The proton spin puzzle

Steven Bass

Proton spin problem:

Where does the spin of the nucleon (proton and neutron) come from ?

E.g. The key difference between ^3He and ^4He in low temperature physics comes from the spin of the extra neutron in ^4He

→ where does this spin come from at the quark level ?

Relativistic quark models → ~ 60% of proton's spin carried by intrinsic spin of quark constituents

Polarized Deep Inelastic Scattering → quark "spin content" ~ 35%

→ Where is the missing "spin" ? (polarized gluons, L_z, … ?)

Progress (theory and experiment)

→ Valence quarks, DChSB (pion cloud) and (perhaps) gluon topology

Cracow, October 2016

Spinning the proton

- Reviews of Modern Physics, April 2013, October 2005
- WSPC book, 2007

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The spin structure of the nucleon

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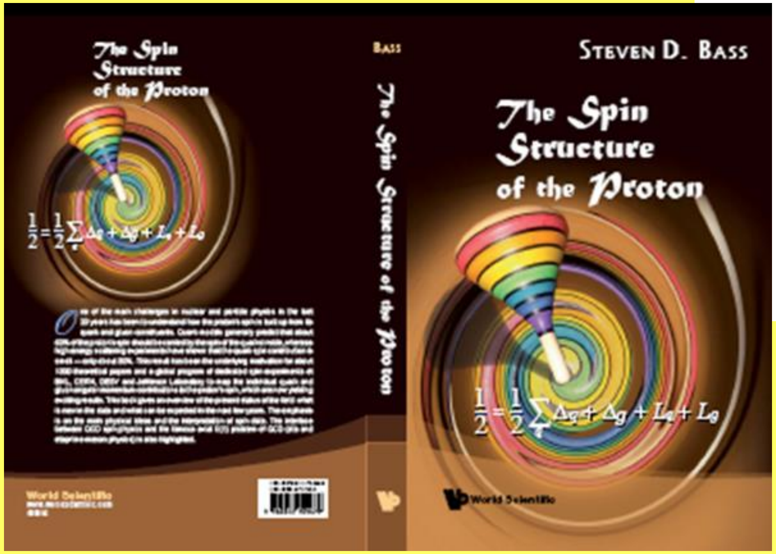
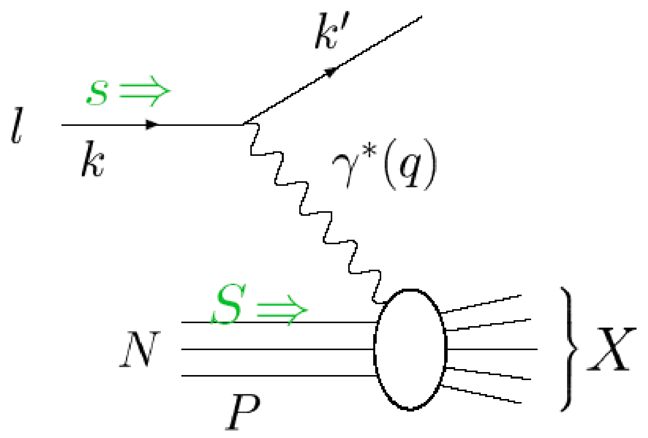
This article reviews our present understanding of QCD spin physics: the proton spin puzzle and new developments aimed at understanding the transverse structure of the nucleon. Present experimental investigations of the nucleon's internal spin structure, the theoretical interpretation of the different measurements, and the open questions and challenges for future investigation are discussed.



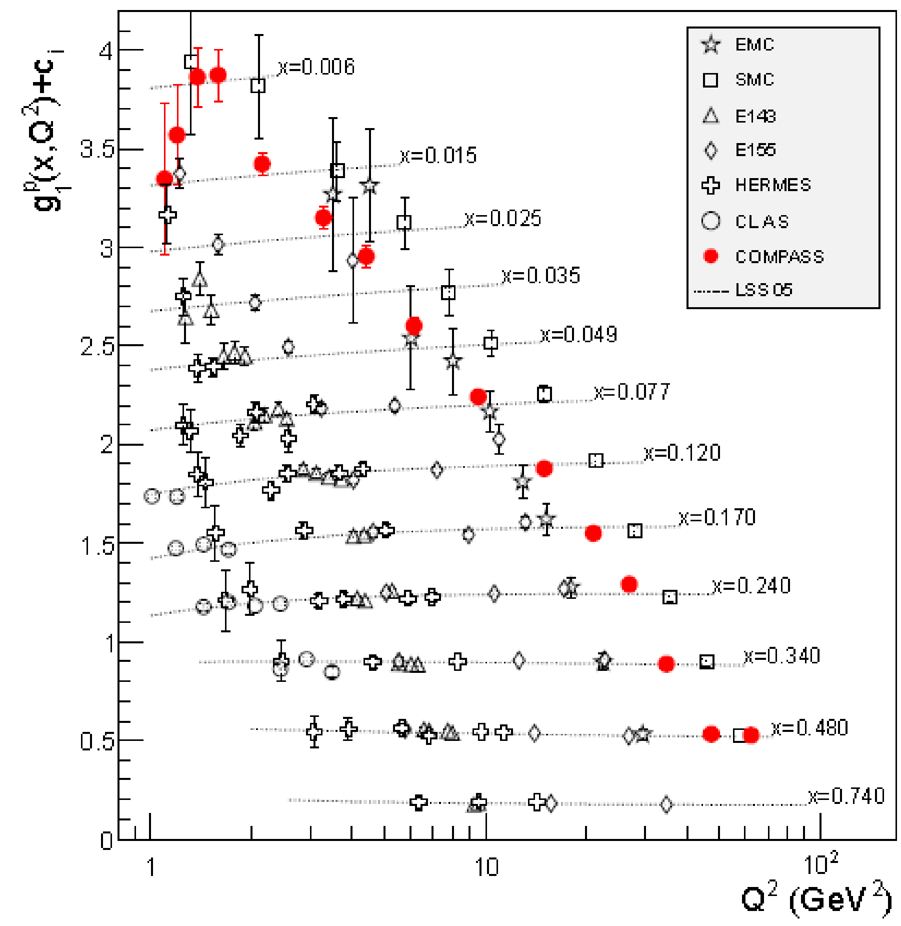
Deep Inelastic Scattering

- Inclusive electron proton scattering



$$Q^2 = -q^2 \gg m^2$$

$$x = \frac{Q^2}{2P\cdot q} = \frac{Q^2}{2m\nu}$$



- Cross-sections behave like incoherent elastic scattering on nearly-free quarks (fermions) inside

$$\left(\frac{d^2\sigma \uparrow\Downarrow}{d\Omega dE'} + \frac{d^2\sigma \uparrow\Uparrow}{d\Omega dE'}\right) = \frac{\alpha^2}{4E^2 M \sin^4\frac{\theta}{2}}\left[2\sin^2\frac{\theta}{2}\; F_1 + \cos^2\frac{\theta}{2}\;\frac{M}{\nu}\; F_2\right]$$

$$\left(\frac{d^2\sigma \uparrow\Downarrow}{d\Omega dE'} - \frac{d^2\sigma \uparrow\Uparrow}{d\Omega dE'}\right) = \frac{4\alpha^2}{MQ^2}\;\frac{E'}{E\nu}\left[(E + E'\cos\theta)\; g_1 - 2xM\; g_2\right]$$

The proton spin puzzle: where it started

Volume 206, number 2 PHYSICS LETTERS B 19 May 1988

A MEASUREMENT OF THE SPIN ASYMMETRY AND DETERMINATION OF THE STRUCTURE FUNCTION g_1 IN DEEP INELASTIC MUON–PROTON SCATTERING

European Muon Collaboration

The spin asymmetry in deep inelastic scattering of longitudinally polarised muons by longitudinally polarised protons has been measured over a large x range $(0.01 < x < 0.7)$. The spin-dependent structure function $g_1(x)$ for the proton has been determined and its integral over x found to be $0.114 \pm 0.012 \pm 0.026$, in disagreement with the Ellis–Jaffe sum rule. Assuming the validity of the Bjorken sum rule, this result implies a significant negative value for the integral of g_1 for the neutron. These values for the integrals of g_1 lead to the conclusion that the total quark spin constitutes a rather small fraction of the spin of the nucleon.

indicating that the quark spins carry $(1 \pm 12 \pm 24)\%$ of the proton spin.

Spin thoughts

- Polarization data has often been the graveyard of fashionable theories. If theorists had their way they might well ban such measurements altogether out of self-protection.

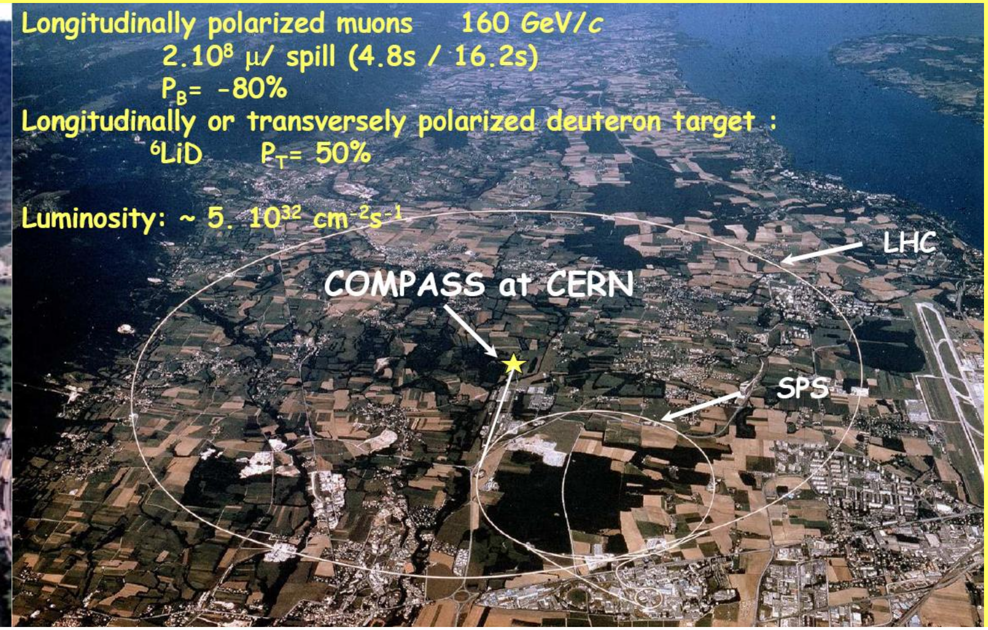
J.D. Bjorken (1987)

