Determination of the $\eta'$-Proton Scattering Length in Free Space

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Taking advantage of both the high mass resolution of the COSY-11 detector and the high energy resolution of the low-emittance proton beam of the cooler synchrotron COSY, we determine the excitation function for the $pp \rightarrow pp\eta'$ reaction close to threshold. Combining these data with previous results, we extract the scattering length for the $\eta'$-proton potential in free space to be $\Re(a_{\eta'p}) = 0 \pm 0.43$ fm and $\Im(a_{\eta'p}) = 0.37^{+0.40}_{-0.16}$ fm.

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In this Letter, we report the determination of the scattering length for the interaction of the $\eta'$ meson with the proton based on the shape of the excitation function for the $pp \rightarrow pp\eta'$ reaction measured close to the kinematic threshold. Using the high mass resolution of the updated COSY-11 detector [1,2] and the low-emittance proton beam of the cooler synchrotron COSY [3], the excitation function was determined down to excess energy $Q = 0.76$ MeV above threshold, with the precision $\Delta Q = 0.1$ MeV improved by more than a factor of 5 with respect to previous measurements. The improved resolution enabled quantitative extraction of the $\eta'$-proton scattering length in free space.

The scattering lengths describing interaction potentials between mesons and nucleons are of fundamental importance in hadron physics. However, they are not well established, especially for those flavor neutral mesons that are characterized by very short lifetimes, making investigations of the meson-nucleon potential in the standard way via scattering experiments impossible. So far, based on the shift and width of the ground state of pionic hydrogen atoms [4], only the scattering length of the $\pi^0$-nucleon potential is accurately determined with a precision of about 0.001 fm. The scattering length for the $\eta$-nucleon potential is determined more than 2 orders of magnitude less precisely, with phenomenological values quoted for the real part between $\sim0.2$ and $\sim1$ fm, depending on the analysis method [5]. Until now, the $\eta'$-nucleon scattering length had been estimated only qualitatively [6].

Measurements of the $\eta$- and $\eta'$-nucleon and nucleus systems are sensitive to dynamical chiral and axial U(1) symmetry breaking in low energy QCD. While pions and kaons are would-be Goldstone bosons associated with chiral symmetry, the isosinglet $\eta$ and $\eta'$ mesons are too massive by about 300–400 MeV for them to be pure Goldstone states. They receive extra mass from nonperturbative gluon dynamics associated with the QCD axial anomaly. This Okubo-Zweig-Iizuka rule violation is also expected to influence the $\eta'$-nucleon interaction [7]. Without the gluonic mass contribution, the $\eta'$ would be a strange quark state after $\eta$-$\eta'$ mixing (and the $\eta'$ would be a light-quark state degenerate with the pion), mirroring the situation with isoscalar $\phi$ and $\omega$ vector mesons. To the extent that coupling to nucleons and nuclear matter is induced by light-quark components in the meson, any observed scattering length and mass shift in a nuclear medium are induced by the QCD axial anomaly that generates part of the $\eta'$ mass [8].

