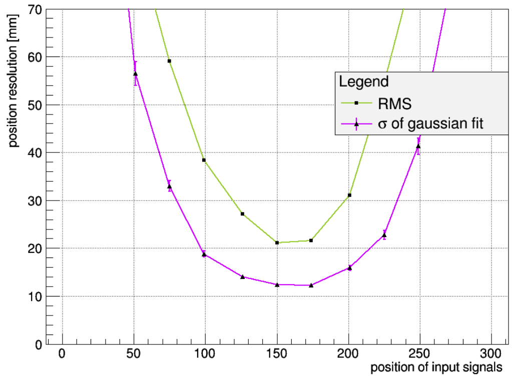


FIGURE 3.26: Sigma and RMS of reconstructed position as a function of position of input signals for the evaluation method: minimum of quadratic function fitted to standard deviation of distance measure per position.

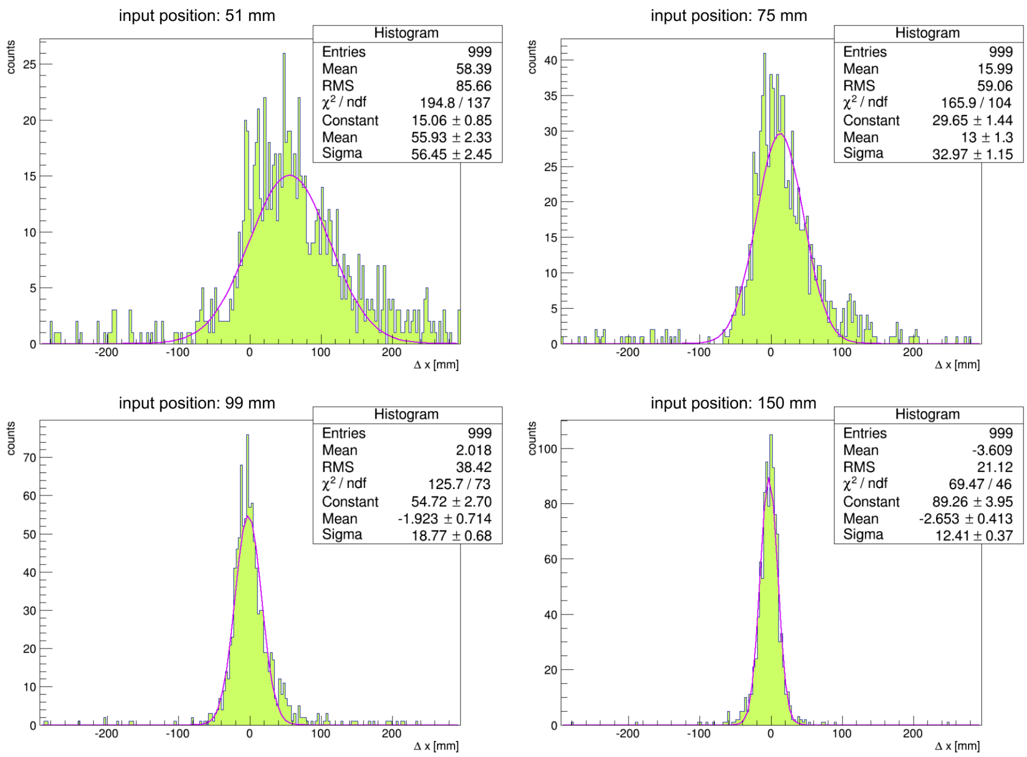
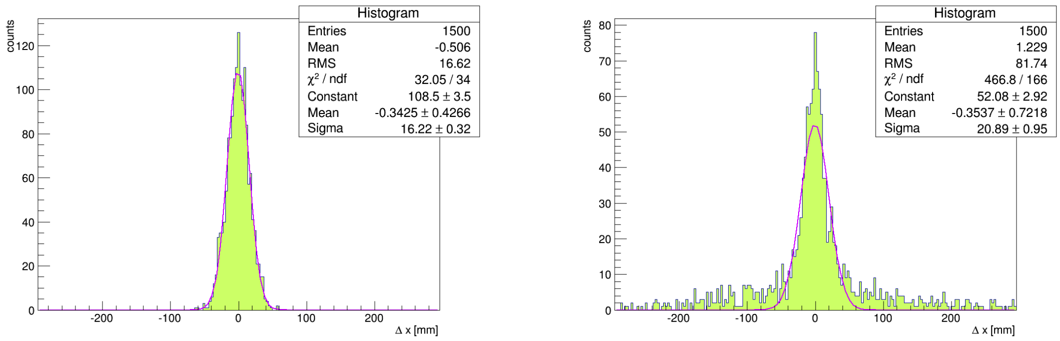


FIGURE 3.27: Example histograms used to obtain the data points for Fig. 3.26, for the positions of input signals: 51 mm (upper left), 75 mm (upper right), 99 mm (bottom left), 150 mm (bottom right).