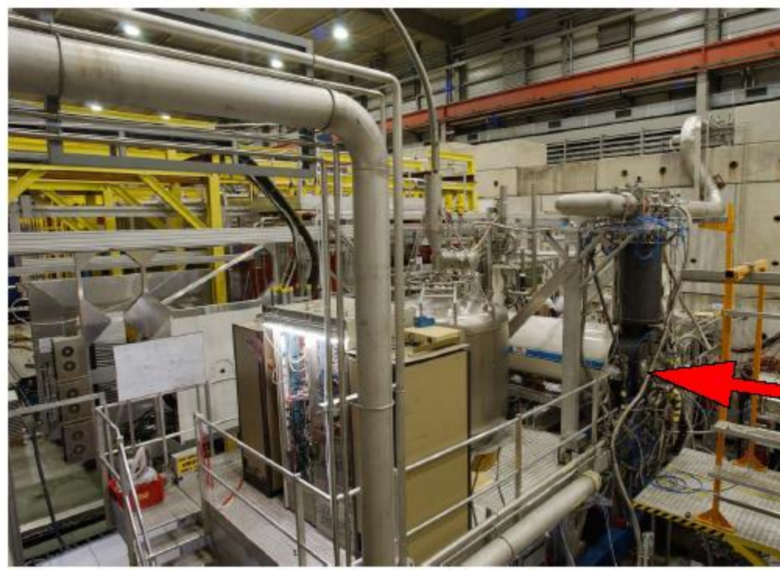
Spin experiments

- Today: many spin experiments ...
- 40 years of proton spin studies

Experiment	Year	Beam	Target	Energy (GeV)	Q^2 (GeV^2)	x
			Completed experiments			
SLAC: E80, E130	1976–1983	e^-	H-butanol	≤ 23	1–10	0.1–0.6
SLAC: E142/3	1992–1993	e^-	NH_3, ND_3	≤ 30	1–10	0.03–0.8
SLAC: E154/5	1995–1999	e^-	NH_3, ^6LiD, ^3He	≤ 50	1–35	0.01–0.8
CERN: EMC	1985	μ^+	NH_3	100, 190	1–30	0.01–0.5
CERN: SMC	1992–1996	μ^+	H/D-butanol, NH_3	100, 190	1–60	0.004–0.5
FNAL: E581/E704	1988–1997	p	p	200	~ 1	$0.1 < x_F < 0.8$
			Analyzing and/or running			
DESY: HERMES	1995–2007	e^+, e^-	H, D, ^3He	~ 30	1–15	0.02–0.7
CERN: COMPASS	2002–2012	μ^+	NH_3, ^6LiD	160, 200	1–70	0.003–0.6
JLab6: Hall A	1999–2012	e^-	^3He	≤ 6	1–2.5	0.1–0.6
JLab6: Hall B	1999–2012	e^-	NH_3, ND_3	≤ 6	$1-5$	0.05–0.6
RHIC: BRAHMS	2002–2006	p	p (beam)	$2\times$ (31–100)	$\sim$1–6	$-0.6 < x_F < 0.6$
RHIC: PHENIX, STAR	2002+	p	p (beam)	$2\times$ (31–250)	$\sim$1–400	$\sim$0.02–0.4
			Approved future experiments (in preparation)			
CERN: COMPASS-II	2014+	μ^+, μ^-	Unpolarized H_2	160	$\sim$1–15	$\sim$0.005–0.2
		π^-	NH_3	190		$-0.2 < x_F < 0.8$
JLab12: Halls A/B/C	2014+	e^-	HD, NH_3, ND_3, ^3He	≤ 12	$\sim$1–10	$\sim$0.05–0.8

Polarized DIS – global programme





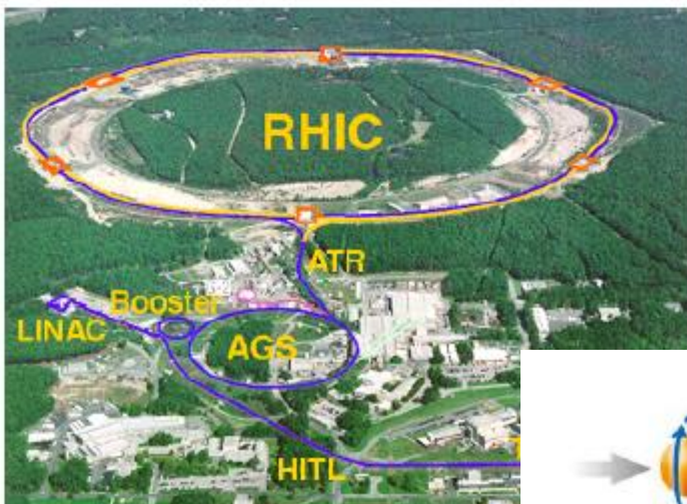
- ^{6}LiD
- ± 50 % polarisation
- 50 % dilution factor
- 2.5 T
- 50 mK

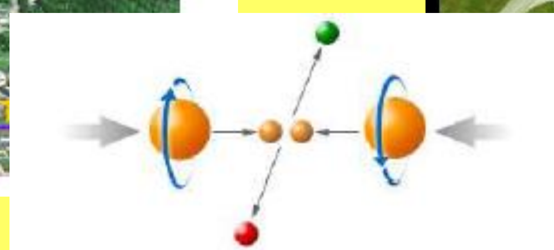
μ

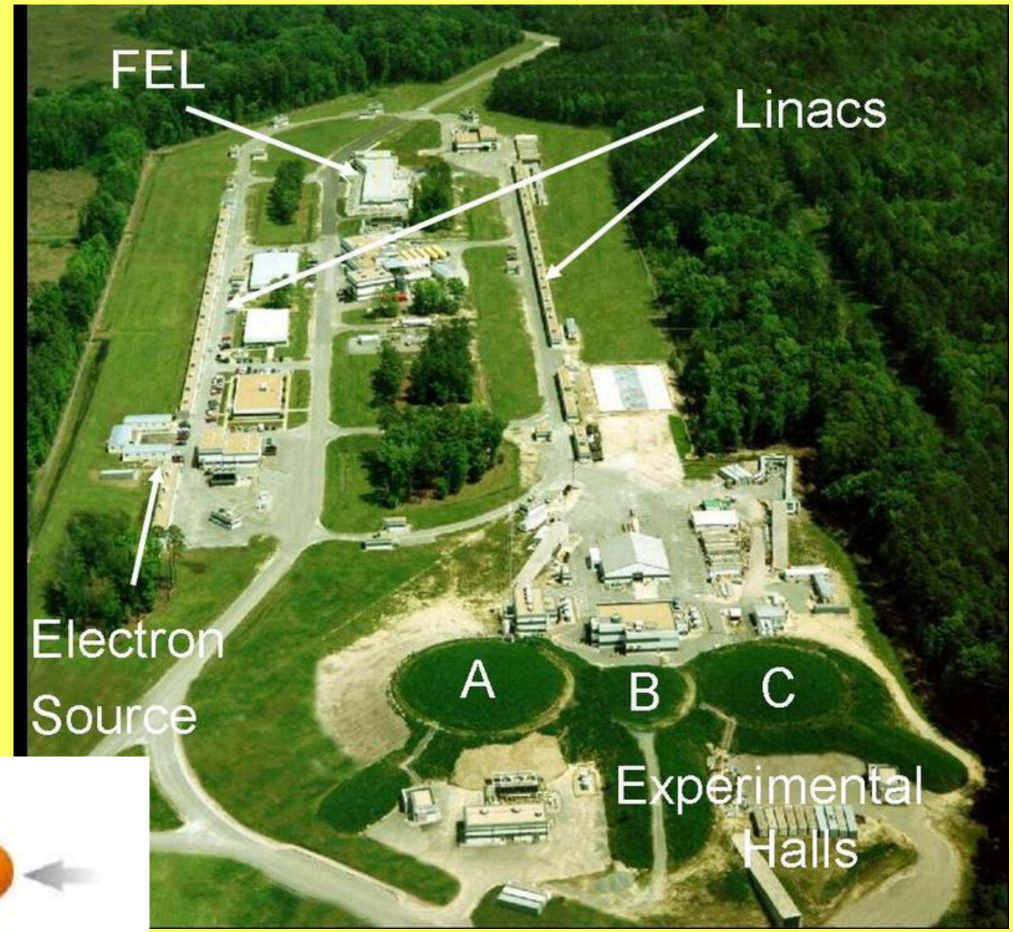
New spin facilities

- RHIC@BNL: the world´s first polarized proton proton collider
- Jefferson Lab: continuous polarized electron beams and fixed target

BROOKHAVEN
NATIONAL LABORATORY







Proton Spin

- **What do we expect for the proton spin content ?**

$$|p\uparrow\rangle = \frac{1}{\sqrt{2}}|u\uparrow(ud)_{S=0}\rangle + \frac{1}{\sqrt{18}}|u\uparrow(ud)_{S=1}\rangle - \frac{1}{3}|u\downarrow(ud)_{S=1}\rangle - \frac{1}{3}|d\uparrow(uu)_{S=1}\rangle + \frac{\sqrt{2}}{3}|d\downarrow(uu)_{S=1}\rangle$$

  - Static quark model → 100 %
  - Relativistic effects → Constituent quark models ~ 60 %

    [e.g. Bag → Lower component of Dirac spinor is in p-wave: Shift of J from intrinsic spin → orbital angular momentum]

$$\psi \sim \begin{pmatrix} f \\ i\sigma.\hat{r}g \end{pmatrix}.$$

- What has been measured ?
  - Quarks seem to contribute just ~30 % of the proton's spin (!)
  - Where is the „missing spin" ?

  - **What does this result tell us about spin and constituent quarks ?**

Partonic spin structure of the proton

- Spin independent

$$F_1(x) = \frac{1}{2x} F_2(x) = \frac{1}{2} \sum_q e_q^2 \{q + \bar{q}\}(x)$$

and spin dependent structure functions

$$g_1(x) = \frac{1}{2} \sum_q e_q^2 \Delta q(x).$$

Have the parton interpretation

$$\begin{aligned} \{q + \bar{q}\}(x) &= (q^{\uparrow} + \bar{q}^{\uparrow})(x) + (q^{\downarrow} + \bar{q}^{\downarrow})(x) \\ \Delta q(x) &= (q^{\uparrow} + \bar{q}^{\uparrow})(x) - (q^{\downarrow} + \bar{q}^{\downarrow})(x) \end{aligned}$$

Where the distributions measure the probability to find an (anti-)quark with given polarization and momentum fraction x of the proton´s momentum

- In QCD the first moment of the spin distributions are measured through axial current matrix elements

$$\Delta q = \int_0^1 dx \ \Delta q(x)$$

$$2M s_\mu \Delta q = \langle p, s | \ \bar{q} \gamma_\mu \gamma_5 q \ | p, s \rangle$$

Deep Inelastic Spin Sum Rule

- Dispersion relation for polarized photon-nucleon scattering + operator product expansion → Sum Rule

$$\int_0^1 dx\ g_1^p(x,Q^2) = \left(\frac{1}{12}g_A^{(3)} + \frac{1}{36}g_A^{(8)}\right)\left\{1+\sum_{\ell\geq 1} c_{\mathrm{NS}\ell}\,\alpha_s^\ell(Q)\right\} + \frac{1}{9}g_A^{(0)}|_{\mathrm{inv}}\left\{1+\sum_{\ell\geq 1} c_{\mathrm{S}\ell}\,\alpha_s^\ell(Q)\right\} + \mathcal{O}(\frac{1}{Q^2})$$

$$\begin{aligned} g_A^{(3)} &= \Delta u - \Delta d \\ g_A^{(8)} &= \Delta u + \Delta d - 2\Delta s \\ g_A^{(0)} &= \Delta u + \Delta d + \Delta s \end{aligned}$$

- Here nature helps us (Bjorken):

  $g_A^{(3)}$ = 1.26 (same matrix element measured in neutron beta decay)

