QED and Fundamental Symmetries in Positronium Decays *

Steven D. Bass

Marian Smoluchowski Institute of Physics, Jagiellonian University,
30-348 Krakow, Poland

Kitzbühel Centre for Physics, Kitzbühel, Austria

We discuss positronium decays with emphasis on tests of fundamental symmetries and the constraints from measurements of other precision observables involving electrons and photons.

1. Introduction

Precision studies of positronium decays are a sensitive probe of QED and allow new tests of fundamental symmetries involving charged leptons. Prime topics for experimental investigation include decay rates, tests of discrete symmetries and looking for possible rare and invisible decays of positronium atoms. With a new generation of positronium experiments coming on-line, it is valuable to review the present status of theory and constraints from other processes on the key observables. This we present here, with main emphasis on physics experiments [1, 2, 3] that can be performed using the new J-PET detector, Jagiellonian Positron Emission Tomograph, being built in Cracow [4]. The J-PET is a new PET device based on plastic scintillators designed for total body scanning in medicine as well as biological applications [5] and fundamental physics research [1] with detection of positronium and (from its decay products) Compton rescattered photons in the detector.

The physics of positronium (an "atom" consisting of an electron and a positron) is described by QED with small radiative corrections from QCD and weak interaction effects in the Standard Model. Positronium comes in two ground states, $^1S_0$ para-positronium, denoted p-Ps, and $^3S_1$ ortho-positronium, denoted o-Ps. Spin-zero p-Ps has lifetime 125 picoseconds and

* Lecture presented at the 3rd Symposium on Positron Emission Tomography, Cracow, September 10-13 2018.

spin-one o-Ps has lifetime 142 nanoseconds. Reviews of positronium physics are given in [6, 7, 8].

Measurements of positronium decay rates are consistent with QED theoretical predictions although the present experimental uncertainties $\sim \mathcal{O}(10^{-4})$ are very much larger than the theoretical uncertainties on the QED calculations, by a factor of 100 for o-Ps and by 10,000 for p-Ps, calling for increased experimental precision.

Precision observables in positronium decays can be used to test discrete symmetries C, CP and CPT with charged leptons. Charge conjugation invariance here has been tested up to the level of 10^{-6} [9, 10, 11]. The symmetries CP [12, 13] and CPT [14, 15] have each been tested up to $O(10^{-3})$. QED final state effects in o-Ps decays can mimic CP, T and CPT violation at the level of 10^{-9} to 10^{-10} [16, 14]. Possible invisible decays of positronium are also an interesting topic of investigation. Mirror matter models of dark matter allow a branching ratio for the invisible decay of o-Ps in vacuum to mirror particles up to about 2×10^{-7}, below the present experimental bound of 5.9×10^{-4} [17].

Possible "new physics scenarios" are strongly constrained by studies of other (QED related) observables including precision measurements of the fine structure constant [18, 19] and the electron electric dipole moment (EDM) [20]. Here we discuss the status of this physics taking into account the latest from theory and experimental investigation. Section 2 describes positronium decays in QED. In Section 3 we discuss precision measurements of α and the electron EDM, and their constraints on possible positronium decays. In Section 4 we explore discrete symmetry tests as well as rare and exotic decays, e.g. involving axions and possible invisible decays. Conclusions are given in Section 5.

2. Positronium decays in QED

2.1. o-Ps decay rate

Positronium properties such as decay rates and energy levels can be calculated using the formalism of non-relativistic QED [21]. The o-Ps decay rate within QED has been evaluated to two-loop level. One finds [22]

$$\begin{aligned}\Gamma(\text{o-Ps} \to 3\gamma, 5\gamma) \;=\; & \frac{2(\pi^2-9)\alpha^6 m_e}{9\pi}\Bigg[1 + A\frac{\alpha}{\pi} + \frac{\alpha^2}{3}\ln\alpha \\ & + B\left(\frac{\alpha}{\pi}\right)^2 - \frac{3\alpha^3}{2\pi}\ln^2\alpha + C\frac{\alpha^3}{\pi}\ln\alpha + D\left(\frac{\alpha}{\pi}\right)^3 + ...\Bigg]. \end{aligned} \tag{1}$$

Here $A = -10.286606(10)$, $B = 44.87(26)$ if just the 3 γ decay is included and $B = 45.06(26)$ if we also include the 5γ decay, $C = A/3 - 229/30 + 8\ln 2 = -5.517025(03)$ and D remains to be calculated. There is good convergence of the perturbation expansion. The terms in Eq. (1) evaluate as 7.211167, -0.172303, -0.000630, 0.001753 (11), -0.000032, 0.000024, 0.00000009 D, – see Table XVII of [22]. That is, the NLO (next-to-leading order) term proportional to A is 2% of the leading Born term and the NNLO term proportional to B is just 0.02%. The 5γ decay contributes $0.19(1)\left(\frac{\alpha}{\pi}\right)^2 \approx 10^{-6}$ through the B parameter in Eq. (1) [23, 24]. QED light-by-light contributions to the positronium decays [25] contribute 0.350(4) to the B parameter for o-Ps in units of $\left(\frac{\alpha}{\pi}\right)^2$ times the respective lowest order rate.

