


# Formation of $\eta'(958)$ bound states by $(\gamma, d)$ reaction



Moeki Miyatani (Nara Women's Univ.)

Natsumi Ikeno (Tottori Univ.)

Hideko Nagahiro (Nara Women's Univ.)

Satoru Hirenzaki (Nara Women's Univ.)

Acknowledgements: Hiroyuki Fujioka (Kyoto Univ.)

Takatsugu Ishikawa (Tohoku Univ.)

Emiko Hiyama (RIKEN)

# 1. Introduction

## Purpose

We like to know the possibility of formation of  $\eta'$ (958) mesic nucleus by  $(\gamma, d)$  reaction

$\phi$  mesic nucleus by  $(\gamma, d)$



by N. Ikeno et al., Phys. Rev. C 84, 054609(2011)

$\eta'$  mesic nucleus by  $(\gamma, d)$

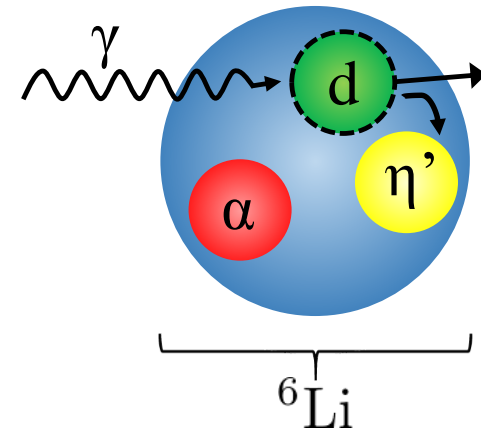
- In-medium  $\eta'$  properties  
⇒ Information on  $U_A(1)$  anomaly effect
- Possible at photon facilities ?
- Formation by  $(\gamma, p)$  and  $(p, d)$

( Hideko Nagahiro, Satoru Hirenzaki, Phys. Rev. Lett. 94 (2005) 232503 )

( Kenta Itahashi et al., Prog. Theor. Phys. 128 (2012) 601-613 )

Improvements from N. Ikeno et al., Phys. Rev. C 84, 054609 (2011)

- Distortion effect
- Elementary cross section
- Realistic  $\alpha$  density distribution
- Recoil effect

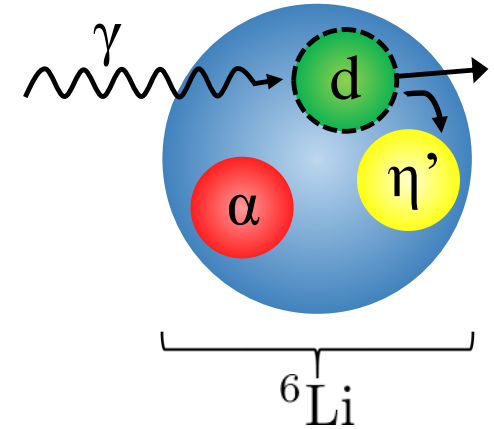


## 2. Two-nucleon pick-up reaction for ${}^6\text{Li}$ target by effective number ( $N_{\text{eff}}$ ) approach

${}^6\text{Li}$  has well-developed cluster structure of  $\alpha+d$

▣ formation cross section

$$\frac{d^2\sigma}{dEd\Omega} = \left(\frac{d\sigma}{d\Omega}\right)^{\text{ele}} \sum_f \frac{\Gamma}{2\pi} \frac{1}{\Delta E^2 + \Gamma^2/4} \underbrace{N_{\text{eff}}}_{\uparrow}$$



$N_{\text{eff}}$  : effective number of deuteron

$$N_{\text{eff}} = \sum_{JM} \left| \int \chi_d^*(\mathbf{r}) \left[ \phi_{l_{\eta'}}^*(\mathbf{r}) \otimes \psi_{l_d}(\mathbf{r}) \right]_{JM} \chi_{\gamma}(\mathbf{r}) d\mathbf{r} \right|^2$$

$\chi_{\gamma}$ ,  $\chi_d$  : incident  $\gamma$ , emitted  $d$  wave function

$\phi_{l_{\eta'}}$  :  $\alpha$ - $\eta'$  relative wave function

$\psi_{l_d}$  :  $\alpha$ - $d$  relative wave function

$$\left(\frac{d\sigma}{d\Omega}\right)^{\text{ele}} : \text{Elementary cross section, } \left[ \begin{array}{l} \Delta E = T_d - E_{\gamma} + S_d - B_{\eta'} + m_{\eta'} \\ \Gamma : \text{width of } \eta'\text{-meson bound states} \end{array} \right.$$

## 2-1. Initial state: $\alpha$ -d relative wave function (2s bound state)

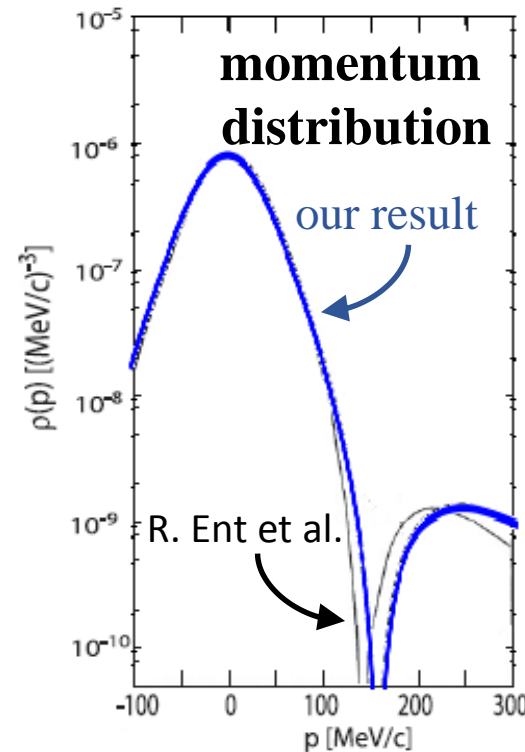
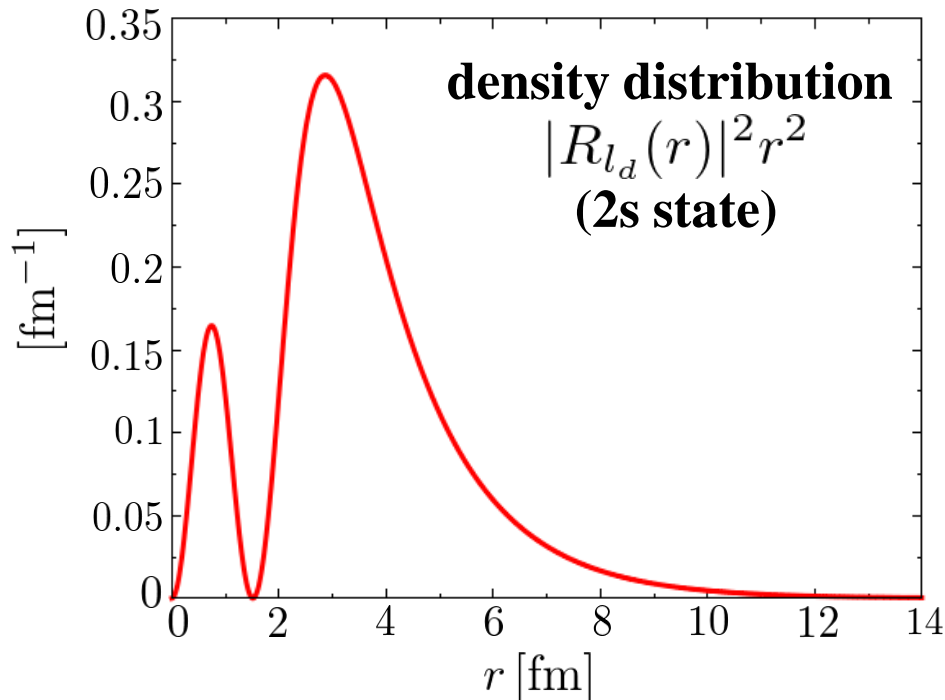
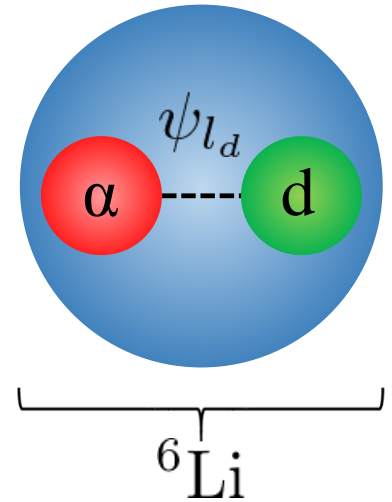
Probability of  $\alpha$ +d cluster structure in  ${}^6\text{Li}$  is reported to be 73%

( R. Ent et al., Phys. Rev. Lett. 57, 2367 (1986) )

Schrödinger eq.

$$\left[ -\frac{1}{2m} \nabla^2 + V(r) \right] \psi_{l_d}(\mathbf{r}) = E \psi_{l_d}(\mathbf{r})$$

$$V(r) = \frac{V_0}{1 + \exp((r - R)/a)} : \text{Woods-Saxon-type potential}$$



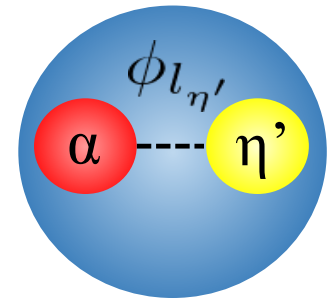
fix two parameters  
to reproduce  $\rho(p)$   
in R. Ent et al.

