On Quasibound N*-Nuclei

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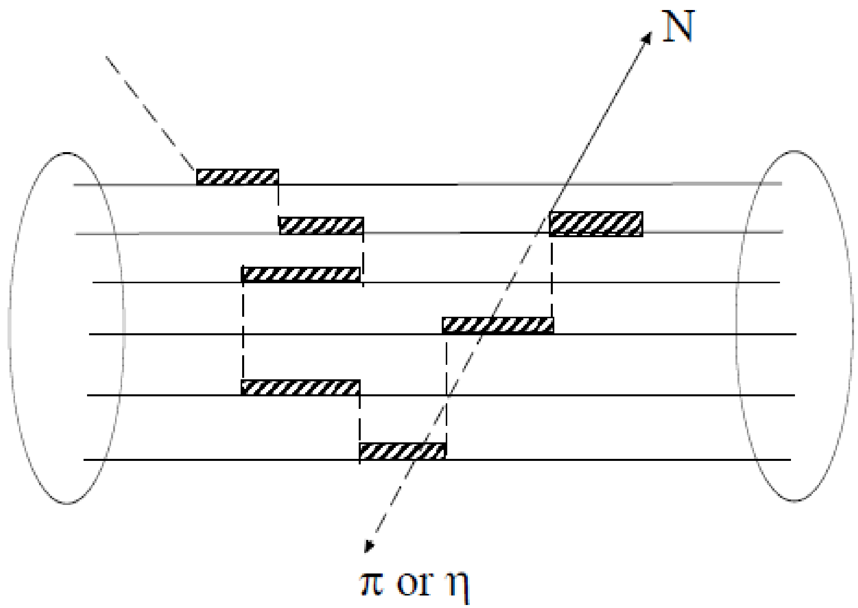
- A developing idea
- Meson-exchange model calculation
- Analytical solutions
- N*-3He and N*-24Mg states

S11 resonance N*(1535) decays in **10^{-23}** seconds

Does it make sense to talk about **quasibound states** of N* and nuclei?



People did investigate Δ-nucleus bound states!

Eur. Phys. J. A **4**, 209–216 (1999)

THE EUROPEAN PHYSICAL JOURNAL A

Short note

Evidence for narrow $\Delta^0(1232)$ states in the $^{12}C(e, e'p\pi\ \ )^{11}C$ Reaction

P. Bartsch[1], D. Baumann[1], J. Bermuth[2], K. Bohinc[3], R. Böhm[1], D. Bosnar[4], N. Clawiter[1], S. Derber[1], M. Ding[1], M.O. Distler[1,a], A. Ebbes[1], I. Ewald[1], J. M. Friedrich[1], J. Friedrich[1], P. Jennewein[1], M. Kahrau[1], M. Kohl[5], A. Kozlov[1], K.W. Krygier[1], M. Kuss[5,b], A. Liesenfeld[1], H. Merkel[1], P. Merle[1], U. Müller[1], R. Neuhausen[1], T. Pospischil[1], M. Potokar[3], D. Rohe[2], G. Rosner[1], H. Schmieden[1], S. Širca[3], A. Wagner[1], T. Walcher[1], M. Weis[1], S. Wolf[1]

$$^{12}C(e, e'\Delta^0)^{11}C \rightarrow \ ^{12}C(e, e'p\pi\ \ )^{11}C$$

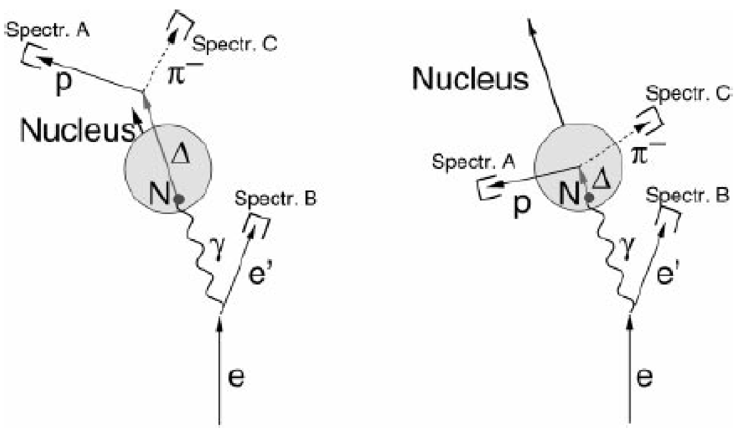
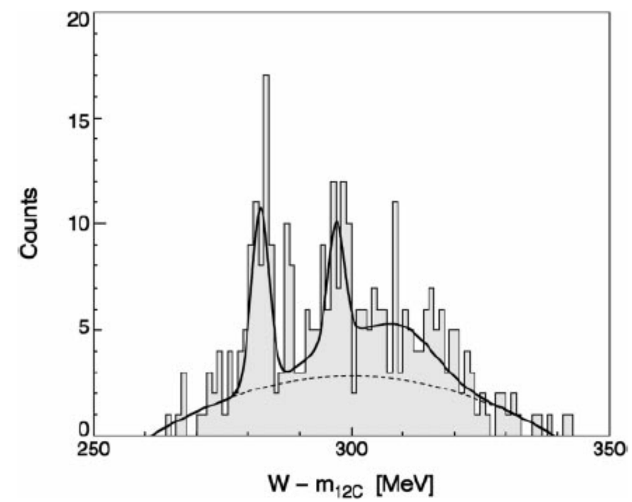


Fig. 1. Kinematics for "quasi free Δ" (left) and "bound Δ" (right), schematically