In COSY-11, the $\eta'$ meson was produced in $p-p$ collisions of the COSY proton beam with an internal hydrogen cluster target. The four-momenta of outgoing protons from the $pp \rightarrow ppX$ reaction were measured in two drift chambers and scintillator detectors and the $\eta'$ meson was identified via the missing mass technique [1,9]. The low-emittance proton beam combined with the high mass resolution of the COSY-11 detector allowed measurements very close to the kinematic threshold where the signal-to-background ratio increases due to the more rapid reduction of the phase space for multimeson than for single meson production [9]. The measurement was conducted at five excess energies in the range $Q = 0.76$ to $Q = 4.78$ MeV. The determination of the absolute value of $Q$ was based on the position of the $\eta'$ signal in the missing mass spectra. (A typical missing mass spectrum for experimental data and Monte Carlo (MC) simulations is shown in the top plot of Fig. 1.) $Q$ was determined with a precision of 0.10 MeV, while 0.06 MeV is due to the uncertainty of the $\eta'$ meson mass [10] and 0.04 MeV comes...
from the possible misalignment of the relative setting of the detection system components and the center of the region of the beam and target overlap. The latter was monitored by the measurement of elastically scattered protons [11]. The experiment was designed to reduce the spread of excess energy to a negligible level by the use of a rectangular collimator in the target setup, so the width of the target stream was equal to 0.90 mm while crossing the proton beam. Because of the known dispersion of the COSY beam, this width is equivalent to an effective beam momentum spread of ±0.06 MeV/c corresponding to a 0.02 MeV spread of excess energy $Q$. The size of the target stream was monitored by a dedicated wire device with an accuracy of 0.05 mm [12], and, in addition, it was controlled independently by measuring elastically scattered protons. The number of registered $pp \rightarrow pp$ events as a function of the

![Graph](image)

**FIG. 1** (color online). Results obtained for a beam momentum of 3210.7 MeV/c corresponding to $Q = 0.76$ MeV. Top: Missing mass spectrum from experimental data (dots) and simulations (histogram). The simulated spectrum was normalized to the data. Bottom: Open points indicate the number of measured events of elastically scattered protons. Solid points denote fit results of differential cross sections determined by the EDDA Collaboration [13] with luminosity as the only free parameter.

<table>
<thead>
<tr>
<th>$Q$ (MeV)</th>
<th>$\sigma(pp \rightarrow pp\eta')$ (nb)</th>
</tr>
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<tbody>
<tr>
<td>0.76 ± 0.10</td>
<td>1.38 ± 0.08 ± 0.17</td>
</tr>
<tr>
<td>1.35 ± 0.10</td>
<td>3.82 ± 0.19 ± 0.47</td>
</tr>
<tr>
<td>1.66 ± 0.10</td>
<td>4.97 ± 0.28 ± 0.61</td>
</tr>
<tr>
<td>2.84 ± 0.10</td>
<td>11.41 ± 0.40 ± 1.39</td>
</tr>
<tr>
<td>4.78 ± 0.10</td>
<td>17.58 ± 0.64 ± 2.15</td>
</tr>
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</table>

**TABLE I.** Cross sections for the $pp \rightarrow pp\eta'$ reaction at the five measured excess energies. The excess energy $Q$ is tabulated with the absolute systematic uncertainty, and the cross section values are given with the statistical and systematic uncertainties, respectively.

Based on the data from previous experiments [14,20–25] and the close to threshold total cross sections reported in Table I, we have extracted the $\eta'$-proton scattering length. To this end, the experimental excitation function for

$3210.7\text{ MeV/c} \rightarrow 0.76\text{ MeV}$.
the \( pp \to pp\eta' \) reaction was compared to the results of calculations taking into account proton-proton and \( \eta'p \) interactions, where the real and imaginary parts of the \( \eta'p \) scattering length were varied as free parameters. At threshold, the distance probed by the \( pp \to pp\eta' \) reaction is determined by the momentum transfer between colliding nucleons and equal to about 0.2 fm, whereas the typical range of the strong nucleon-nucleon or meson-nucleon interaction is of the order of a few Fermi. In addition, the energy range considered in this Letter is 2 orders of magnitude smaller than the four-momentum transfer (1 GeV) governing the production amplitude. Therefore, the calculations were carried out using a Watson-Migdal approximation [26] and the complete transition matrix element of the \( pp \to pp\eta' \) reaction was factorized as

\[
|M_{pp \to pp\eta'}|^2 \approx |M_0|^2 |M_{\text{FSI}}|^2. \tag{1}
\]