$$V_0 = -75 \text{ [MeV]}$$

$$R = 2.0 \text{ [fm]}$$

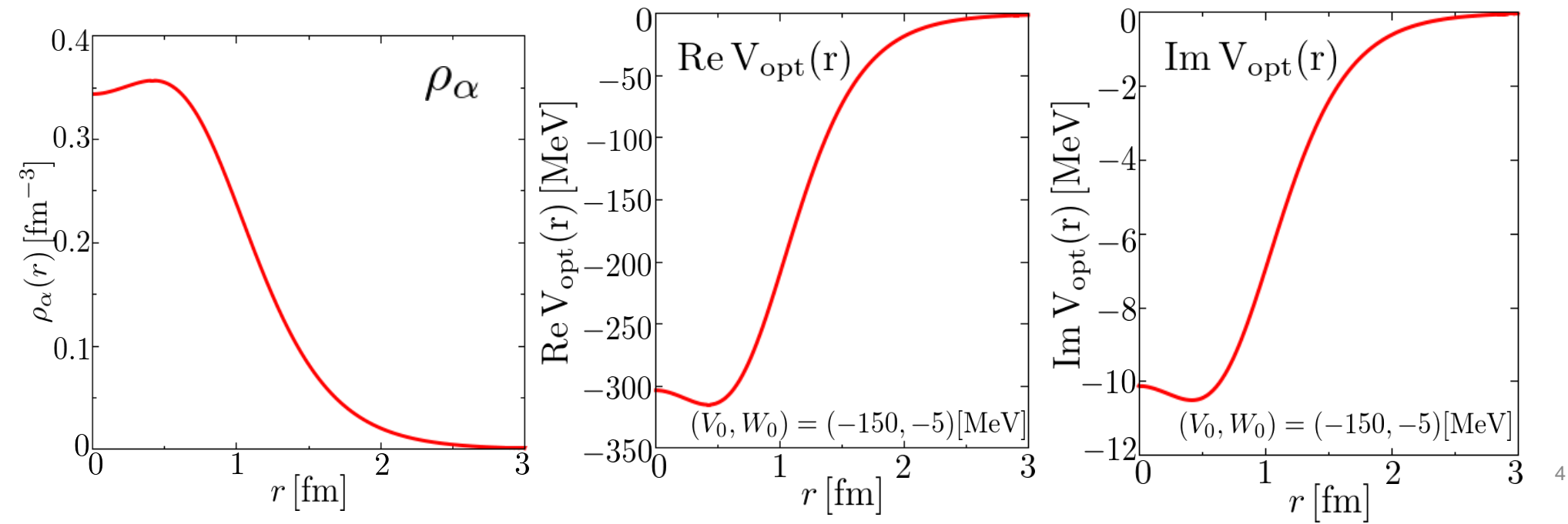
## 2-2. Final state: $\alpha$ - $\eta'$ relative wave function

$$[-\nabla^2 + \mu^2 + 2\mu V_{\text{opt}}(r)] \phi_{l_{\eta'}}(\mathbf{r}) = E_{\text{KG}}^2 \phi_{l_{\eta'}}(\mathbf{r})$$

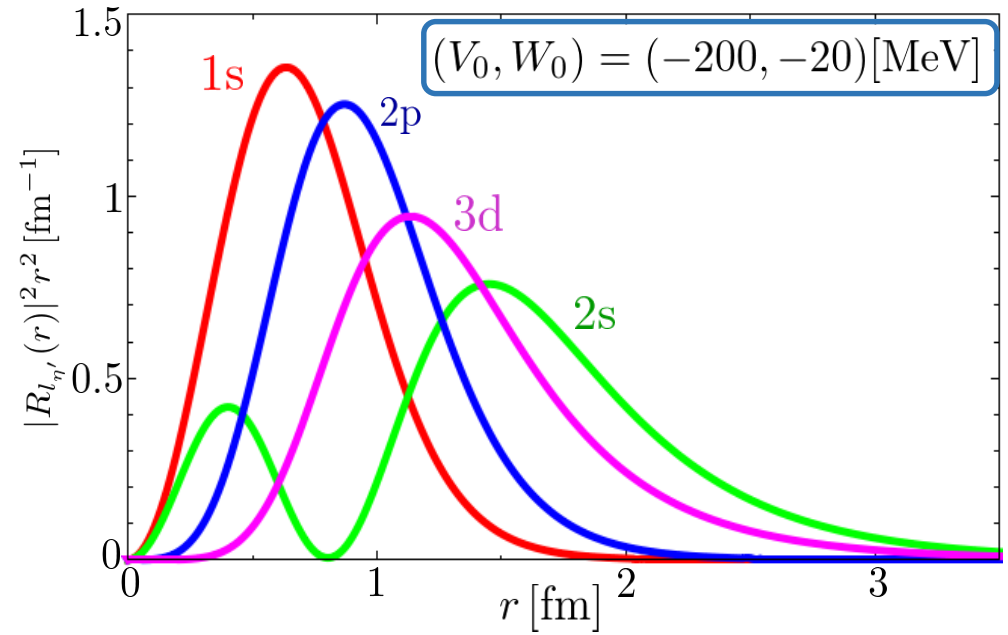
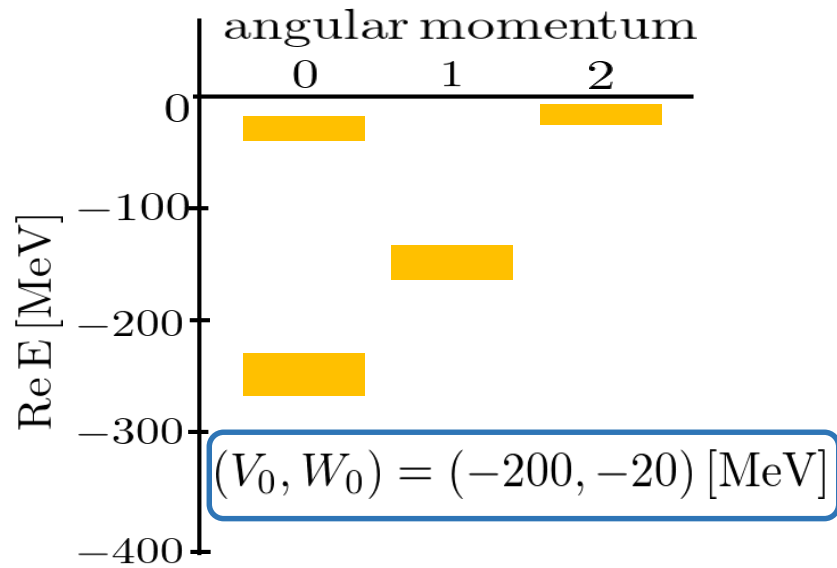


$$V_{\text{opt}}(r) = \frac{(V_0 + iW_0) \rho_\alpha(r)}{\rho_0}$$

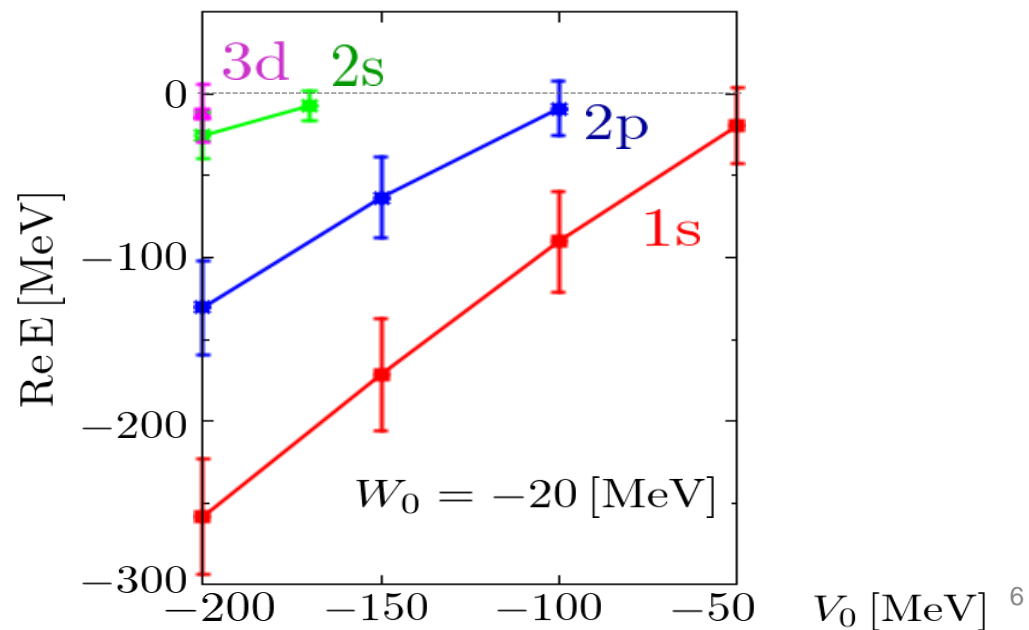
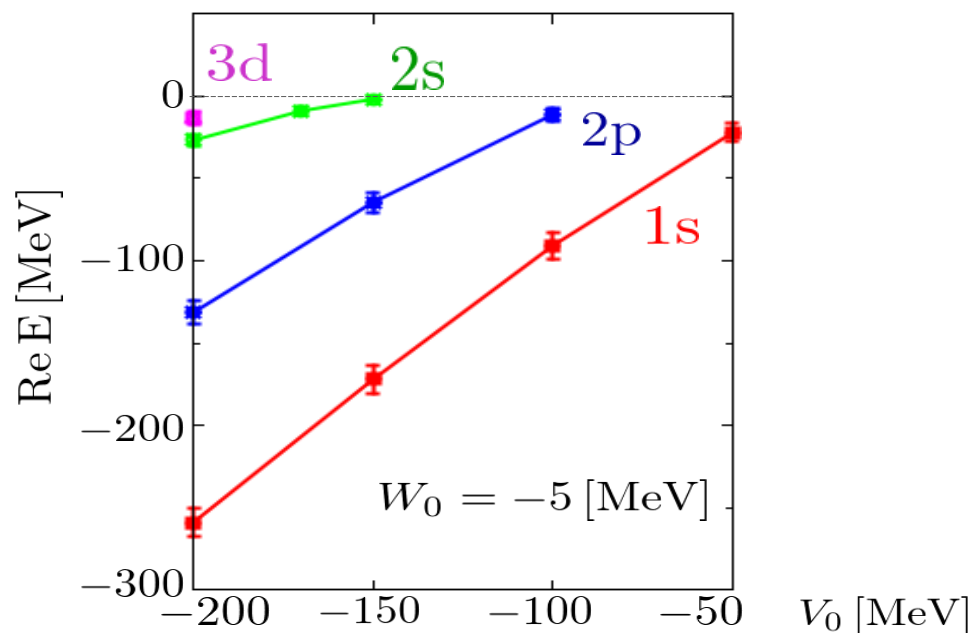
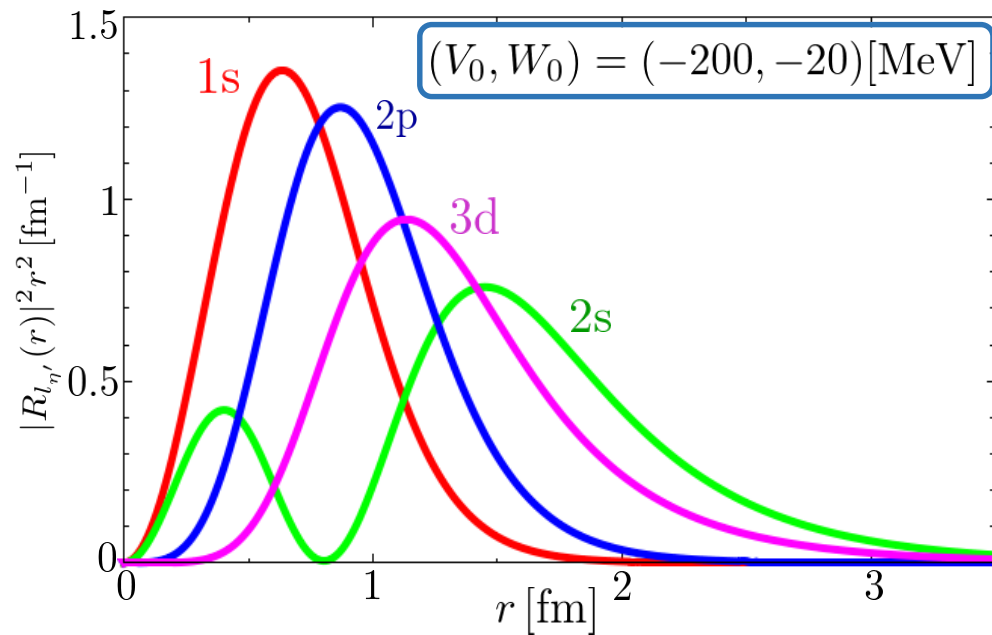
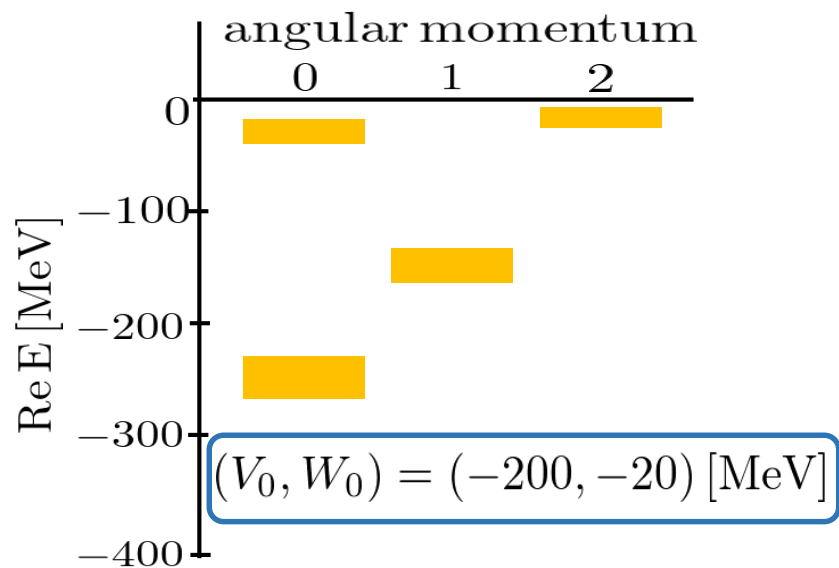
$V_0 = -50, -100, -150, -200$  [MeV],  $\rho_0 = 0.17$  [fm<sup>-3</sup>]  
 $W_0 = -5, -20$  [MeV] (H. Nagahiro et al., Phys. Rev. C 87, 045201 (2013))  
 $\rho_\alpha$ : Realistic  $\alpha$  density distribution  
 Gaussian expansion method by Emiko Hiyama



