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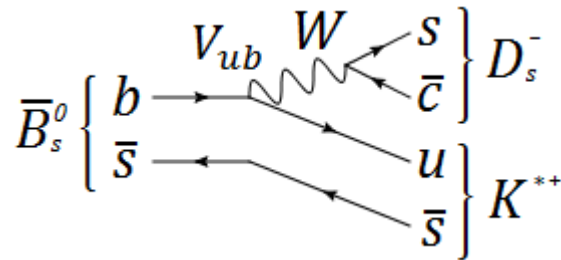
The CKM parameters sensitivity studies using the time dependent analysis for $B^0 \rightarrow DK$ decay channels

Piotr Bednarski, Agnieszka Obłąkowska-Mucha

The CKM Matrix

- The CKM Matrix describes weak interactions using 4 parameters.
- The least known parameter is γ angle

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix} = \begin{pmatrix} |V_{ud}| & |V_{us}| & |V_{ub}|e^{-i\gamma} \\ -|V_{cd}| & |V_{cs}| & |V_{cb}| \\ |V_{td}|e^{-i\beta} & -|V_{ts}|e^{i\beta_s} & |V_{tb}| \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

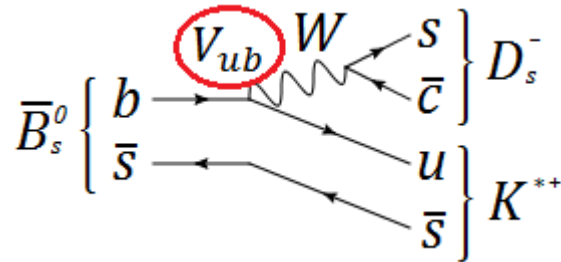


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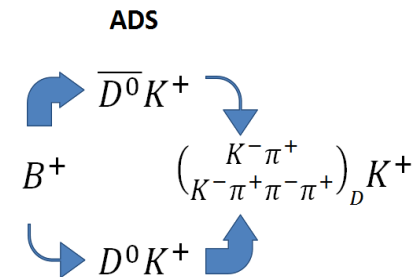
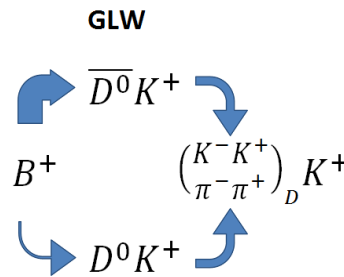
\uparrow
weak eigenstates
 \uparrow
mass eigenstates



Methods for the CKM γ Angle Extraction

a) $B^+ \rightarrow DK^+$

- Theoretically clean
- GLW and ADS methods

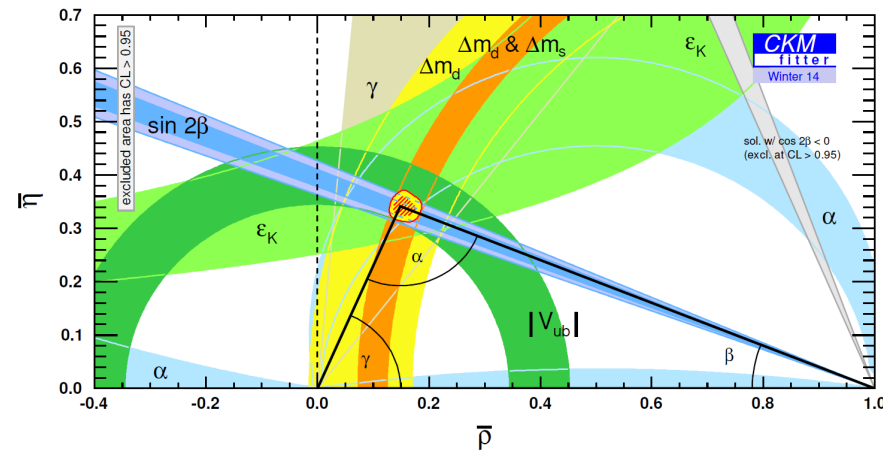


b) $B^0 \rightarrow hh$

- Rather big loop diagram contribution
- Knowledge of hadronic factors is necessary

c) Time dependent methods

- Measurement of $B^0 - \overline{B}^0$ oscillations,
- Experimentally difficult,
- No penguin pollution,
- High sensitivity for γ angle,
- Can be used as a reference point for New Physics studies.





AGH B Meson Decay Rates Equations

$$\Gamma_{B_S^0 \rightarrow f}(t) = |A_f|^2 \left(1 + |\lambda_f|^2\right) \frac{e^{-\Gamma_s t}}{2} \cdot \left(\cosh \frac{\Delta\Gamma_s t}{2} + D_f \sinh \frac{\Delta\Gamma_s t}{2} + C_f \cos \Delta m_s t - S_f \sin \Delta m_s t \right)$$

$$|B_{H,L}\rangle = p|B_S^0\rangle \mp q|\bar{B}_S^0\rangle$$

$$D_f = \frac{2\text{Re}\lambda_f}{1 + |\lambda_f|^2}, \quad D_{\bar{f}} = \frac{2\text{Re}\bar{\lambda}_{\bar{f}}}{1 + |\bar{\lambda}_{\bar{f}}|^2},$$

$$C_f = \frac{1 - |\lambda_f|^2}{1 + |\lambda_f|^2}, \quad C_{\bar{f}} = \frac{1 - |\bar{\lambda}_{\bar{f}}|^2}{1 + |\bar{\lambda}_{\bar{f}}|^2},$$