Two narrow states with a width of about 5 MeV

PHYSICAL REVIEW C, VOLUME 63, 064605

Schematic model for narrow $\Delta(1232)$ resonances bound in a nucleus

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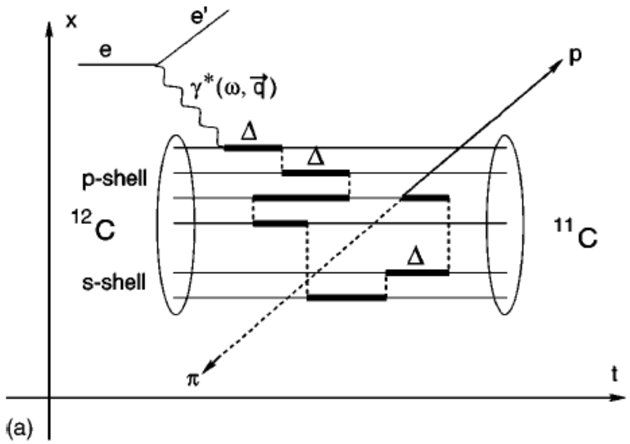
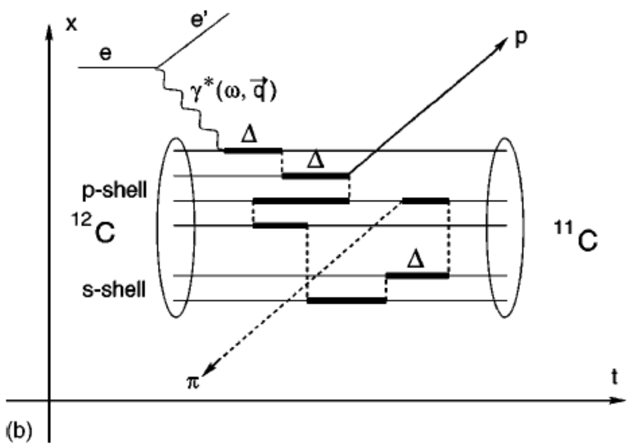
(Received 16 February 2001; published 4 May 2001)



$$e + {}^{12}C \rightarrow e' + \left[\Delta^{0}\text{-}{}^{11}C\right] \rightarrow {}^{11}C + p + \pi^{-}$$



$$e + {}^{12}C \rightarrow e' + \left[\Delta^{0}\text{-}{}^{11}C\right] \rightarrow \left[\Delta\text{-}{}^{10}C\right] + p \rightarrow {}^{11}C + p + \pi^{-}$$

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Nuclear Physics A 791 (2007) 329–341
NUCLEAR PHYSICS A

The $\Delta N \to NN$ transition in finite nuclei

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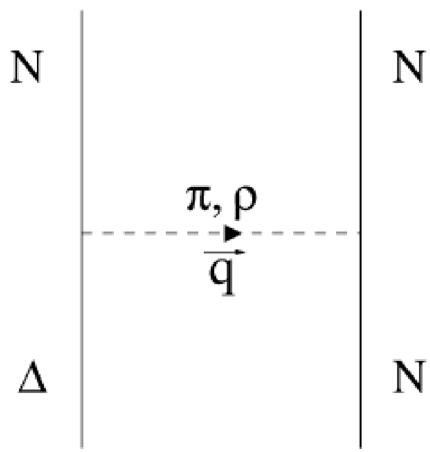
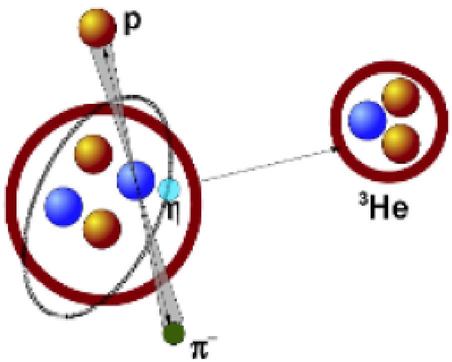


Considering also the partial width of the mesonic decay mode Δ → N π

Total width of a bound Δ in nuclei is of the order of 100 MeV

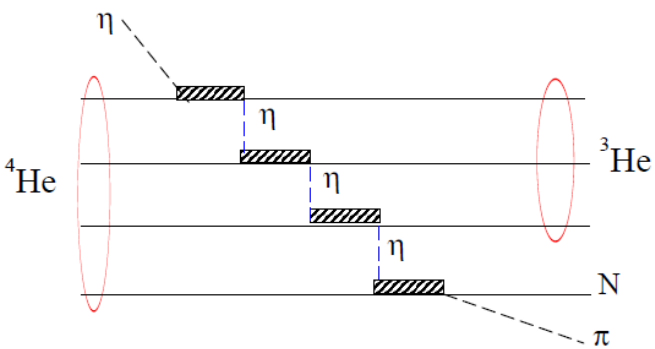
No narrow Δ bound nuclei!

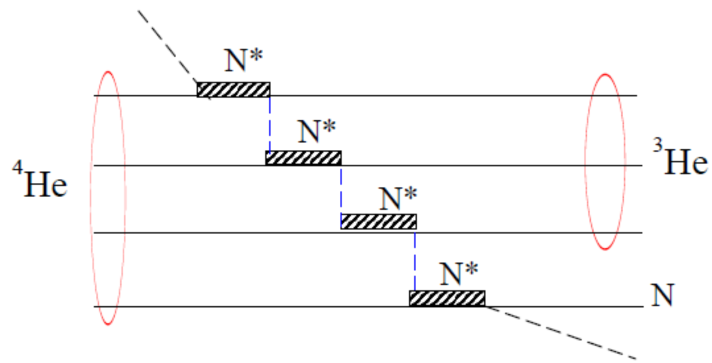
Eta-mesic nuclei and N* nuclei



W. Krzemien, P.Moskal, J. Smyrski and M. Skurzok,
Search for the η mesic 4He with WASA at COSY,
EPJ Web of Conferences 66, 09009 (2014)