Including both the 3 and 5 photon contributions gives the QED decay rate prediction

$$\Gamma = 7.039979(11) \times 10^6 s^{-1} \tag{2}$$

where we here neglect the $\mathcal{O}(\alpha^3)$ term proportional to the unknown constant D [22].

The calculations in [22] are pure QED. There will also be hadronic QCD light-by-light and photon self-energy corrections. Hadronic light-by-light corrections are known to be important in the muon $g-2$ puzzle [26]. In positronium decays these corrections will enter at $\mathcal{O}(\alpha^2)$ in the decay rate with extra suppression factor $(m_e/\mu)^2 \sim 10^{-6}$ in the B parameter with μ a typical hadronic scale, and are well beyond present experimental reach.

The most accurate measurements of o-Ps decays are consistent with each other and with the theoretical prediction, Eq. (2). Kataoka et al. [27] found

$$\Gamma = 7.0401 \pm 0.0007 \times 10^6 s^{-1} \tag{3}$$

with the o-Ps produced in SiO_2 powder, whereas Vallery et al. [28] found

$$\Gamma = 7.0404 \pm 0.0010 \pm 0.0008 \times 10^6 s^{-1} \tag{4}$$

working in vacuum.

These results are consistent with the QED theory prediction, Eq.(1) with the caveat that the present experimental uncertainties on the decay rate are about 100 times greater than the theoretical error. The leading $\mathcal{O}(\alpha)$ correction in Eq. (1) is needed to agree with the data, the $\mathcal{O}(\alpha^2)$ terms are of order the same size as the experimental error and the $\mathcal{O}(\alpha^3)$ terms are well within the experimental uncertainties. Five photon decay measurements are consistent both with zero and with theoretical expectations. Matsumoto *et al.* [29] found

$$Br(\text{o-Ps} \to 5\gamma) = \left[2.2^{+2.6}_{-1.6} \pm 0.5\right] \times 10^{-6} \tag{5}$$

and Vetter and Freedman [30] found

$$Br(\text{o-Ps} \to 5\gamma) = \left[1.67 \pm 0.99 \pm 0.37\right] \times 10^{-6}. \tag{6}$$

The energy spectrum for the o-Ps to 3γ decay is discussed [31, 32] with formulas presented at leading order in α. A first test of the $\mathcal{O}(\alpha)$ correction for the energy spectrum for orthopositronium decay is discussed in [33], where the NLO term is needed at 92% C.L.

2.2. p-Ps decay rate

For p-Ps the QED prediction is [34, 35, 36]

$$\begin{aligned}\Gamma(\text{p-Ps} \to 2\gamma) \; = \; & \frac{\alpha^5 m_e}{2}\Bigg[1 + \frac{\alpha}{\pi}\left(\frac{\pi^2}{4} - 5\right) + \left(\frac{\alpha}{\pi}\right)^2 \left[-2\pi^2 \ln\alpha + 5.1243(33)\right] \\ & + \frac{\alpha^3}{\pi}\left[-\frac{3}{2}\ln^2\alpha + 7.9189 \ln\alpha + \frac{D_p}{\pi^2}\right]\Bigg] + ...\end{aligned} \tag{7}$$

The 4γ decay contributes an extra $0.274290(8)\left(\frac{\alpha}{\pi}\right)^2$ in units of $\frac{\alpha^5 m_e}{2}$ [36]. Summing these contributions gives

$$\Gamma_p = 7989.6178(2) \times 10^6 s^{-1} \tag{8}$$

where we neglect the term proportional to the unknown D_p coefficient. This value compares with the experimental result [37]

$$\Gamma_p = 7990.9(1.7) \times 10^6 s^{-1}. \tag{9}$$

The experimental error is 10,000 times the size of the QED theoretical error.

While not needed by present data, new physics possibilities might be explored within the uncertainties on the total decay rates, presently corresponding to branching ratios at the $\mathcal{O}(10^{-4})$ level. There are strong constraints from other processes including precision measurements of the fine structure constant α and the electron EDM, which we discuss in Section 3.

3. QED tests and α measurements

The most accurate determinations of α come from precision measurements of the electron's anomalous magnetic moment, atom interferometry measurements with Caesium, Cs, and Rubidium, Rb, and the Quantum Hall Effect.

