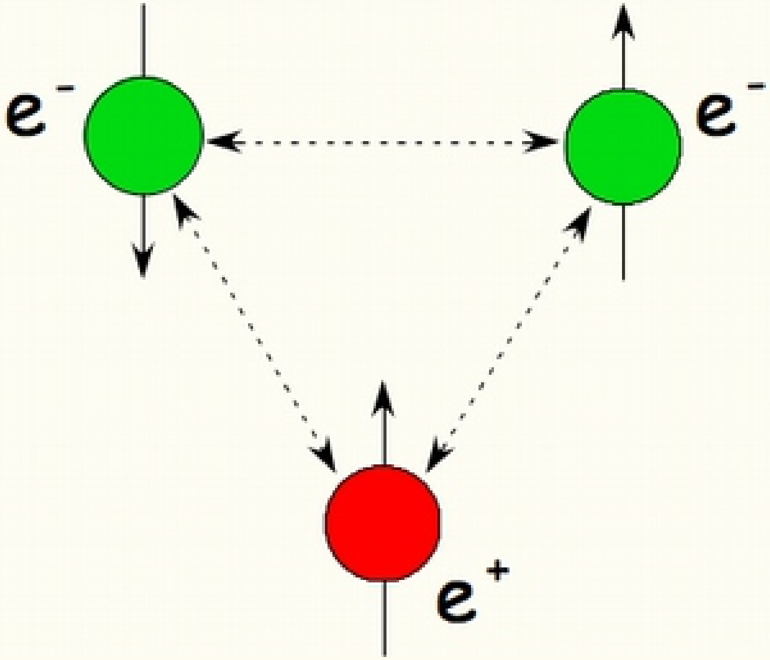
Positronium ion and other exotic bound states



Jagiellonian Symposium
June 10, 2015

Andrzej Czarnecki University of Alberta

Outline

Three-body Ps ion

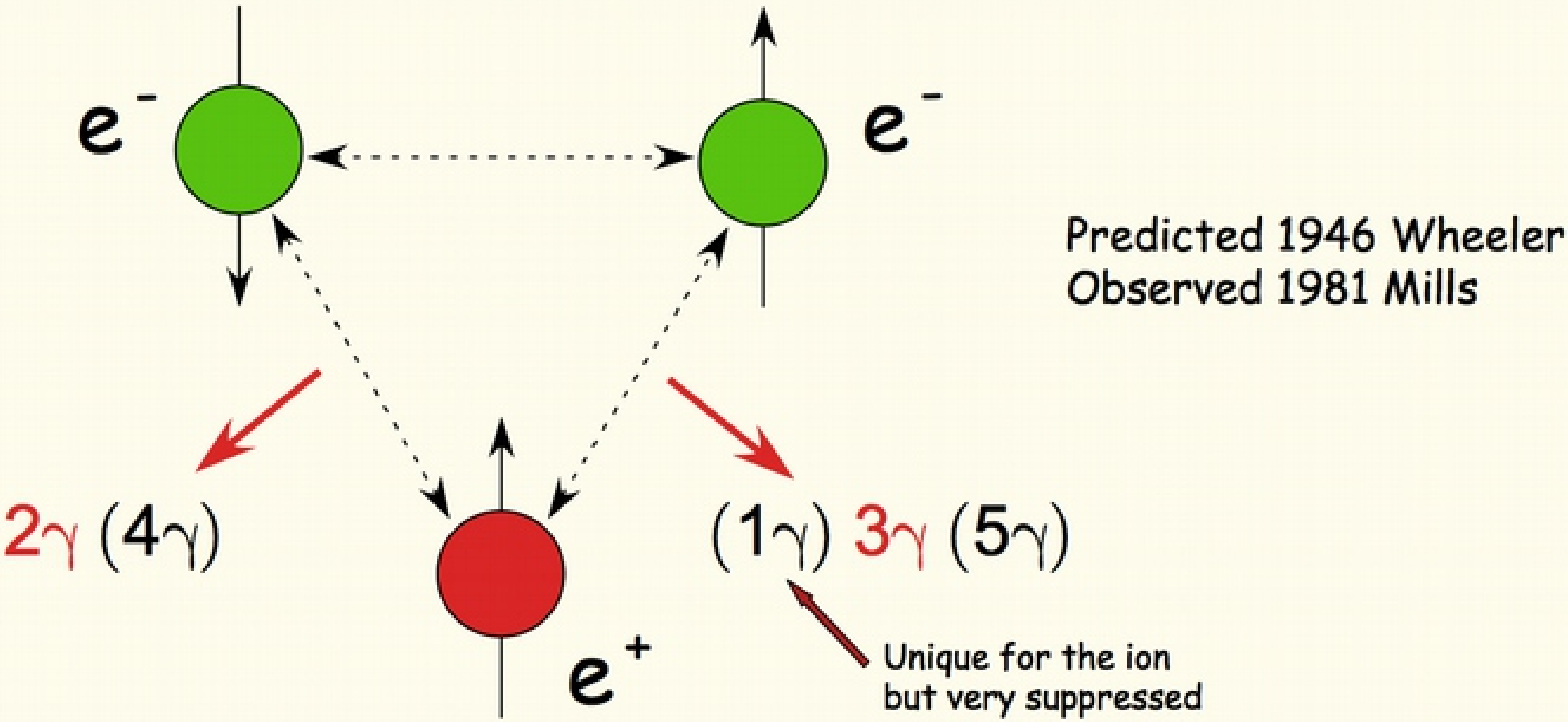
- decay rate
- anomalous magnetic moment

