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On the possibility to measure the $\pi^0 \to \gamma\gamma$ decay width and the $\gamma^*\gamma \to \pi^0$ transition form factor with the KLOE-2 experiment

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Abstract A possibility of KLOE-2 experiment to measure the width $\Gamma_{\pi^0\to\gamma\gamma}$ and the $\pi^0\gamma\gamma^*$ form factor $F(Q^2)$ at low invariant masses of the virtual photon in the space-like region is considered. This measurement is an important test of the strong interaction dynamics at low energies. The feasibility is estimated on the basis of a Monte-Carlo simulation. The expected accuracy for $\Gamma_{\pi^0\to\gamma\gamma}$ is at a per cent level, which is better than the current experimental world average and theory. The form factor will be measured for the first time at $Q^2 \le 0.1$ GeV^2 in the space-like region. The impact of these measurements on the accuracy of the pion-exchange contribution to the hadronic light-by-light scattering part of the anomalous magnetic moment of the muon is also discussed.

1 Introduction

The QCD Green function $\langle VVA\rangle$ exhibits the axial anomaly of Adler, Bell and Jackiw [1, 2] (non-conservation of the axial vector current), which is responsible for the decay $\pi^0 \to \gamma\gamma$. The anomaly is a pure one-loop effect (triangle diagram) and receives corrections neither perturbatively [3] nor non-perturbatively [4]. It bridges in QCD the strong dynamics of infrared physics at low energies (pions) with the perturbative description in terms of quarks and gluons at high energies. The anomaly allows therefore to gain insights into the strong interaction dynamics of QCD and has received great attention from theorists over many years. Due to the recent advances, the decay width $\Gamma_{\pi^0\to\gamma\gamma}$ is now predicted with a 1.4% accuracy: $\Gamma^{\text{theor}}_{\pi^0\to\gamma\gamma} = 8.09 \pm 0.11$ eV [5, 6]. The major experimental information on this decay comes from the photo-production of pions on a nuclear target via the Primakoff effect [7]. The most precise value of the pion lifetime cited by PDG [8] comes from a direct decay measurement [9]. It can be related to the two-photon width via the $\pi^0 \to \gamma\gamma$ branching fraction. Until recently, the experimental world average of $\Gamma^{\text{PDG}}_{\pi^0\to\gamma\gamma} = 7.74 \pm 0.48$ eV [8] was only known to 6.2% precision. Due to the poor agreement between the existing data, the PDG error of the width average is inflated (scale factor 2.6) and it gives an additional motivation for new precise measurements. The PrimEx Collaboration, using a Primakoff effect experiment at JLab, has achieved 2.8% precision, reporting the value $\Gamma_{\pi^0\to\gamma\gamma} = 7.82 \pm 0.14 \pm 0.17$ eV [10], but this result is not yet included in the PDG average. There are plans to further reduce the uncertainty to the per cent level.

Though theory and experiment are in a fair agreement, a better experimental precision is needed to really test the theory predictions. The Primakoff effect-based experiments suffer from model dependence due to the contamination by the coherent and incoherent conversions in the strong field of a nucleus [11]. Therefore, a measurement using a completely different method (Sect. 2), which can reach a similar accuracy, is highly desirable.

The first aim of this letter is to demonstrate that a per cent level of precision can be achieved in the measurement of $\Gamma_{\pi^0\to\gamma\gamma}$ by the KLOE-2 experiment at Frascati (Sect. 3),

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where the first phase of data taking (step-0) is expected to have the integrated luminosity of 5 fb^{-1} [12].

Putting the pion on-shell in the QCD Green function $\langle VVA\rangle$, one defines the $\pi^0\gamma^*\gamma^*$ form factor $\mathcal{F}_{\pi^0\gamma^*\gamma^*}(q_1^2, q_2^2)$

$$i\int d^4x\, e^{iq_1\cdot x}\langle 0|T\{j_\mu(x)j_\nu(0)\}|\pi^0(q_1+q_2)\rangle = \varepsilon_{\mu\nu\rho\sigma}q_1^\rho q_2^\sigma \mathcal{F}_{\pi^0\gamma^*\gamma^*}(q_1^2, q_2^2), \tag{1}$$

where j_μ is the electromagnetic current of the light quarks (u, d, s), $\varepsilon_{\mu\nu\rho\sigma}$ is the Levi-Civita symbol and q_1 and q_2 are the 4-momenta of the off-shell photons. The form factor for real photons is related to the $\pi^0 \to \gamma\gamma$ decay width:

$$\mathcal{F}^2_{\pi^0\gamma^*\gamma^*}(q_1^2 = 0, q_2^2 = 0) = \frac{4}{\pi\alpha^2 m_\pi^3}\Gamma_{\pi^0\to\gamma\gamma}. \tag{2}$$

The form factor $\mathcal{F}_{\pi^0\gamma^*\gamma^*}(q_1^2, q_2^2)$ as a function of both photon virtualities has never been studied experimentally in the space-like region, and studied with very limited accuracy in the time-like region [13]. The pion-photon transition form factor $F(Q^2)$ with one on-shell and one off-shell photon

$$F(Q^2) \equiv \mathcal{F}_{\pi^0\gamma^*\gamma^*}(-Q^2, q_2^2 = 0), \quad Q^2 \equiv -q_1^2 \tag{3}$$

has been measured in the experiments CELLO [14], CLEO [15] and BaBar [16] at large space-like momenta ($Q^2 \geq 0.5\ \text{GeV}^2$).