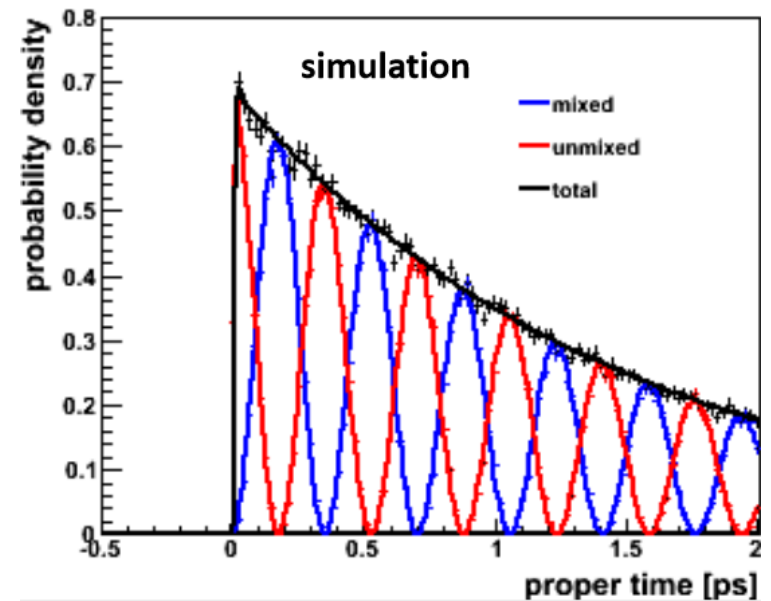
$$S_f = \frac{2\text{Im}\lambda_f}{1 + |\lambda_f|^2}, \quad S_{\bar{f}} = \frac{2\text{Im}\bar{\lambda}_{\bar{f}}}{1 + |\bar{\lambda}_{\bar{f}}|^2},$$

where:

$$A_f = \langle f|T|B_S^0\rangle, \quad \bar{A}_{\bar{f}} = \langle \bar{f}|T|\bar{B}_S^0\rangle,$$

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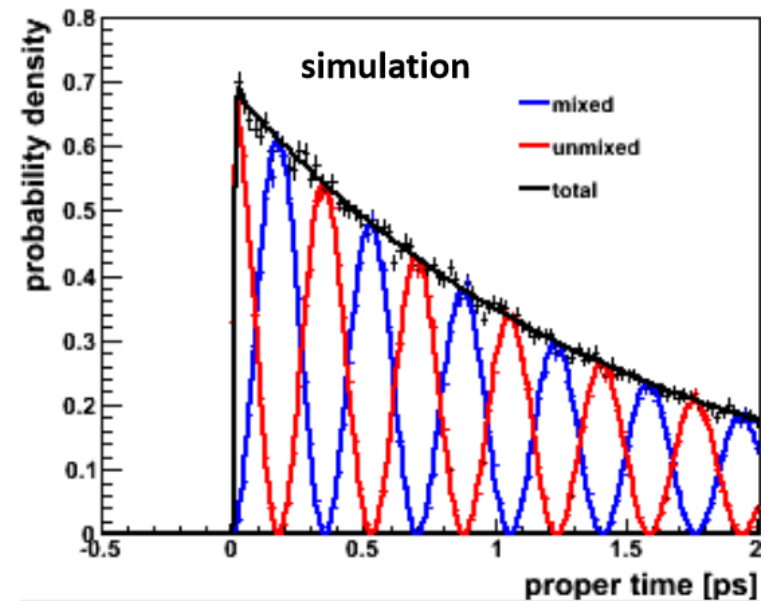
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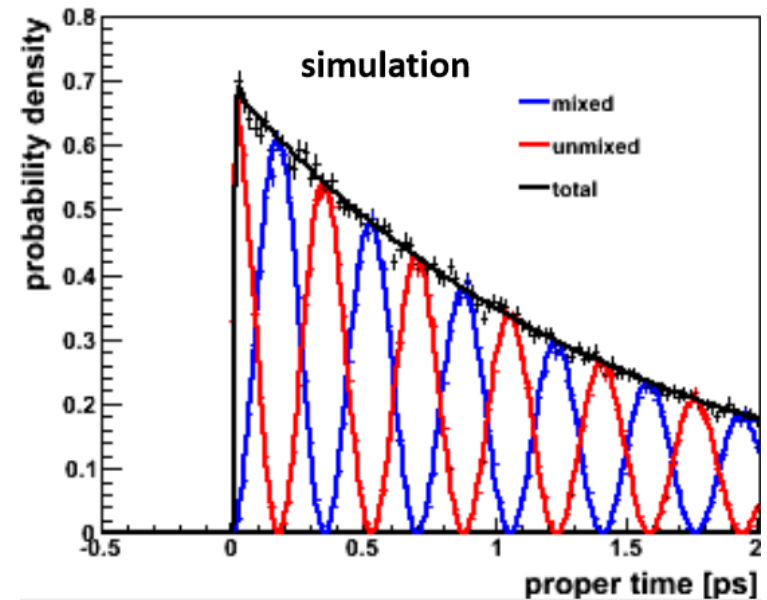
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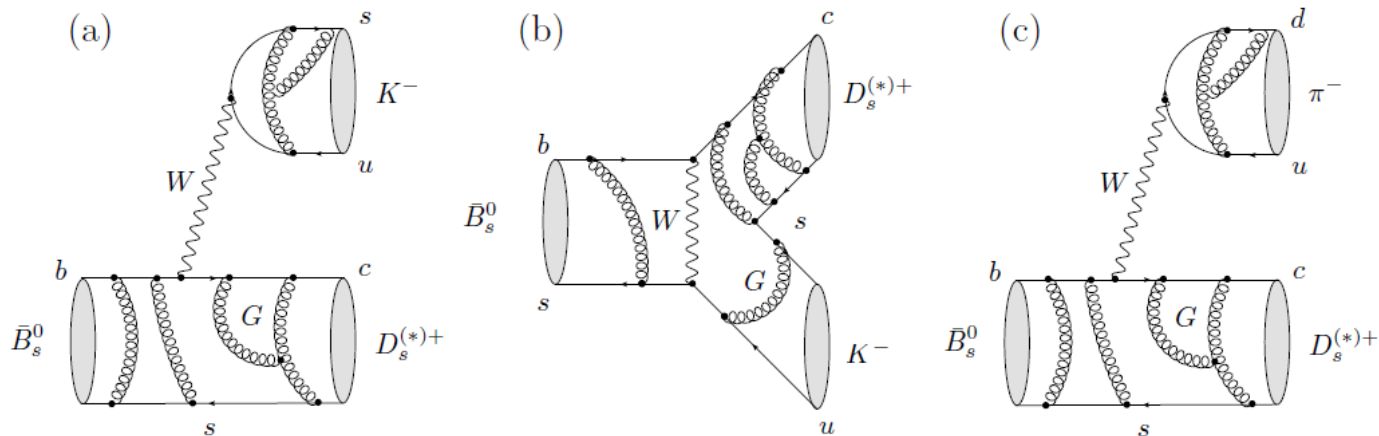
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$B^0 \rightarrow Dh$ time dependent study