The electron anomalous magnetic moment $a_e = (g-2)/2$ is generated by radiative corrections, which have been evaluated to tenth-order in QED perturbation theory plus small QCD and weak contributions. The electron a_e value gives a precision measurement of α (modulo any radiative corrections from new physics beyond the Standard Model). Atom interferometry experiments with Cs and Rb and Quantum Hall Effect measurements provide a more direct determination (less sensitive to details of radiative corrections) but also involve a combination of parameters measured in experiments. The Cs measurements are presently the most accurate. Comparing these different determinations of α gives a precision test of QED as well as constraining possible new physics scenarios. Any "beyond the Standard Model" effects involving new particles active in radiative corrections will enter a_e but not the Cs, Rb and Quantum Hall Effect measurements.

The anomalous magnetic moment a_e is related to α through [38]

$$\begin{aligned} a_e^{\rm QED,e} &= \frac{\alpha}{2\pi} - 0.328478965579193...\Big(\frac{\alpha}{\pi}\Big)^2 + 1.181241456587...\Big(\frac{\alpha}{\pi}\Big)^3 \\ &\quad -1.912245764...\Big(\frac{\alpha}{\pi}\Big)^4 + 6.675(192)\Big(\frac{\alpha}{\pi}\Big)^5 + ... \end{aligned} \qquad (10)$$

from Feynman diagrams involving electrons and photons. Contributions from heavy leptons sum to

$$a_e(\text{QED: mass-dependent}) = 2.7475719(13) \times 10^{-12} \qquad (11)$$

with extra electroweak and QCD corrections

$$a_e^{\rm SM} = a_e^{\rm QED} + 0.03053(23) \times 10^{-12}\ (\text{weak}) + 1.6927(120) \times 10^{-12}\ (\text{hadronic}). \qquad (12)$$

The most accurate measurement of a_e comes from the Harvard group, Gabrielse et al. [18]

$$a_e^{\rm exp} = 0.00115965218073(28). \qquad (13)$$

Plans for new, even more accurate, measurements are discussed in [39]. Using the theoretical formulae (10–12), the α value extracted from $a_e^{\rm SM}$ is [38]

$$1/\alpha|_{a_e^{\rm SM}} = 137.0359991491(331). \qquad (14)$$

For direct measurements of α, the most accurate comes from Cs interferometry [19]

$$1/\alpha|_{\rm Cs} = 137.035999046(27) \qquad (15)$$

with

$$1/\alpha|_{\rm Rb} = 137.035998996(85) \qquad (16)$$

from ^{87}Rb atom interferometry [40, 41] quoted by CODATA [42, 43]. Note that these Cs and Rb results rely on a number of other experimental quantities involving the Rydberg constant R_∞, the ratio of the atom to electron

Process	$1/\alpha$	Reference
a_e^{SM}	137.035 999 1491 (331)	[38]
$h/m(^{133}\mathrm{Cs})$	137.035 999 046 (27)	[19]
$h/m(^{87}\mathrm{Rb})$	137.035 998 995 (85)	[43]
QHE	137.036 003 7(33)	[42]

Table 1. Values of α extracted from different experiments.

mass m_{atom}/m_e and new precision measurements of the Cs or Rb masses from recoil of a Cs or Rb atom in an atomic lattice, *viz.*

$$\alpha^2 = \frac{2R_\infty}{c} \frac{m_{\mathrm{atom}}}{m_e} \frac{h}{m_{\mathrm{atom}}}. \tag{17}$$

Here c is the speed of light and h is Planck's constant. Quantum Hall Effect experiments yield the value (CODATA [42])

$$1/\alpha|_{\mathrm{QHE}} = 137.0360037(33). \tag{18}$$

The Hall conductivity of two dimensional electron systems is quantised in integral multiples of e^2/h [44]. Interest in the ratio e^2/h lies not only in its application as an "atomic" resistance standard based on fundamental constants but also as a method to measure α. The different measurements of $1/\alpha$ are collected in Table 1.

The new most accurate Cs atomic physics measurement corresponds to

$$a_e^{\mathrm{exp}} - a_e^{\mathrm{th}}|_{\mathrm{Cs}} = (-88 \pm 36) \times 10^{-14} \tag{19}$$

when we substitute the α value in Eq. (15) into Eqs. (10–12) to obtain the value $a_e^{\mathrm{th}}|_{\mathrm{Cs}}$. Suppose we interpret this 2.5 σ "discrepancy" as an upper limit on contributions coming from new physics, $\Delta a_e^{\mathrm{New\ Physics}} \equiv a_e^{\mathrm{exp}} - a_e^{\mathrm{th}}$. If this originates from new heavy exchanges with coupling constant g_X

$$|\Delta a_e^{\mathrm{New\ Physics}}| \approx \frac{1}{2\pi} \frac{g_X^2}{4\pi} m_e^2 \ / \ \Lambda^2 \tag{20}$$

fixes Λ bigger than ~ 40 GeV (much below collider constraints) with $\frac{g_X^2}{4\pi} \sim 4/137 \simeq 0.03$ (that is, taking coupling constants of order the Standard Model ones). If one instead assumes a new light particle with mass $m_X^2 \ll m_e^2$, then [8]

$$|\Delta a_e^{\mathrm{New\ Physics}}| \approx \frac{1}{2\pi} \frac{g_X^2}{4\pi}. \tag{21}$$