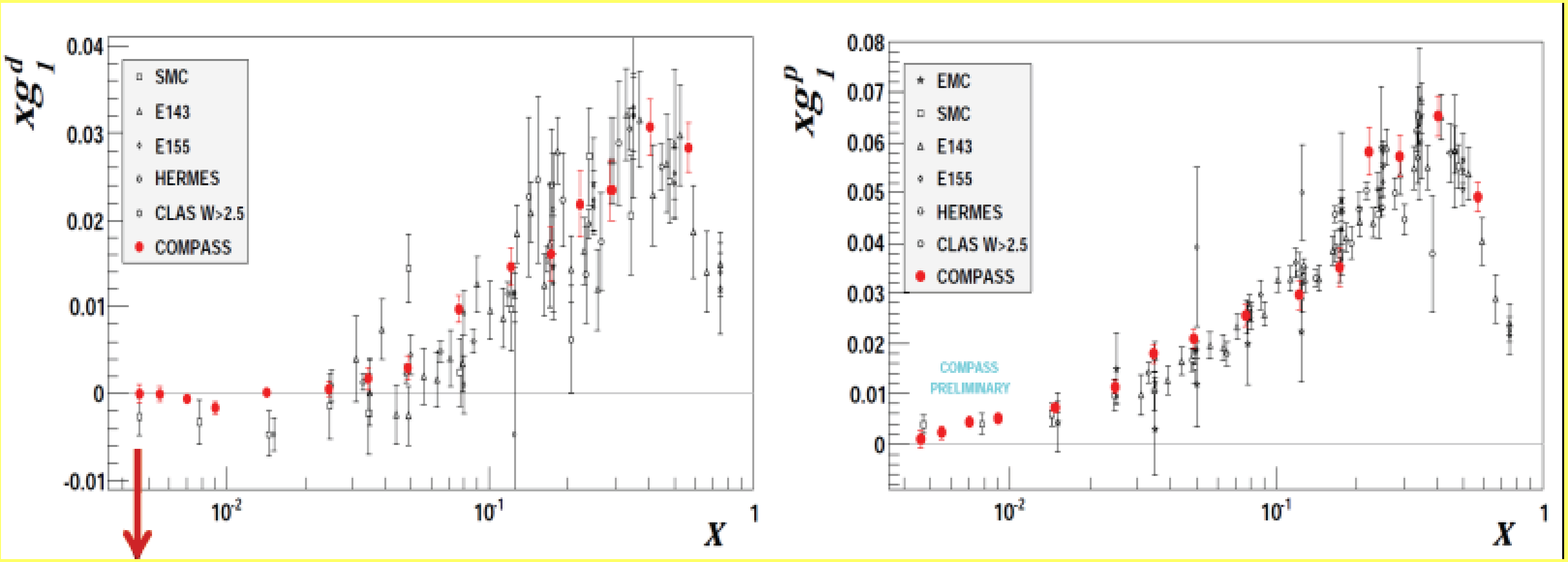
  $g_A^{(8)}$ = 0.58 +/- 0.03 (extracted from hyperon beta decays + SU(3))
- Perturbative QCD corrections calculated to high precision (Larin et al)
- Guess (Ellis-Jaffe hypothesis): Strangeness contribution ~ 0

→ $g_A^{(8)} = g_A^{(0)} \sim 0.6$

TEST THIS IN EXPERIMENT

The Spin Structure of the Proton

Polarized Deep Inelastic Scattering



Measure g_1 spin structure function

First moment →

$$g_A^{(0)}|_{\mathrm{pDIS},Q^2\to\infty} = 0.33 \pm 0.03(\mathrm{stat.}) \pm 0.05(\mathrm{syst.})$$

WHERE IS THE „MISSING SPIN" ?

→ „Strangeness polarization"

$$\Delta s_{Q^2\to\infty} = \frac{1}{3}(g_A^{(0)}|_{\mathrm{pDIS},Q^2\to\infty} - g_A^{(8)}) = -0.08 \pm 0.01(\mathrm{stat.}) \pm 0.02(\mathrm{syst.})$$

Quark spins in the proton

- Taking SU(3)

$$\Delta u_{Q^2\to\infty} = 0.84 \pm 0.01(\text{stat}) \pm 0.02(\text{syst}),$$
$$\Delta d_{Q^2\to\infty} = -0.43 \pm 0.01(\text{stat}) \pm 0.02(\text{syst}).$$

$$\Delta s_{Q^2\to\infty} = \tfrac{1}{3}(g_A^{(0)}|_{\text{pDIS},Q^2\to\infty} - g_A^{(8)})$$
$$= -0.08 \pm 0.01(\text{stat}) \pm 0.02(\text{syst}),$$

- With Cloudy Bag, pion cloud violates SU(3)
  - » Also lattice

$$\Delta s \sim -0.03 \pm 0.03.$$

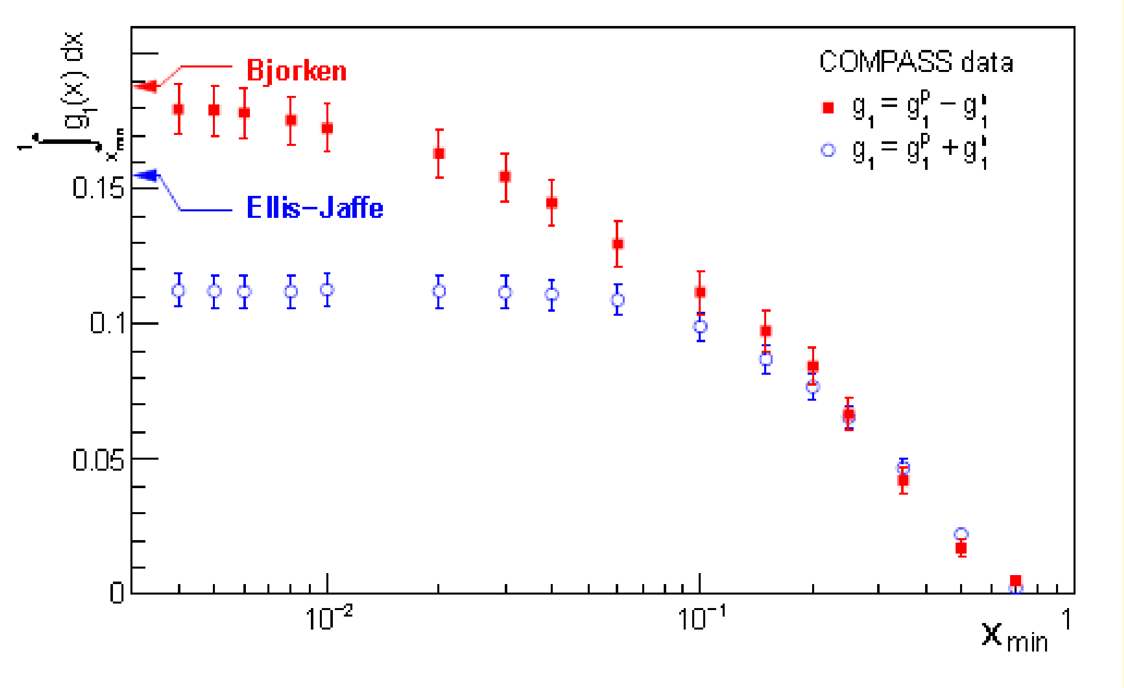
Convergence of the first moment integrals

Isosinglet integral converges at
x ~ 0.03

Spin problem associated with „collapse“ of the singlet structure function at small x

Bjorken SR for g1(p-n) works!

... Accurate to 9% [COMPASS data, NLO result]



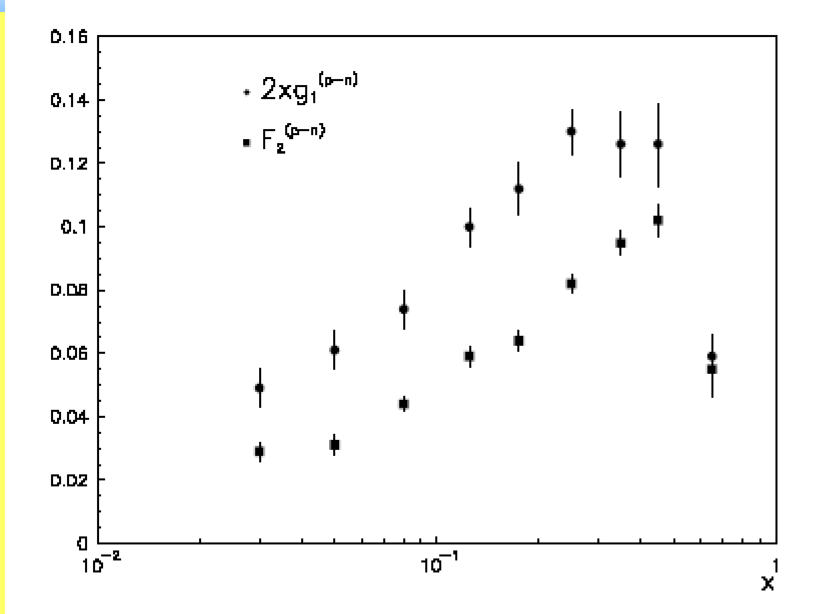
$$|g_A/g_V| = 1.22 \pm 0.05\ (\text{stat.}) \pm 0.10\ (\text{syst.})$$

Isovector structure functions

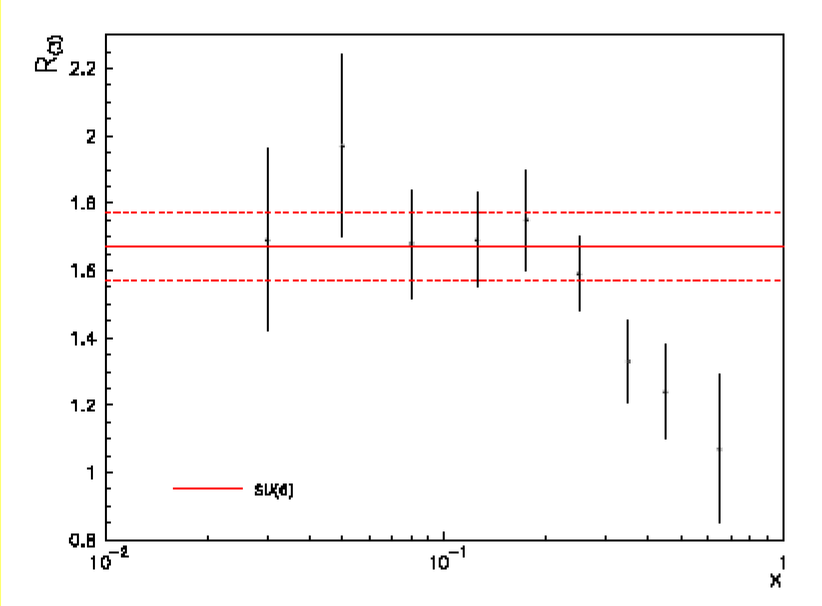
Plot the spin dependent and spin independent isovector s.f.s together

$$2x(g_1^p - g_1^n) = \frac{1}{3}x\left[(u+\overline{u})^{\uparrow} - (u+\overline{u})^{\downarrow} - (d+\overline{d})^{\uparrow} + (d+\overline{d})^{\downarrow}\right]$$

$$(F_2^p - F_2^n) = \frac{1}{3}x\left[(u+\overline{u})^{\uparrow} + (u+\overline{u})^{\downarrow} - (d+\overline{d})^{\uparrow} - (d+\overline{d})^{\downarrow}\right]$$