1. First succesful time dependent analysis were performed in B-factories
 - The measurements were done using $B^0 \rightarrow J/\Psi K_s^0$ (measurement of β angle)
 - Thanks to large statistics in LHCb $B_s^0 \rightarrow J/\Psi \Phi$ channel can also be used (for measurement of weak phase Φ_s)
2. Nowadays different $B^0 \rightarrow Dh$ decays are used:



$B^0 \rightarrow Dh$ time dependent study

a) $B_d^0 \rightarrow D^\pm \pi^\mp, D^{*\pm} \pi^\mp$

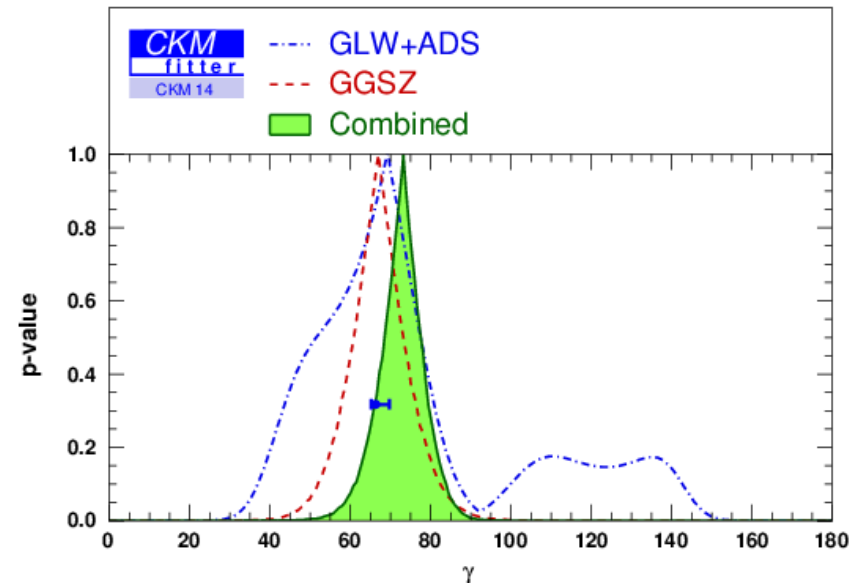
- Sensitive to γ
- Large BR

b) $B_s^0 \rightarrow D_s^+ \pi^-$

- Large BR
- Clear reconstruction
- Flavour specific (no tagging required)
- Information on $\Delta\Gamma_s$ and $\left|\frac{q}{p}\right|$

c) $B_s^0 \rightarrow D_s^- K^+, D_s^{(*)-} K^{(*)+}$

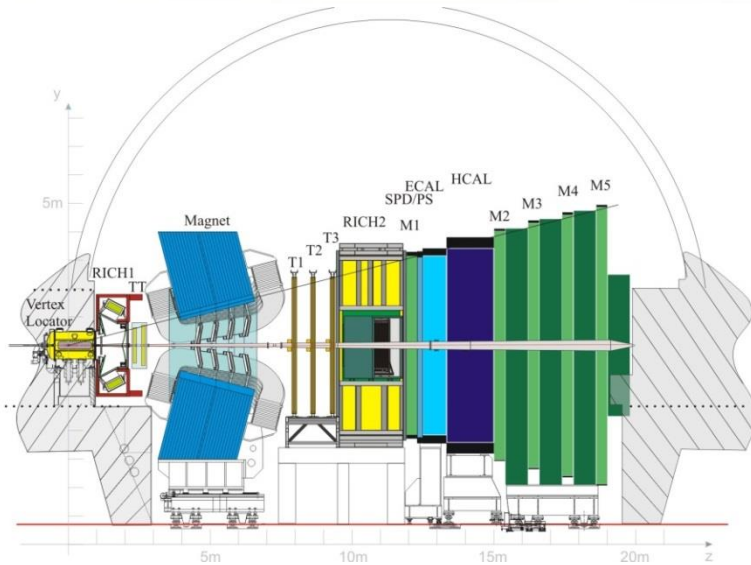
- Sensitive to $(\gamma + \Phi_s)$
- Tree dominated – precise calculations
- Small BR
- Tagging and good PID required
- Angular analysis for vector mesons
- Due to my interest in this particular decay most of the examples will be based on it





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The Experiment - LHCb



LHCb – modern B-factory:

- 10-300 mrad of space acceptance
- VELO (VERTex LOcator) – precise vertices location
- Calorimeters
- RICH – Cherenkov Radiation Detectors
- Track detectors
- Mion detectors
- 10fb^{-1} of data in Run 2!