## 2-2. Final state: $\alpha$ - $\eta'$ relative wave function



## 2-2. Final state: $\alpha$ - $\eta'$ relative wave function



## 2-3. Scattering waves $\chi_\gamma$ and $\chi_d$

$$N_{\text{eff}} = \sum_{JM} \left| \int \underline{\chi_d^*(\mathbf{r})} \left[ \phi_{l_{\eta'}}^*(\mathbf{r}) \otimes \psi_{l_d}(\mathbf{r}) \right]_{JM} \underline{\chi_\gamma(\mathbf{r})} d\mathbf{r} \right|^2$$

### 1. Distortion effect (DWIA)

$$\chi_d^*(\mathbf{r}) \chi_\gamma(\mathbf{r}) = e^{i\mathbf{q}\cdot\mathbf{r}} \rightarrow e^{i\mathbf{q}\cdot\mathbf{r}} D(\mathbf{b}, z)$$

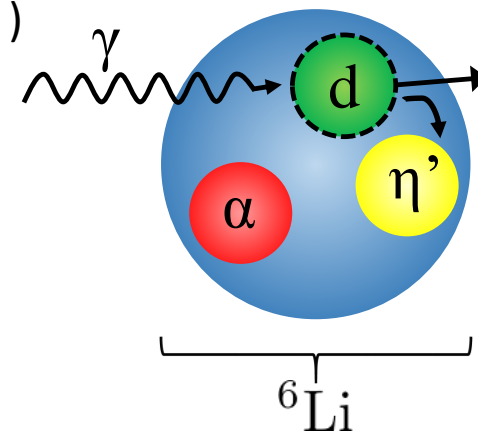
$$D(\mathbf{b}, z) = \exp \left[ -\frac{\sigma_{\gamma N}}{2} \int_{-\infty}^z \rho_{6\text{Li}}(\mathbf{b}, z') dz' - \frac{\sigma_{dN}}{2} \int_z^{+\infty} \rho_\alpha(\mathbf{b}, z') dz' \right]$$

$$\begin{cases} \sigma_{\gamma N} = 0 \text{ [mb]} \\ \sigma_{dN} = 60 \text{ [mb]} \end{cases} \quad (\text{taken from } \sigma_{pd} \text{ in PDG (2012)})$$

### 2. Recoil effect

$$\vec{r} \rightarrow \frac{M_\alpha}{m_{\eta'} + M_\alpha} \vec{r}$$

correction factor

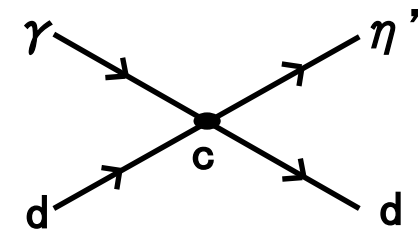


( the same prescription as in T. Koike, T. Harada, Nucl. Phys. A 804 (2008) 231-273 )



## 2-4. Elementary cross section

$$\left(\frac{d\sigma}{d\Omega}\right)_{\text{CM}}^{\text{ele}} = \frac{1}{2} \frac{|c|^2}{4\pi} \frac{M_d^2}{\lambda^{\frac{1}{2}}(s, M_d^2, 0)} \frac{1}{p_\gamma} \frac{1}{E_{d'} + \omega_{\eta'}} |F_d(\mathbf{q})|^2$$

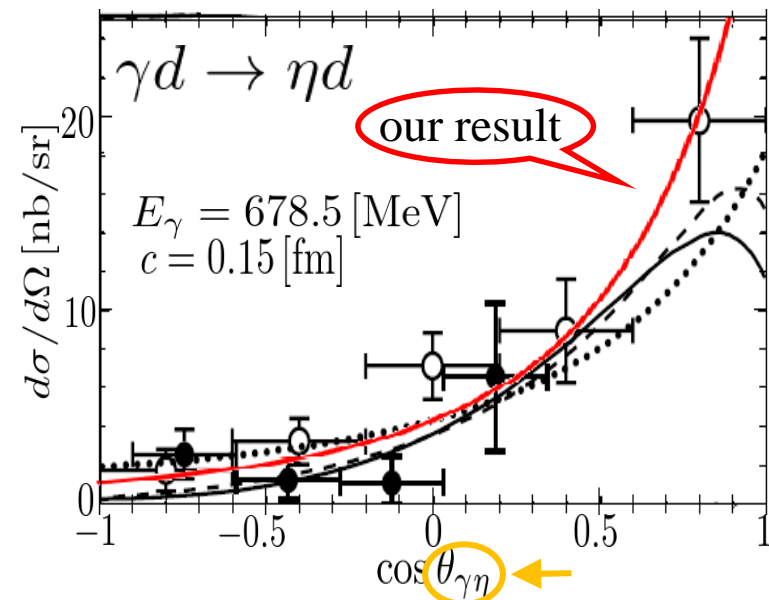


$$F_d(\mathbf{q}) = \int \psi_d^*(\mathbf{r}) e^{i\mathbf{q}\cdot\frac{\mathbf{r}}{2}} \psi_d(\mathbf{r}) d\mathbf{r} : \text{Form factor}$$

$\psi_d(\mathbf{r})$  : proton-neutron relative wave function in deuteron  
by Bonn potential  
( R. Machleidt et al., Phys. Rep. 149, No.1 (1987) 1-89 )

$\mathbf{q}$  : momentum transfer in deuteron rest frame

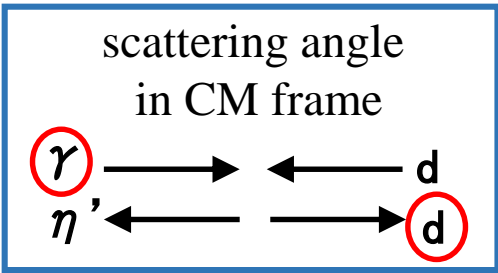
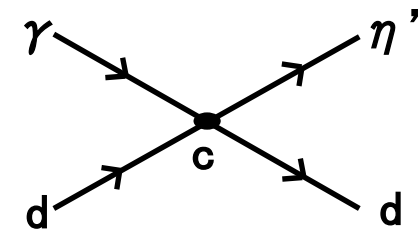
angular distribution of  $\eta(548)$  in CM



( J. Weiß et al., Eur. Phys. J. A 11, 371-374 (2001) )

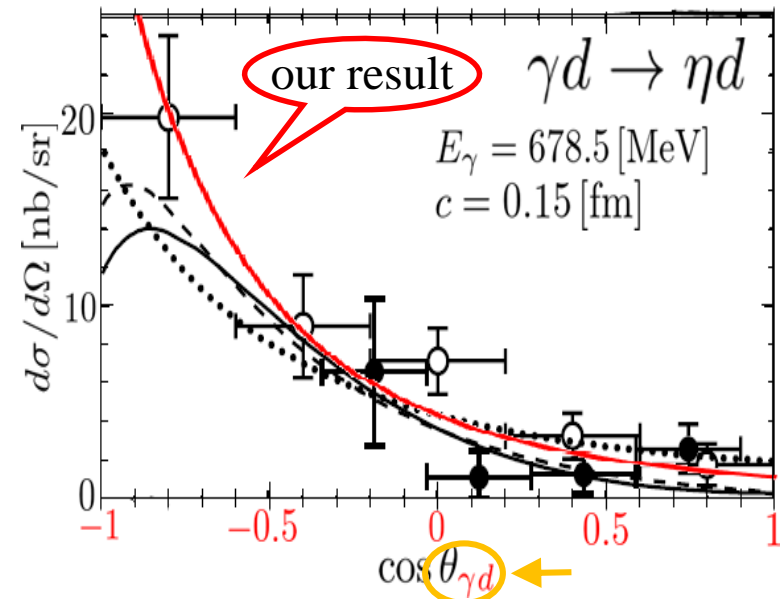
## 2-4. Elementary cross section

$$\left(\frac{d\sigma}{d\Omega}\right)_{\text{CM}}^{\text{ele}} = \frac{1}{2} \frac{|c|^2}{4\pi} \frac{M_d^2}{\lambda^{\frac{1}{2}}(s, M_d^2, 0)} \frac{1}{p_\gamma} \frac{1}{E_{d'} + \omega_{\eta'}} |F_d(\mathbf{q})|^2$$