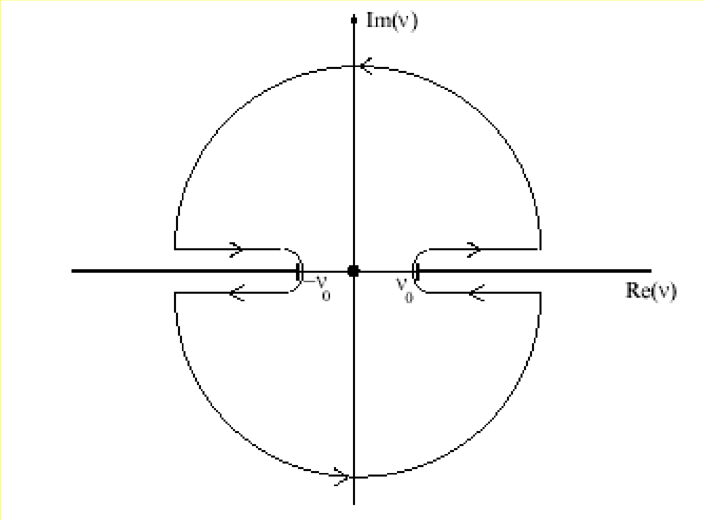
- Ratio $R_{(3)} = 2x\ g_1^{(p-n)}/F_2^{(p-n)}$ measures ratio of spin to non-spin isovector distributions.
- Interesting: Essentially constant in the measured region for $x < 0.2$ at the value predicted by simple quark models.
- Why should quark model expectations work for $x < 0.1$?
- NOWHERE ELSE TO PUT THE AREA UNDER THE BJORKEN SUM RULE !!



Is there a catch ?

- SU(3) and hyperon beta-decays (pion chiral corrections can be large)
- Sum Rule is derived starting from the dispersion relation …
- Assumes no contribution from the „circle at infinity“
- Otherwise we get a (finite) correction to the first moment sum rule !

[SDB, Zakopane lectures 03 and RMP]



$$\int_0^1 dx\; g_1^p(x,Q^2) = \left(\frac{1}{12}g_A^{(3)} + \frac{1}{36}g_A^{(8)}\right)\left\{1+\sum_{\ell\geq 1} c_{\mathrm{NS}\ell}\,\alpha_s^\ell(Q)\right\} + \frac{1}{9}g_A^{(0)}|_{\mathrm{inv}}\left\{1+\sum_{\ell\geq 1} c_{\mathrm{S}\ell}\,\alpha_s^\ell(Q)\right\} + \mathcal{O}\left(\frac{1}{Q^2}\right)$$
$$- \beta_1(Q^2)\frac{Q^2}{4M^2}$$

Sum rules

- Sum rules for

  $g_A^{(0)}$ singlet axial charge (… e.g. Ellis-Jaffe #2)

  what dynamics separates its value from $g_A^{(8)}$?

  and

  $\frac{1}{2}$ = Quark spin + Glue spin + Orbital

QCD and $g_A^{(0)}$

- How does the polarized DIS measurement fit with the constituent quark picture ?

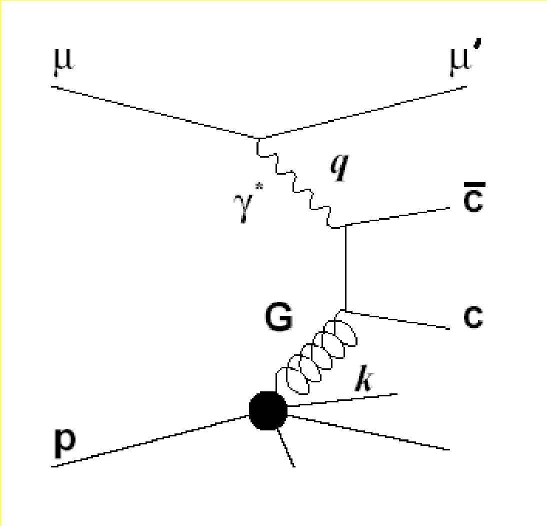
- What have we really measured ?

Polarized glue

- Attempts to understand the polarized DIS values of $g_A^{(0)}$ and Delta s
  - Gluon polarization
  - Sea and valence quark polarization
  - → measure through hard processes in (semi-inclusive) DIS, jets, polarized pp collisions at RHIC …

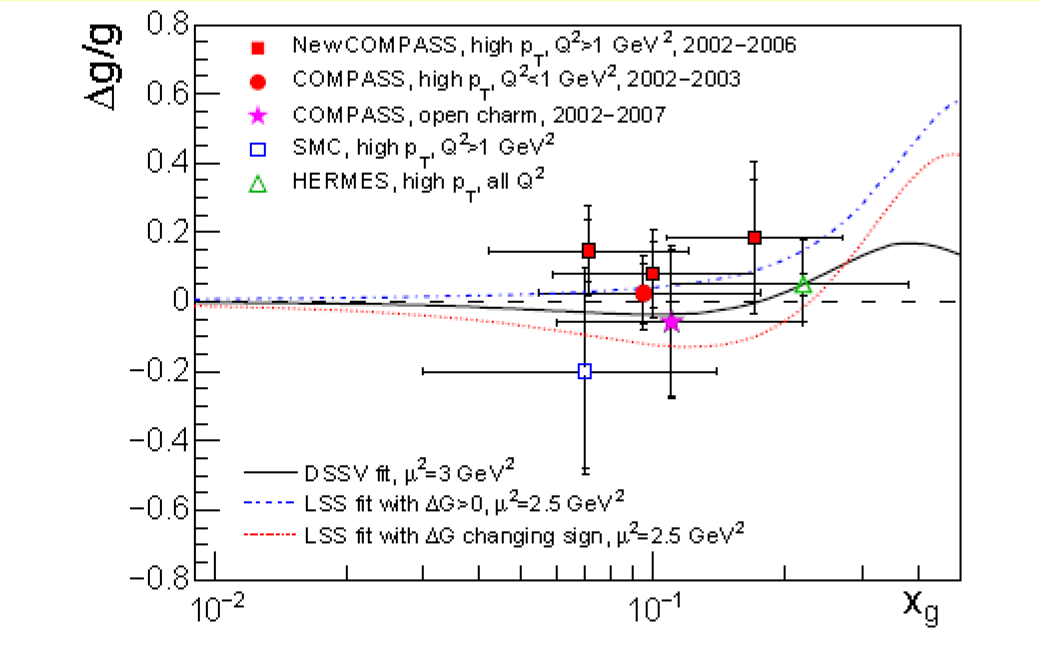
$$g_A^{(0)} = \left(\sum_q \Delta q - 3\frac{\alpha_s}{2\pi}\Delta g \right)_{\mathrm{partons}} + C_\infty$$

$$\alpha_s \Delta g \sim \mathrm{constant}, \quad Q^2 \to \infty$$



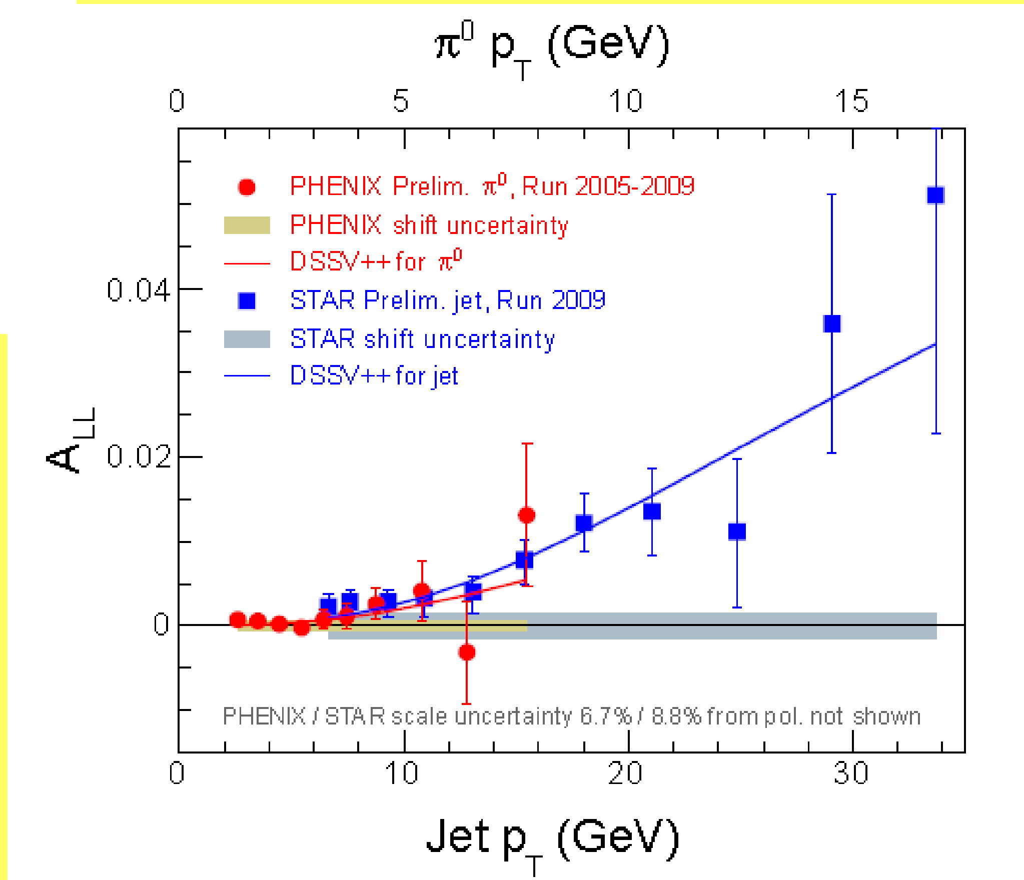
- Measurements at COMPASS, RHIC, HERMES: Delta g < 0.5, Q^2 ~ 3 GeV^2

Gluon polarization appears small !