Taking the numbers in Eq. (19), the upper bound on $g_X^2/4\pi$ is about 6 $\times 10^{-12}$, a factor of 50 reduction from the earlier analysis in [8]. This upper bound constrains the branching ratios for possible decays of o-Ps to a photon and very-light mass pseudoscalar and to two photons and a new light vector boson to be $< 10^{-6}$ and $< 10^{-9}$ respectively. Further constraints on pseudoscalar axion models come from astrophysics and laboratory experiments, see Section 4.3 below. In a recent paper [45] the difference in Eq.(19) and the muon $g-2$ anomaly [46] are interpreted together through introduction of a new scalar with mass bigger than about 250 MeV and couplings to the muon and electron of $\sim 10^{-3}$ and a few times 10^{-4}.

New, most accurate, measurements of the electron EDM [20] give

$$|d_e| < 1.1 \times 10^{-29} e\mathrm{cm}. \tag{22}$$

A finite value of d_e would correspond to some new CP violating interaction. Within typical extensions of the Standard Model involving new heavy particles, the electron EDM constraint puts limits on the mass scales of this new physics as 30 TeV in one-loop calculations and 3 TeV at two-loops, strong constraints on new physics models which are competitive with the constraints from the LHC [20]. (These numbers are obtained assuming similar size couplings to the Standard Model ones and $\sin\phi_{\rm CP} \sim 1$, where $\phi_{\rm CP}$ is the CP violating phase). Changing from possible new heavy particles to exchanges involving new near-massless particles corresponds to an upper bound on their coupling to electrons of $g_X^2/4\pi \sim 8 \times 10^{-18}$ in the leading-order calculation and $\sim 5 \times 10^{-9}$ within two-loop calculations for CP violating interactions (evaluated by rescaling the heavy mass scale in the calcuations to the electron mass and assuming no special phase cancellation in the EDM). The latter bound on $g_X^2/4\pi$ corresponds to an upper bound on the branching ratio for CP violating o-Ps decays of about 10^{-9}.

4. Rare and exotic decays and new physics

4.1. C and P violating decays

Experimental bounds on possible C-violating decays of positronium have been reported by earlier experiments

$$BR(\text{p-Ps} \to 3\gamma/\text{p-Ps} \to 2\gamma) < 2.8 \times 10^{-6} \text{ at } 68\%\text{C.L. [9]} \tag{23}$$

$$BR(\text{o-Ps} \to 4\gamma/\text{o-Ps} \to 3\gamma) < 2.6 \times 10^{-6} \text{ at } 90\%\text{C.L. [10]} \tag{24}$$

$$BR(\text{p-Ps} \to 5\gamma/\text{p-Ps} \to 2\gamma) < 2.7 \times 10^{-7} \text{ at } 90\%\text{C.L. [11]} \tag{25}$$

J-PET will push these limits. With a 10 MBq positronium source and upgraded 4 layer detector geometry, one expects to measure 9.4×10^{10} o-Ps to 3 photon decays and 3×10^{11} p-Ps to two photon decays in 365 days of data taking [47].

QED forbidden decays can proceed through weak interactions but with very small branching ratios because of the massive W and Z boson propagators that appear in these reactions. For example, the three photon decay of p-Ps is forbidden in QED but can occur through weak interactions involving a W-boson loop with branching ratio $BR(\text{p-Ps} \rightarrow 3\gamma) = 4.4 \times 10^{-77}$ [48]. Tiny branching ratios are found for decays into a photon and two neutrinos [49], less than about 10^{-21} for o-Ps and 10^{-24} for p-Ps. Exotic decays to a single photon and possible light mass "dark photon" (dark matter candidate) have been postulated with branching ratio up to $\mathcal{O}(10^{-10})$ [50].

4.2. CP violation

After the electron EDM, Eq. (22), o-Ps decays are an experimentally clean system to look for CP violation with charged leptons [51]. The observed matter antimatter asymmetry in the Universe requires some extra source of CP violation beyond the quark mixing described by the Cabbibo-Kobayashi-Maskawa (CKM) matrix in the electroweak Standard Model. Recent measurements by the T2K Collaboration in Japan are consistent with CP violation in the neutrino sector at the level of two standard deviations [52]. So far there is no hint from experiments for CP violation with charged leptons. The electron EDM places strong constraints on any new effect. Searching for CP violation in positronium decays is an active topic of investigation. New effects will need to be large to see a signal since usual Standard Model CP violations are very much suppressed.

