Measurements of the total cross section of the reaction \( pp \rightarrow ppK^+K^- \) were performed at the cooler synchrotron COSY near the kinematical threshold [1, 2] by the COSY-11 collaboration [3], and for higher energies by the ANKE collaboration [4]. The experiment was conducted also by the DISTO collaboration [5] at \( Q = 114 \text{ MeV} \) at the SATURN accelerator. The results indicate that near threshold data points lie significantly above any theoretical expectations.

The enhancement may be due to the influence of \( K^+K^- \) or \( Kp \) interaction which was neglected in the calculations. This interaction should manifest itself also in the distributions of the differential cross-sections [6]. A significant effect observed for the excitation function for the \( pp \rightarrow ppK^+K^- \) reaction encourages us to take the effort to carry out the analysis of double differential cross sections in spite of the fact that the available statistics is quite low. In our investigation we use experimental data obtained from two COSY-11 measurements at excess energies of \( Q = 10 \text{ MeV} \) (27 events) and 28 MeV (30 events) [2]. At present we are, however, at the early stage of the analysis.

For three particles in the final state the process of analysing elementary-particle-reaction data by plotting them in the space of two internal parameters is a very convenient way to study the interaction. I was originated by Dalitz in a non-relativistic application. In the original paper [7] Dalitz let the distances to the sides of an equilateral triangle be the energies of the three particles in the centre-of-mass frame. More instructive coordinates than energies are the squared invariant masses of the two-body subsystems. Using such coordinates we obtain event distribution bounded by well defined smooth closed curve [8]. The area of the Dalitz Plot is proportional to the phase space volume. Moreover, in case of no dynamics and the absence of any final state interaction, the occupation of the Dalitz plot would be fully homogeneous because the creation in any phase space interval would be equally probable. Thus, final state interaction should show up as a modification of the event density on Dalitz plot.

In case of four particles in final state the analysis is more complex, because one need five variables to fully describe a relative movement of particles. Nevertheless, there are many different types of generalization of the Dalitz plot for four-particle final states, which make possible to study interaction between particles in the exit channel. For studying of the \( K^+K^- \) or \( Kp \) interaction we use two convenient generalizations proposed by Chodrow [9] and Goldhaber [10, 11].

In his work [9] Chodrow assumed, that the invariant matrix element for the process depends only on energies of the particles. Consider four particles with masses \( m_i \) and total energy \( E \) in centre-of-mass frame. The probability of a reaction yielding a state with \( i \)-th particle in energy range \( dE_i \) (when two of the particles are identical) is:

\[
\frac{d^3P}{dM_1dM_2dM_3} = \frac{\pi^3}{8E^2M_{12}^2} \left( \frac{M_1}{M_{12}} \right) \left( \frac{M_2}{M_{12}} \right) \left( \frac{M_3}{M_{12}} \right) dM_1 dM_2 dM_3
\]

where \( dF_1 = \sqrt{E_1^2 - m_1^2} dE_1 \) and \( M \) denotes the invariant matrix element for the process. This implies that 

\[
F_1 = \frac{1}{4} \left( E_1 \sqrt{E_1^2 - m_1^2} - m_1 \cos \theta \right) \left( \frac{E_1}{m_1} \right).
\]

The distribution of events can then be plotted in the \( F_1E_2 \)-plane, and resonances may be directly read off this part of the plot, where \( E_1 < E_2 \). Like in case of three particle final states the physically allowed region on Chodrow plot is bounded by well defined curve, but the event density is not homogeneous and the area of the plot is not proportional to the phase space volume.

According to Nyborg several others extensions of Dalitz plot can be obtained if one assumes, that the matrix element \( M \) depends only on invariant masses of two and three particle subsystems [8]. Let suppose that \( M \) depends at most on \( M_{12}, M_{34}, \) and \( M_{124} \), which correspond to situation were only two two-particle or one three-particle resonances are present [8]. The distribution of events is then given by:

\[
d^3P = \frac{\pi^3}{8E^2M_{12}^2} \left( \frac{M_1}{M_{12}} \right) \left( \frac{M_2}{M_{12}} \right) \left( \frac{M_3}{M_{12}} \right) \left( \frac{M_4}{M_{12}} \right) dM_1 dM_2 dM_3 dM_4
\]

where:

\[
g \left(M_{12}^2, m_1^2, m_2^2\right) = \sqrt{M_{12}^2 - \left(m_1 + m_2\right)^2} \left[M_{12}^2 - \left(m_1 - m_2\right)^2\right].
\]

The projection of the physical region on the \( (M_{12}, M_{34}) \) plane gives a right isosceles triangle within which the area is not proportional to the phase space volume. It is worth mentioning that the event density on Goldhaber plot is not homogeneous and goes to zero on entire boundary of the plot given by following equations:

\[
M_{12}^2 + M_{34}^2 = E, \quad M_{12}^2 = m_1^2 + m_2^2, \quad M_{34}^2 = m_3^2 + m_4^2.
\]

So far we conducted Monte Carlo simulations of the \( pp \rightarrow ppK^+K^- \) reaction using a FORTRAN-based code, called GENBOD [12]. The simulations were first made assuming that there is no final state interaction, then the pp-FSI was included [13]. The pp-FSI was taken into account as a weights proportional to the inverse of a squared Jost-function of the Bonn potential [14]. In order to compare these spectra with the experimental distributions we need to correct them for the acceptance and detection efficiency of the COSY-11 facility. This will be a next step of the investigations which we intend to perform in the near future.

References:

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