Two, three, many body systems involving mesons

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Systems of two mesons and a baryon Three body systems with strangeness Three body with hidden strangeness Nuclei made from mesons? Multirho states K*-multirho states Pseudotensor mesons, J^{PC}=2⁻⁺

ρD* D*bar

D* -multirho states

DNN system

Dynamically generated resonances in three-body systems

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$\Sigma(1480)$ Bumps

 $I(J^{P}) = 1(?^{?})$ Status: *



$$I(J^P) = 1(\frac{1}{2}^-)$$
 Status: **

$$\Sigma(1620)$$
 Production Experiments

 $I(J^P) = 1(?^?)$ OMITTED FROM SUMMARY TABLE

ps

Production experiments: Partial-wave analyses of course separate partial waves, whereas a peak in a cross section or an invariant mass distribution usually cannot be disentangled from background and analyzed for its quantum numbers; and more than one resonance may be contributing to the peak. Results from partial-wave analyses and from production experiments are generally kept separate in the Listings, and in



$$I(J^P) = 1(\frac{1}{2}^+)$$
 Status: * $\Sigma(1560)$ Bum

$$I(J^P) = 1(?^?)$$
 Status: **



N(1710) DECAY MODES

The following branching fractions are our estimates, not fits or averages.

	Mode	Fraction (Γ_i/Γ)
Γ_1	$N\pi$	10-20 %
Γ_2	Nη	(6.2±1.0) %
Γ ₃	Nω	(13.0±2.0) %
Γ4	ΛΚ	5–25 %
Γ_5	ΣΚ	
T _s	$N\pi\pi$	40–90 %
Γ ₇	$\Delta\pi$	15-40 %
Г ₈	$arDelta(1232)\pi$, $\mathit{P} ext{-wave}$	
Г9	Nρ	5–25 %
Γ ₁₀	N ho, S=1/2, P-wave	
Γ_{11}	Nρ, S=3/2. P-wave	
Г	$N(\pi\pi)^{I=0}_{S-wave}$	10-40 %
Γ_{13}	$p\gamma$	0.002-0.05%
Γ_{14}	$p\gamma$, helicity= $1/2$	0.002-0.05%
Γ ₁₅	nγ	0.0-0.02%
Γ ₁₆	$n\gamma$, helicity=1/2	0.0–0.02%

One starts from a two nucleon potential and draws all possible diagrams with any amount of interactions between two particles



The iterative potential exchanges between two same lines sum up to give the two nucleon t-matrix

Formalism

$$T = T^1 + T^2 + T^3$$





The input of the Faddeev equations,

$$T^{1} = t^{1} + t^{1}G\left[T^{2} + T^{3}\right]$$
$$T^{2} = t^{2} + t^{2}G\left[T^{1} + T^{3}\right]$$
$$T^{3} = t^{3} + t^{3}G\left[T^{1} + T^{2}\right]$$

i.e., the two-body t-matrices are calculated by solving the Bethe-Salpeter equation \rightarrow potential from Unitary Chiral Dynamics ^{9,10,11}.

In general, one has to solve the Faddeev equations with t *off-shell*.

Kaiser, Siegel and Weise, Nucl. Phys A594 (1995); Kaiser , Waas and Weise Nucl. Phys A612 (1997) ⁹ E. Oset, A. Ramos, Nucl. Phys. A 635 (1998) 99, J. A. Oller, E. Oset, Nucl. Phys. A 620 (1997) 438 ¹⁰ T. Inoue, E. Oset, M. J. Vicente Vacas, Phys. Rev. C 65 035204 ¹¹ J.A. Oller, Ulf-G. Meissner, Phys. Lett. B 500 (2001) 263-272, J.A. Oller, Ulf-G. Meissner, Phys. Rev. D,1999.



There are six diagrams with two successive interactions.



This is not the only source of three body forces.

 \Box They also arise directly from the chiral Lagrangian¹².



Hence, the diagrams containing two t-matrices, like



can be written mathematically as tⁱg^(ij)t^j where,

$$g^{(ij)} = \frac{N_k}{2E_k} \frac{1}{\sqrt{s} - E_i(\vec{k}_i) - E_j(\vec{k}_j) - E_k(\vec{k}_i' + k_j) + i\epsilon}$$

 $N_l = \begin{cases} 1 & \text{meson-meson interaction} \\ 2M_l & \text{meson-baryon interaction.} \end{cases}$

If we add another interaction to this diagram, we have a loop of three particles. □ Let us consider the diagram shown below.