Spin-one o-Ps decays are sensitive to CP and CPT odd correlations through the spin vector of the o-Ps. Previous experiments have focused on the correlations

$$A_{CP} = \langle (\vec{S}.\vec{k}_1)(\vec{S}.(\vec{k}_1 \times \vec{k}_2)) \rangle \quad (26)$$

$$A_{CPT} = \langle \vec{S}.(\vec{k}_1 \times \vec{k}_2) \rangle \quad (27)$$

measuring the T-odd integrated moments between the polarisation vector $\vec{S}$ of the o-Ps and the momenta of the emitted photons with magnitude $k_1 \geq k_2 \geq k_3$.

New CP and CPT observables enter if we can also measure observables related to Compton scattering of photons from the positronium decay in the detector [1]. The Compton scattering cross section is peaked perpendicular to the electric field and polarisation axis of the incident photon [53, 54]. This leads to defining the polarisation related quantities $\vec{\epsilon_i} = \vec{k_i} \times \vec{k_i}'/k_i k_i'$, where $\vec{k_i}$ and $\vec{k_i}'$ are the momenta of a photon from

Operator	C	P	T	CP	CPT
$\vec{S}\cdot\vec{k}_1$	+	-	+	-	-
$\vec{S}\cdot(\vec{k}_1\times\vec{k}_2)$	+	+	-	+	-
$(\vec{S}\cdot\vec{k}_1)(\vec{S}\cdot(\vec{k}_1\times\vec{k}_2))$	+	-	-	-	+
$\vec{k}_1.\vec{\epsilon}_2$	+	-	-	-	+
$\vec{S}\cdot\vec{\epsilon}_1$	+	+	-	+	-
$\vec{S}\cdot(\vec{k}_2\times\vec{\epsilon}_1)$	+	-	+	-	-

Table 2. Operators for the o-Ps $\to 3\gamma$ process, and their properties with respect to C, P, T, CP and CPT symmetries. Here $\vec{k_1}$ and $\vec{k_2}$ denote momentum vectors of photons ordered according to their magnitude, $k_1 \geq k_2$, $\vec{S}$ is the spin vector for the o-Ps, and $\vec{\epsilon_i} = \vec{k_i}\times\vec{k_i}'/k_i k_i'$ where $\vec{k_i}$ and $\vec{k_i}'$ denote the momentum of the $i^{\rm th}$ photon before and after Compton scattering in the detector [1].

the positronium decay and the rescattered photon from Compton scattering in the detector [1]. These $\vec{\epsilon_i}$ vectors are peaked to lie along the axis of the incident photon polarisation vector. They are even under P and T transformations. One can form new CP and CPT correlations between the o-Ps spin vector, the momenta of the radiated photons and the $\vec{\epsilon_i}$ vectors, see Table 2. The first three rows in Table 2 refer to correlations involving just the o-Ps decay and the second three rows involve correlations between the positronium system and Compton scattering processes in the detector.

We briefly comment on the relation between the $\vec{\epsilon_i}$ and the polarisation vectors $\varepsilon_\mu^{(j)}$ for circularly polarised photons [55, 56]. These polarisation vectors transform under P as $\vec{\varepsilon}^{\,(3)} = \vec{k}/|k| \to -\vec{\varepsilon}^{\,(3)}$ for longitudinally polarised photons and $\vec{\varepsilon}^{\,L}(\vec{k}) = \vec{\varepsilon}^{\,R}(-\vec{k})$ for left- and right-handed circularly polarised photons with definite helicity. That is, parity transformations change the sense of rotation about the flipped momentum axis. Of the two transverse direction components, one is parity even, like $\vec{\epsilon_i}$, with the other parity odd.

The present most accurate measurements are

$$
\begin{aligned}
C_{CP} &= 0.0013 \pm 0.0022, \qquad \text{Ref. [12]} \\
C_{CPT} &= 0.0071 \pm 0.0062, \qquad \text{Ref. [15].}
\end{aligned} \tag{28}
$$

J-PET aims to improve the accuracy here to $\mathcal{O}(10^{-5})$ for CP and CPT observables [1, 57]. Standard Model QED final state interactions can mimic CP and CPT violation at the level of $\mathcal{O}(10^{-9}) - \mathcal{O}(10^{-10})$ [16, 14].

The mass scale for weak interference effects in positronium given by $G_F m_e^2 \sim 10^{-11}$, so one needs a dramatic enhancement to obtain an observable effect. Theoretical conditions needed for observation of CP violation in positronium decays are discussed in [16]. The electron EDM measurements strongly constrain any new sources of CP violation coupled to the electron. Any observation of CP violation in positronium decays in the next generation of experiments would point to some cancellation of CP phases in the EDM.