The second aim of this letter is to show that the KLOE-2 experiment can perform the first measurement of $F(Q^2)$ in the space-like region in the vicinity of the origin, namely for $0.01 < Q^2 < 0.1\ \text{GeV}^2$ (Sects. 2, 4).

The third aim of this letter is to estimate the impact of the proposed KLOE-2 measurements on the evaluation of the Standard Model prediction for the anomalous magnetic moment of the muon, a_μ (Sect. 5). The theoretical value of a_μ is currently limited by uncertainties from the hadronic vacuum polarization and the hadronic light-by-light (LbyL) scattering contribution. The value of the latter is currently obtained using hadronic models (e.g., [17–20]) and leads to an uncertainty in a_μ of $(26\text{–}40) \times 10^{-11}$ (the Dyson–Schwinger approach [21] is still far from this value), which is almost as large as the one from hadronic vacuum polarization $\sim (40\text{–}50) \times 10^{-11}$ [22, 23]. For comparison, the precision of the Brookhaven $g-2$ experiment is 63×10^{-11} [24]. In view of the proposed new $g-2$ experiments at Fermilab [25] and JPARC [26] with a precision of 15×10^{-11}, the hadronic LbyL contribution needs to be controlled much better, in order to fully profit from these new experiments to test the Standard Model and constrain New Physics. According to model calculations, the exchange of neutral pions yields the numerically dominant contribution, $a_\mu^{\text{LbyL};\pi^0}$, to the final result for hadronic LbyL scattering.

The conclusions are given in Sect. 6.

2 The basics of width and form factor measurement

Since the original proposal of Low [27] to measure the width of a neutral pion decay into two photons using the $e^+e^- \to e^+e^-\pi^0$ process, only at DESY this measurement has been done using this method, with the result $\Gamma_{\pi^0\to\gamma\gamma} = 7.7 \pm 0.5 \pm 0.5$ eV [28]. It was stressed in [29] and [30] that for a precision measurement of π^0 width via the "$\gamma\gamma$ fusion" ($\gamma\gamma \to \pi^0$) process, one needs to improve the original Low proposal. Namely, instead of a no-tag experiment (like [28]), one should perform a lepton double tagging at small angles. For the ϕ-factory DAΦNE a detailed study was performed in [31], where a lepton tagging system was proposed, which, however, was not installed. This proposal was reconsidered for the KLOE-2 [12] experiment at DAΦNE. The two Low Energy Taggers (LET) [32] and two High Energy Taggers (HET) [33] were specially designed for this experiment and will allow detection of electrons and positrons, scattered at very small polar angles ($\theta < \theta_{\max} \approx 1^\circ$) in two domains of lepton energy. For the present letter only information coming from HET is considered. The effect of the LET detectors and of those to be installed in the near future (see Ref. [12]) will be the subject of a forthcoming investigation.

One can extract the value of the partial decay width from data, using the formula

$$\Gamma_{\pi^0\to\gamma\gamma} = \frac{N_{\pi^0}}{\varepsilon\,\mathcal{L}}\,\frac{\tilde{\Gamma}_{\pi^0\to\gamma\gamma}}{\tilde{\sigma}_{e^+e^-\to e^+e^-\pi^0}}, \tag{4}$$

where N_{π^0} is the number of detected pions, ε accounts for the detection acceptance and efficiency, $\mathcal{L}$ is the integrated luminosity, $\tilde{\Gamma}_{\pi^0\to\gamma\gamma}$ is the model π^0 width and $\tilde{\sigma}_{e^+e^-\to e^+e^-\pi^0}$ is the cross section obtained with a Monte Carlo simulation using the same model as for the $\tilde{\Gamma}_{\pi^0\to\gamma\gamma}$ calculation.

The form factor $F(Q^2)$ can be evaluated through the relation

$$\frac{F^2(Q^2)}{F^2(Q^2)_{\text{MC}}} = \frac{(\frac{d\sigma}{dQ^2})_{\text{data}}}{(\frac{d\sigma}{dQ^2})_{\text{MC}}}, \tag{5}$$

where $(\frac{d\sigma}{dQ^2})_{\text{data}}$ is the experimental differential cross section, and $(\frac{d\sigma}{dQ^2})_{\text{MC}}$ is the Monte Carlo one obtained with the form factor $F(Q^2)_{\text{MC}}$.

3 Feasibility of the π^0 width measurement

The π^0 production in the process $e^+e^- \to e^+e^-\pi^0$ is simulated with EKHARA [34] Monte Carlo event generator. The simulated signal is given by the t-channel amplitude ($\gamma^*\gamma^* \to \pi^0$).