3.2.4 Resolution obtained using Frechet measure

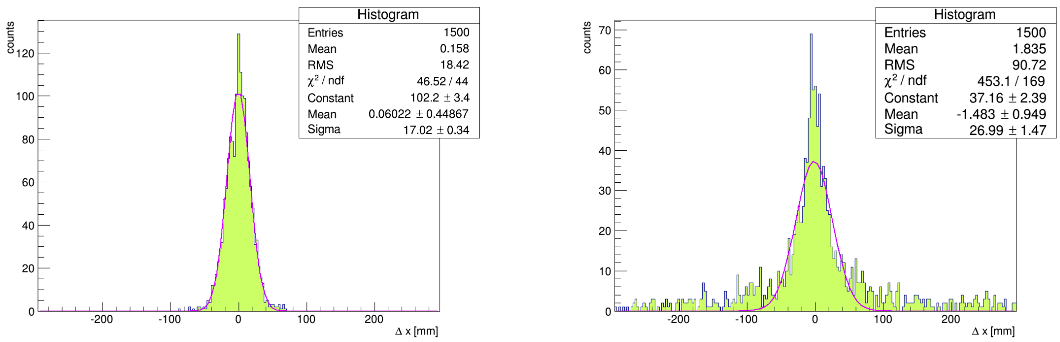
The presented results were calculated using Frechet measure on signals maximally aligned using the approximate method (See section 1.3 Distance Metrics). For each database position, 50 events were compared with the incoming event.



(A) Minimum-valued point designates the reconstructed position.

(B) Minimum of quadratic fit to all points designates the reconstructed position.

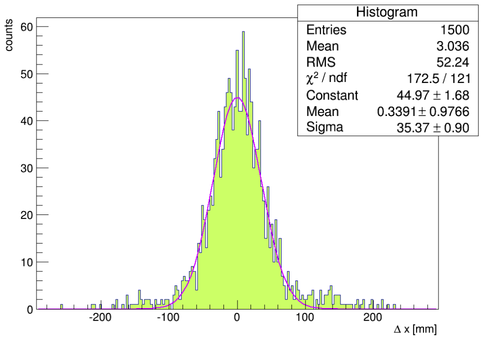
FIGURE 3.28: Histogram of obtained differences between reconstructed and expected positions for the evaluation method: global minimum.



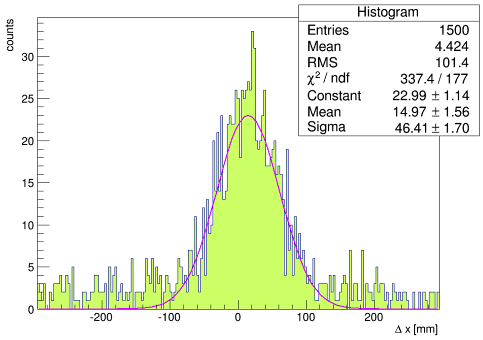
(A) Minimum-valued point designates the reconstructed position.

(B) Minimum of quadratic fit to all points designates the reconstructed position.

FIGURE 3.29: Histogram of obtained differences between reconstructed and expected positions for the evaluation method: minimum of arithmetic means of distance per position.



(A) Minimum-valued point designates the reconstructed position.



(B) Minimum of quadratic fit to all points designates the reconstructed position.

FIGURE 3.30: Histogram of obtained differences between reconstructed and expected positions for the evaluation method: minimum of standard deviation of distance per position.

3.2.5 Using minimization versus using only approximate maximal alignment

Minimization of distance measure is more computationally complex when compared to using the approximate distance method to align signals together. In the second case, the distance measure is calculated only once per comparison, while when using minimization it is computed 40-50 times. The results presented below were obtained from test designed to determine the possibility of improvement of the resolution of reconstruction when using minimization of measure distance.