4.3. Visible exotic decays of o-Ps to a photon and axion

A possible resolution of the strong CP puzzle in QCD is to postulate the existence of a new very-light mass pseudoscalar called the axion [58] which couples through the Lagrangian term

$$\mathcal{L}_a = -\frac{1}{2}\partial_\mu a \partial^\mu a + \frac{a}{M}\frac{\alpha_s}{8\pi}G_{\mu\nu}\tilde{G}^{\mu\nu} + \frac{a}{M}\frac{\alpha}{3\pi}F_{\mu\nu}\tilde{F}^{\mu\nu} + \frac{i f_\psi}{M}\partial_\mu a \; \bar{\psi}\gamma^\mu\gamma_5\psi - ... \quad (29)$$

Here the second and third terms denote coupling to gluons and photons, and the term in ψ denotes possible fermion couplings to the axion a with $f_\psi \sim \mathcal{O}(1)$. The mass scale M plays the role of the axion decay constant and sets the scale for this new physics.

One finds [59, 60]

$$\Gamma(\text{o-Ps} \rightarrow \gamma a) = \frac{8}{3}\; g_a^2 \; \alpha \; |\Psi_n(0)|^2 \; \frac{m^2_{\text{o-Ps}} - m_a^2}{m^4_{\text{o-Ps}}} \quad (30)$$

with $g_a = -2 m_e f_e / M$. Here $\Psi_n(0)$ is the wave function at the origin,

$$|\Psi_n(0)|^2 = (\alpha m_e)^3 / 8\pi n^3 \quad (31)$$

where $n = 1$ for the ground state [61]; m_e, m_a and $m_{\text{o-Ps}}$ are the electron, axion and positronium masses. Eq. (30) becomes

$$\Gamma(\text{o-Ps} \rightarrow \gamma a) = \frac{1}{3\pi}\; \alpha^4 \; m_e^3 \frac{f_e^2}{M^2}\left(1 - \frac{m_a^2}{m^2_{\text{o-Ps}}}\right). \quad (32)$$

Axions are possible dark matter candidates. Constraints from experiments tells us that M must be very large. Laboratory based experiments together with astrophysics and cosmology constraints suggest a favoured QCD axion mass between 1 μeV and 3 meV [62, 63] corresponding to M between 6×10^9 and 6×10^{12} GeV. Taking $f_e \sim \mathcal{O}(1)$ and dividing by the leading order decay rate term from QED in Eq. (1) gives an expected branching ratio for the decay to a photon and axion between about 10^{-28} and 10^{-22}.

In order to see a signal with branching ratio of $\mathcal{O}(10^{-8})$ one would need M close to the TeV scale. The axion mass $m_a \sim 1/M$. If the axion is too heavy it would lead to axions carrying too much energy out of supernova explosions, thereby observably shortening the neutrino arrival pulse length recorded on Earth in contradiction to Sn 1987a data [62].

4.4. Invisible decays

The search for invisible decays is an interesting topic of experimental investigation. Invisible decays can occur in mirror matter models [64]. Here, the o-Ps can oscillate into a virtual photon which then oscillates into an invisible "mirror photon" and "mirror positronium" (with no interaction with the detector). Mirror matter was first proposed in connection with parity violation. The idea is that under spatial inversion particles should transform into parity reflected new mirror states [65], which then restore parity symmetry in nature. The mirror particles would interact with normal particles mainly through gravity and, as such, are dark matter candidates. Oscillations between photons and their mirror partners could proceed through the interaction term [64]

$$\mathcal{L} = \epsilon \; F^{\mu\nu} F^{m}_{\mu\nu}. \tag{33}$$

The upper limit on the mixing term deduced from successful prediction of the primordial ^{4}He abundance by the Standard Model is [66]

$$\epsilon \leq 3 \times 10^{-8}. \tag{34}$$

A value of ϵ between 10^{-10} and 4×10^{-9} has been suggested in models aiming to explain the DAMA anomaly in dark matter physics [67, 68].

The most accurate constraint on a possible invisible decay of o-Ps in vacuum comes from the ETH Zürich group [17]

$$BR(\text{o-Ps} \rightarrow \text{invisible}) < 5.9 \times 10^{-4}, \quad 90\% \text{ C.L.} \tag{35}$$

which is interpreted as a constraint on the mixing parameter, $\epsilon \lesssim \mathcal{O}(10^{-7})$. For experiments in medium one also has to take into account o-Ps collisions within the apparatus which act as a source of decoherence and dilute the accuracy of the measurement of ϵ [69]. Next generation measurements aim to probe possible branching ratios of $\mathcal{O}(10^{-8})$ corresponding to mixing with $\epsilon \sim 4 \times 10^{-9}$.