We write its contribution as

$$t^{1}G^{121}t^{2}g^{21}t^{1} = t^{1}(\sqrt{s_{23}})G^{121}t^{2}(\sqrt{s_{13}})g^{21}(\vec{k}_{2}',\vec{k}_{1})t^{1}(\sqrt{s_{23}})$$

where

$$G^{121} = \int \frac{d\vec{q_1}}{(2\pi)^3} \frac{1}{2E_2(\vec{q_1})} \frac{M_3}{E_3(\vec{q_1})} \frac{1}{\sqrt{s_{23}} - E_2(\vec{q_1}) - E_3(\vec{q_1}) + i\epsilon} \times F^{121}(\vec{q_1}, \vec{k'_2}, \vec{k_1}, s_{13})$$

$$F^{121}(\vec{q_1}, \vec{k'_2}, \vec{k_1}, s_{13}) = t^2(s_{13}^{q_1}) \times g^{21}(\vec{q_1}, \vec{k_1}) \times [g^{21}(\vec{k'_2}, \vec{k_1})]^{-1} \times [t^2(\sqrt{s_{13}})]^{-1}$$

Note that s₂₃ is defined in the diagram from the external variables, the argument s₁₃ of the t² t-matrix in F¹²¹ depend on the loop variable

$$s_{13}^{q_1} = s - m_2^2 - 2\sqrt{s} \, \frac{E_2(\vec{q}_1)(\sqrt{s} - E_3(\vec{k}_3))}{\sqrt{s_{12}}}$$

Reformulation of the Faddeev Equations

• We re-write the Faddeev equations in terms of the G and g functions as

$$\begin{split} T_R^{(12)} &= t^1 g^{(12)} t^2 + t^1 \left[G^{(121)} T_R^{(21)} + G^{(123)} T_R^{(23)} \right] \\ T_R^{(13)} &= t^1 g^{(13)} t^3 + t^1 \left[G^{(131)} T_R^{(31)} + G^{(132)} T_R^{(32)} \right] \\ T_R^{(21)} &= t^2 g^{(21)} t^1 + t^2 \left[G^{(212)} T_R^{(12)} + G^{(213)} T_R^{(13)} \right] \\ T_R^{(23)} &= t^2 g^{(23)} t^3 + t^2 \left[G^{(231)} T_R^{(31)} + G^{(232)} T_R^{(32)} \right] \\ T_R^{(31)} &= t^3 g^{(31)} t^1 + t^3 \left[G^{(312)} T_R^{(12)} + G^{(313)} T_R^{(13)} \right] \\ T_R^{(32)} &= t^3 g^{(32)} t^2 + t^3 \left[G^{(321)} T_R^{(21)} + G^{(323)} T_R^{(23)} \right] \end{split}$$

Note that the Faddeev partitions have been redefined as

$$T^{i} = \left[(2\pi)^{3} \tilde{N}_{i} \delta^{3} (\vec{k}_{i} - \vec{k}_{i}') \right] t^{i} + T_{R}^{(ij)} + T_{R}^{(ik)}$$

The terms denoted as T_R contain only connected diagrams.

The πKN system and its coupled channels

A. Martinez Torres, K. Khemchandani, E. O, PRC77 (2008)

- We study the πKN system by solving the Faddeev equations in the coupled channel approach.

total energy. Later we vary these two variables.

• The three pairs (πN , KN, $K\pi$) couple strongly to many other channels.



$\Lambda(1600) P_{01}$

$$I(J^P) = 0(\frac{1}{2}^+)$$
 Status: ***

There are quite possibly two P_{01} states in this region



□ We find two peaks at 1568 MeV (width 70 MeV) and 1700 MeV (width 136 MeV) in the $\pi \overline{K}N$ amplitude .

Solving these equations for the π KN system and its coupled channels, we find four Σ 's and two Λ 's as dynamically generated resonances in this system.

	$\Gamma (PDG)$	Peak position (this work)	Γ (this work)
	(MeV)	$({ m MeV})$	(MeV)
		Isospin = 1	
$\Sigma(1560)$	10 - 100	1590	70
$\Sigma(1620)$	10 - 100	1630	39
$\Sigma(1660)$	40 - 200	1656	30
$\Sigma(1770)$	60 - 100	1790	24
		Isospin = 0	
$\Lambda(1600)$	50 - 250	1568,1700	60, 136
$\Lambda(1810)$	50 - 250	1740	20

- It is rewarding that the widths for all these resonances are smaller than the total ones listed by the PDG.
 - It should be emphasized that all the Σ 's and all the Λ 's 1/2⁺ up to 1810 MeV get dynamically generated as three body resonances.