$F_d(\mathbf{q}) = \int \psi_d^*(\mathbf{r}) e^{i\mathbf{q} \cdot \frac{\mathbf{r}}{2}} \psi_d(\mathbf{r}) d\mathbf{r}$  : Form factor  
 $\psi_d(\mathbf{r})$  : proton-neutron relative wave function in deuteron  
 by Bonn potential  
 ( R. Machleidt et al., Phys. Rep. 149, No.1 (1987) 1-89 )  
 $\mathbf{q}$  : momentum transfer in deuteron rest frame

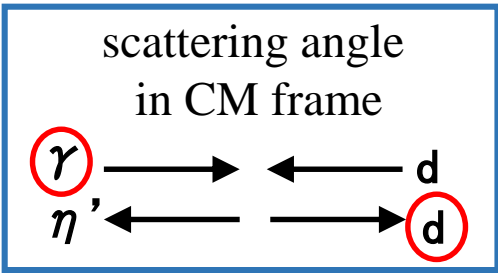
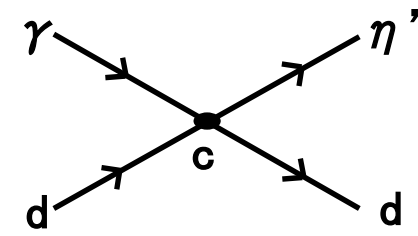
angular distribution of  $\eta(548)$  in CM



( J. Weiß et al., Eur. Phys. J. A 11, 371-374 (2001) )

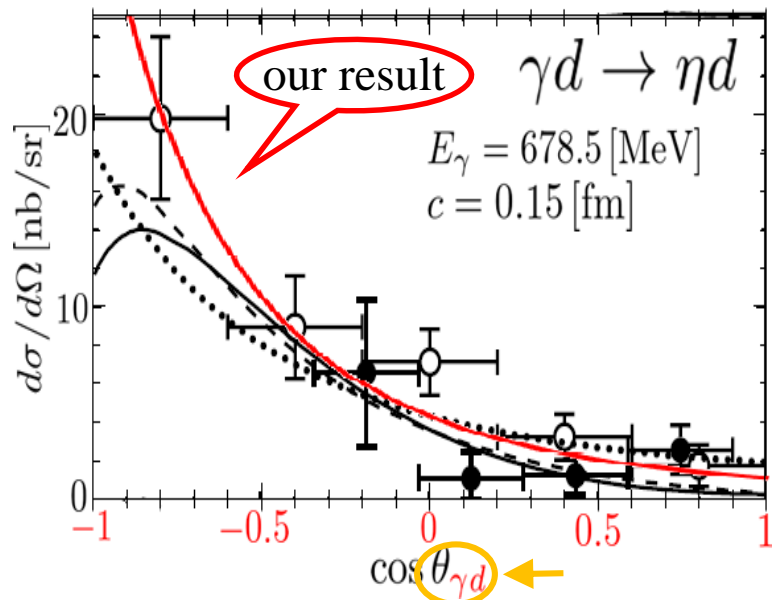
## 2-4. Elementary cross section

$$\left(\frac{d\sigma}{d\Omega}\right)_{\text{CM}}^{\text{ele}} = \frac{1}{2} \frac{|c|^2}{4\pi} \frac{M_d^2}{\lambda^{\frac{1}{2}}(s, M_d^2, 0)} \frac{1}{p_\gamma} \frac{1}{E_{d'} + \omega_{\eta'}} |F_d(\mathbf{q})|^2$$

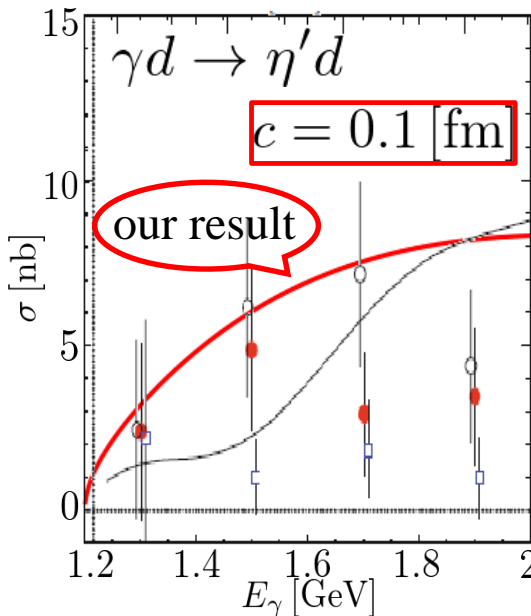


$F_d(\mathbf{q}) = \int \psi_d^*(\mathbf{r}) e^{i\mathbf{q}\cdot\frac{\mathbf{r}}{2}} \psi_d(\mathbf{r}) d\mathbf{r}$  : Form factor  
 $\psi_d(\mathbf{r})$  : proton-neutron relative wave function in deuteron  
 by Bonn potential  
 ( R. Machleidt et al., Phys. Rep. 149, No.1 (1987) 1-89 )  
 $\mathbf{q}$  : momentum transfer in deuteron rest frame

angular distribution of  $\eta(548)$  in CM

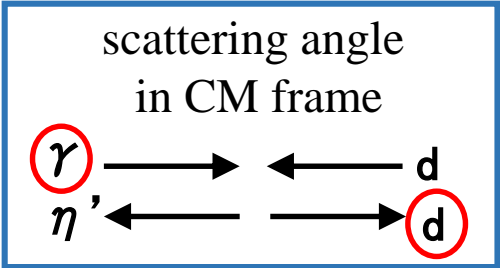
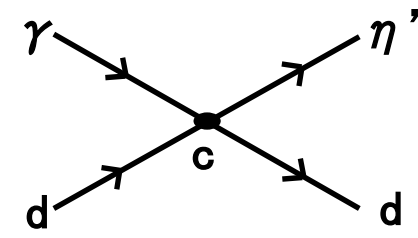


total cross section of  $\eta'$



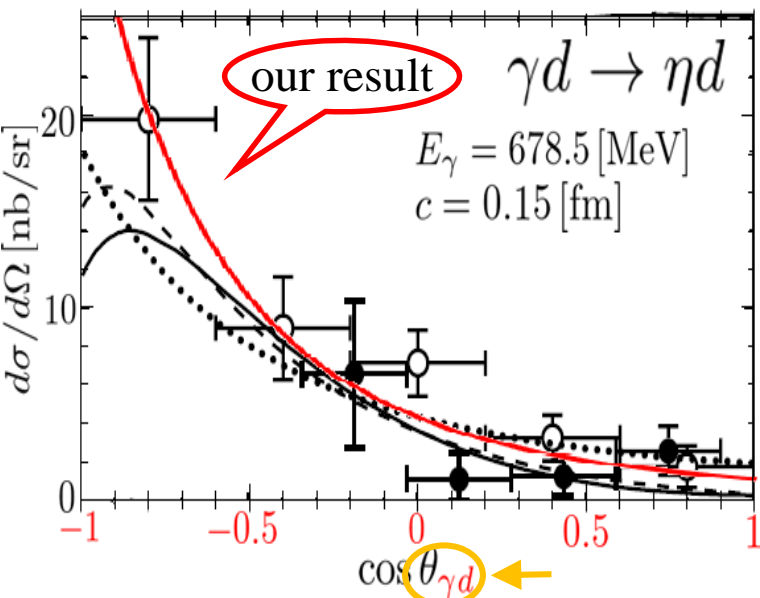
## 2-4. Elementary cross section

$$\left(\frac{d\sigma}{d\Omega}\right)_{\text{CM}}^{\text{ele}} = \frac{1}{2} \frac{|c|^2}{4\pi} \frac{M_d^2}{\lambda^{\frac{1}{2}}(s, M_d^2, 0)} \frac{1}{p_\gamma} \frac{1}{E_{d'} + \omega_{\eta'}} |F_d(\mathbf{q})|^2$$

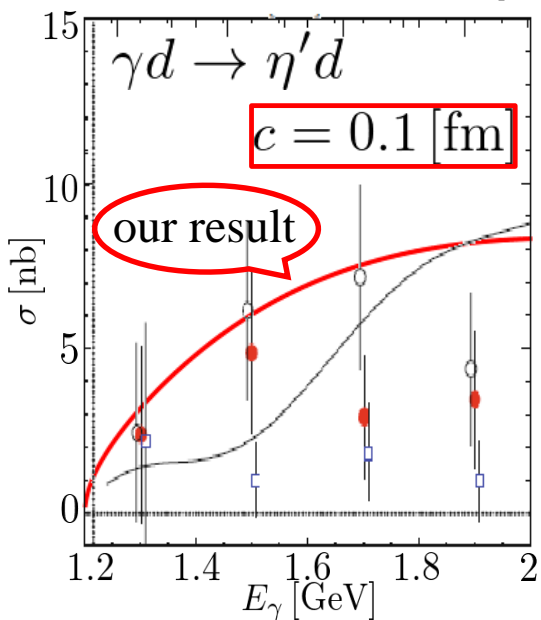


$F_d(\mathbf{q}) = \int \psi_d^*(\mathbf{r}) e^{i\mathbf{q}\cdot\frac{\mathbf{r}}{2}} \psi_d(\mathbf{r}) d\mathbf{r}$  : Form factor  
 $\psi_d(\mathbf{r})$  : proton-neutron relative wave function in deuteron  
 by Bonn potential  
 ( R. Machleidt et al., Phys. Rep. 149, No.1 (1987) 1-89 )  
 $\mathbf{q}$  : momentum transfer in deuteron rest frame