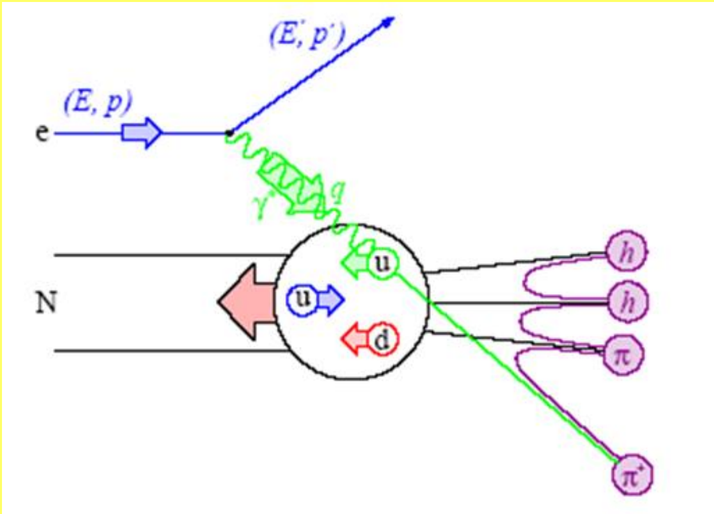
$$|\Delta g(m_c^2)| < 0.3$$

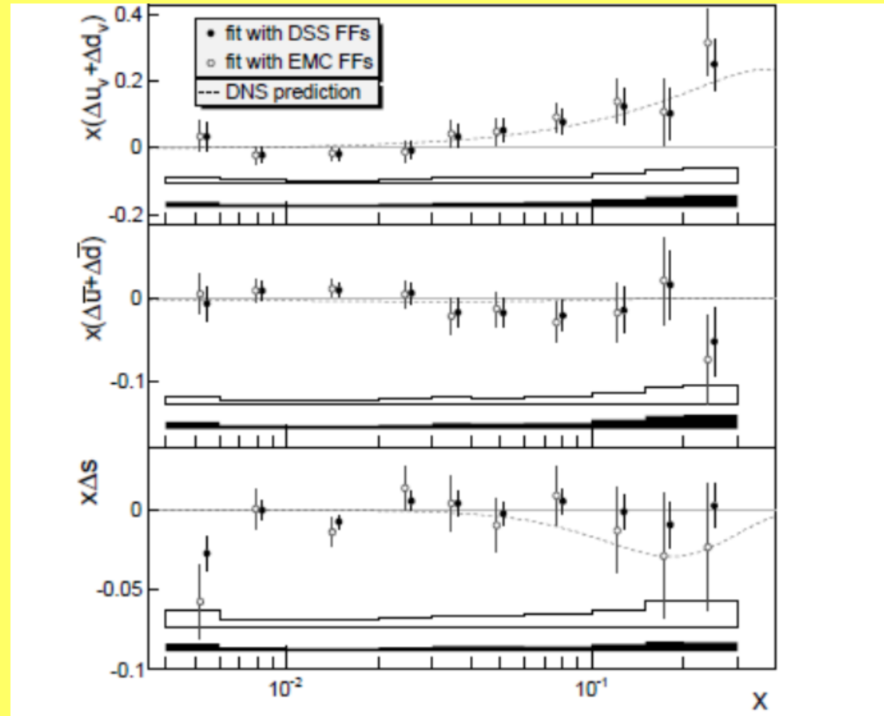
$$|-3(\alpha_s/2\pi)\Delta g| < 0.06$$



What about polarized strangeness ?

- Tag on final state pion or kaon → Spin-Flavour separation





$$A_{1p}^h(x,Q^2) \simeq \frac{\sum_{q,h} e_q^2 \Delta q(x,Q^2) \int_{z_{min}}^1 D_q^h(z,Q^2)}{\sum_{q,h} e_q^2 q(x,Q^2) \int_{z_{min}}^1 D_q^h(z,Q^2)}$$

$$D_q^h(z,Q^2) = \int dk_t^2 D_q^h(z,k_t^2,Q^2)$$

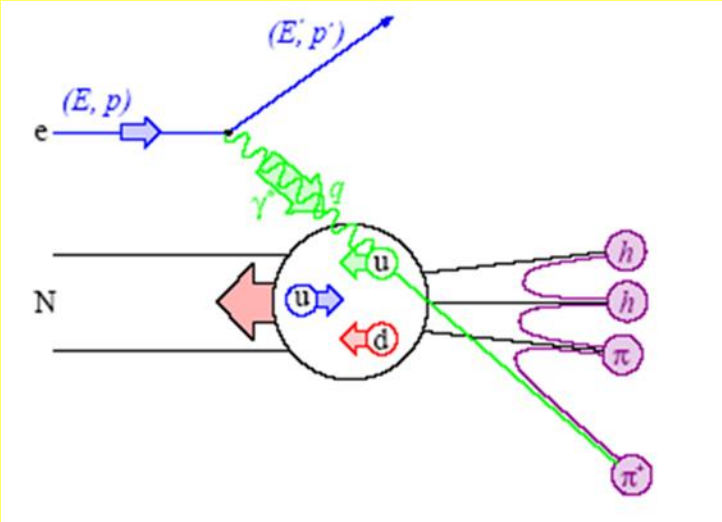
Experiment	x-range	Q^2 (GeV^2)	$\Delta u_v + \Delta d_v$	$\Delta \bar{u} + \Delta \bar{d}$
SMC98	0.003–0.7	10	$0.26 \pm 0.21 \pm 0.11$	$0.02 \pm 0.08 \pm 0.06$
HERMES05	0.023–0.6	2.5	$0.43 \pm 0.07 \pm 0.06$	$-0.06 \pm 0.04 \pm 0.03$
COMPASS	0.006–0.7	10	$0.40 \pm 0.07 \pm 0.05$	$0.0 \pm 0.04 \pm 0.03$

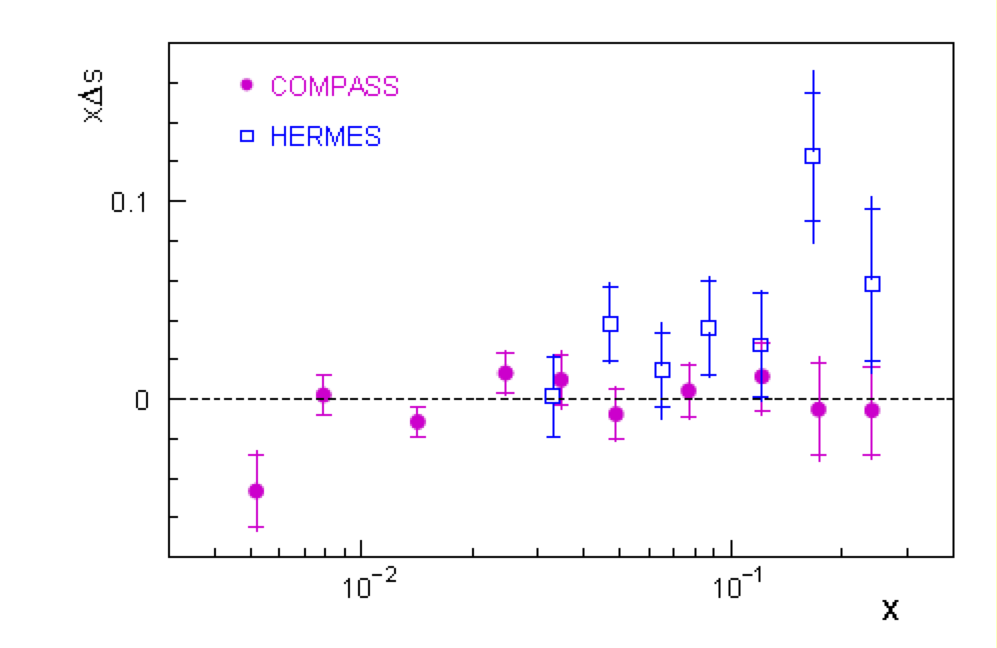
- No evidence of negative strangeness polarization

  Delta s = -0.02 +/- 0.02 (stat) +/- 0.02 (syst) @ 0.003 < x < 0.3

What about polarized strangeness ?

- Tag on final state pion or kaon → Spin-Flavour separation





$$A^h_{1p}(x,Q^2) \simeq \frac{\sum_{q,h} e_q^2 \Delta q(x,Q^2) \int_{z_{min}}^1 D_q^h(z,Q^2)}{\sum_{q,h} e_q^2 q(x,Q^2) \int_{z_{min}}^1 D_q^h(z,Q^2)}$$

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- No evidence of negative strangeness polarization
  Delta s = -0.02 +/- 0.02 (stat) +/- 0.02 (syst) @ 0.003 < x < 0.3

Elastic neutrino proton scattering

Independent weak interaction measurement of $g_A^{(0)}$

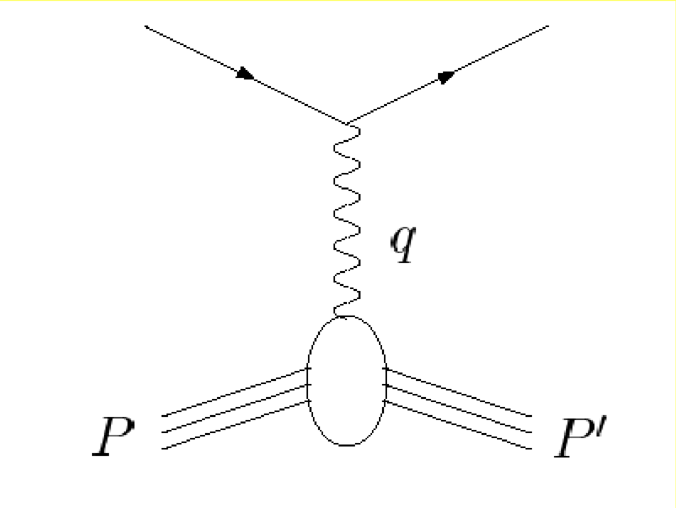
- Measure matrix element of non-singlet current

$$J^Z_{\mu 5} = \frac{1}{2}\left\{ \sum_{q=u,c,t} - \sum_{q=d,s,b} \right\} \bar{q}\gamma_\mu\gamma_5 q$$

- → Axial charge

$$2g_A^{(Z)} = (\Delta u - \Delta d - \Delta s) + (\Delta c - \Delta b + \Delta t)$$

- Measures strangeness up to heavy quark corrections



$$2g_A^{(Z)} = (\Delta u - \Delta d - \Delta s)_{\mathrm{inv}} + \mathcal{H}(\Delta u + \Delta d + \Delta s)_{\mathrm{inv}} + O(m^{-1}_{t,b,c})$$

- NLO heavy quark calculation [SDB,Crewther,Steffens,Thomas]

$$\mathcal{H} = \frac{6}{23\pi}(\widetilde{\alpha}_b - \widetilde{\alpha}_t)\left\{1 + \frac{125663}{82800\pi}\widetilde{\alpha}_b + \frac{6167}{3312\pi}\widetilde{\alpha}_t - \frac{22}{75\pi}\widetilde{\alpha}_c\right\} - \frac{6}{27\pi}\widetilde{\alpha}_c - \frac{181}{648\pi^2}\widetilde{\alpha}_c^2 + O(\widetilde{\alpha}^3_{t,b,c})$$

- Small … = -0.02
- Direct measurement of Delta s
  (independent of potential subtraction constants)

QCD and $g_A^{(0)}$

- How does the polarized DIS measurement fit with the constituent quark picture ?

- What have we really measured ?

What's left ?

… where are we ?