Other states obtained with $\pi \pi N$ and coupled channels:

The π N amplitudes at energies bigger than 1600 MeV have been taken from experiment,

K. Khemchandani, A. Martinez Torres, E. O, EPJA37 (2008)

$I(J^P)$	Theory			PDG data		
	$\operatorname{channels}$	mass	width	name	mass	width
		(MeV)	(MeV)		(MeV)	(MeV)
$1/2(1/2^+)$	only $\pi\pi N$	1704	375	$N^{*}(1710)$	1680 - 1740	90-500
	$\pi\pi N, \pi K\Sigma, \pi K\Lambda, \pi\eta N$	\sim no change	\sim no change			
$1/2(1/2^+)$	only $\pi\pi N$	2100	250	$N^{*}(2100)$	1885 - 2270	80-400
	$\pi\pi N, \pi K\Sigma, \pi K\Lambda, \pi\eta N$	2080	54			
$3/2(1/2^+)$	$\pi\pi N, \pi K\Sigma, \pi K\Lambda, \pi\eta N$	2126	42	$\Delta(1910)$	1870 - 2152	190-270
$1/2(1/2^+)$	$N\pi\pi, N\pi\eta, NKK$	1924	20	$N^*(?)$?	?

First predicted by Jido and Eny'o, PRC78, 2008

Martinez Torres, Khemchandani, Meissner, Oset, EPJA 2009, claim this state is responsible for peak of $\gamma p \rightarrow K^+ \Lambda$ around 1920 MeV

Mart and Bennhold PRC 2000



FIG. 1. Total cross section for $K^+\Lambda$ photoproduction on the proton. The dashed line shows the model without the $D_{13}(1960)$ resonance, while the solid line is obtained by including the $D_{13}(1960)$ state. The new SAPHIR data [6] are denoted by the solid squares, old data [22] are shown by the

Multirho states:

The vector vector interaction can be studied using the local hidden gauge formalism, Bando et al.

$$\mathcal{L}^{(4V)} = \frac{g^2}{2} \langle V_\mu V_\nu V^\mu V^\nu - V_\nu V_\mu V^\mu V^\nu \rangle , \qquad g = M_v / 2f_\pi$$

$$\mathcal{L}^{(3V)} = ig\langle (V^{\mu}\partial_{\nu}V_{\mu} - \partial_{\nu}V_{\mu}V^{\mu})V^{\nu}\rangle ,$$

$$V^{(I=0,S=2)}(s) = -4g^2 - 8g^2 \left(\frac{3s}{4m_{\rho}^2} - 1\right) \sim -20g^2$$
$$V^{(I=2,S=2)}(s) = 2g^2 + 4g^2 \left(\frac{3s}{4m_{\rho}^2} - 1\right) \sim 10g^2$$

$$T = \frac{V}{1 - VG},$$

$$G(s) = i \int \frac{d^4p}{(2\pi)^4} \frac{1}{p^2 - m_{\rho}^2 + i\epsilon} \frac{1}{(Q-p)^2 - m_{\rho}^2 + i\epsilon} ,$$

Rho-rho interaction in the hidden gauge approach





We would like to construct states of many ρ with parallel spins, so as to have maximum binding for any pair



This is like a ferromagnet of ρ mesons

Fixed center approximation to ρf_2 scattering







a)





+







This interaction generates the ρ_3

d)

+

$$T_{1} = t_{1} + t_{1}G_{0}T_{2}$$
$$T_{2} = t_{2} + t_{2}G_{0}T_{1}$$
$$T = T_{1} + T_{2}$$

$$G_0 \equiv \frac{1}{M_{f_2}} \int \frac{d^3 q}{(2\pi)^3} F_{f_2}(q) \frac{1}{q^{0^2} - \vec{q}^2 - m_{\rho}^2 + i\epsilon}$$

$$F_{f_2}(q) = \frac{1}{\mathcal{N}} \int_{\substack{p < \Lambda \\ |\vec{p} - \vec{q}| < \Lambda}} d^3 p \, \frac{1}{M_{f_2} - 2\omega_\rho(\vec{p})} \, \frac{1}{M_{f_2} - 2\omega_\rho(\vec{p} - \vec{q})}$$

where the normalization factor ${\cal N}$ is

$$\mathcal{N} = \int_{p < \Lambda} d^3 p \frac{1}{(M_{f_2} - 2\omega_\rho(\vec{p}))^2}$$

One then continues and makes scattering of f_2 with f_2 to get the f_4 Then ρ interaction with f_4 to give ρ_5 and finally f_2 with f_4 to give f_6





On the nature of the $K_2^*(1430)$, $K_3^*(1780)$, $K_4^*(2045)$, $K_5^*(2380)$ and K_6^* as K^* -multi- ρ states