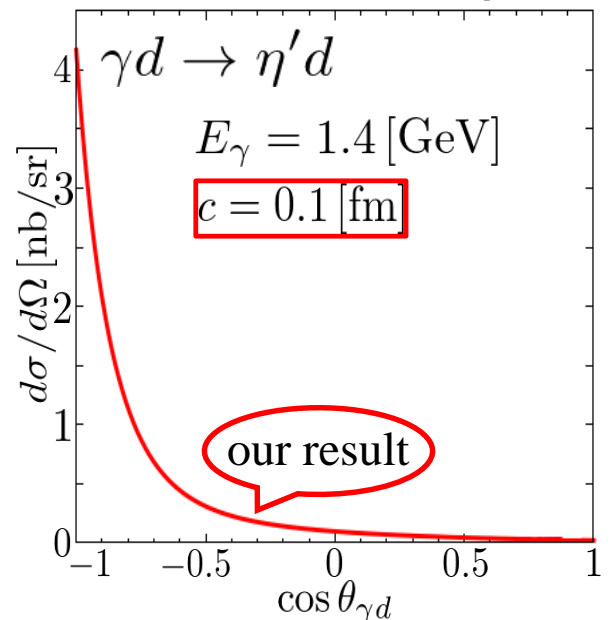
angular distribution of  $\eta(548)$  in CM



total cross section of  $\eta'$



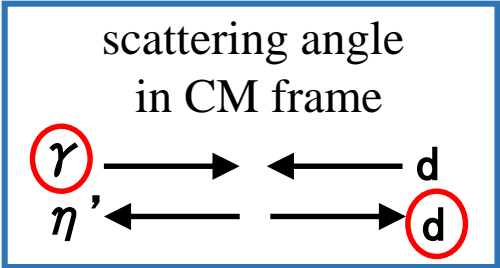
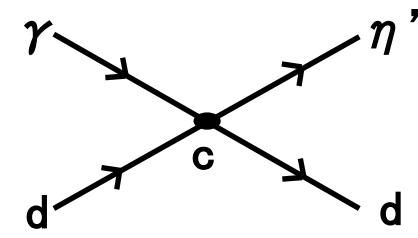
angular distribution of  $\eta'$  in CM



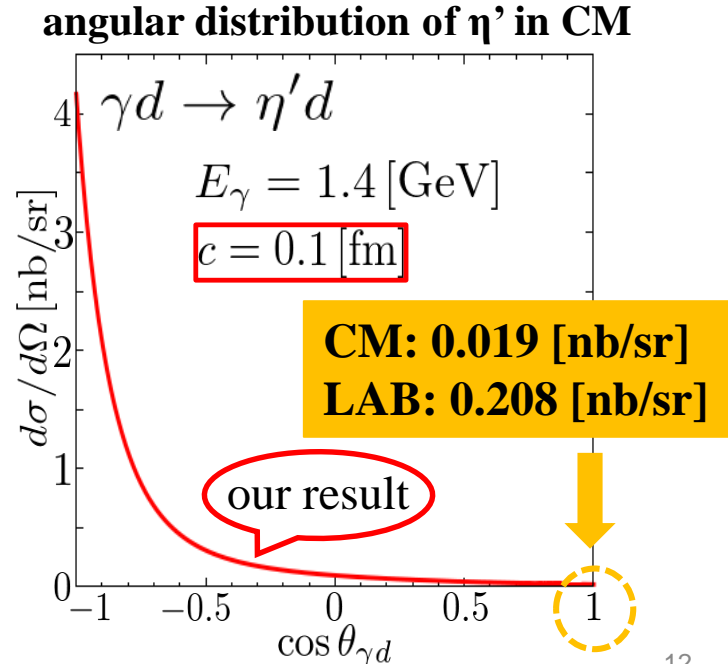
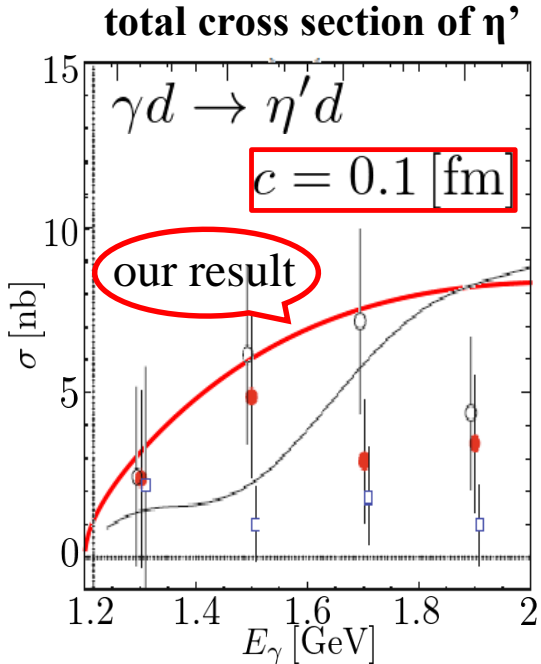
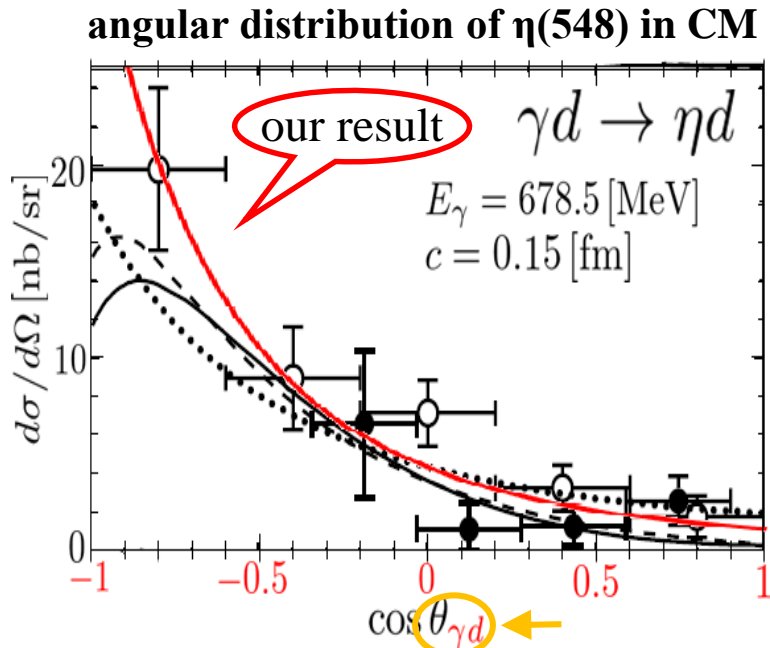
( J. Weiß et al., Eur. Phys. J. A 11, 371-374 (2001) ) ( I. Jaegle et al., Eur. Phys. J. A (2011) 47: 11 )

## 2-4. Elementary cross section

$$\left(\frac{d\sigma}{d\Omega}\right)_{\text{CM}}^{\text{ele}} = \frac{1}{2} \frac{|c|^2}{4\pi} \frac{M_d^2}{\lambda^{\frac{1}{2}}(s, M_d^2, 0)} \frac{1}{p_\gamma} \frac{1}{E_{d'} + \omega_{\eta'}} |F_d(\mathbf{q})|^2$$



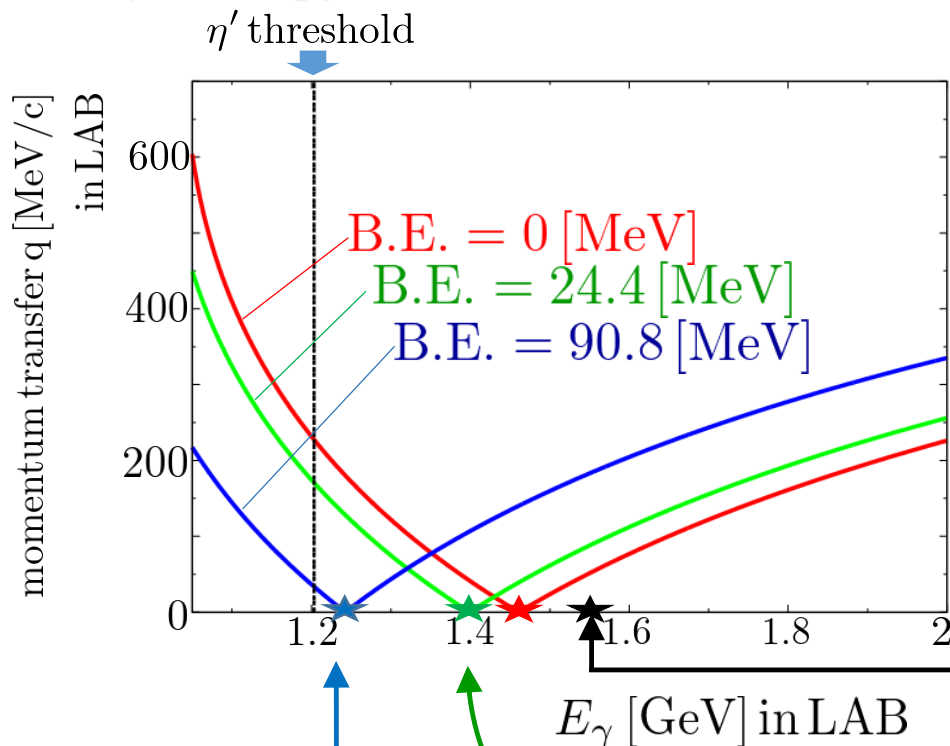
$F_d(\mathbf{q}) = \int \psi_d^*(\mathbf{r}) e^{i\mathbf{q}\cdot\frac{\mathbf{r}}{2}} \psi_d(\mathbf{r}) d\mathbf{r}$  : Form factor  
 $\psi_d(\mathbf{r})$  : proton-neutron relative wave function in deuteron by Bonn potential  
 ( R. Machleidt et al., Phys. Rep. 149, No.1 (1987) 1-89 )  
 $\mathbf{q}$  : momentum transfer in deuteron rest frame



( J. Weiß et al., Eur. Phys. J. A 11, 371-374 (2001) ) ( I. Jaegle et al., Eur. Phys. J. A (2011) 47: 11 )

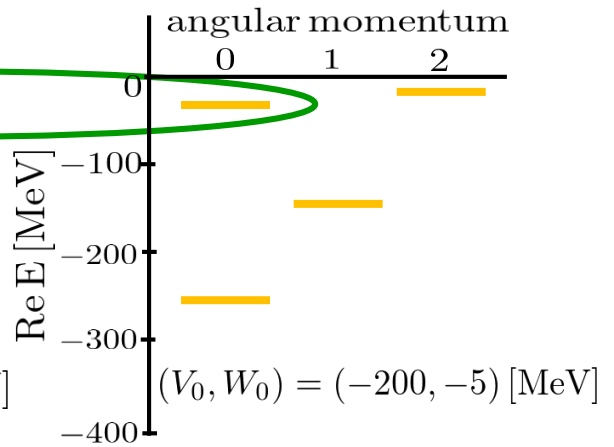
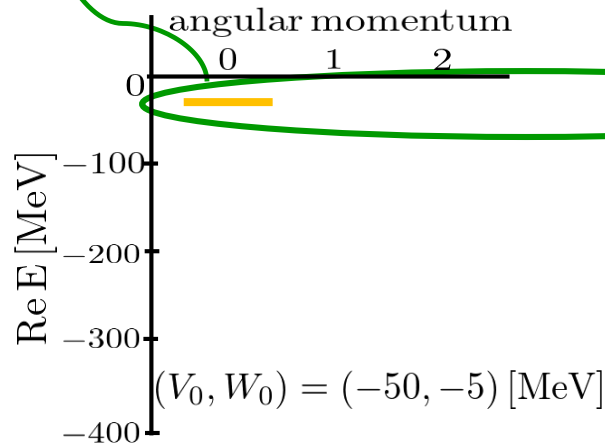
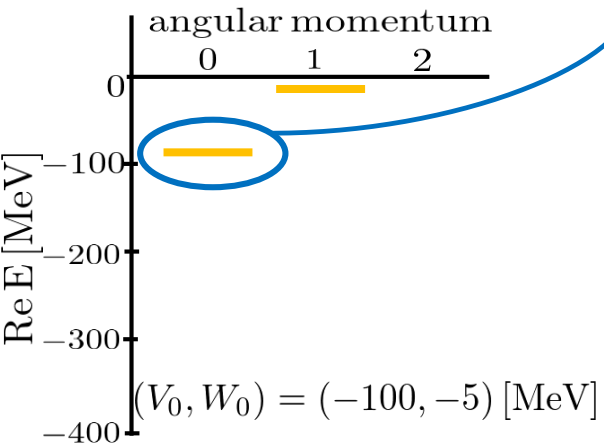
### 3. Result of $\eta'$ mesic nucleus formation reaction