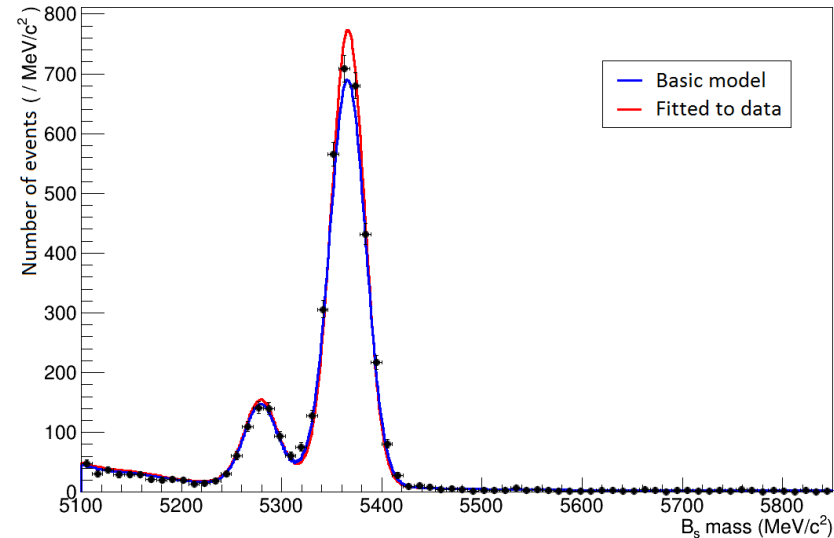


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Toy Monte Carlo study

35 000	signal events $B_s \rightarrow D_s K^*$
3 400	$B_d \rightarrow D_s^* K^*$ physical background events
3 500	$B_d \rightarrow D_s K^*$ physical background events
3 000	$B_d \rightarrow DK^*$ physical background events
500	$B_s \rightarrow D_s X$ physical background events
10 000	combinatorial background events

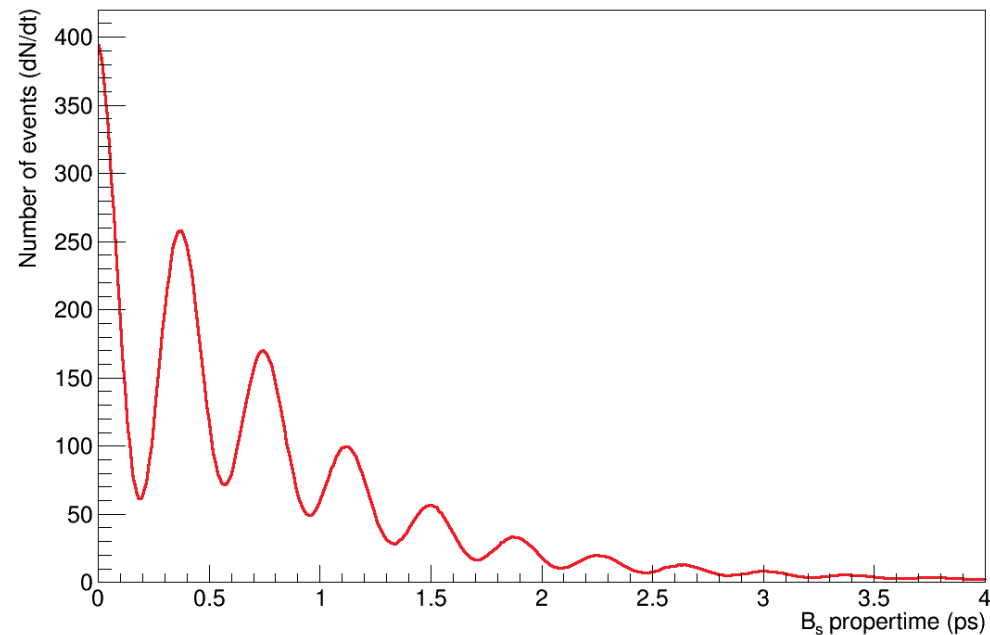
- **Much faster than full simulation**
- **Almost as accurate if used properly - remember about the background and detector effects!**
- **The $B_s \rightarrow D_s K^*$ decay simulation number of events (per 10 fb^{-1}) are shown in the table above**



Detector effects have huge impact on the shape of the lifetime curve
(mass curve stays almost intact).

Three main effects are:

- time resolution
- time acceptance,
- mistagging.