J. Yamagata-Sekihara¹ L. Roca² and E. Oset¹ Phys.Rev. D82 (2010) 094017



	A	$B(b_1b_2)$
two-body	ρ	K^*
three body	K^*	$f_2 \ (\rho \rho)$
tillee-body	ho	$K_2^*~(\rho K^*)$
four-body	f_2	$K_2^* \ (\rho K^*)$
five body	K^*	$f_4 \ (f_2 f_2)$
nve-body	ho	$K_4^* \ (f_2 K_2^*)$
siy body	K_2^*	$f_4 \ (f_2 f_2)$
SIX-DOUY	f_2	$K_4^* (f_2 K_2^*)$



generated	amplituda	magg DDC [91]	${ m mass}$	\max
resonance	ampirtude	mass, PDG [21]	only single scatt.	full model
$K_2^*(1430)$	$ ho K^*$	1429 ± 1.4	_	1430
$K_3^*(1780)$	K^*f_2	1776 ± 7	1930	1790
$K_4^*(2045)$	$f_2 K_2^*$	2045 ± 9	2466	2114
$K_5^*(2380)$	K^*f_4	$2382 \pm 14 \pm 19$	2736	2310
K_6^*	$K_2^* f_4 - f_2 K_4^*$	—	3073-3310	2661-2698

Description of $\rho(1700)$ as a $\rho K \bar{K}$ system with the fixed center approximation

Bayar, Liang, Uchino and Xiao, EPJA 2014



63.7

160.8

 250 ± 100

Width (MeV) 144.4

The cluster is assumed to be the interacting K Kbar pair that forms the $f_0(980)$

 $|T|^2$

Pseudotensor mesons as three body resonances

Luis Roca, Phys.Rev. D84(2011) 094006

Systems with $J^{PC} = 2^{-+}$ can be regarded as molecules made of a pseudoscalar (P) 0^{-+} and a tensor 2^{++} meson

with the 2⁺⁺ state made out of two vector mesons

assigned	dominant	mass	mass, only	mass
resonance	$\operatorname{channel}$	PDG [47]	single scatt.	full model
$\pi_2(1670)$	$\eta a_2(1320)$	1672 ± 3	1800	1660
$\eta_2(1645)$	$\eta f_2(1270)$	1617 ± 5	1795	1695
$K_2^*(1770)$	$Ka_2(1320)$	1773 ± 8	1775	1775

The era of charm

States of $\rho D^* \overline{D}^*$ with J = 3 within the Fixed Center Approximation to the Faddeev equations

M. Bayar, X. L. Ren, E. O, Eur. Phys. J. A 2015

In the D* D*bar interaction one state with $J^P=2^+$ is generated around 3920 MeV, which could be the X(3915) or the Z(3940) (with I=0)

We let the p interact with the D* D*bar cluster and obtain a new state



FIG. 3: Modulus squared of the $\rho(D^*\bar{D}^*)$ scattering amplitude with total isospin I = 1.

A prediction of D^* -multi- ρ states

Xiao, Bayar, E. O, PRD 2012

particles:	3	R (1,2)	$\operatorname{amplitudes}$
Two-body	ho	D^*	$t_{ ho D^*}$
	ho	ho	$t_{ ho ho}$
Three-body	D^*	$f_2 (\rho \rho)$	$T_{D^*-f_2}$
	ho	$D_2^* \left(\rho D^* \right)$	$T_{\rho-D_2^*}$
Four-body	D_2^*	$f_2 (\rho \rho)$	$T_{D_2^*-f_2}$
	f_2	$D_2^* \ (\rho D^*)$	$T_{f_2 - D_2^*}$
Five-body	D^*	$f_4 \ (f_2 f_2)$	$T_{D^*-f_4}$
	ho	$D_4^* (f_2 D_2^*)$	$T_{\rho-D_4^*}$
Six-body	D_2^*	$f_4 \ (f_2 f_2)$	$T_{D_2^*-f_4}$
	f_2	$D_4^* (f_2 D_2^*)$	$T_{f_2 - D_4^*}$

TABLE I: The cases considered in the $D^*\mbox{-multi-}\rho$ interactions.



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A narrow DNN quasi-bound state

Bayar, Xiao, Hyodo, Dote, Oka, E.O., PRC 2012

Calculations done with variational method and with FCA

Bound state and narrow around 3500 -3530 MeV, Γ=30-40 MeV

Could be considered as a $\Lambda_c(2595)$ N bound state.

Interesting : Narrower than the Kbar NN system

Conclusions

--The combination of chiral unitary dynamics with Faddeev equations has proved very useful.

--Conceptually it removes the ambiguities tied to the off shell 2 body t-matrix.

--One finds that all 1/2 ⁺ low lying states (except the Roper) qualify largely as states of two mesons and one baryon.

--Multimeson states are also emerging:

-- $\Phi(2170)$ appears as resonance of $\phi(1020)$ K Kbar

--Multirho states could be identified with meson states of increasing spin

--K*-multirho states can also be identified with K* states of increasing spin --Low lying pseudotensor states also can be constructed from a pseudoscalar and two vectors coupled to a tensor.

-- In the charm sector the pattern is repeated, the widths are usually smaller. --Many other states are under investigation by several groups , Jido, Martinez Torres, Roca, Oller, Bayar, Sun, Xie

-- The beauty sector is coming