#### 3-1. Incident $\gamma$ energy and momentum transfer

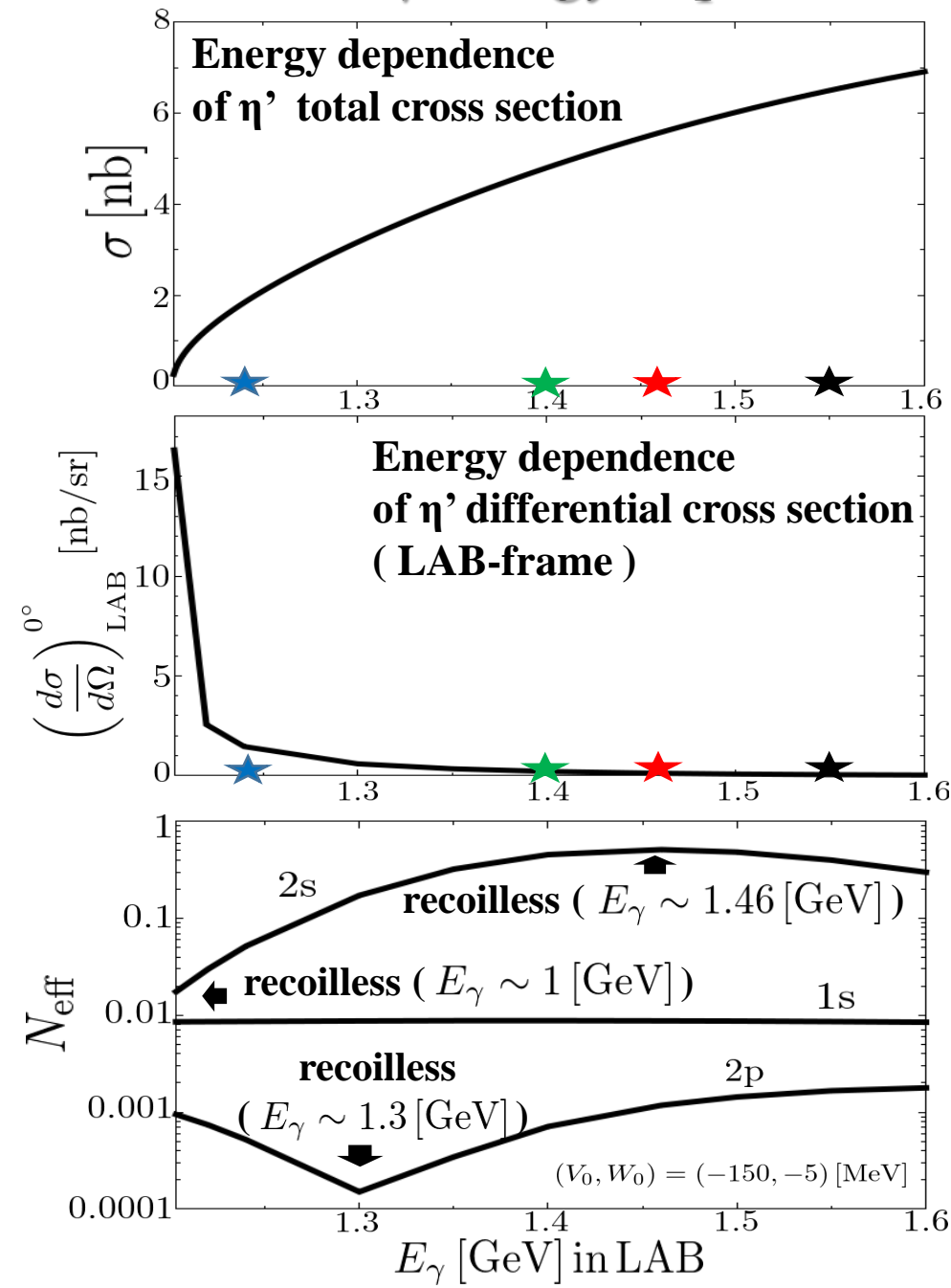


- ★  $E_\gamma = 1.24$  [GeV]
- ★  $E_\gamma = 1.4$  [GeV]
- ★  $E_\gamma = 1.46$  [GeV]
- ★  $E_\gamma = 1.55$  [GeV]

expected energy range  
 for experiment  
 ( around Recoilless )  
 1.35 ~ 1.55 [GeV]  
 private communication with  
 Takatsugu Ishikawa

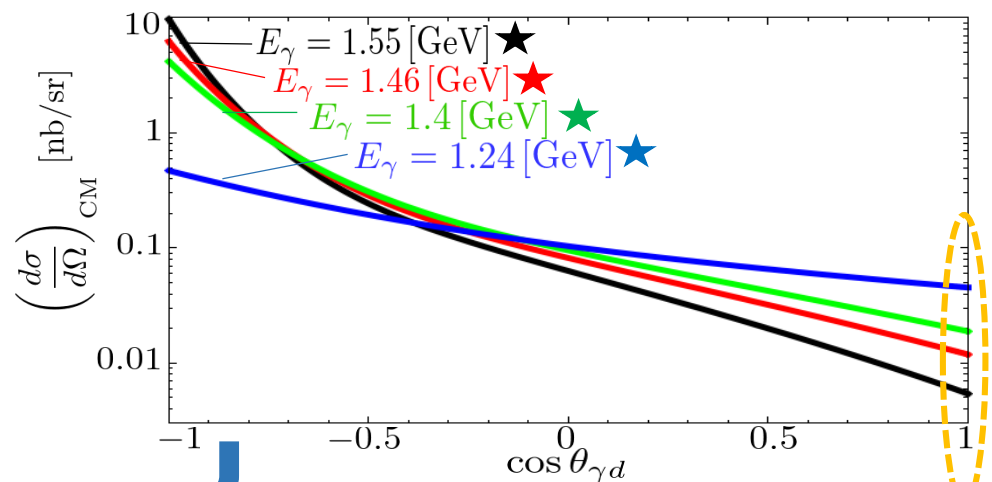


## 3-2. Incident $\gamma$ energy dependence



### Formation cross section

$$\frac{d^2\sigma}{dEd\Omega} = \left(\frac{d\sigma}{d\Omega}\right)^{\text{ele}} \sum_f \frac{\Gamma}{2\pi} \frac{1}{\Delta E^2 + \Gamma^2/4} N_{\text{eff}}$$