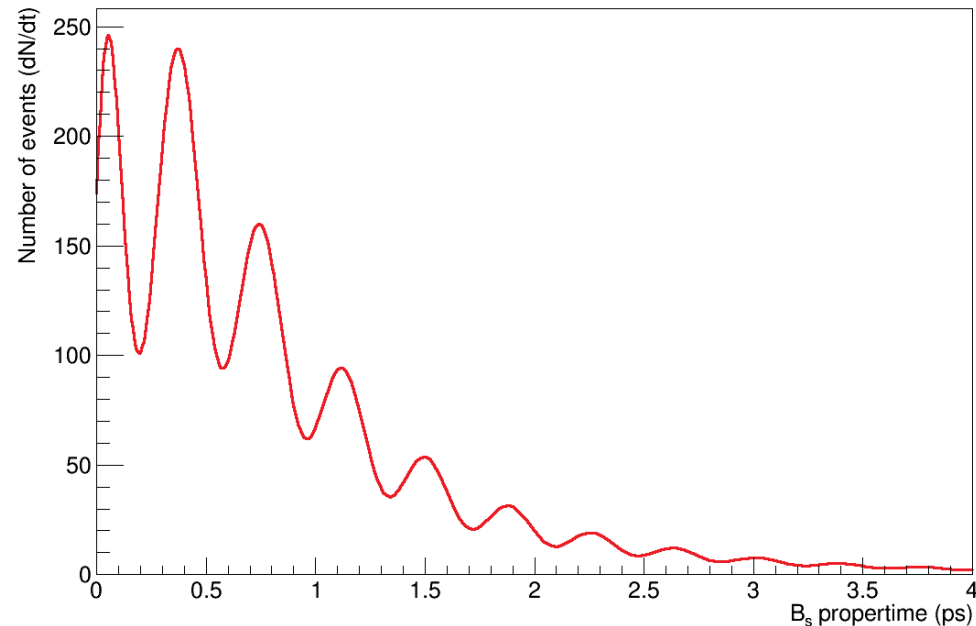
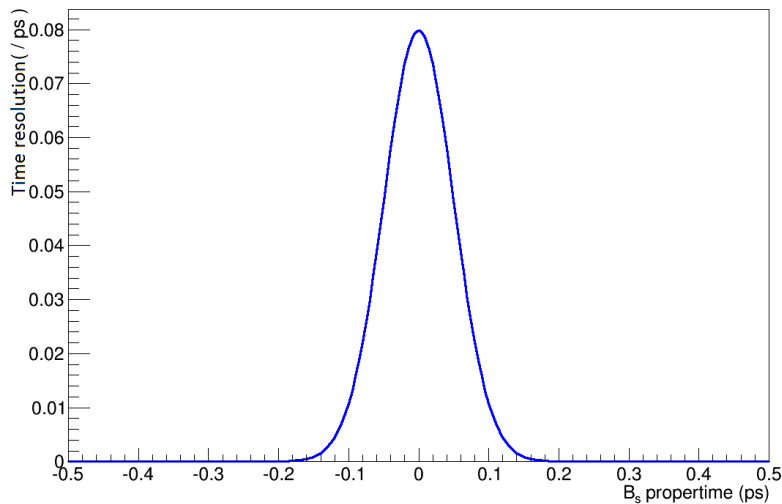


Detector Effects – Time Resolution

- Time Resolution has a finite value
- The effect can be estimated as a sum of 3 gaussians:

$$R(t) = f_1 G(t; \mu, \sigma_1) + f_2 G(t; \mu, \sigma_2) + (1 - f_1 - f_2) G(t; \mu, \sigma_3)$$

- Observed PDF is theoretical PDF convoluted with $R(t)$





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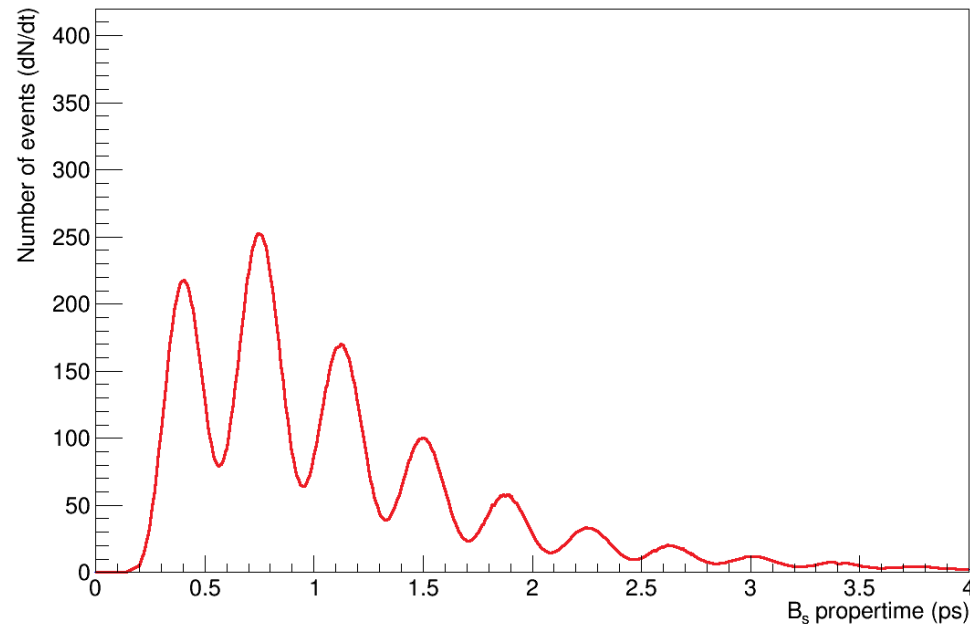
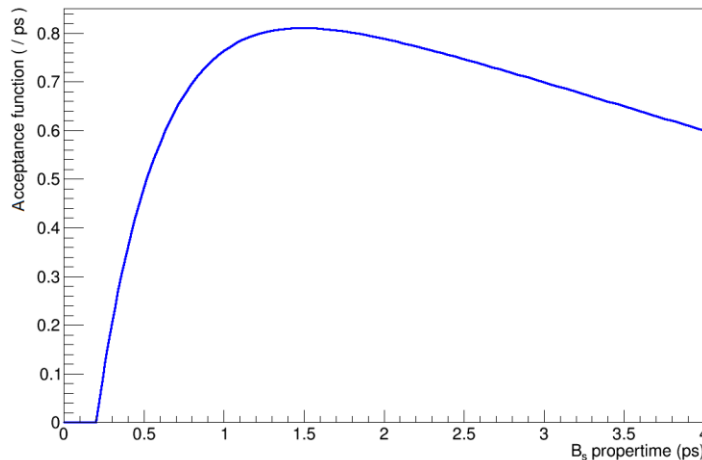
Detector Effects – Time Acceptance