Since a stand-alone EKHARA version works in the CM (center of mass) frame of incident leptons and does not simulate the pion decays, it has been modified to take into account the DAΦNE crossing angle between the incoming beams ($\theta_{e^+e^-} \approx 51.3$ mrad) and the decay of the π^0 into two photons.

In order to simulate the DAΦNE optics, EKHARA has been interfaced with the BDSIM package [35] which allows to trace the emitted electron (positron) through the magnetic elements of DAΦNE layout. In this way only few of them (∼2%) reach the HET detectors, providing a realistic estimate of its acceptance. In the following the coincidence of the HET detectors will be required, which selects the energy of the final leptons to be between 420 and 460 MeV.

Figure 1 shows the energy of the emitted π^0 in the $\gamma\gamma$ process: as can be seen, the request of the HET-HET coincidence allows us to select π^0 almost at rest (dark region), compared with the no-tag case (light-gray). Since the π^0 decays almost at rest, most of the photons from its decay are emitted with large polar angle (defined as the angle between the direction of the photon and the beam axis), as shown in Fig. 2. In particular, about 95% of the photons are emitted above 25° (and below 155°), resulting in a large acceptance for photons reaching the KLOE Electromagnetic Calorimeter (EMC) [36].

By requiring both photons in the barrel of the EMC (between 50° and 130°) and the HET-HET coincidence, a value

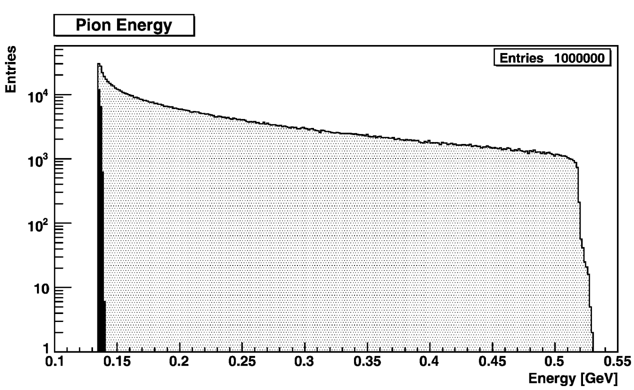

Fig. 1 The π^0 energy (in the laboratory frame) distribution with (*dark*) and without (*light-gray*) HET-HET coincidence

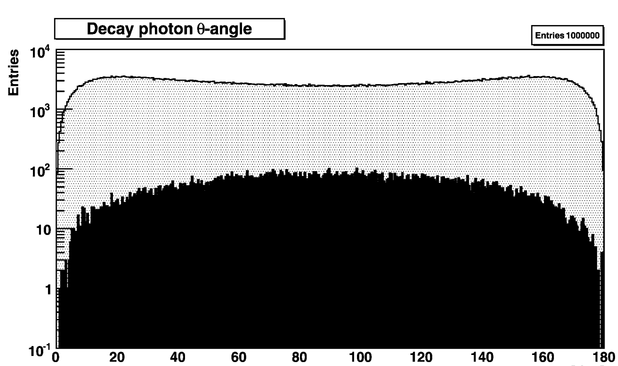

Fig. 2 Polar angle (in the laboratory frame) distribution of decay photons from π^0 with (*dark*) and without (*light-gray*) the HET-HET coincidence

for the acceptance ϵ_{acc} of 1.2% is obtained. Since the total cross section of $e^+e^- \to e^+e^-\pi^0$ at $\sqrt{s} = 1020$ MeV is $\sigma_{tot} \approx 0.28$ nb, a cross section of about 3.4 pb is obtained within the acceptance cuts. The integrated luminosity $\mathcal{L}$ at DAΦNE required to reach a 1% statistical error is

$$\mathcal{L} = \frac{10\,000}{\sigma_{tot}\,\epsilon_{acc}\,\epsilon_{det}} \approx \frac{3}{\epsilon_{det}}\ \mathrm{fb}^{-1}, \qquad (6)$$

where the efficiency ϵ_{det} due to trigger, reconstruction and analysis criteria is estimated to be about 50%. Therefore, the required data sample can be obtained during the first phase (about one year) of data taking.

Extraction of the width $\Gamma(\pi^0 \to \gamma\gamma)$ with ∼1% accuracy requires a very good control of the systematic errors. From the experimental side, the clean signature of the process, the use of the KLOE detector and the HET-HET coincidence should allow to keep the systematic effects under control at the required level of precision. A possible background to this measurement comes from the double radiative Bhabha scattering, which has the same signature as our signal. An extensive simulation of this background (based on 10^8 events generated with Babayaga MC [37, 38]) shows that no events survive the coincidence of HET for electrons and positrons and the KLOE acceptance for photon, and therefore the expected contribution is negligible. From the theoretical side, the systematic errors can arise also from accuracy of the generator, e.g., due to missing radiative corrections and the uncertainty in the modeled $\pi^0\gamma^*\gamma^*$ transition form factor. In order to reduce the former effect, the radiative corrections are planned to be introduced in the EKHARA generator. For the latter effect, a numerical simulation with different formulas for the form factor can be performed. The HET-HET coincidence, imposed in such a simulation, leads to a significant restriction on the photon virtuality in $\gamma^*\gamma^* \to \pi^0$: for most of the events one has $|q^2| < 10^{-4}$ GeV^2, as shown in Fig. 3. Thus, for the KLOE-2 case the possible effect of the photon virtualities which can influence