Consider non-perturbative aspects:
pion cloud physics plus the axial anomaly …

Modelling the spin

- Colour hyperfine interaction (OGE) and pions shift total angular momentum into orbital angular momentum

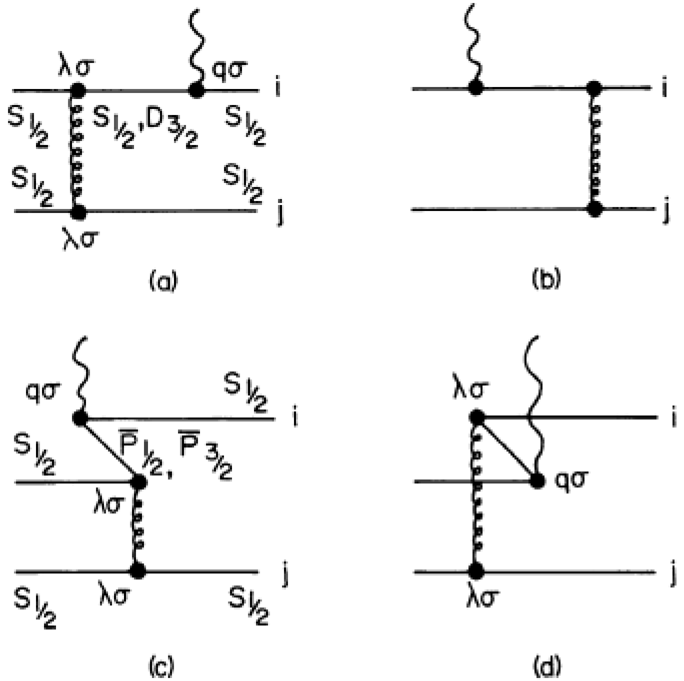


Fig. 1 We illustrate the quark-quark hyperfine contributions which involve an excited intermediate quark state. In the figures the external probe (top vertical wavy

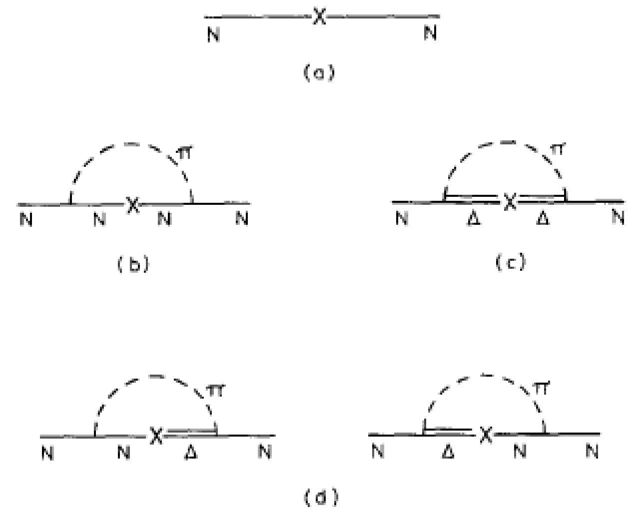


Fig. 1. Diagrams contributing to $|g_A/g_V|$ and hence to the sum rule for g_1 in the cloudy bag model. (The cross represents the action of the axial current operator.)

The nucleon's octet axial-charge $g_A^{(8)}$ with chiral corrections

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Physics Letters B 684 (2010) 216–220

SU(3) breaking

- SU(3) for axial charges works at zeroth order in chiral expansion

Process	measurement	SU(3) combination	Fit value	MIT + OGE
$n \to p$	1.270 ± 0.003	$F + D$	1.26	$\frac{5}{3}B' + G$
$\Lambda^0 \to p$	0.718 ± 0.015	$F + \frac{1}{3}D$	0.73	B'
$\Sigma^- \to n$	-0.340 ± 0.017	$F - D$	-0.34	$-\frac{1}{3}B' - 2G$
$\Xi^- \to \Lambda^0$	0.25 ± 0.05	$F - \frac{1}{3}D$	0.19	$\frac{1}{3}B' - G$
$\Xi^0 \to \Sigma^+$	1.21 ± 0.05	$F + D$	1.26	$\frac{5}{3}B' + G$

- Pion chiral corrections induce SU(3) breaking – we use Cloudy Bag Model [SDB+AW Thomas, PLB 684 (2010) 216]

$$g_A^{(3)} = g_A^{(3)}|_{\rm MIT} \times Z_{\rm CBM} \times \left(1 - \frac{8}{9}P_{N\pi} - \frac{4}{9}P_{\Delta\pi} + \frac{8}{15}P_{N\Delta\pi}\right)$$

$$g_A^{(8)} = g_A^{(8)}|_{\rm MIT} \times Z_{\rm CBM} \times \left(1 - \frac{4}{3}P_{N\pi} + \frac{2}{3}P_{\Delta\pi}\right).$$

Cloudy Bag results

- Leading order model result depends on representation chosen for chiral symmetry
- Surface coupling model

	$g_A^{(3)}$	S_z (singlet axial-charge)
Non-relativistic	+1.66	+1.00
Relativistic	+1.09	+0.65
+ OGE	+1.13	+0.54
+ Pions	+1.06	+0.43
+ centre of mass	+1.27	+0.52

- Volume coupling model

$$\delta A_i^\lambda = -\frac{1}{2f_\pi} f_{ijk} \bar{q} \gamma^\lambda \lambda_j q \phi^k \theta_V ,$$

	$g_A^{(3)}$	$g_A^{(8)}$	$g_A^{(0)}$
Non-relativistic	+1.66	+1.00	+1.00
Relativistic	+1.09	+0.65	+0.65
+ OGE	+1.13	+0.54	+0.54
Volume CBM	+1.29	+0.45	+0.40
rescale	+1.27	+0.44	+0.39

Octet axial charge

- Leading order results depend on representation of chiral symmetry and whether couple quarks to pions at the Bag surface or through out the Bag volume

$$0.49 \pm 0.02$$

- Surface version

$$0.35 \pm 0.03(\text{stat.}) \pm 0.05(\text{syst.})$$

Still a spin puzzle to solve

$$0.42 \pm 0.02$$

- Volume version

$$0.37 \pm 0.03(\text{stat.}) \pm 0.05(\text{syst.})$$

Pions enough to resolve the spin puzzle

- Combine a la PDG: Octet axial charge = 0.46 ± 0.05

$$g_A^{(0)}|_{\text{pDIS}} = 0.36 \pm 0.03 \pm 0.05.$$

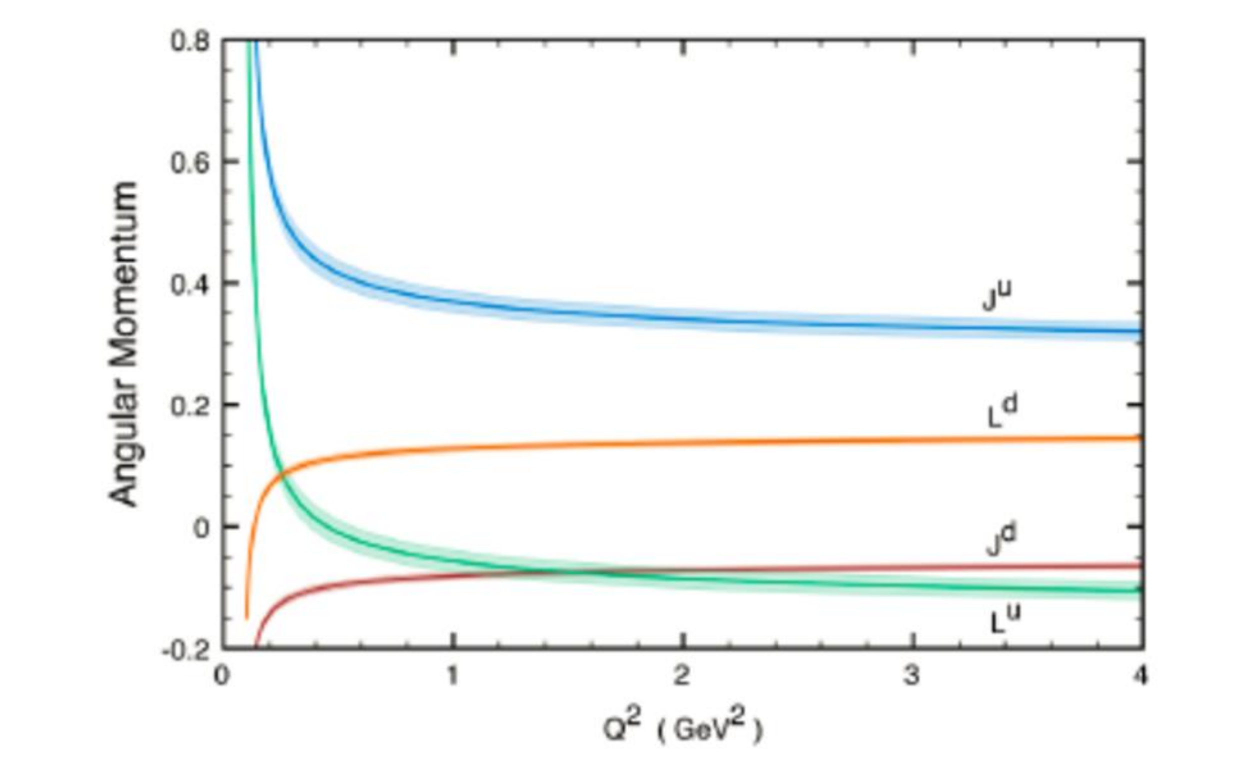
compares with model prediction 0.42 +/- 0.07 (before polarized glue) $\Delta s \sim -0.01$

and reduces the „polarized strangeness" needing to be explained

$|-3(\alpha_s/2\pi)\Delta g| < 0.06$ $\Delta s \sim -0.03 \pm 0.03$

Evolution and comparison with theory

	Cloudy bag	Lattice	GPD	TMD
Δu	0.85 ± 0.06	0.82 ± 0.07		
Δd	-0.42 ± 0.06	-0.41 ± 0.07		
J_u	0.30	0.24 ± 0.05	0.24	0.24
J_d	−0.04	0.00 ± 0.05	0.02	0.02



Non pertubative gluon topology

Constituent quark =

„(topological) condensate"

+ partons

QCD axial anomaly and $g_A^{(0)}$

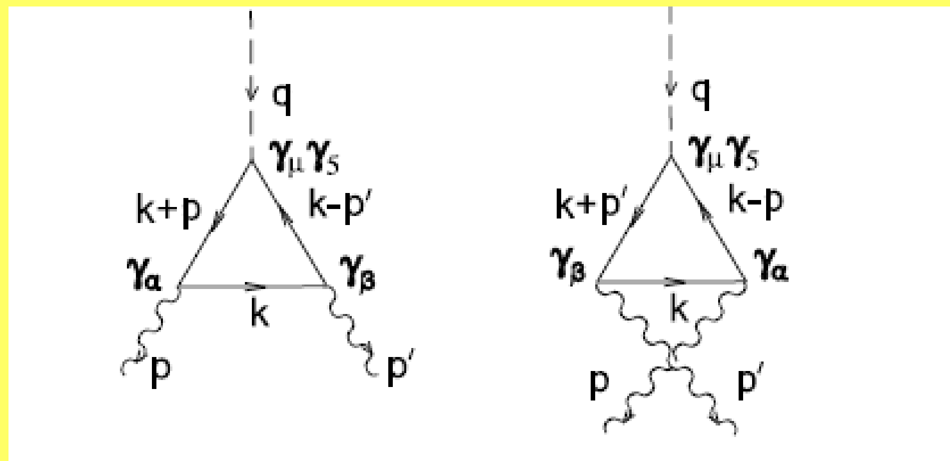
- The QCD Axial Anomaly in the singlet axial current

$$J^{GI}_{\mu 5} = \left(\bar{u}\gamma_\mu\gamma_5 u + \bar{d}\gamma_\mu\gamma_5 d + \bar{s}\gamma_\mu\gamma_5 s\right)_{GI}$$