- Short living particles are harder to observe
- Many different empirical models

$$acc(t) = \begin{cases} 0, & t - b < 0 \\ (1 - \exp[-a(t - b)])(1 - \beta t), & a(t - b) < 0 \\ 1 - \beta t, & \text{otherwise} \end{cases}$$

$$\begin{aligned} & t - b < 0 \\ & a(t - b) < 0 \\ & \text{otherwise} \end{aligned}$$

with $a = 2.36$, $b = 0.2\text{ps}$, $\beta = 0.1\text{ps}^{-1}$

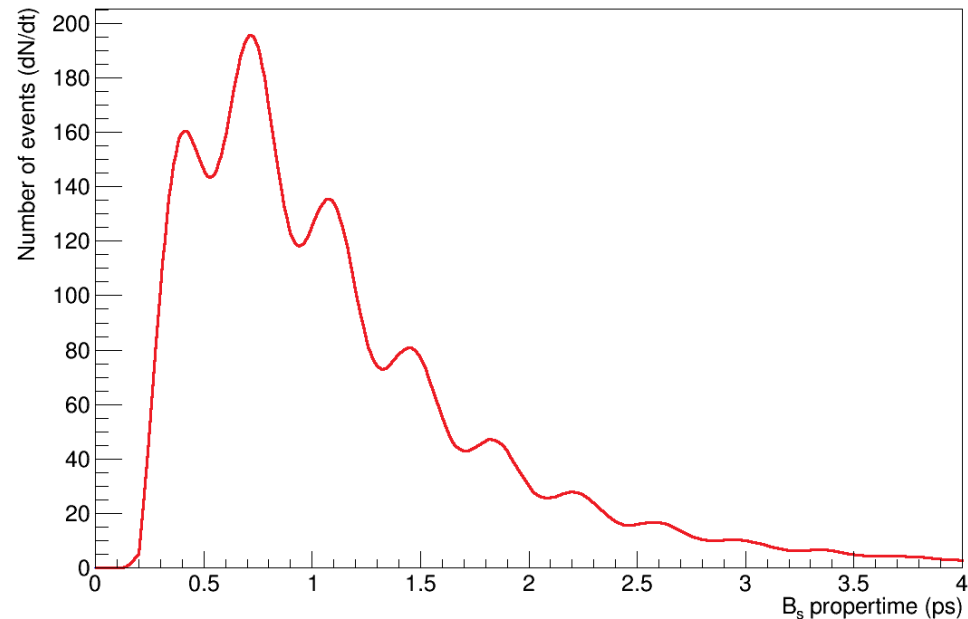




Detector Effects – Mistagging

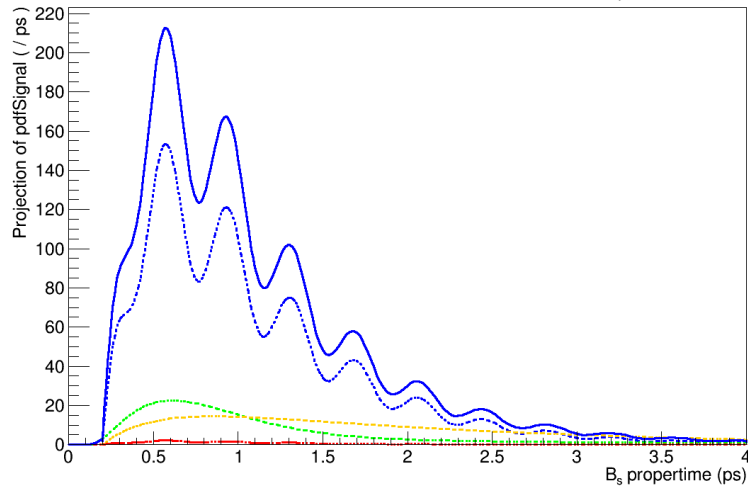
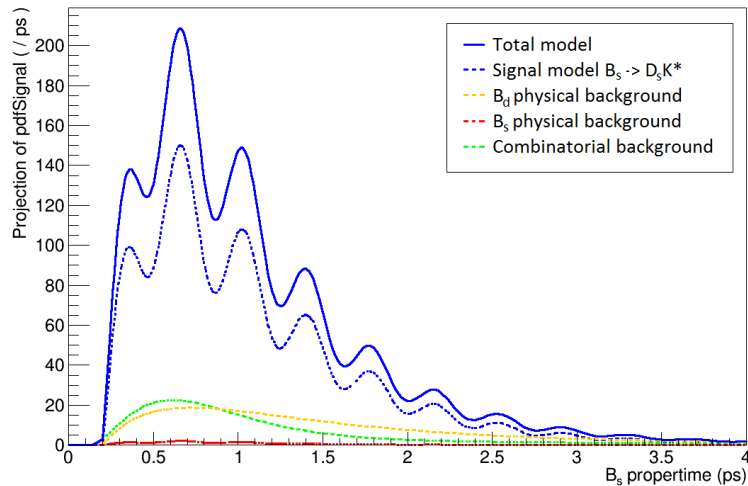
- Mistagging – decrease of amplitude by a factor of $(1 - 2\omega)$
 - A couple of different tagging methods – OS tagger, SSK tagger
 - Both initial and final states need to be tagged
 - Tagging can be expressed as follows:

$$\Gamma_{obs}(tagged B(\tau) \rightarrow f) = \epsilon(\tau)\epsilon_{tag}[(1 - \omega_{tag})\Gamma'_{obs}(B(\tau) \rightarrow f) + \omega_{tag}\Gamma'_{obs}(\bar{B}(\tau) \rightarrow f)]$$