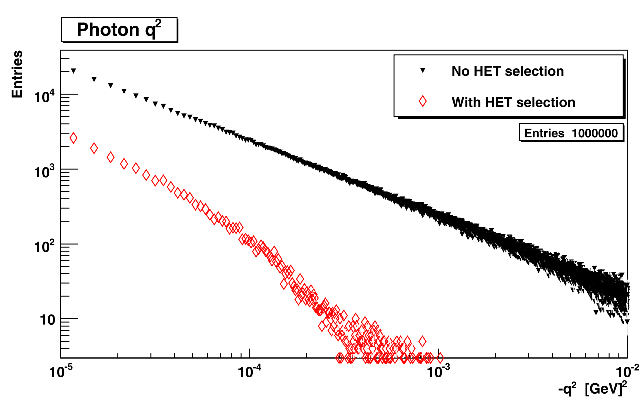

Fig. 3 (Color online) Distribution of the photon virtuality in $\gamma^*\gamma^* \to \pi^0$. The lepton double tagging (HET-HET) selects the events (*red diamonds*) with small virtuality of the photons

the accuracy of (4) is negligible. Our simulation shows that the uncertainty in the measurement of $\Gamma(\pi^0 \to \gamma\gamma)$ due to the form factor parametrization in the generator is expected to be less than 0.1%.

4 Feasibility of the $\gamma^*\gamma\pi^0$ transition form factor measurement

By requiring one lepton inside the KLOE detector ($20° < \theta < 160°$, corresponding to $0.01 < |q_1^2| < 0.1$ GeV^2) and the other lepton in the HET detector (corresponding to $|q_2^2| \lesssim 10^{-4}$ GeV^2 for most of the events) one can measure the differential cross section $(d\sigma/dQ^2)_{\text{data}}$, where $Q^2 \equiv -q_1^2$. Using (5), the form factor $|F(Q^2)|$ can be extracted from this cross section.

The simulation has been performed using a lowest meson dominance ansatz with two vector multiplets (LMD+V) for the form factor $\mathcal{F}_{\pi^0\gamma^*\gamma^*}$, which is available in EKHARA. The LMD+V ansatz is based on large-N_C QCD matched to short-distance constraints from the operator-product expansion (OPE), see the Ref. [39]. In the following we use the definition of the LMD+V parameters $\bar{h}_5 = h_5 + h_3 m_\pi^2$ and $\bar{h}_7 = h_7 + h_6 m_\pi^2 + h_4 m_\pi^4$. Figure 4 shows the expected experimental uncertainty (statistical) on $F(Q^2)$ achievable at KLOE-2 with an integrated luminosity of 5 fb^{-1}. In this measurement the detection efficiency is different and is estimated to be about 20%. From our simulation we conclude that a statistical uncertainty of less than 6% for every bin is feasible.

Having measured the form factor, one can evaluate also the slope parameter a of the form factor at the origin[1]

$$a \equiv m_\pi^2 \frac{1}{\mathcal{F}_{\pi^0\gamma^*\gamma^*}(0,0)} \frac{d\mathcal{F}_{\pi^0\gamma^*\gamma^*}(q^2,0)}{dq^2}\bigg|_{q^2=0}. \tag{7}$$

Though for *time-like* photon virtualities ($q^2 > 0$), the slope can be measured directly in the rare decay $\pi^0 \to e^+e^-\gamma$, the current experimental uncertainty is very big [40, 41]. The PDG average value of the slope parameter is quite precise, $a = 0.032 \pm 0.004$ [8], and it is dominated by the CELLO result [14]. In the latter, a simple vector-meson dominance (VMD) form factor parametrization was fitted to the data [14] and then the slope was calculated according to (7). Thus the CELLO procedure for the slope calculation suffers from model dependence not accounted for in the error estimation. The validity of such a procedure has never been verified, because there were no data at $Q^2 < 0.5$ GeV^2.

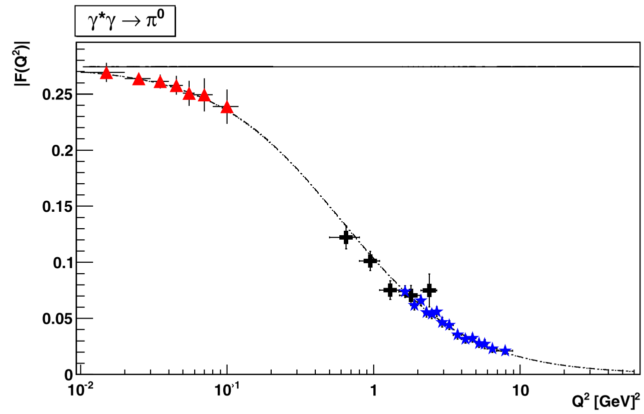

Fig. 4 (Color online) Simulation of KLOE-2 measurement of $F(Q^2)$ (red triangles) with statistical errors for 5 fb^{-1}. *Dashed line* is the $F(Q^2)$ form factor according to LMD+V model [39], *solid line* is $F(0)$ given by Wess–Zumino–Witten term, (8). CELLO [14] (*black crosses*) and CLEO [15] (*blue stars*) data at high Q^2 are also shown for illustration

Therefore, filling of this gap in Q^2 by the KLOE-2 experiment can provide a valuable test of the form factor parametrizations.