- Current sensitive to gluonic degrees of freedom

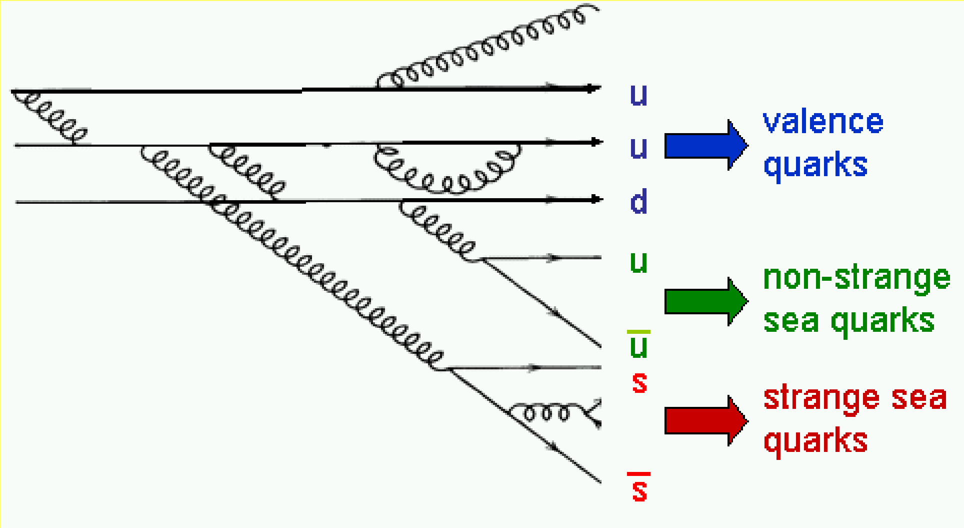
$$\partial^\mu J^{GI}_{\mu 5} = 2f\,\partial^\mu K_\mu + \sum_{i=1}^{f} 2im_i\bar{q}_i\gamma_5 q_i$$

$$K_\mu = \frac{g^2}{32\pi^2}\epsilon_{\mu\nu\rho\sigma}\left[A^\nu_a\left(\partial^\rho A^\sigma_a - \frac{1}{3}gf_{abc}A^\rho_b A^\sigma_c\right)\right]$$

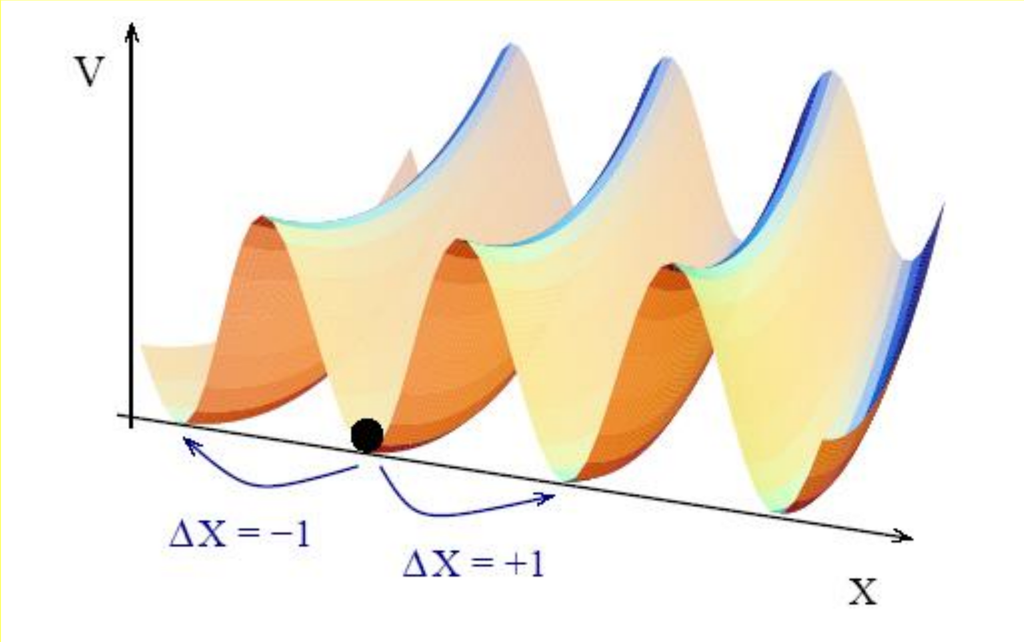


The proton in QCD

- Quarks, antiquarks and gluon partons

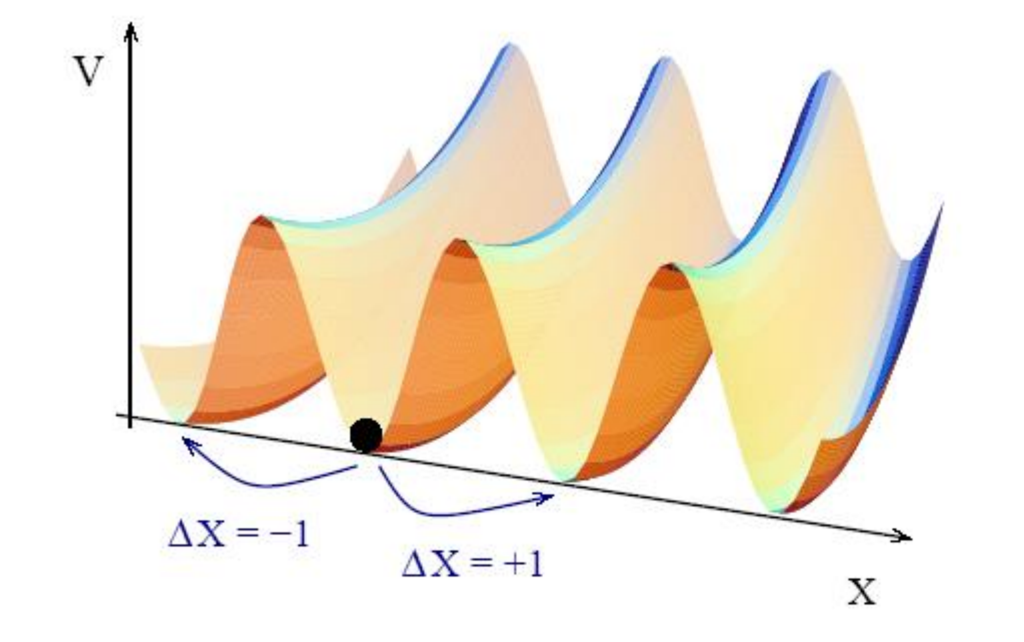


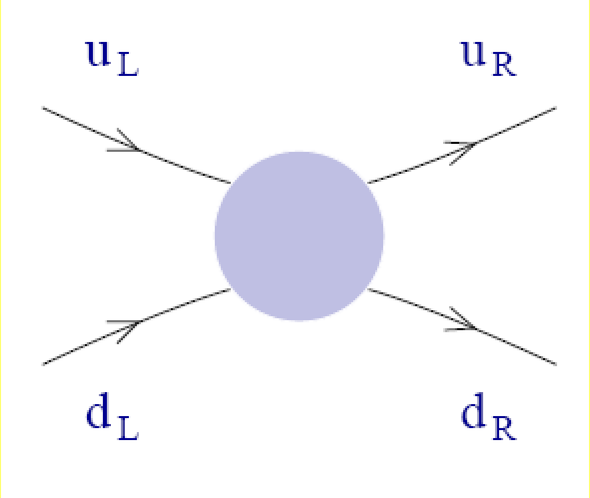
- Beyond pQCD, QCD vacuum is Bloch superposition of vacuum states with different topological winding numbers, and different chiralities



Example of x=0 polarization

- Vacuum tunneling processes (instantons, … and interaction with QCD theta vacuum)





- Chirality of moving quarks gets flipped but the total is conserved

  → Is absorbed into the vacuum and

  → Spin asymmetry of moving partons gets washed out with spin shifted to x=0

  → „x=0" carries some of the spin !

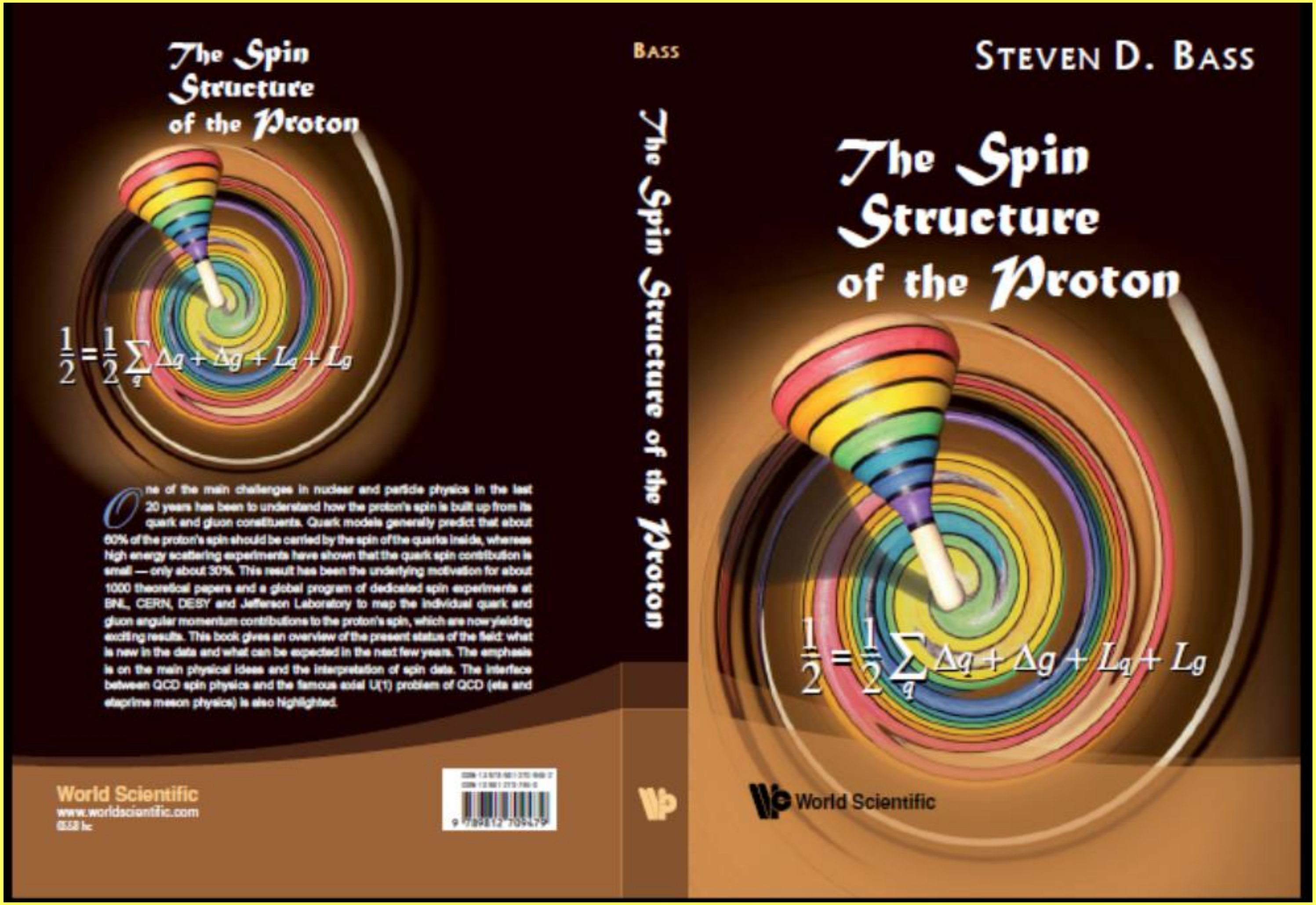
  (corresponds to subtraction constant in dispersion relation for g_1)

Understanding the proton spin

- Non-perturbative physics is important !
  - SU(3) breaking through pion cloud
  - Role of gluon topology in dynamical symmetry breaking and the transition from current to constituent quarks
  - Spin transferred from (valence) quarks to the QCD „theta-vacuum“
- SIDIS data + RHIC Spin → Glue and sea polarization appears small
- Proton spin puzzle is „valence like“

  → connected with chiral dynamics and complex vacuum structure of QCD in (iso-)singlet channel

Spinning the proton



Subtraction at infinity (?)