$E_\gamma \rightarrow$  larger

✓  $\eta'$  total cross section ( $\sigma$ ) increases

✓ elementary cross section  $\left(\frac{d\sigma}{d\Omega}\right)$

$\rightarrow$  decreases at  $\theta_{\gamma d} = 0^\circ$

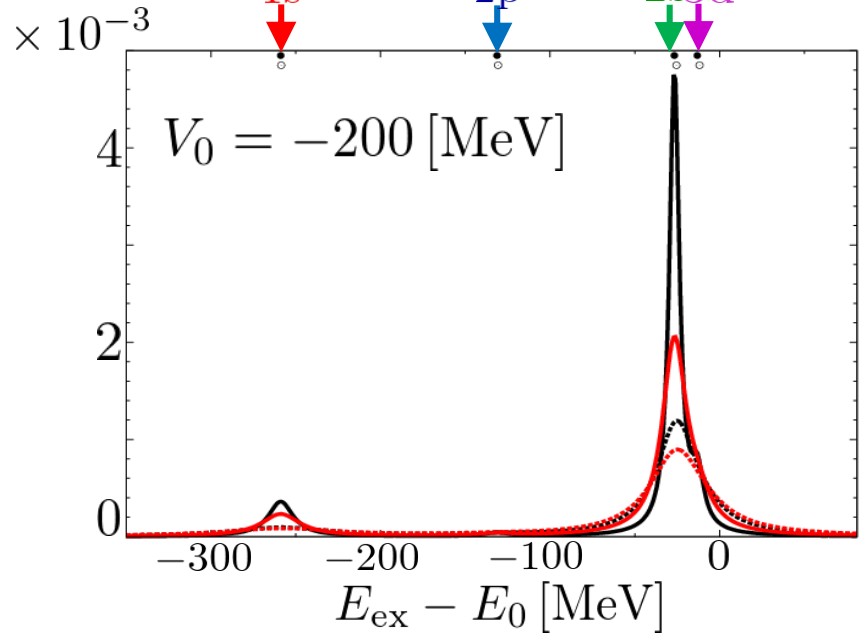
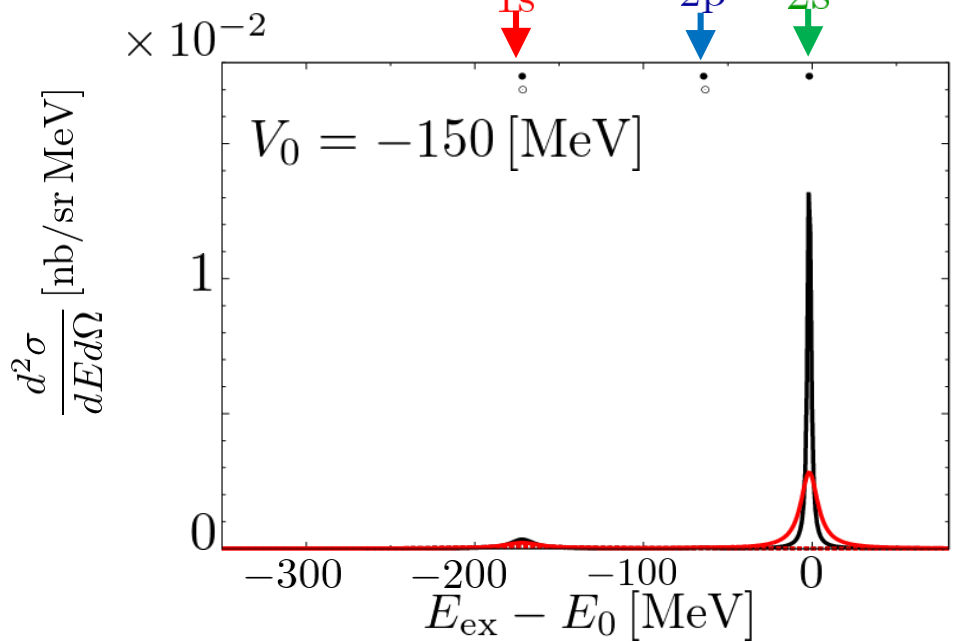
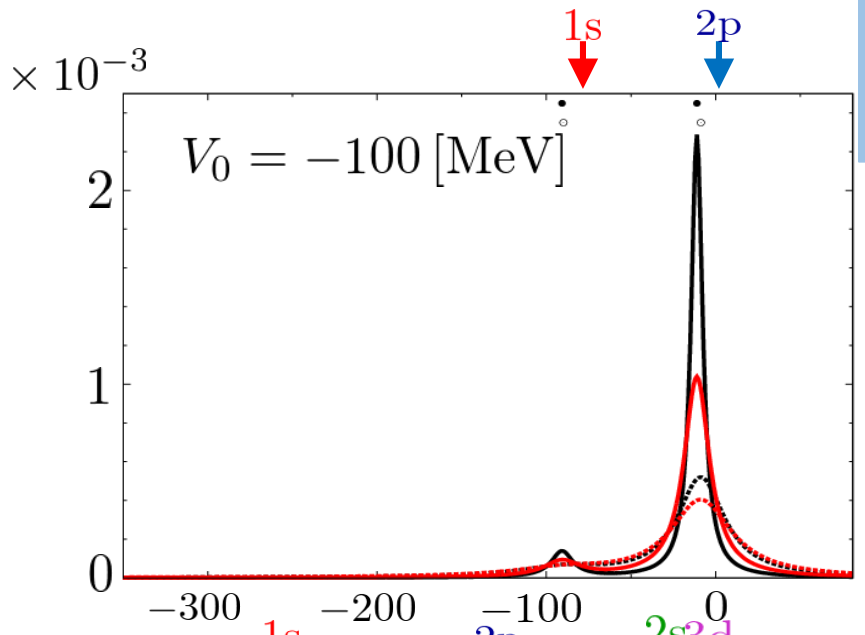
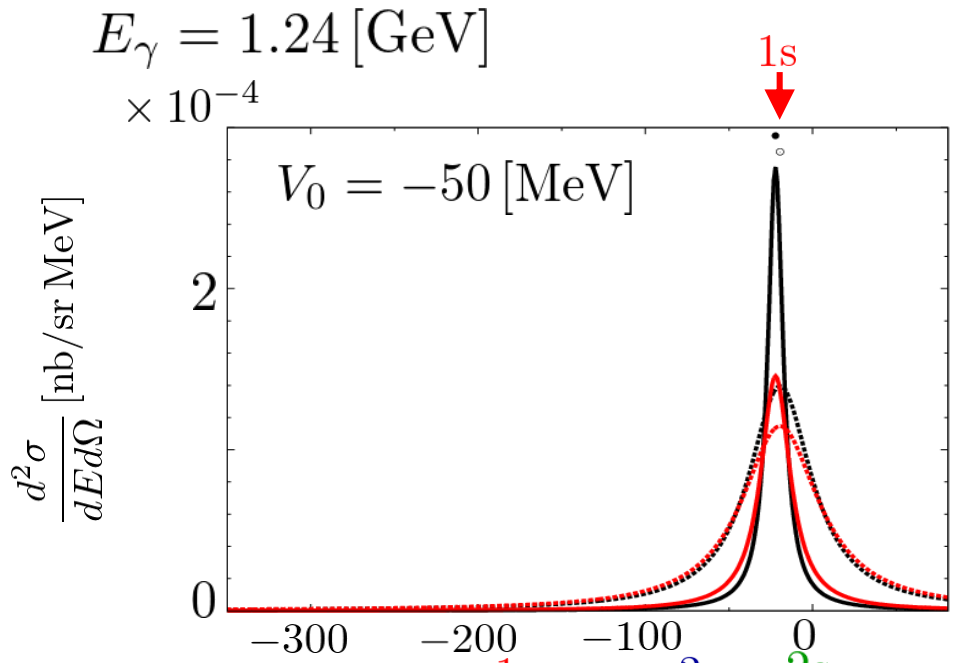
✓  $N_{\text{eff}}$  vary

according to the matching condition

### 3-3. Formation cross section of $\eta'$ bound state

$W_0 = -5$  [MeV] ———  
 $W_0 = -20$  [MeV] ·····

Red Lines  
 experimental  
 energy  
 resolution  
 ( 10 [MeV] )  
 is assumed



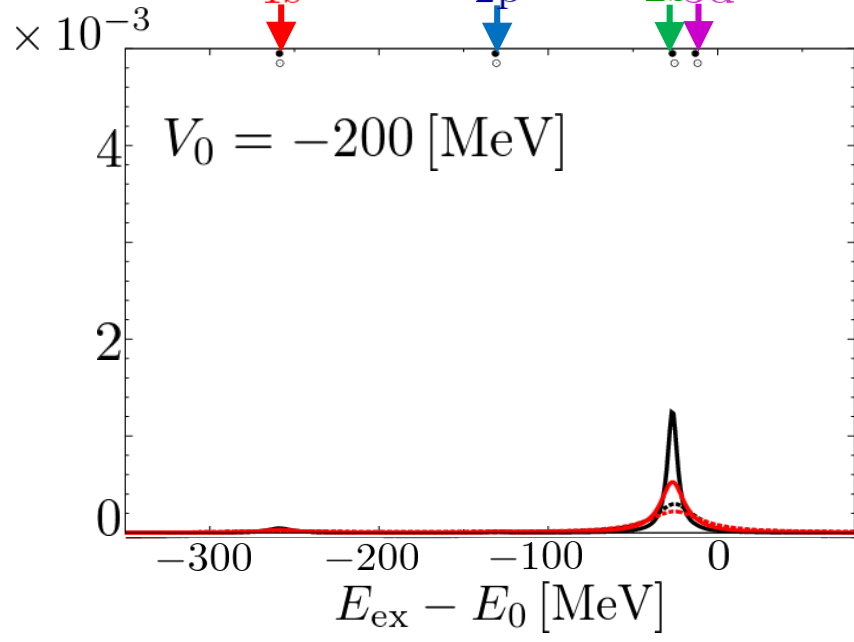
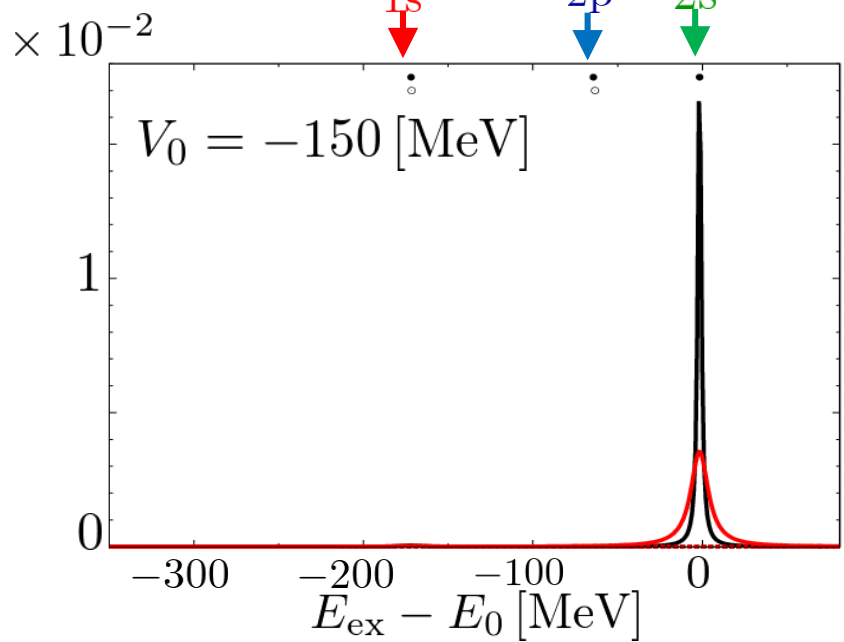
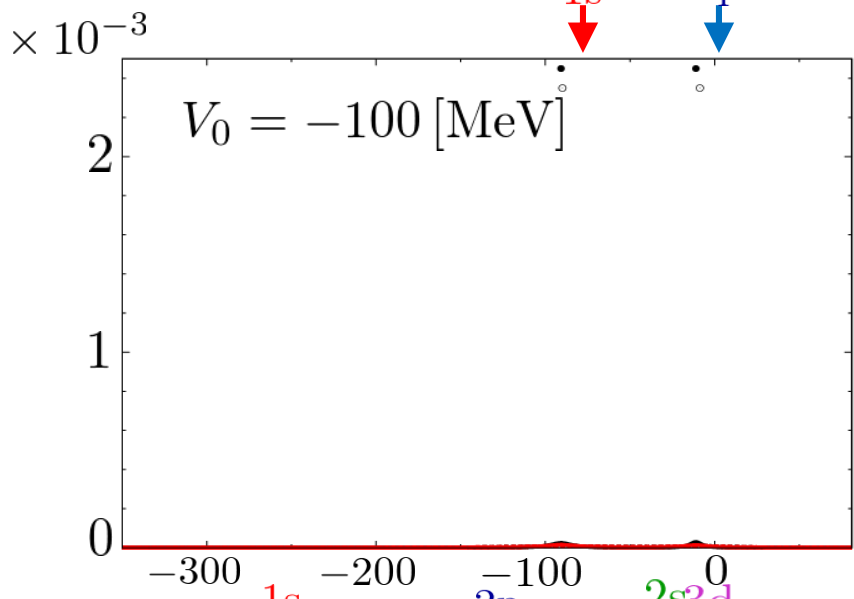
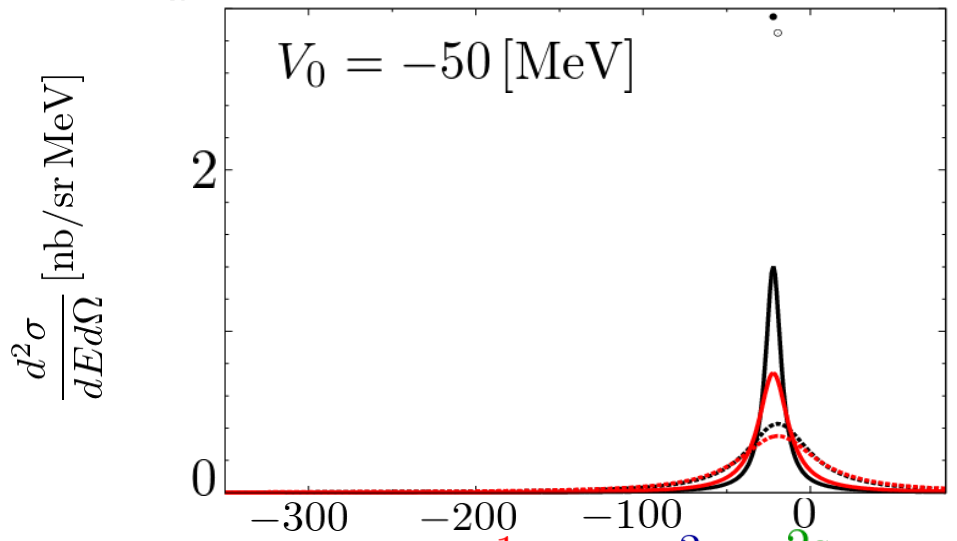