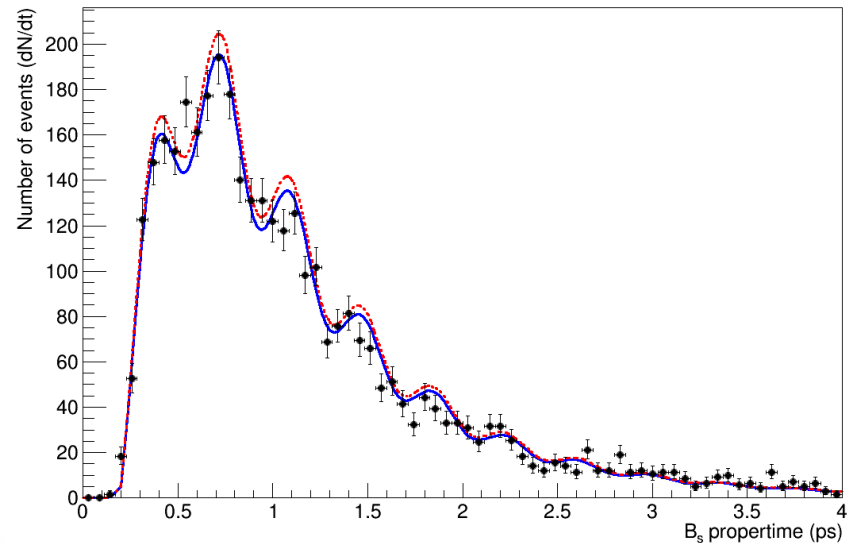


$B_s^0 \rightarrow D_s K^*$ Decay Model and sensitivity

Different final states:

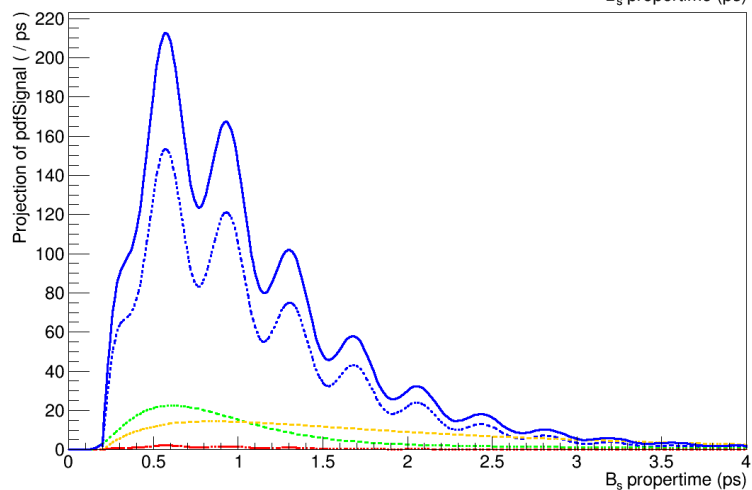
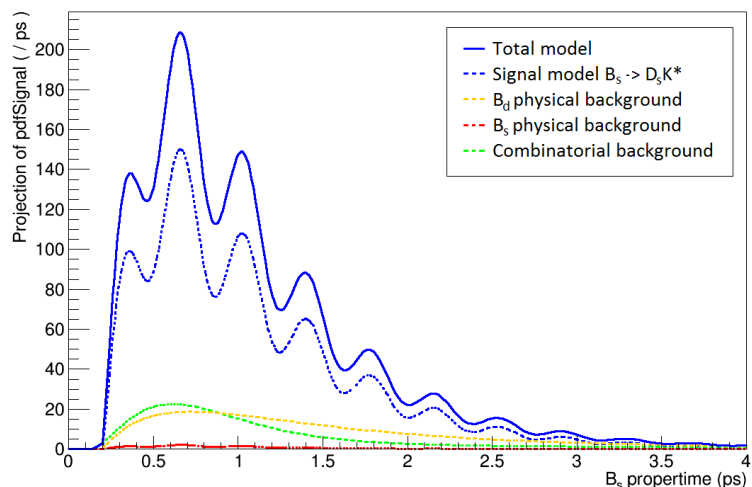


- Data points are generated according to created model
- Performing a fit to the generated data is an ,experiment'
- One can obtain fitted parameters and calculate γ
- Having done multiple such experiments one can create a pull distribution
- The pull distribution translates to sensitivity to γ

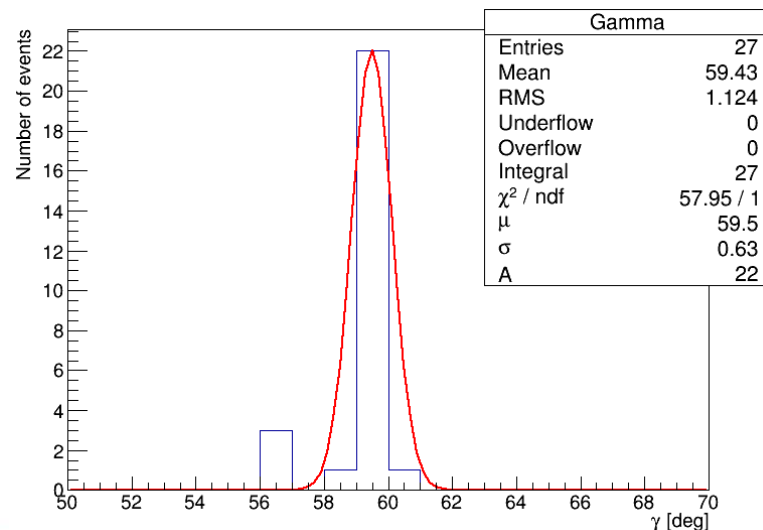


$B_S^0 \rightarrow D_S K^*$ Decay Model and sensitivity

A RooPlot of "B_s proptime"