$({}^{4}\mathrm{He} - \eta)_{bound} \rightarrow {}^{3}\mathrm{He} + p + \pi^{-}$

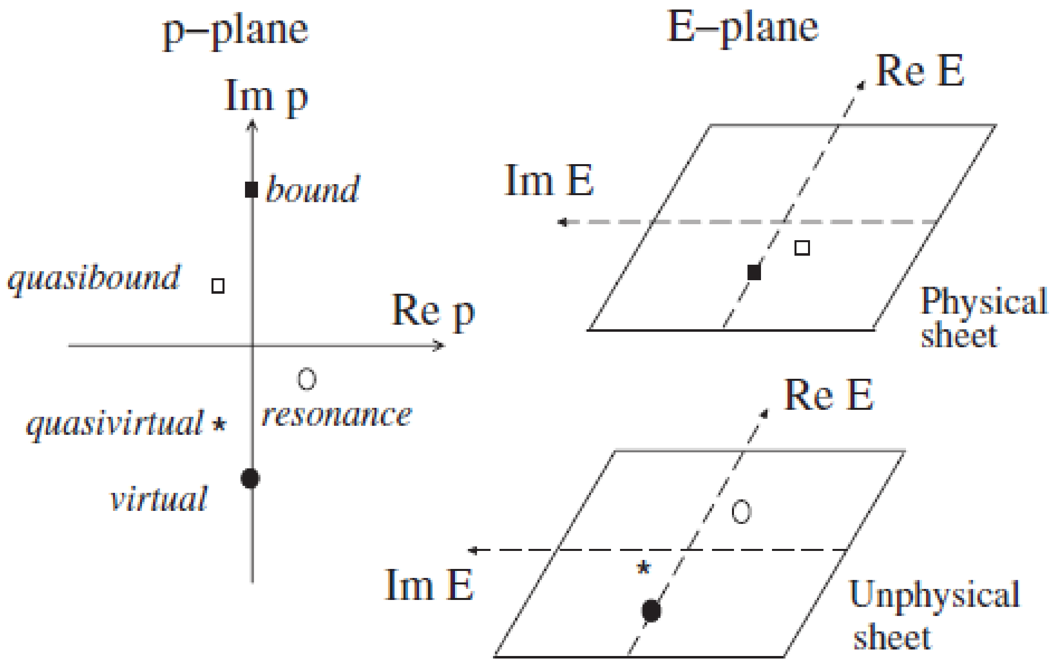




$({}^{4}\mathrm{He} - \eta)_{bound}$ ⟺ $({}^{3}\mathrm{He} - N^{*})_{bound}$??

Are they related?

Poles in the complex energy and momentum plane



$$E = p^2/2\mu \qquad E = \frac{1}{2\mu}\left(p_R^2 - p_I^2 + 2\,i\,p_R\,p_I\right)$$

Quasibound state → real part of the energy is negative
decays exponentially, hence pole at (-|E| - iΓ/2)

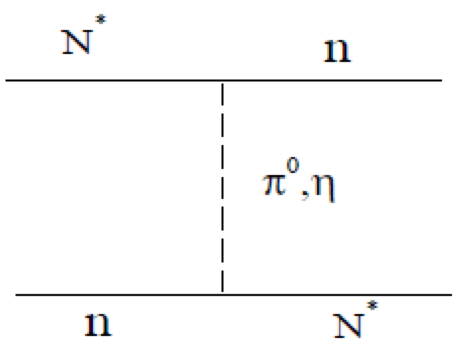
The N*-nucleus potential

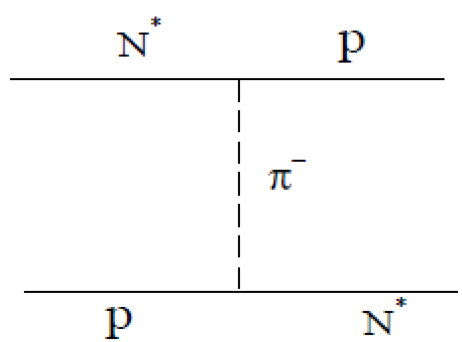
The potential corresponding to the elementary N N* → N N* reaction is written within a one-meson excahnge model with the exchange of π and η mesons.

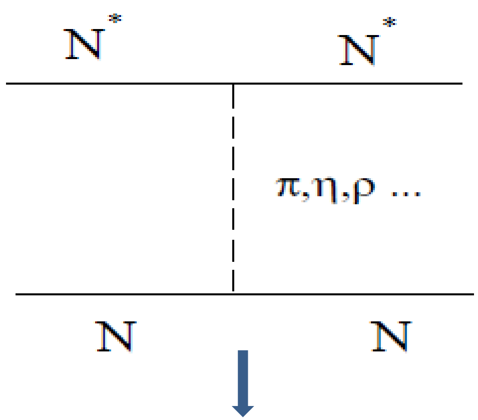
The interaction Hamiltonians are given by

$$\delta H_{\pi NN^*} = g_{\pi NN^*} \bar{\Psi}_{N^*} \boldsymbol{\tau} \Psi_N \cdot \boldsymbol{\Phi_\pi} + \mathrm{h.c.}$$

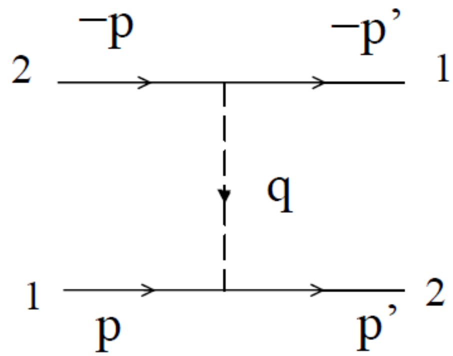
$$\delta H_{\eta NN^*} = g_{\eta NN^*} \bar{\Psi}_{N^*} \Psi_N \cdot \Phi_\eta + \mathrm{h.c.}$$