When the normalization of the form factor is fixed to the decay width $\pi^0 \to \gamma\gamma$ or to some effective pion decay constant F_π, the VMD and (on-shell) LMD+V models have only one free parameter.[2] For VMD this parameter is the vector-meson mass M_V (sometimes denoted by Λ_{π^0}) and for LMD+V this is $\bar{h}_5$, once we put $h_1 = 0$ to get the $1/Q^2$ behavior for large Q^2, as expected from theoretical arguments [42–44]. It is a priori not clear why only one parameter should be sufficient to describe the behavior of the form factor simultaneously at low momenta (slope at the origin) and at large momenta (asymptotic behavior, related to perturbative QCD / OPE near the light-cone). Since the available data [14–16] cover only the relatively high $Q^2 > 0.5$ GeV^2 region, a new measurement by KLOE-2 at $Q^2 < 0.1$ GeV^2 would help to verify the consistency of the parametrizations of the form factor $F(Q^2)$.

5 Impact on the hadronic light-by-light scattering contribution to the muon $g - 2$

The value of the pion-exchange part $a_\mu^{\text{LbyL};\pi^0}$ of the hadronic LbyL contribution to a_μ is currently obtained using hadronic models and any experimental information on the transition form factor is important in order to constrain the models. However, having a good description for the transition form

[1] We would like to stress that the q^2 range of KLOE-2 measurement is not small enough to use the linear approximation $\mathcal{F}_{\pi^0\gamma^*\gamma^*}(q^2,0) = \mathcal{F}_{\pi^0\gamma^*\gamma^*}(0,0)(1 + q^2 a/m_\pi^2)$ because the higher order terms are not negligible.

[2] In the Brodsky–Lepage ansatz [42–44] the parameter F_π fixes the normalization *and* the asymptotic behavior at the same time. Comparison with data from CELLO and CLEO shows that the asymptotic behavior is off by about 20%, once the normalization is fixed from $\pi^0 \to \gamma\gamma$.

Table 1 Estimate of KLOE-2 impact on the accuracy of $a_\mu^{\mathrm{LbyL};\pi^0}$ in case of one year of data taking (5 fb^{-1}). For calculation we used the Jegerlehner–Nyffeler (JN) [19, 20] and Melnikov–Vainshtein (MV) [17] approaches. The values marked with asterisk (*) do not contain additional uncertainties coming from the "off-shellness" of the pion (see the text). Data sets used for fits (A0, A1, A2, B0, B1, B2)—see the text, (9)

Model	Data	χ^2/d.o.f.		Parameters		$a_\mu^{\mathrm{LbyL};\pi^0}\times 10^{11}$
VMD	A0	6.6/19	$M_V=0.778(18)$ GeV	$F_\pi=0.0924(28)$ GeV		$(57.2\pm 4.0)_{JN}$
VMD	A1	6.6/19	$M_V=0.776(13)$ GeV	$F_\pi=0.0919(13)$ GeV		$(57.7\pm 2.1)_{JN}$
VMD	A2	7.5/27	$M_V=0.778(11)$ GeV	$F_\pi=0.0923(4)$ GeV		$(57.3\pm 1.1)_{JN}$
VMD	B0	77/36	$M_V=0.829(16)$ GeV	$F_\pi=0.0958(29)$ GeV		–
VMD	B1	78/36	$M_V=0.813(8)$ GeV	$F_\pi=0.0925(13)$ GeV		–
VMD	B2	79/44	$M_V=0.813(5)$ GeV	$F_\pi=0.0925(4)$ GeV		–
LMD+V, $h_1=0$	A0	6.5/19	$\bar{h}_5=6.99(32)$ GeV^4	$\bar{h}_7=-14.81(45)$ GeV^6		$(72.3\pm 3.5)^*_{JN}$
						$(79.8\pm 4.2)_{MV}$
LMD+V, $h_1=0$	A1	6.6/19	$\bar{h}_5=6.96(29)$ GeV^4	$\bar{h}_7=-14.90(21)$ GeV^6		$(73.0\pm 1.7)^*_{JN}$
						$(80.5\pm 2.0)_{MV}$
LMD+V, $h_1=0$	A2	7.5/27	$\bar{h}_5=6.99(28)$ GeV^4	$\bar{h}_7=-14.83(7)$ GeV^6		$(72.5\pm 0.8)^*_{JN}$
						$(80.0\pm 0.8)_{MV}$
LMD+V, $h_1=0$	B0	65/36	$\bar{h}_5=7.94(13)$ GeV^4	$\bar{h}_7=-13.95(42)$ GeV^6		–
LMD+V, $h_1=0$	B1	69/36	$\bar{h}_5=7.81(11)$ GeV^4	$\bar{h}_7=-14.70(20)$ GeV^6		–
LMD+V, $h_1=0$	B2	70/44	$\bar{h}_5=7.79(10)$ GeV^4	$\bar{h}_7=-14.81(7)$ GeV^6		–
LMD+V, $h_1\neq 0$	A0	6.5/18	$\bar{h}_5=6.90(71)$ GeV^4	$\bar{h}_7=-14.83(46)$ GeV^6	$h_1=-0.03(18)$ GeV^2	$(72.4\pm 3.8)^*_{JN}$
LMD+V, $h_1\neq 0$	A1	6.5/18	$\bar{h}_5=6.85(67)$ GeV^4	$\bar{h}_7=-14.91(21)$ GeV^6	$h_1=-0.03(17)$ GeV^2	$(72.9\pm 2.1)^*_{JN}$
LMD+V, $h_1\neq 0$	A2	7.5/26	$\bar{h}_5=6.90(64)$ GeV^4	$\bar{h}_7=-14.84(7)$ GeV^6	$h_1=-0.02(17)$ GeV^2	$(72.4\pm 1.5)^*_{JN}$
LMD+V, $h_1\neq 0$	B0	18/35	$\bar{h}_5=6.46(24)$ GeV^4	$\bar{h}_7=-14.86(44)$ GeV^6	$h_1=-0.17(2)$ GeV^2	$(71.9\pm 3.4)^*_{JN}$
LMD+V, $h_1\neq 0$	B1	18/35	$\bar{h}_5=6.44(22)$ GeV^4	$\bar{h}_7=-14.92(21)$ GeV^6	$h_1=-0.17(2)$ GeV^2	$(72.4\pm 1.6)^*_{JN}$
LMD+V, $h_1\neq 0$	B2	19/43	$\bar{h}_5=6.47(21)$ GeV^4	$\bar{h}_7=-14.84(7)$ GeV^6	$h_1=-0.17(2)$ GeV^2	$(71.8\pm 0.7)^*_{JN}$