- Forward Compton amplitude

$$T_{\mu\nu}(q,p) = i\int d^4z\ e^{iq.z}\langle p,s|\ T(J_\mu(z)J_\nu(0))\ |p,s\rangle$$

$$\begin{aligned} T^A_{\mu\nu} &= \frac{1}{2}(T_{\mu\nu} - T_{\nu\mu}) \\ &= \frac{i}{M^2}\epsilon_{\mu\nu\lambda\sigma}q^\lambda\left[s^\sigma(A_1 + \frac{\nu}{M}A_2) - \frac{1}{M^2}s.qp^\sigma A_2\right]. \end{aligned}$$

- Hadron tensor for polarized deep inelastic

$$W_{\mu\nu} = \frac{1}{\pi}\mathrm{Im}T_{\mu\nu} = \frac{1}{2\pi}\int d^4z\ e^{iq.z}\langle p,s|\ [J_\mu(z),J_\nu(0)]\ |p,s\rangle.$$

$$W^A_{\mu\nu} = \frac{i}{M^2}\epsilon_{\mu\nu\lambda\sigma}q^\lambda\left[s^\sigma(G_1 + \frac{\nu}{M}G_2) - \frac{1}{M^2}s.qp^\sigma G_2\right]$$

- Scaling structure functions

$$\begin{aligned} \frac{\nu}{M}\ G_1(\nu,Q^2) &\rightarrow g_1(x,Q^2) \\ \frac{\nu^2}{M^2}\ G_2(\nu,Q^2) &\rightarrow g_2(x,Q^2). \end{aligned}$$

Dispersion relation for polarized DIS

- Crossing symmetry

$$A_1^*(Q^2,-\nu) = A_1(Q^2,\nu)$$
$$A_2^*(Q^2,-\nu) = -A_2(Q^2,\nu).$$

$$G_1(Q^2,-\nu) = -G_1(Q^2,\nu)$$
$$G_2(Q^2,-\nu) = +G_2(Q^2,\nu).$$

$$g_1(x,Q^2) = +g_1(-x,Q^2)$$
$$g_2(x,Q^2) = +g_2(-x,Q^2).$$

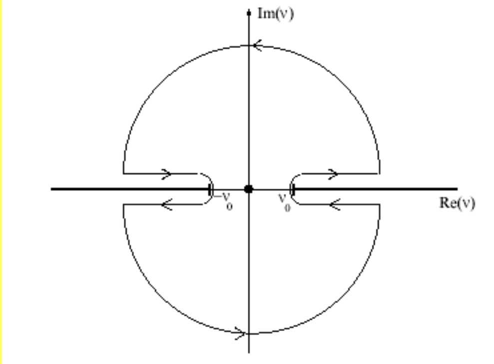
- Define

$$\alpha_1(\omega,Q^2) = \frac{\nu}{M} A_1$$
$$\alpha_2(\omega,Q^2) = \frac{\nu^2}{M^2} A_2.$$

$$\frac{\nu}{M} G_1(\nu,Q^2) \rightarrow g_1(x,Q^2)$$
$$\frac{\nu^2}{M^2} G_2(\nu,Q^2) \rightarrow g_2(x,Q^2).$$

- Dispersion relation with possible finite subtraction at infinity

$$A_1(Q^2,\nu) = \mathcal{P}_1(\nu,Q^2) + \frac{2}{\pi}\int_{Q^2/2M}^{\infty} \frac{\nu' d\nu'}{\nu'^2-\nu^2}\,\mathrm{Im}A_1(q^2,\nu')$$



- … or in terms of g_1

$$\alpha_1(\omega,Q^2) = \frac{Q^2}{2M^2}\beta_1(Q^2)\,\omega + 2\omega\int_1^{\infty}\frac{d\omega'}{\omega'^2-\omega^2}\,g_1(\omega',Q^2)$$

$$\mathcal{P}_1(\nu,Q^2) = \beta_1(Q^2)$$

- Subtraction constant must be non-polynomial in Q^2

Moments and the OPE

- Analytic for $|\omega| \leq 1$ → Taylor series expansion

$$\alpha_1(x,Q^2) = \frac{Q^2}{2M^2}\,\beta_1(Q^2)\,\frac{1}{x} + \frac{2}{x}\sum_{n=0,2,4,..}\left(\frac{1}{x^n}\right)\int_0^1 dy\; y^n g_1(y,Q^2)$$

- Compare result with light-cone OPE

$$T^A_{\mu\nu} = i\epsilon_{\mu\nu\lambda\sigma}q^\lambda \sum_{n=0,2,4,..}\left(-\frac{2}{q^2}\right)^{n+1} q^{\mu_1}q^{\mu_2}...q^{\mu_n} \sum_{i=q,g}\Theta^{(i)}_{\sigma\{\mu_1...\mu_n\}}E^i_n\left(\frac{Q^2}{\mu^2},\alpha_s\right)$$

$$\Theta^{(q)}_{\sigma\{\mu_1...\mu_n\}} \equiv i^n\bar{\psi}\gamma_\sigma\gamma_5 D_{\{\mu_1}...D_{\mu_n\}}\psi - \text{traces}$$

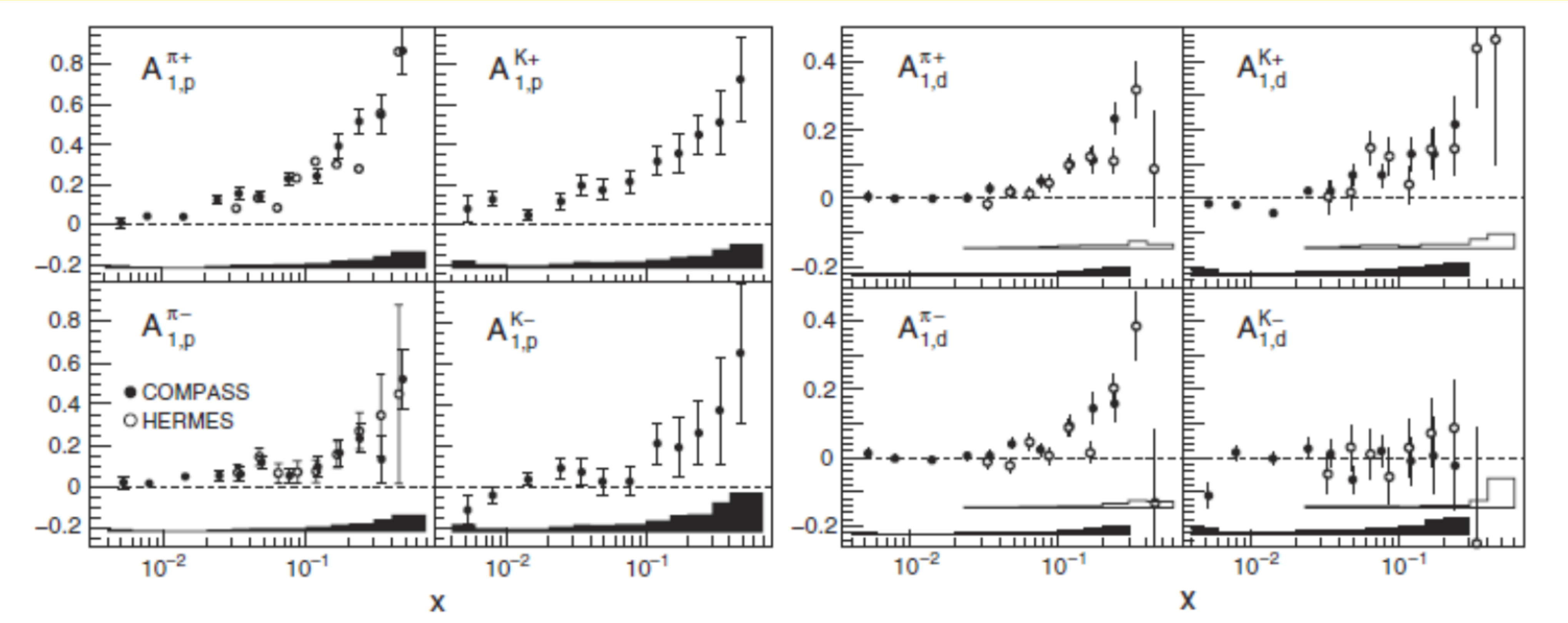
$$\Theta^{(g)}_{\sigma\{\mu_1...\mu_n\}} \equiv i^{n-1}\epsilon_{\alpha\beta\gamma\sigma}G^{\beta\gamma}D_{\{\mu_1}...D_{\mu_{n-1}}G^{\alpha}_{\mu_n\}} - \text{traces.}$$

$$\langle p,s|\Theta_{\{\sigma\mu_1...\mu_n\}}|p,s\rangle = \{s_\sigma p_{\mu_1}...p_{\mu_n} + s_{\mu_1}p_\sigma p_{\mu_2}...p_{\mu_n} + ...\}\frac{a_n}{n+1}$$

- Equating powers of 1/x, the subtraction constant affects just the first moment

$$\int_0^1 dx x^n g_1 = \frac{1}{2}\tilde{a}_n - \delta_{n0}\,\frac{1}{2}\frac{Q^2}{2M^2}\beta_1(Q^2)$$

Semi-inclusive polarised DIS



Experiment	x range	Q^2 (GeV^2)	Δu_v	Δd_v	$\Delta \bar{u}$	$\Delta \bar{d}$
SMC	0.003–0.7	10	$0.73 \pm 0.10 \pm 0.07$	$-0.47 \pm 0.14 \pm 0.08$	$0.01 \pm 0.04 \pm 0.03$	$0.01 \pm 0.04 \pm 0.03$
HERMES	0.023–0.6	2.5	$0.60 \pm 0.07 \pm 0.04$	$-0.17 \pm 0.07 \pm 0.05$	$0.00 \pm 0.04 \pm 0.02$	$-0.05 \pm 0.03 \pm 0.01$
COMPASS	0.006–0.7	10	$0.67 \pm 0.03 \pm 0.03$	$-0.28 \pm 0.06 \pm 0.03$	$0.02 \pm 0.02 \pm 0.01$	$-0.05 \pm 0.03 \pm 0.02$