Not considered here!



$$u_i = \sqrt{2m_i}\begin{pmatrix} w_i \\ \frac{\vec{\sigma}_i\cdot\vec{p}_i}{2m_i c}\, w_i \end{pmatrix}$$

The amplitude for the process N* n → n N* for example can be written as

$$\frac{g^2_{xNN^*}\,\bar{u}_2(\vec{p}')\,u_1(\vec{p})\,\bar{u}_1(-\vec{p}')\,u_2(-\vec{p})}{q^2 - m_x^2}$$

where *x* is the π or η exchanged meson

$$\bar{u}_1(-\vec{p}')\,u_2(-\vec{p}) = N\left(1 - \frac{\vec{\sigma}_2\cdot\vec{p}'\vec{\sigma}_1\cdot\vec{p}}{4m_N m_N^* c^2}\right)$$

Dropping the $1/c^2$ suppressed, spin dependent term the N N* potential in momentum space:

Form factor off shell mesons

$$v_x(q) = \frac{g^2_{xNN^*}}{q^2 - m_x^2}\left(\frac{\Lambda_x^2 - m_x^2}{\Lambda_x^2 - q^2}\right)^2$$

The Foruier transform of the N N* potential gives

$$v_x(r) = \frac{g^2_{xNN^*}}{4\pi}\left[\frac{1}{r}\left(e^{-\Lambda_x r} - e^{-m_x r}\right) + \frac{\Lambda_x^2 - m_x^2}{2\Lambda_x}\, e^{-\Lambda_x r}\right]$$

Folding this potential on to the nuclear density

$$V(R) = \int d^3r\, \rho(r)\, v(|\vec{r} - \vec{R}|)$$

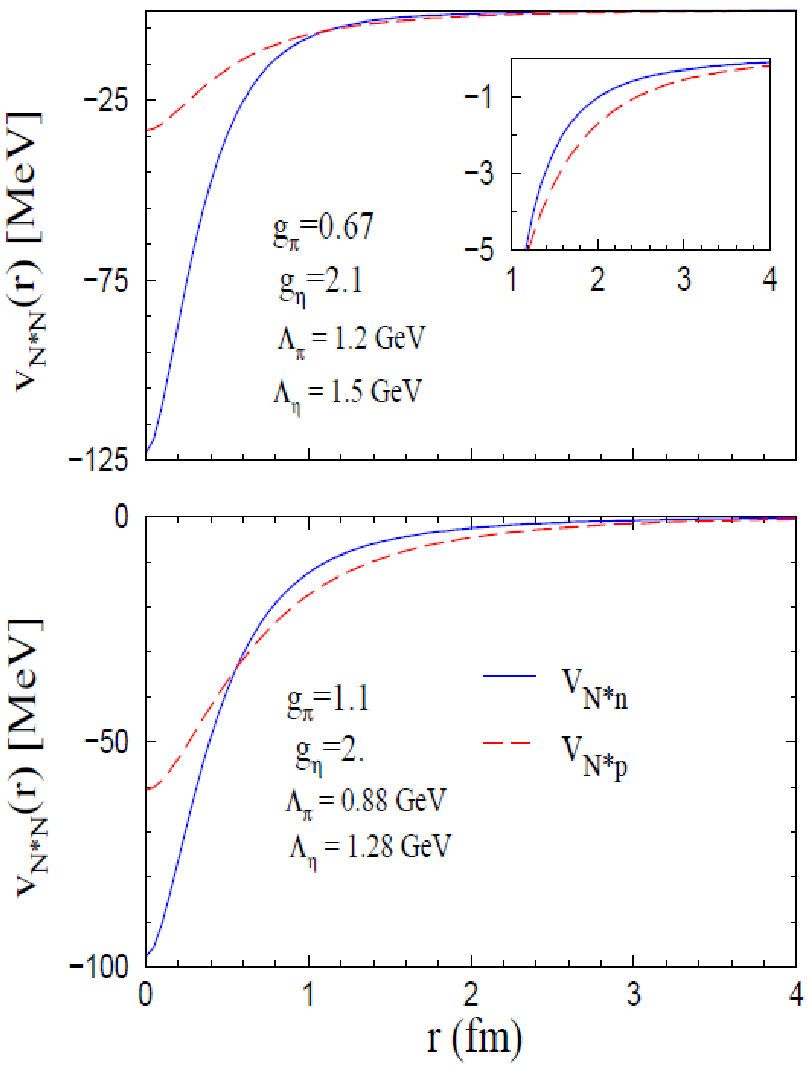
the N*-nucleus potential is written as

$$\begin{aligned} V(R) &= V_n(R) + V_p(R) \\ &= Z\int d^3r\, \rho_p(r)\, v_p(|\vec{r} - \vec{R}|) + N\int d^3r\, \rho_n(r)\, v_n(|\vec{r} - \vec{R}|)\,, \end{aligned}$$

$$v_n(r) = v_{\pi^0}(r) + v_\eta(r) \qquad\qquad v_p(r) = v_{\pi^-}(r)\vec{\tau}_1\cdot\vec{\tau}_2$$

We assume $\rho(r) = \rho_n(r) = \rho_p(r)$

Potentials are sensitive to the choice of the coupling parameters



Parameters taken from:

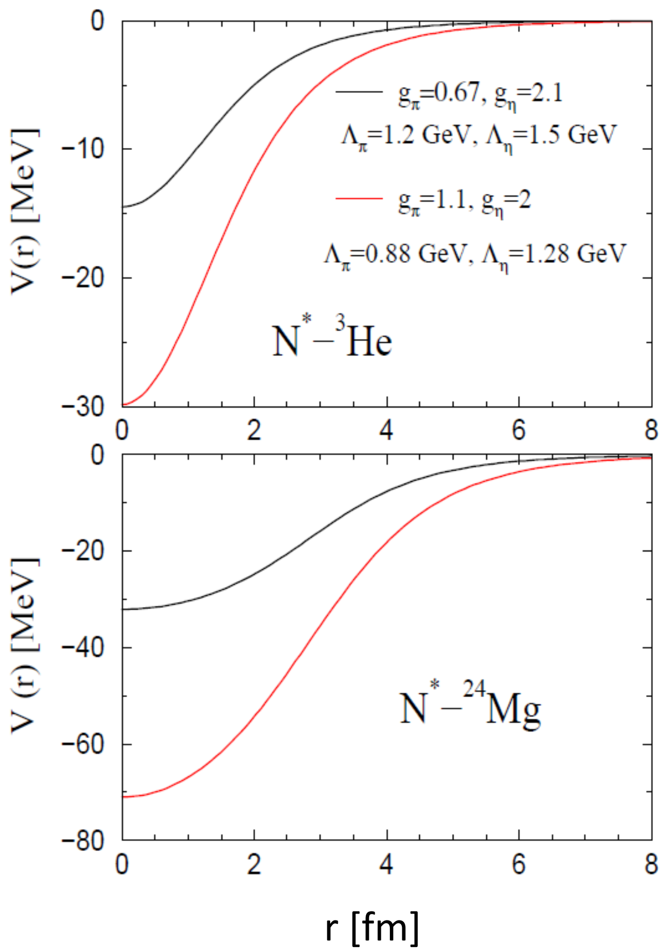
A.B. Santra and B. K. Jain, Nucl. Phys. A 634, 309 (1998)

A. Fix and H. Arenhoevel, Nucl. Phys. A 697, 277 (2002).

$$V_n(R) = \frac{-2\pi A}{R}\int r\,dr\,\rho(r)\Big\{\frac{e^{-m_x(|r-R|)} - e^{-m_x(r+R)}}{m_x} - \frac{e^{-\Lambda_x(|r-R|)} - e^{-\Lambda_x(r+R)}}{\Lambda_x}$$

$$+B\Big[\Big(\frac{r+R}{\Lambda_x}+\frac{1}{\Lambda_x^2}\Big)e^{-\Lambda_x(r+R)} - \Big(\frac{|r-R|}{\Lambda_x}+\frac{1}{\Lambda_x^2}\Big)e^{-\Lambda_x|r-R|}\Big]\Big\}, \quad A = g_{xNN^*}^2/4\pi$$

$$B = (\Lambda_x^2 - m_x^2)/2\Lambda_x$$



The ^{3}He density

$$\rho(r) = \frac{1}{8\pi^{3/2}}\left[\frac{1}{a^3}e^{-r^2/4a^2} - \frac{b^2(6c^2-r^2)}{4c^7}e^{-r^2/4c^2}\right]$$

The ^{24}Mg density

$$\rho(r) = \frac{\rho_0}{1+\exp\left(\frac{r-c}{a}\right)},$$

$$c = r_A\,[1-(\pi^2 a^2/3r_A^2)] \qquad a = 0.54 \text{ fm}$$

$$r_A = 1.13A^{1/3}$$

Bound states of a Woods – Saxon potential

S. Fluegge, Practical Quantum Mechanics; M. Ghominejad, Eur. Phys. J Plus 128, 59 (2013)

For a Woods-Saxon potential of the type $V(r) = -\frac{V_0}{1+e^{\frac{r-R}{a}}}$

The Schroedinger equation $\frac{d^2u}{dr^2} + \frac{2}{r}\frac{du}{dr} + \frac{2m}{\hbar^2}(E-V)u = 0$

may be transformed to the independent variable $y = \frac{1}{1+e^{\frac{r-R}{a}}}$

to give

$$y(1-y)\frac{d^2\chi}{dy^2} + (1-2y)\frac{d\chi}{dy} + \frac{-\beta^2+\gamma^2 y}{y(1-y)}\chi = 0.$$

This equation is to be solved with the boundary conditions

$$\chi = 0 \text{ at } y = 0 \;\; (r = \infty) \qquad\qquad y \simeq 1 - e^{-R/a} \simeq 1 \;\; (r = 0).$$

Writing $\chi(y) = y^{\nu}(1-y)^{\mu} f(y)$

one can get the hypergeometric differential equation

$$y(1-y)f'' + [(2\nu+1) - y(2\nu+2\mu+2)f' - (\nu+\mu)(\nu+\mu+1)f = 0.$$

Eventually after performing some algebra, we obtain the condition for bound states

$$\boxed{\frac{\lambda R}{a} + \Psi - 2\phi - \arctan\frac{\lambda}{\beta} = (2n-1)\frac{\pi}{2} \quad n = 0, \pm 1, \pm 2, \ldots}$$

$$\frac{2mE}{\hbar^2} a^2 = -\beta^2; \quad \frac{2mV_0}{\hbar^2} a^2 = -\gamma^2; \quad \lambda = \sqrt{\gamma^2 - \beta^2}$$

$$\phi = arg\Gamma(\beta = i\lambda); \; \Psi = arg\Gamma(2i\lambda).$$

Bound states for the two sets of parameters:

Set 1: $g_{\pi NN^*} = 0.67$, $g_{\eta NN^*} = 2.1$, $\Lambda_\pi = 1.2$ GeV and $\Lambda_\eta = 1.5$ GeV

Set 2: $g_{\pi NN^*} = 1.1$, $g_{\eta NN^*} = 2$, $\Lambda_\pi = 0.88$ GeV, and $\Lambda_\eta = 1.28$ GeV

	Set 1	Set 2
N*-^{3}He	-0.028172 MeV $V_0 = 18.12, a = 0.79, R = 1.27$	-3.88736 MeV $V_0 = 37.27, a = 0.84, R = 1.37$
N*-^{24}Mg	-17.1039 MeV and -1.82417 MeV $V_0 = 34.03, a = 0.91, R = 2.88$	-47.6003 MeV and -20.7584 MeV and -2.63927 MeV $V_0 = 76.4, a = 0.98, R = 2.87$

An estimate for the width

T. Walcher, Phys. Rev. C 63, 064605 (2001)

Assuming an average mean free path of the meson inside the nucleus to be

$$\langle l(\omega)\rangle = (\rho\,\sigma(\omega))^{-1}$$

and also assuming that the N* was produced at the centre of the nucleus, the number of times that the meson rescatters is given by

$$N(\omega) = g_{corr}\left(\frac{R}{\langle l(\omega)\rangle}\right)^2 = g_{corr}[R\rho\sigma(\omega)]^2\,,$$

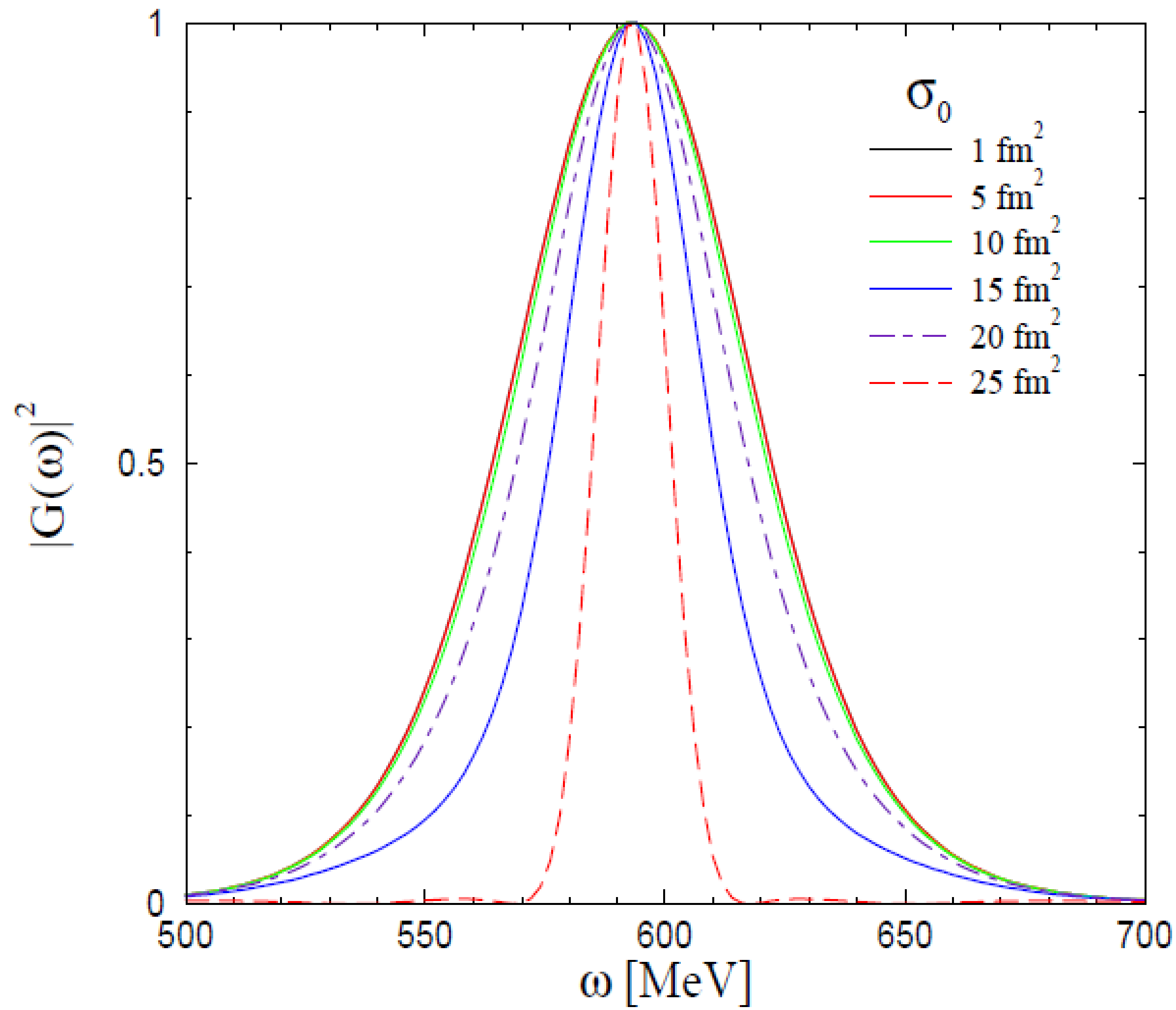
Starting with the amplitude

$$G(\omega) = G_0\frac{\hbar}{2\pi}\frac{-i}{(\omega-\omega_0-\epsilon)+i(\Gamma/2)}$$