factor is only necessary, not sufficient, in order to uniquely determine $a_\mu^{\mathrm{LbyL};\pi^0}$.

As pointed out in Refs. [45, 46], what enters in the calculation of the pion-exchange contribution $a_\mu^{\mathrm{LbyL};\pi^0}$ is the fully off-shell form factor $\mathcal{F}_{\pi^{0*}\gamma^*\gamma^*}((q_1+q_2)^2,q_1^2,q_2^2)$ (vertex function), where also the pion is off-shell with 4-momentum q_1+q_2. The form factor defined in (1) with on-shell pions is then given by $\mathcal{F}_{\pi^0\gamma^*\gamma^*}(q_1^2,q_2^2)\equiv\mathcal{F}_{\pi^{0*}\gamma^*\gamma^*}(m_\pi^2,q_1^2,q_2^2)$. A measurement of the transition form factor $\mathcal{F}_{\pi^{0*}\gamma^*\gamma^*}(m_\pi^2,q^2,0)$ can only be sensitive to a subset of the model parameters and in general does not allow to reconstruct the full off-shell form factor. Therefore, within any given approach, the uncertainty of the calculated $a_\mu^{\mathrm{LbyL};\pi^0}$ related to the off-shell pion can be different and the complete error on $a_\mu^{\mathrm{LbyL};\pi^0}$ should take into account this model dependence.

For instance, the estimate in the LMD+V model $a_{\mu;\mathrm{LMD+V}}^{\mathrm{LbyL};\pi^0}=(72\pm 12)\times 10^{-11}$ given in Ref. [19] is based on the variation of all model parameters, where $\bar{h}_5=(6.93\pm 0.26)$ GeV^4 has been used, which was obtained in Ref. [39] from a fit to the CLEO data for the transition form factor $F(Q^2)$. The variation of ± 0.26 GeV^4 in $\bar{h}_5$ only leads to a variation in $a_{\mu;\mathrm{LMD+V}}^{\mathrm{LbyL};\pi^0}$ of $\pm 0.6\times 10^{-11}$. Within the off-shell LMD+V model the variation of the parameters related to the off-shellness of the pion completely dominate the total uncertainty and will *not* be shown in Table 1.

In contrast to the off-shell LMD+V model, many models do not have these additional sources of uncertainty (the VMD model, the ansätze for the transition form factor used in Ref. [47], etc.). Therefore, the precision of the KLOE-2 measurement can dominate the total accuracy of $a_\mu^{\mathrm{LbyL};\pi^0}$ in such models.

We would like to stress that a realistic calculation of $a_\mu^{\mathrm{LbyL};\pi^0}$ is *not* the purpose of this letter. The estimates given below are performed to demonstrate, within several approaches, an improvement of uncertainty, which will be possible when the KLOE-2 data appear. Discussion of the validity of these approaches as well as the form factor modeling is beyond the scope of this letter.