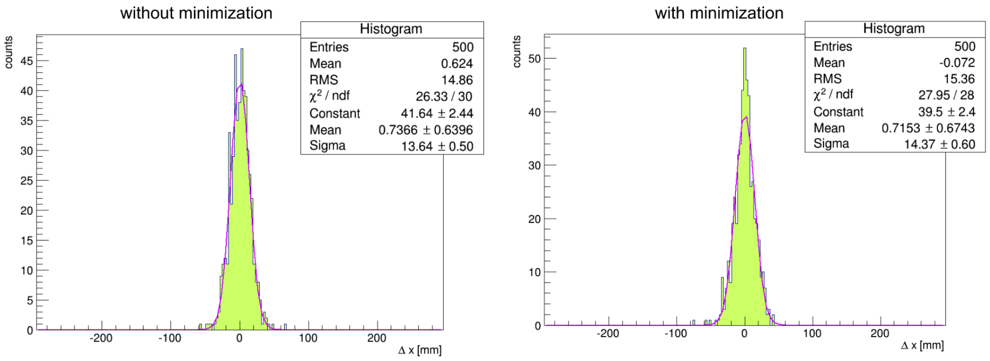


FIGURE 3.31: Histograms of obtained differences between reconstructed and expected positions for the evaluation method: global minimum, for computation without minimization (left) and with minimization (right).

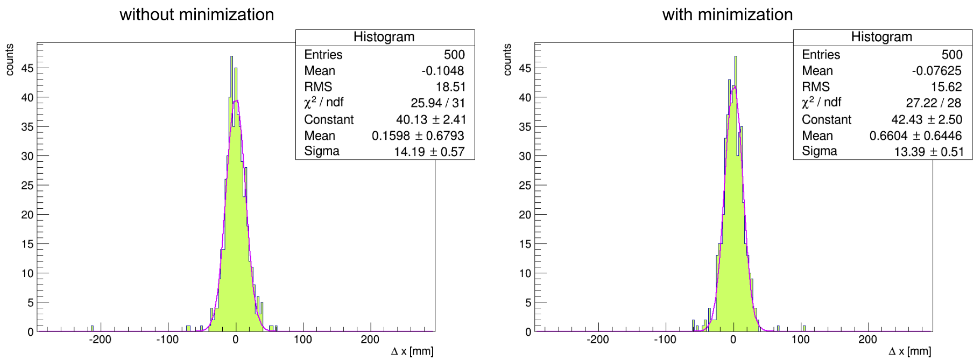


FIGURE 3.32: Histograms of obtained differences between reconstructed and expected positions for the evaluation method: minimum of quadratic fit to minimal distance per position, for computation without minimization (left) and with minimization (right).

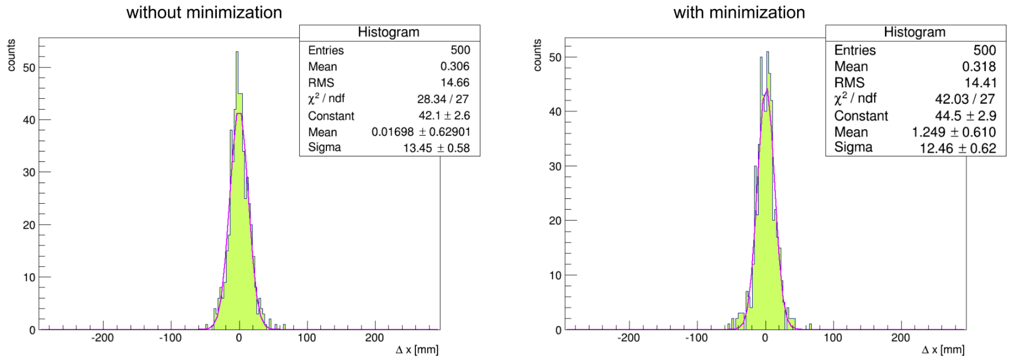


FIGURE 3.33: Histograms of obtained differences between reconstructed and expected positions for the evaluation method: minimum of arithmetic means of distance per position, for computation without minimization (left) and with minimization (right).

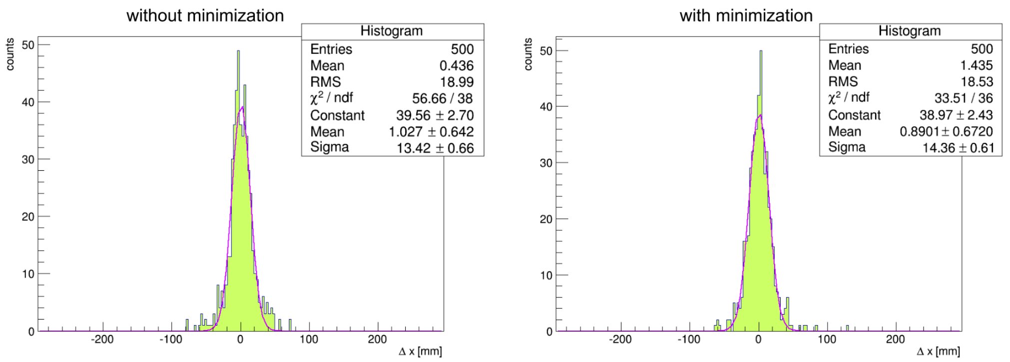


FIGURE 3.34: Histograms of obtained differences between reconstructed and expected positions for the evaluation method: minimum of quadratic fit to arithmetic means of distance per position, for computation without minimization (left) and with minimization (right).