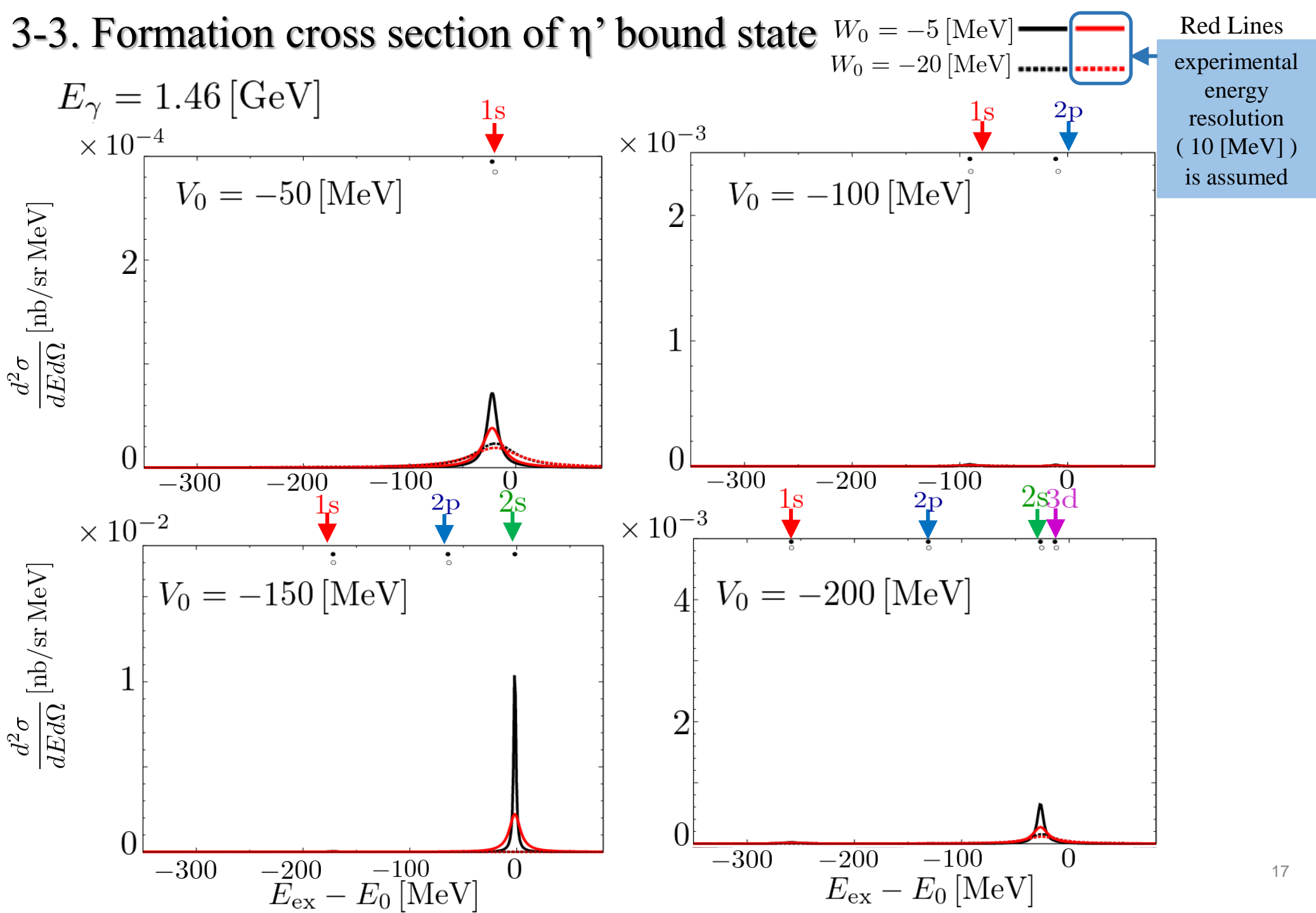
### 3-3. Formation cross section of $\eta'$ bound state

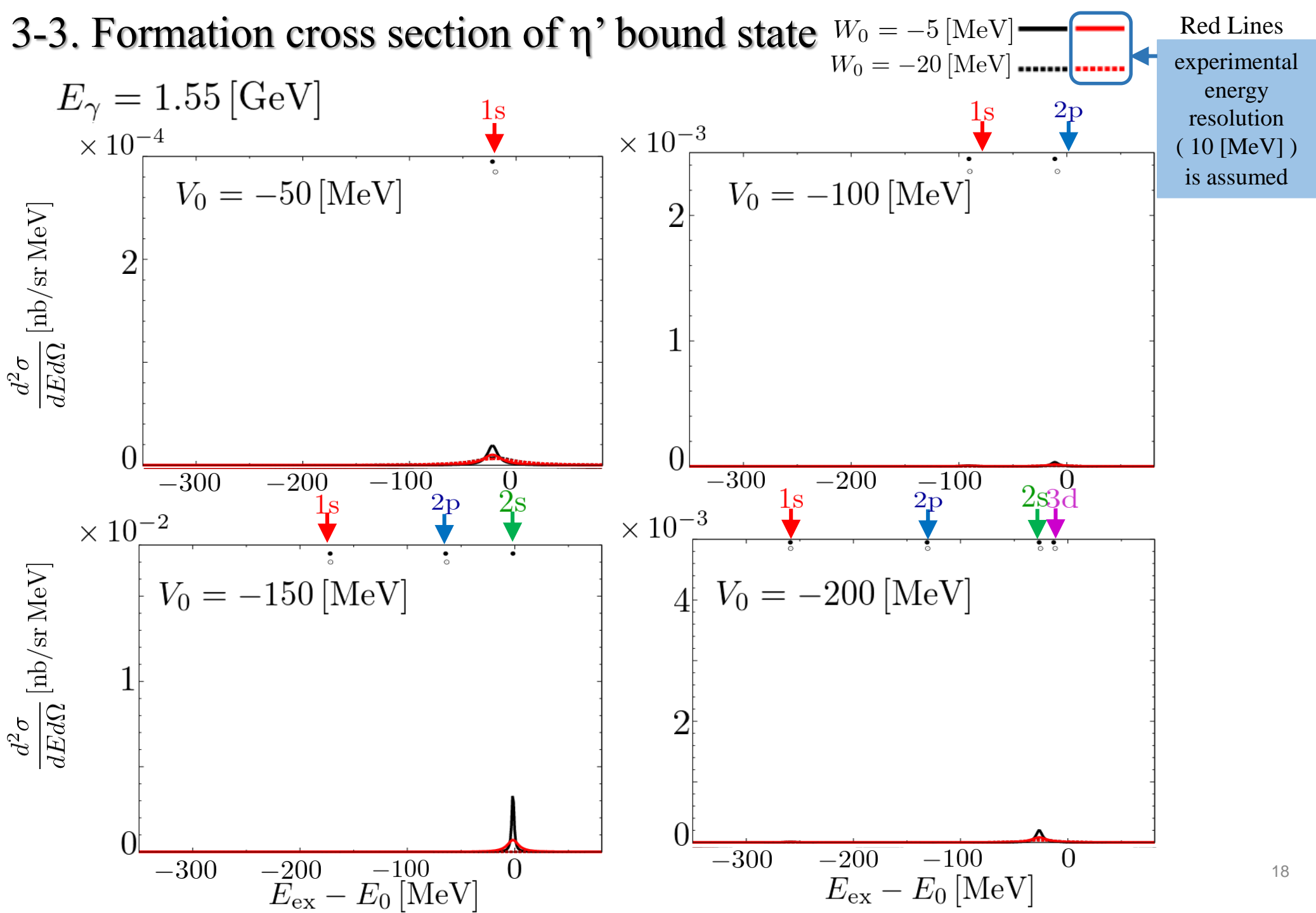
$W_0 = -5$  [MeV] ————  
 $W_0 = -20$  [MeV] ······

Red Lines  
 experimental  
 energy  
 resolution  
 ( 10 [MeV] )  
 is assumed

$E_\gamma = 1.4$  [GeV]  
 $\times 10^{-4}$







## 4. Summary

### Purpose of this work

To know the possibility of formation of  $\eta'$ (958) mesic nucleus by  $(\gamma, d)$  reaction

### Formalism

- Effective number approach
  - Improvements from N. Ikeno et al., Phys. Rev. C 84, 054609 (2011)
    - Distortion effect
    - Elementary cross section
    - Realistic  $\alpha$  density distribution
    - Recoil effect
- ‘ Formation of  $\phi$  mesic nucleus ’

### Numerical results

- Formation of the  $\eta'$  mesic nucleus in recoilless kinematics is possible
- Formation cross section
  - Peak height is smaller than 0.01 [nb/sr MeV] for almost all cases
  - Larger cross sections at  $E_\gamma = 1.24$  [GeV] than other energies considered here
  - The bound  $\eta'$  states form well-separated peak structures in the spectra

## Discussions

### ➤ Effects of the deuteron form factor

Large effects for the formation cross section of  $\eta'$  mesic nucleus

⇒ For larger  $E_\gamma$ ,  $\left(\frac{d\sigma}{d\Omega}\right)^{\text{ele}}$  decreases at  $\theta_{\gamma d} = 0^\circ$

⇒ Experiments with photon energy  
around  $\eta'$  production threshold ( $E_\gamma \sim 1.2$  [GeV]) could be better



### ➤ Incident photon energy

- for larger elementary cross section
- for recoilless kinematics ( $E_\gamma \sim 1.46$  [GeV])

are different !!

⇒ We need to consider optimum incident  $\gamma$  energy

## Future plan

- Discussion for the feasibility of the experiments with experimentalists  
(Ishikawa, Fujioka, ...)
- Systematic theoretical calculations
- Optimum condition determination
- .....