- Data points are generated according to created model
- Performing a fit to the generated data is an ,experiment'
- One can obtain fitted parameters and calculate γ
- **Having done multiple such experiments one can create a pull distribution**
- **The pull distribution translates to sensitivity to γ**





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Conclusions

- **The γ angle in the CKM Matrix is least known parameter,**
- **Knowing its value with more precision will allow to investigate the New Physics,**
- **Many different methods of γ extraction were proposed – each of them with pros and cons,**
- **Method presented here has big advantage of no penguin pollution and high sensitivity,**
- **Fitter can be used when the data is collected in Run 2,**
- **γ angle value obtained with this method might be used as a starting point to study the Physics Behind the Standard Model.**



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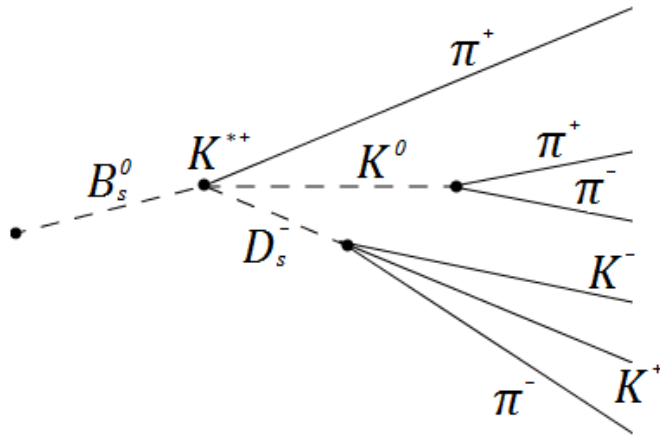
Thank you!



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Backup slides

The kinematics of $B_s \rightarrow D_s K^*$ decay



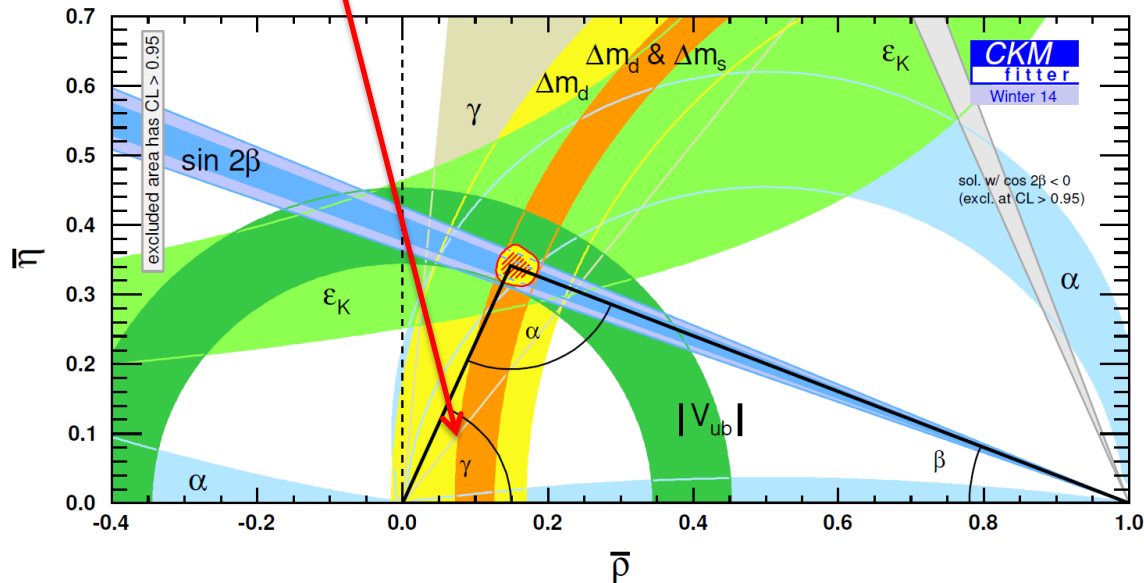
Decay	Branching Ratio
$B_s^0 \rightarrow D_s^- K^{*+}$	$4,7 \cdot 10^{-5}$
$K^{*+} \rightarrow \bar{K}^0 \pi^+$	65%
$\bar{K}^0 \rightarrow \pi^+ \pi^-$	66%
$D_s^- \rightarrow K^+ K^- \pi^-$	11,6% (including resonance states)
Total	$2,33 \cdot 10^{-6}$

- Six particles in final state
- Rarely measured
- $B_s - \bar{B}_s$ mixing can provide information about New Physics
- Good tagging is required

Unitarity Triangles

$$\begin{pmatrix} |V_{ud}| & |V_{us}| & |V_{ub}|e^{-i\gamma} \\ -|V_{cd}| & |V_{cs}| & |V_{cb}| \\ |V_{td}|e^{-i\beta} & -|V_{ts}|e^{i\beta_s} & |V_{tb}| \end{pmatrix}$$

γ – least known CKM parameter



- **Big part of my work is based on B2DXFitters package.**
- **Thanks to Eduardo Rodriguez I managed to understand basics of the implementation.**
- **B2DXFitters is a part of Urania (v2r4 in my case).**
- **The study is being developed basing on two scripts:**
 - *runBdMassFitterToyMC*
 - *runBdDeltaMFitterToyMC*
- **My main part of analysis is 2-dimensional fit – simultaneous in mass and time.**