Here, \( |M_0|^2 \) represents the total short range production amplitude and \( |M_{\text{FSI}}|^2 \) denotes the final state interaction enhancement factor. Exact Faddeev calculations for the dynamics of three-body \( pp\eta' \) final states are unavailable. Therefore, the enhancement factor for the \( pp\eta' \) system was approximated assuming the factorization of \( M_{\text{FSI}} \) into two-particle scattering amplitudes [6]:

\[
M_{\text{FSI}} = M_{pp}(k_1) \times M_{p\eta'}(k_2) \times M_{p\eta'}(k_3). \tag{2}
\]

Here, \( k_1 \) denotes the momentum in the proton-proton rest frame and \( k_2 \) and \( k_3 \) are the three-momenta of the \( \eta' \) and proton in the proton-\( \eta' \) subsystems. For the estimation of the proton-proton enhancement factor, we have used the inverse of the squared Jost function [18]. To estimate the model dependence of the result, two other extreme solutions for the proton-proton enhancement factor were considered: the Niskanen-Goldberger-Watson parametrization [19] and the square of the on-shell amplitude of the proton-proton scattering calculated in the frame of the optical potential, with phase shift including strong and Coulomb interactions [15–17].

The proton-proton and \( \eta' \)-proton invariant mass distributions determined for the \( pp \to pp\eta' \) reaction at an excess energy of \( Q = 16.4 \) MeV show an enhancement which may indicate a non-negligible \( P \)-wave contribution from the proton-proton subsystem [25,27]. Therefore, in order to avoid a bias on the result from distortion of higher partial waves, we restrict the extraction of the scattering length only to the range \( Q < 11 \) MeV. This limitation

FIG. 2 (color online). The total cross sections for the \( pp \to pp\eta' \) reaction as a function of the excess energy. Solid circles represent new results reported in this Letter, and results from previous experiments are shown with symbols, as indicated in the legend. The statistical and systematic errors are separated by dashes. The superimposed curves show results of fits with the \( \eta' \)-proton scattering length as a free parameter and parametrizing the \( pp \) FSI enhancement factor as in Refs. [15–17] (thick dashed line), the inverse of the squared Jost function [18] (thin solid line), and the Niskanen-Goldberger-Watson model [19] (thin dashed line). The thick dashed line is shown only in the range of applicability of the formula used for the enhancement factor [15]. For comparison, the thick solid line shows the result of the fit obtained for the whole \( Q \) range with \( pp \) FSI parametrization from Ref. [18]. The small plot shows the excitation function up to \( Q = 180 \) MeV, where the thin solid and thin dashed curves overlap.

FIG. 3. Square of the proton-proton scattering amplitude calculated as a function of \( k \) and of the proton three-momentum in the proton-proton subsystem, parametrized as in Refs. [15–17] (thick dashed curve), as the inverse of the squared Jost function [18] (thin solid line), and using the Niskanen-Goldberger-Watson model [19] (thin dashed line). The thick solid curve shows the phase-space \( k \) distribution for \( Q = 11 \) MeV. All the curves are arbitrarily normalized to unity at the maximum.
minimizes also the dependence of the result on the \( pp \) FSI model and reduces the corresponding systematic uncertainty. Moreover, the low energy range used in the analysis allowed us to parametrize the \( \eta'p \) FSI enhancement factor with the scattering length approximation

\[
M_{\eta'p} = \frac{1}{1 - i k a_{\eta'p}},
\]

where \( a_{\eta'p} \) is the scattering length of the \( \eta'p \) interaction treated as a free parameter in the analysis.

To determine \( a_{\eta'p} \), we have constructed the following Neyman \( \chi^2 \) statistics:

\[
\chi^2(\text{Re}(a_{\eta'p}), \text{Im}(a_{\eta'p}), \alpha) = \sum_{i=1}^{17} \frac{(\sigma_i^{\text{exp}} - \sigma_i^{\text{calc}}(a_{\eta'p}))^2}{(\Delta \sigma_i^{\text{exp}})^2},
\]