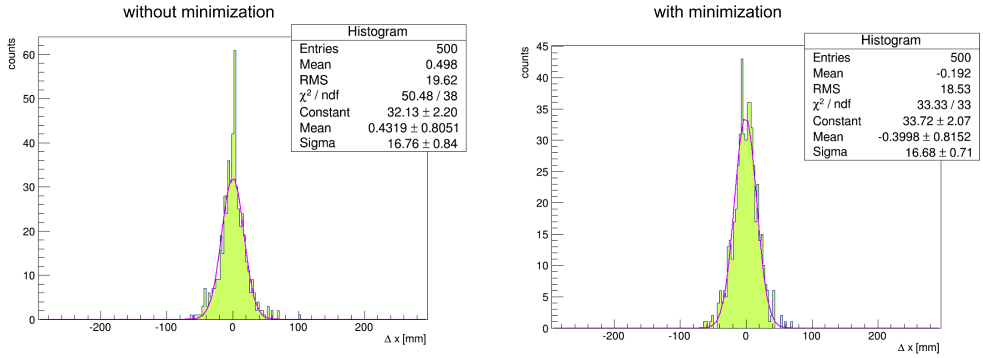


FIGURE 3.35: Histograms of obtained differences between reconstructed and expected positions for the evaluation method: minimum of standard deviation of distance per position, for computation without minimization (left) and with minimization (right).

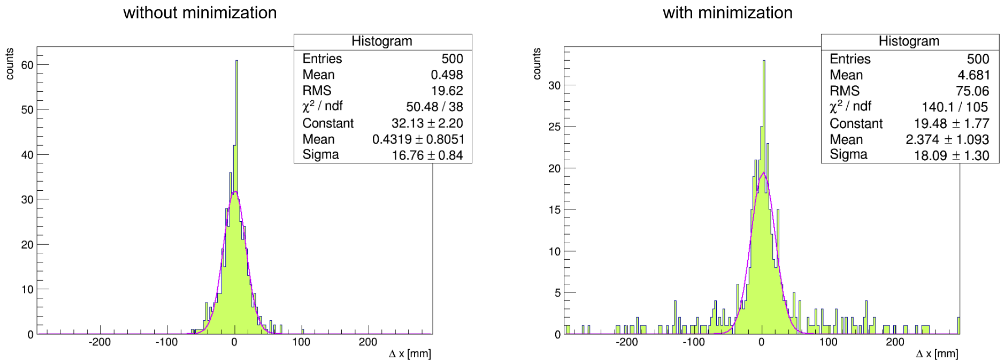


FIGURE 3.36: Histograms of obtained differences between reconstructed and expected positions for the evaluation method: minimum of quadratic fit to standard deviation of distance per position, for computation without minimization (left) and with minimization (right).

Summary

The main aim of this thesis was the development and testing of a method for the reconstruction of gamma quanta hit position in polymer scintillators based on comparing incoming signals with signals from previously created database.

The similarity of two signals was computed using either a set of times corresponding to a set of voltage thresholds common for the two compared signals (Chi-square method) or two sets of points designating two curves representing the signals (Frechet distance method). The theoretical basis of the concept, its general idea and individual steps of the proposed algorithm, as well as the computer program which was created to test the proposed method were explained in detail in this thesis.

The results obtained from conducted calculations suggest that at present the best resolution that can be achieved by the proposed method is ~13.5 mm. The resolution is showing little improvement with increasing number of signals in the reference database (the improvement of ~1 mm was observed when comparing 300 signals per positions versus 10 signals per position). Among the evaluation methods, the global minimum method and the minimum of arithmetic means method proved to show best results, the difference between their respective resolutions being in the error range. The spatial resolution shows no significant dependence on the position along the scintillator when using evaluation methods without quadratic fitting, and slight influence for evaluation methods with quadratic fitting.

The resolution achieved from tests using chi-square similarity measure was better by ~4 mm than the resolution obtained using Frechet distance measure.

The obtained results are comparable with the results of other independently developed methods for reconstructing the position of gamma quanta hit in a single polymer scintillator strip [16].

Appendix A

CD with software source code

This thesis is supplemented with a CD containing the source code of the computer program which is the realization of the described method.

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