5. Conclusions

Precision measurements of positronium decays provide tests of fundamental symmetries with charged leptons. The new generation experiments will have several orders of magnitude improvement on precision compared to previous measurements. The large relative error on the experimental decay rates compared to the QED theory uncertainties demands improved experimental accuracy. Constraints from other observables tell us that any new effects will be small with branching ratios for processes beyond the most simple QED decays most likely starting at $\mathcal{O}(10^{-6})$.

Acknowledgements

I thank C. Aidala, N. Krawczyk, W. Krzemien, P. Moskal, J. Raj and M. Silarski for stimulating discussions.

REFERENCES

[1] P. Moskal *et al.*, Acta Phys. Polon. B **47** (2016) 509

[2] D. Kaminska *et al.*, Eur. Phys. J. C **76** (2016) 445

[3] A. Gajos *et al.*, Nucl. Instrum. Meth. A **819** (2016) 54

[4] P. Moskal *et al.*, Nucl. Instrum. Meth. A **764** (2014) 317; P. Moskal *et al.*, Nucl. Instrum. Meth. A **775** (2015) 54; M. Palka *et al.*, JINST **12** (2017) P08001; G. Korcyl *et al.*, IEEE Transactions On Medical Imaging **37** (2018) 2526

[5] P. Moskal *et al.*, Phys. Med. Biol. **61** (2016) 2025; Sz. Niedzwiecki *et al.*, Acta Phys. Polon. B **48** (2017) 1567; P. Kowalski *et al.*, Phys. Med. Biol. **63** (2018) 165008; P. Moskal *et al.*, `arXiv:1805.11696 [physics.ins-det]` (to appear in Phys. Med. Biol.)

[6] D. B. Cassidy, Eur. Phys. J. D **72** (2018) 53.

[7] S. G. Karshenboim, Phys. Rept. **422** (2005) 1

[8] S. N. Gninenko, N. V. Krasnikov and A. Rubbia, Mod. Phys. Lett. A **17** (2002) 1713.

[9] A. P. Mills and S. Berko, Phys. Rev. Lett. **18**, 420 (1967).

[10] J. Yang *et al.*, Phys. Rev. A **54**, 1952 (1996).

[11] P. A. Vetter and S. J. Freedman, Phys. Rev. A **66**, 052505 (2002).

[12] T. Yamazaki, T. Namba, S. Asai and T. Kobayashi, Phys. Rev. Lett. **104** (2010) 083401; Erratum: [Phys. Rev. Lett. **120** (2018) 239902]

[13] M. Skalsey and J. Van House, Phys. Rev. Lett. **67**, 1993 (1991).

[14] B. K. Arbic et al., Phys. Rev. **A 37**, 3189 (1988)

[15] P. A. Vetter and S. J. Freedman, Phys. Rev. Lett. **91**, 263401 (2003).

[16] W. Bernreuther, U. Low, J. P. Ma and O. Nachtmann, Z. Phys. C **41** (1988) 143.

[17] C. Vigo, L. Gerchow, L. Liszkay, A. Rubbia and P. Crivelli, Phys. Rev. D **97** (2018) 092008

[18] D. Hanneke, S. Fogwell and G. Gabrielse, Phys. Rev. Lett. **100** (2008) 120801

[19] R. H. Parker, C. Yu, W. Zhong, B. Estey and H. Müller, Science **360** (2018) 191

[20] V. Andreev *et al.* [ACME Collaboration], Nature **562** (2018) 355.

[21] P. Labelle, hep-ph/9209266.

[22] G. S. Adkins, R. N. Fell and J. Sapirstein, Ann. Phys. **295** (2002) 136.

[23] G. P. Lepage, P. B. Mackenzie, K. H. Streng and P. M. Zerwas, Phys. Rev. A **28** (1983) 3090.

[24] G. S. Adkins and F. R. Brown, Phys. Rev. A **28** (1983) 1164.

[25] G. S. Adkins, R. N. Fell and J. Sapirstein, Phys. Rev. A **63** (2001) 032511.

[26] F. Jegerlehner, *The Anomalous Magnetic Moment of the Muon*, Springer Tracts Mod. Phys. **274** (2017) pp.1.

[27] Y. Kataoka, S. Asai and T. Kobayashi, Phys. Lett. B **671** (2009) 219

[28] R. S. Vallery, P. W. Zitzewitz and D. W. Gidley, Phys. Rev. Lett. **90** (2003) 203402.

[29] T. Matsumoto, M. Chiba, R. Hamatsu, T. Hirose, J. Yang and J. Yu, Phys. Rev. A **54** (1996) 1947.

[30] P. A. Vetter, S. J. Freedman, Phys. Rev. Lett. **91** (2003) 263401.

[31] V. B. Berestetskii, E. M. Lifshitz and L. P. Pitaevskii, *Quantum Electrodynamics*, Section 89 (Pergamon Press, 1982)

[32] A. Ore and J. L. Powell, Phys. Rev. **75** (1949) 1696.