where \( \sigma_i^{\text{exp}} \) denotes the \( i \)th experimental total cross section measured with the statistical uncertainty \( \Delta \sigma_i^{\text{exp}} \) and \( \sigma_i^{\text{calc}} \) stands for the calculated total cross section normalized with a factor \( \alpha \) which is treated as an additional parameter of the fit. \( \sigma_i^{\text{calc}}(a_{\eta'p}) \) was calculated for each excess energy \( Q \) integrating Eq. (1) over the available phase space [28]. The best fit to the experimental data corresponds to

\[
\begin{align*}
\text{Re}(a_{\eta'p}) &= 0.00 \pm 0.43_{\text{stat}} \pm 0.43_{\text{syst}} \text{ fm} \quad \text{(systematic error negligible)} \\
\text{Im}(a_{\eta'p}) &= 0.37_{-0.11_{\text{stat}}}^{+0.02_{\text{stat}}} + 0.38_{-0.05_{\text{syst}}} \text{ fm}.
\end{align*}
\]

The statistical uncertainties in this case were determined at the 70% confidence level, taking into account that we have varied three parameters [29]. The systematic uncertainties due to the parameterization of the proton-proton interaction used in the analysis were estimated as the maximal difference between the result obtained in Eq. (5) and that determined using the two other \( pp \) FSI models. For the real part of \( a_{\eta'p} \), the differences obtained by applying different models are negligible.

It is interesting to compare these results with theoretical expectations and with recent studies based on the \( \eta' \)-nucleus optical potential. In the quark meson coupling (QMC) model [30], one calculates the in-medium meson masses and corresponding effective in-medium meson-nucleon scattering lengths through coupling the light quarks in the meson to the scalar isoscalar \( \sigma \) (and also \( \omega \) and \( \rho \)) mean fields in the nucleus. For a 20° \( \eta-\eta' \) mixing angle, the QMC model predicts the \( \eta' \) mass shift to be \(-37 \) MeV at nuclear matter density \( \rho_0 \) corresponding to the real part of the effective \( \eta' \)-nucleon scattering length being \( 0.5 \) fm. This mass shift is very similar to the recent determination of the \( \eta' \)-nucleus optical potential by the CBELSA/TAPS Collaboration from studies of \( \eta' \) photoproduction from carbon [31]. The \( \eta' \)-nucleus optical potential \( V_{\text{opt}} = V_{\text{real}} + i W \) deduced from these photoproduction experiments is \( V_{\text{real}}(\rho_0) = -37 \pm 10(\text{stat}) \pm 10(\text{syst}) \) MeV, which is equal to the meson mass shift in medium and \( W(\rho_0) = -10 \pm 2.5 \) MeV. Larger mass shifts, downwards by up to 80–150 MeV, were found in the Nambu—Jona-Lasinio [32] and linear sigma model calculations [33]. Each of these theoretical models prefers a positive sign for the real part of \( a_{\eta'N} \) in medium. A chiral coupled channel calculation performed with possible scattering lengths with a real part between 0 and 1.5 fm is reported in Ref. [34]. A free-space scattering length close to 0 was found in a coupled channel fit to \( \eta' \) scattering processes [35]. The energy and density dependence of the \( \eta' \) (and also \( \eta \)) nucleon scattering lengths is an open topic of investigation [36]. If one assumes no density and energy dependence of the \( \eta' \)-nucleon scattering length, then the value obtained in Eq. (5) is consistent with the QMC result [30] and disfavors the expectations in Refs. [32,33].

In summary, the close to threshold excitation function for the \( pp \rightarrow pp\eta' \) reaction was determined down to an excess energy of \( Q = 0.76 \) MeV with the precision \( \Delta Q = 0.10 \) MeV improved by more than a factor of 5 with respect to previous measurements. The achieved resolution enabled the first quantitative extraction of the scattering length for the \( \eta' \)-proton interaction in free space. Most importantly, the extracted value of the real part of the scattering length is found to be independent of the proton-proton FSI model in the close to threshold energy range (up to 11 MeV) used in the fit.

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