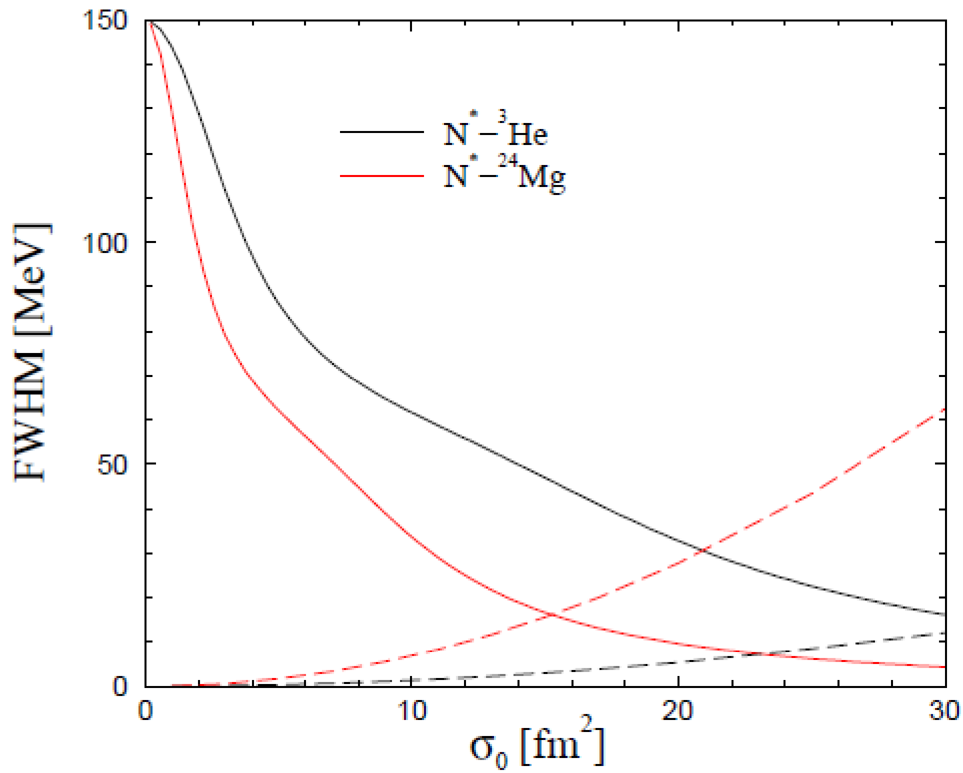
where ϵ is the binding energy of the N* in the nucleus and taking care of the fact that the meson does not propagate as a plane wave between rescatterings in the nucleus

$$|G(\omega)|^2 = G_0^2\frac{\hbar^2}{2\pi}\frac{1}{(\omega-\omega_0-\epsilon)^2+(\Gamma/2)^2}\frac{\sin^2\left((N(\omega)+1)\phi(\omega)/2\right)}{\sin^2\left(\phi(\omega)/2\right)}$$

$$\phi(\omega) = \arctan\left(\frac{\omega_0+\epsilon-\omega}{\Gamma/2}\right) \qquad \sigma(\omega) = \sigma_0\,\frac{(\Gamma/2)^2}{(\omega-\omega_0-\epsilon)^2+(\Gamma/2)^2}\,.$$



1
0.5
0
500
550
600
650
700
$|G(\omega)|^2$
ω [MeV]
σ_0
1 fm^2
5 fm^2
10 fm^2
15 fm^2
20 fm^2
25 fm^2



Dashed lines are the number of rescatters

$$N(\omega) = g_{corr}\left(\frac{R}{\langle l(\omega)\rangle}\right)^2 = g_{corr}[R\rho\sigma(\omega)]^2 \,,$$

$$\sigma(\omega) = \sigma_0 \, \frac{(\Gamma/2)^2}{(\omega - \omega_0 - \epsilon)^2 + (\Gamma/2)^2} \,.$$

Widths are insensitive to the change in binding energy, parameters of N N* interaction

Cross sections for $\pi^- p \rightarrow \pi^- p$, $\pi^0 n$ or $\eta N \rightarrow \eta N$ are of the order of 3 fm^2 in the region of N* → Broad width and hardly one or two rescatters!

Coming back to the question:

$(A - \eta)_{bound}$ ⟺ $([A\text{-}1] - N^*)_{bound}$??

Let us have a look at the predictions for η-mesic nuclei

Table 1. Pole values of eta-mesic light nuclear states.

	Complex Pole (E,Γ/2) in (MeV)	State	Ref.
ηd	$-2 - i10$	Quasibound	[25]
	$-i10.317$	Quasibound	[32]
	$28.06 - i24.976$	Resonance	[32]
	$8.24 - i4.575$	Resonance	[32]
	$3.73 - i3.405$	Resonance	[32]
	$i0.743$	Quasivirtual	[29]
	$-24 + i27.93$	Quasivirtual	[26]
	$-0.87 + i0.95$	Quasivirtual	[26]
	$-17.1 - i17.5$	Quasibound	Missed in [26] noted in [41]
	$-15 - i20$	Quasibound	[41]
η–^{3}He	$7.03 - i13.1$	Resonance	[32]
	$-i11.15$	Quasibound	[32]
	$0.5 - i0.65$	Resonance	[44]
	$-5 - i8, -i\,1.95$	Quuasibound	[44]
η–^{4}He	$-4.44 - i6.37, -i5.725$	Quasibound	[32]
	$-2 - i1.75$	Quasibound	[44]

N. G. Kelkar, K. P. Khemchandani, N. J. Upadhyay and B. K. Jain, Rep. Prog. Phys. 76, 066301 (2013).

Table 2. Quasibound η-mesic states.