As pointed out in Ref. [48], essentially all evaluations of the pion-exchange contribution (or the pion-pole contribution with on-shell form factors) use the following normalization for the form factor:

$$\mathcal{F}_{\pi^{0*}\gamma^*\gamma^*}\big(m_\pi^2,0,0\big)=1/\big(4\pi^2F_\pi\big) \tag{8}$$

derived from the Wess–Zumino–Witten (WZW) term [49, 50] and the value $F_\pi = 92.4$ MeV is used without any error attached to it.[3] Instead, if one uses the decay width $\Gamma_{\pi^0\to\gamma\gamma}$ for the normalization of the form factor, see eq. (2), an additional source of uncertainty enters. This uncertainty has not been taken into account so far, except in the very recent paper [52].[4] In our calculations we account for this normalization issue, using in the fit:

- $\Gamma^{\mathrm{PDG}}_{\pi^0\to\gamma\gamma} = 7.74 \pm 0.48$ eV [8] for a pre-PrimEx case.
- $\Gamma^{\mathrm{PrimEx}}_{\pi^0\to\gamma\gamma} = 7.82 \pm 0.22$ eV [10] for a pre-KLOE-2 case.
- $\Gamma^{\mathrm{KLOE-2}}_{\pi^0\to\gamma\gamma} = 7.73 \pm 0.08$ eV for a KLOE-2 simulation (assuming a 1% precision, see Sect. 3).

In this section we assume that the KLOE-2 measurement will be consistent with the LMD+V and VMD models. This allows us to use the simulation (Sects. 3 and 4) as new "data" and evaluate the impact of such "data" on the precision of the $a_\mu^{\mathrm{LbyL};\pi^0}$ calculation. In order to do that, we fit the LMD+V and VMD models to the following data sets:

$$\begin{aligned}
&\mathrm{A0: CELLO, CLEO, PDG;}\\
&\mathrm{A1: CELLO, CLEO, PrimEx;}\\
&\mathrm{A2: CELLO, CLEO, PrimEx, KLOE\text{-}2;}\\
&\mathrm{B0: CELLO, CLEO, BaBar, PDG;}\\
&\mathrm{B1: CELLO, CLEO, BaBar, PrimEx;}\\
&\mathrm{B2: CELLO, CLEO, BaBar, PrimEx, KLOE\text{-}2;}
\end{aligned} \tag{9}$$

and evaluate $a_\mu^{\mathrm{LbyL};\pi^0}$.

Some comments about the BaBar data [16] are in order here. This measurement of the pion transition form factor does not show the $1/Q^2$ behavior as expected from earlier theoretical considerations [42–44] and as seen in the CELLO and CLEO data (although in the latter experiments the Q^2 was maybe not yet large enough). The situation is puzzling, since BaBar observes for the η that $Q^2F(Q^2)$ rises about three times slower than for the pion, whereas the transition form factor of the η' shows a $1/Q^2$ fall-off, see Ref. [53]. Though several approaches exist, which claim to be able to reconcile the data of Refs. [16] and [53] (see, e.g., the results of Ref. [54]), the strong obstacles for the theory to confront the data [16] are being widely discussed lately (see [55, 56]).

The VMD model always shows a $1/Q^2$ fall-off and therefore is not compatible with the BaBar data. The LMD+V model has another parameter, h_1, which determines the behavior of the transition form factor for large Q^2. To get the $1/Q^2$ behavior according to Brodsky–Lepage [42–44], one needs to set $h_1 = 0$. However, one can simply leave h_1 as a free parameter and fit it to the BaBar data, yielding $h_1 \neq 0$ [48]. In this case the form factor does not vanish at $Q^2 \to \infty$. Since VMD and LMD+V with $h_1 = 0$ are not compatible with the BaBar data (as can be seen from the large χ^2 per degree of freedom of the fits of the data sets B0, B1 and B2 below), we will not evaluate $a_\mu^{\mathrm{LbyL};\pi^0}$ for these cases.

For illustration, we use the following two approaches to calculate $a_\mu^{\mathrm{LbyL};\pi^0}$:

- Jegerlehner–Nyffeler (JN) approach [19, 20] with the off-shell pion form factor.
- Melnikov–Vainshtein (MV) approach [17], where one uses the on-shell pion form factor in one vertex and the other vertex is constant (WZW).

Table 1 shows the impact of the (existing) PrimEx and the (future) KLOE-2 measurements on the model parameters (e.g., the normalization of the form factor) and, consequently, on the $a_\mu^{\mathrm{LbyL};\pi^0}$ uncertainty. The errors of the fitted parameters are the `MINOS` (`MINUIT` from `CERNLIB`) parabolic errors. The other parameters of the (on-shell and off-shell) LMD+V model have been chosen as in the papers [17, 19, 20]. We would like to stress again that our estimate of the $a_\mu^{\mathrm{LbyL};\pi^0}$ uncertainty is given only by the propagation of the errors of the newly fitted parameters listed in Table 1 and therefore we may not reproduce the total uncertainties of $a_\mu^{\mathrm{LbyL};\pi^0}$ given in the original papers.

We can clearly see from Table 1 that for each given model and each approach (JN or MV), there is a trend of reduction in the error for $a_\mu^{\mathrm{LbyL};\pi^0}$ (related only to the given model parameters) by about half when going from A0 (PDG) to A1 (including PrimEx) and by about another half when going from A1 to A2 (including KLOE-2). This is mainly due to the improvement in the normalization of the form factor (decay width $\pi^0 \to \gamma\gamma$), controlled by the parameters F_π or $\bar{h}_7$, respectively, but more data also better constrain the other model parameters M_V or $\bar{h}_5$, respectively.