[33] S. Adachi, T. Yamaji, A. Ishida, T. Namba, S. Asai and T. Kobayashi, J. Phys. Conf. Ser. **618** (2015) 012007

[34] B. A. Kniehl and A. A. Penin, Phys. Rev. Lett. **85** (2000) 1210; Erratum: [Phys. Rev. Lett. **85** (2000) 3065]

[35] K. Melnikov and A. Yelkhovsky, Phys. Rev. D **62** (2000) 116003

[36] G. S. Adkins, N. M. McGovern, R. N. Fell and J. Sapirstein, Phys. Rev. A **68** (2003) 032512

[37] A. H. Al-Ramadhan and D. W. Gidley, Phys. Rev. Lett. **72** (1994) 1632.

[38] T. Aoyama, T. Kinoshita and M. Nio, Phys. Rev. D **97** (2018) 036001

[39] G. Gabrielse, S. F. Hoogerheide, J. Dorr and E. Novitski, Springer Tracts Mod. Phys. **256** (2014) 1.

[40] R. Bouchendira, P. Clade, S. Guellati-Khelifa, F. Nez and F. Biraben, Phys. Rev. Lett. **106** (2011) 080801

[41] R. Bouchendira, P. Clad, S. Guellati-Khlifa, F. Nez and F. Biraben, Annalen Phys. **525** (2013) 484

[42] P. J. Mohr, D. B. Newell and B. N. Taylor, Rev. Mod. Phys. **88** (2016) 035009

[43] P. J. Mohr, D. B. Newell, B. N. Taylor and E. Tiesinga, Metrologia **55** (2018) 125

[44] K. von Klitzing, G. Dorda and M. Pepper, Phys. Rev. Lett. **45** (1980) 449

[45] H. Davoudiasl and W. J. Marciano, Phys. Rev. D **98** (2018) 075011

[46] G. W. Bennett *et al.* [Muon g-2 Collaboration], Phys. Rev. D **73** (2006) 072003

[47] P. Moskal, private communication

[48] A. Pokraka and A. Czarnecki, Phys. Rev. D **96** (2017) 093002

[49] A. Pokraka and A. Czarnecki, Phys. Rev. D **94** (2016) 113012

[50] J. Prez-Ros and S. T. Love, Eur. Phys. J. D **72** (2018) 44.

[51] I. I. Bigi and A. I. Sanda, *CP violation*, Camb. Monogr. Part. Phys. Nucl. Phys. Cosmol. (Cambridge U.P., 2009)

[52] K. Abe *et al.* [T2K Collaboration], Phys. Rev. Lett. **121** (2018) 171802

[53] P. Moskal *et al.*, Eur. Phys. J. C **78** (2018) 970

[54] O. Klein and Y. Nishina, Z. Physik **52** (1929) 853.

[55] J. D. Bjorken and S. D. Drell, *Relativistic Quantum Fields* (McGraw-Hill, 1965).

[56] M. Sozzi, *Discrete Symmetries and CP Violation* (Oxford UP, 2008).

[57] E, Czerwinski *et al.*, Acta Phys. Polon. B **48** (2017) 1961

[58] S. Weinberg, Phys. Rev. Lett. **40** (1978) 223; F. Wilczek, Phys. Rev. Lett. **40** (1978) 279.

[59] W. Bernreuther and O. Nachtmann, Z. Phys. C **11** (1981) 235.

[60] J. Cleymans and P. S. Ray, Lett. Nuovo Cim. **37** (1983) 569.

[61] K. O. Mikaelian, Phys. Lett. **77B** (1978) 214; Erratum: [Phys. Lett. **105B** (1981) 489].

[62] M. Kawasaki and K. Nakayama, Ann. Rev. Nucl. Part. Sci. **63** (2013) 69

[63] L. Baudis, European Review **26** (2018) 70

[64] S. L. Glashow, Phys. Lett. **167B** (1986) 35.

[65] T. D. Lee and C. N. Yang, Phys. Rev. **104** (1956) 254; A. Salam, Nuovo Cim. **5** (1957) 299; I. Y. Kobzarev, L. B. Okun and I. Y. Pomeranchuk, Sov. J. Nucl. Phys. **3** (1966) 837 [Yad. Fiz. **3** (1966) 1154].

[66] E. D. Carlson and S. L. Glashow, Phys. Lett. B **193** (1987) 168.

[67] R. Cerulli, P. Villar, F. Cappella, R. Bernabei, P. Belli, A. Incicchitti, A. Addazi and Z. Berezhiani, Eur. Phys. J. C **77** (2017) 83

[68] R. Foot, Int. J. Mod. Phys. A **29** (2014) 1430013; Phys. Lett. B **789** (2019) 592

[69] A. Badertscher *et al.*, Phys. Rev. D **75** (2007) 032004