Nucleus		Pole values in MeV	Ref.
^{6}He	1s	−10.7 − i7.25, −8.75 − i14.95	[23]
^{11}B	1s	−24.5 − i11.4, −22.9 − i23.05	[23]
^{12}C	1s	−1.19 − i3.67	[20]
		−9.71 − i17.5	[22]
		−5 − i8, −6 − i16	[21]
^{16}O	1s	−3.45 − i5.38	[20]
		−32.6 − i13.35, −31.2 − i26.95	[23]
	1p	−7.72 − i9.15, −5.25 − i19.1	[23]
^{24}Mg	1s	−12.57 − i16.7	[22]
^{26}Mg	1s	−6.39 − i6.6	[20]
		−38.8 − i14.25, −37.6 − i28.65	[23]
^{27}Al	1s	−16.65 − i17.98	[22]
	1p	−2.9 − i20.47	[22]
^{28}Si	1s	−16.78 − i17.93	[22]
	1p	−3.32 − i20.35	[22]
^{40}Ca	1s	−8.91 − i6.8	[20]
		−14 − i43, −18 − i21, −14 − i11.5	[21]
		−46 − i15.85, −44.8 − i31.8	[23]
		−17.88 − i17.19	[22]
	1p	−3 − i16.5	[21]
		−7.04 − i19.3	[22]
		−26.8 − i13.4, −25.2 − i27.1	[23]
	2s	−4.61 − i8.85, −1.24 − i19.25	[23]
^{90}Zr	1s	−14.8 − i8.87	[20]
		−52.9 − i16.6, −51.8 − i33.2	[23]
	1p	−4.75 − i6.7	[20]
		−40 − i15.25, −38.8 − i30.6	[23]
	2s	−21.7 − i13.05, −19.9 − i26.55	[23]

Predictions of broad eta-mesic states I

Table 3
Calculated η meson single-particle energies, $E = Re(E_\eta - m_\eta)$, and full widths, Γ, (both in MeV), in various nuclei, where the complex eigenenergies are, $E_\eta = E + m_\eta - i\Gamma/2$. See Eq. (13) for the definition of γ_η. Note that the free space width of the η is 1.18 keV, which corresponds to $\gamma_\eta = 0$

		$\gamma_\eta = 0$		$\gamma_\eta = 0.5$		$\gamma_\eta = 1.0$	
		E	Γ	E	Γ	E	Γ
${}^{16}_{\eta}O$	1s	−33.1	0	−32.6	26.7	−31.2	53.9
	1p	−8.69	0	−7.72	18.3	−5.25	38.2
${}^{40}_{\eta}Ca$	1s	−46.5	0	−46.0	31.7	−44.8	63.6
	1p	−27.4	0	−26.8	26.8	−25.2	54.2
	2s	−6.09	0	−4.61	17.7	−1.24	38.5
${}^{90}_{\eta}Zr$	1s	−53.3	0	−52.9	33.2	−51.8	66.4
	1p	−40.5	0	−40.0	30.5	−38.8	61.2
	2s	−22.3	0	−21.7	26.1	−19.9	53.1
${}^{208}_{\eta}Pb$	1s	−56.6	0	−56.3	33.2	−55.3	66.2
	1p	−48.7	0	−48.3	31.8	−47.3	63.5
	2s	−36.3	0	−35.9	29.6	−34.7	59.5
${}^{6}_{\eta}He$	1s	−11.4	0	−10.7	14.5	−8.75	29.9
${}^{11}_{\eta}B$	1s	−25.0	0	−24.5	22.8	−22.9	46.1
${}^{26}_{\eta}Mg$	1s	−39.2	0	−38.8	28.5	−37.6	57.3
	1p	−18.5	0	−17.8	23.1	−15.9	47.1

Predictions of the Quark-Meson Coupling (QMC) model

K. Tsushima et al., Phys. Lett. B 443, 26 (1998)

Predictions of broad eta-mesic states II

C. García-Recio, T. Inoue, J. Nieves, E. Oset, Phys. Lett. B 550, 47 (2002).

Table 1
$(B, -\Gamma/2)$ for η–nucleus bound states calculated with the energy-dependent potential

	^{12}C	^{24}Mg	^{27}Al	^{28}Si	^{40}Ca	^{208}Pb
1s	(−9.71, −17.5)	(−12.57, −16.7)	(−16.65, −17.98)	(−16.78, −17.93)	(−17.88, −17.19)	(−21.25, −15.88)
1p			(−2.90, −20.47)	(−3.32, −20.35)	(−7.04, −19.30)	(−17.19, −16.58)
1d						(−12.29, −17.74)
2s						(−10.43, −17.99)
1f						(−6.64, −19.59)
2p						(−3.79, −19.99)
1g						(−0.33, −22.45)

Table 2
$(B, -\Gamma/2)$ for η–nucleus bound states calculated with the energy-independent potential

	^{12}C	^{24}Mg	^{27}Al	^{28}Si	^{40}Ca	^{208}Pb
1s	(−17.71, −25.42)	(−22.69, −25.78)	(−33.80, −30.63)	(−34.01, −30.36)	(−35.42, −30.12)	(−39.71, −28.65)
1p			(−5.28, −23.20)	(−6.07, −23.45)	(−13.02, −25.19)	(−31.97, −27.61)
1d						(−22.69, −26.30)
2s						(−19.11, −25.55)
1f						(−12.16, −24.69)
2p						(−6.81, −23.12)
1g						(−0.60, −22.74)

Unitarized Chiral Perturbation Theory Predictions

SUMMARY

- N*-nuclei may exist, however, mostly broad
- If we can relate Eta-mesic nuclei ⟺ N* nuclei then the η-mesic nuclei would be expected to be broad too!
- An experiment similar to the one at MAMI for the Δ-nucleus could possibly be planned to look for N*-nuclei
- However, a better estimate for N*-nuclei required!

Dziękuję