This trend of improvement is also visible in the last part of the table (LMD+V, $h_1 \neq 0$), when we fit the sets B0, B1 and B2 which include the BaBar data. Furthermore, since we now have even more data to fit, the final error on $a_\mu^{\mathrm{LbyL};\pi^0}$ is improved further, compared to the fits of LMD+V with $h_1 \neq 0$ of the data sets A0, A1 and A2 only. This can be seen in the errors of $\bar{h}_5$ and h_1. On the other hand, the parameter $\bar{h}_7$, related to the normalization of the form factor at the origin, is essentially unchanged by the inclusion of the BaBar

[3] Note that this value of F_π is close to $F_\pi = (92.2 \pm 0.14)$ MeV, as derived from $\pi^+ \to \mu^+\nu_\mu(\gamma)$ with 0.15% precision [8]. It leads, according to (8) and (2), to $\Gamma(\pi^0 \to \gamma\gamma) = 7.73$ eV, which is consistent with the current PDG average. For a detailed discussion of F_π and $\Gamma(\pi^0 \to \gamma\gamma)$ see, e.g., [5, 6, 51].

[4] In [52] the experimental uncertainty of the decay width on $a_\mu^{\mathrm{LbyL};\pi^0}$ has been estimated within the context of the nonlocal chiral quark model.

data at high Q^2. The central values of the final results for $a_\mu^{\mathrm{LbyL};\pi^0}$ are only slightly changed, if we include the BaBar data. They shift only by about -0.5×10^{-11} compared to the corresponding data sets A0, A1 and A2. This is due to a partial compensation in $a_\mu^{\mathrm{LbyL};\pi^0}$, when the central values for $\bar{h}_5$ and h_1 are changed, as already observed in Ref. [48].

Since the data sets A0, A1 and A2 without BaBar show the $1/Q^2$ fall-off, fitting h_1 as a free parameter in the LMD+V model leads to a value compatible with zero, but with quite some large error. Similarly, the value of $\bar{h}_5$ is shifted a bit and its error gets doubled compared to the fit of the data sets A0, A1 and A2 with the LMD+V model with $h_1 = 0$ in the second part of the table. The final result for LbyL, however, only shifts by about $\pm 0.1 \times 10^{-11}$.

Finally, note that both VMD and LMD+V with $h_1 = 0$ can fit the data sets A0, A1 and A2 for the transition form factor very well with essentially the same χ^2 per degree of freedom for a given data set (see first and second part of the table). Nevertheless, the results for the pion-exchange contribution to hadronic LbyL scattering differ by about 20% in these two models. For VMD the result is about $a_\mu^{\mathrm{LbyL};\pi^0} \sim 57.5 \times 10^{-11}$ and for LMD+V with $h_1 = 0$ it is about 72.5×10^{-11} with the JN approach and about 80×10^{-11} with the MV approach. This is due to the different behavior, in these two models, of the fully off-shell form factor $\mathcal{F}_{\pi^{0*}\gamma^*\gamma^*}((q_1+q_2)^2, q_1^2, q_2^2)$ on all momentum variables, which enters for the pion-exchange contribution in hadronic LbyL scattering [45, 46].

We conclude that the KLOE-2 data with a total integrated luminosity of 5 fb^{-1} will give a reasonable improvement in the part of the $a_\mu^{\mathrm{LbyL};\pi^0}$ error associated with the parameters accessible via the $\Gamma_{\pi^0\to\gamma\gamma}$ width and the $\pi^0\gamma\gamma^*$ form factor $F(Q^2)$. As stressed above, depending on the modeling of the off-shellness of the pion, there might be other, potentially larger sources of uncertainty which cannot be improved by the KLOE-2 measurements.

6 Conclusions

A simulation of the KLOE-2 experiment with 1 year of data taking was performed. Numerical results indicate a feasibility of $\sim$1% statistical error in the measurement of $\Gamma_{\pi^0\to\gamma\gamma}$. Such a precision is better than the current experimental world average and the theoretical accuracy. The π^0 electromagnetic transition form factor $F(Q^2)$ in the region $0.01 < Q^2 < 0.1\,\mathrm{GeV}^2$ can be measured with a statistical error of <6% in each bin. This low Q^2 measurement can test the consistency of the models which have been fitted so far to the data from CELLO, CLEO and BaBar at higher Q^2 and will serve as an important test of the strong interaction dynamics at low energies. The proposed measurements with the KLOE-2 experiment can also have an impact on the value and precision of the contribution of a neutral pion exchange to the hadronic light-by-light scattering in the muon $g-2$. We would like to stress that a realistic calculation of this contribution is *not* the purpose of this letter. The given estimates for $a_\mu^{\mathrm{LbyL};\pi^0}$ should only demonstrate, within several approaches, an improvement of uncertainty, which will be possible when the KLOE-2 data appear.

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Appendix: The KLOE-2 Collaboration

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