Four-body Ps2 molecule

- spectrum
- evidence for discovery

(Muonic atoms)

Positronium ion

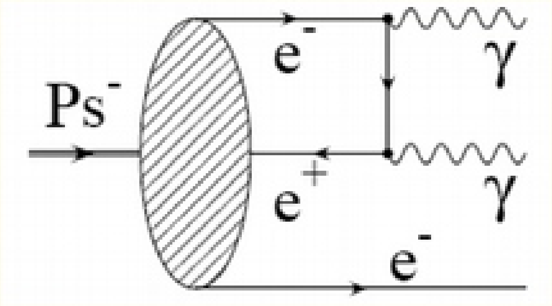


New positronium-ion source
at the FRM II reactor in Munich:
measurements of branching ratios.

New production method: Nagashima et al 2008-12, about 2% efficiency!

Lifetime of the ion

Connection with Paweł Moskal's talk: morphometric imaging.
How the Ps lifetime is modified by the environment.



+ corrections

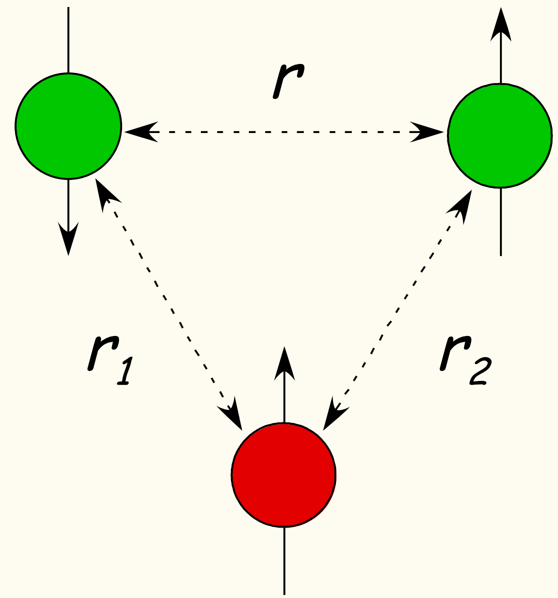
$$\Gamma(\mathrm{Ps}^-) = 2\pi \frac{\alpha^5 m_e c^2}{\hbar}(1+C)\left\langle \delta^3(r_{12})\right\rangle$$

$$\Gamma\left(\mathrm{Ps}^-\right) = 2.087963(12)\,\mathrm{ns}^{-1}$$

with M. Puchalski and S. Karshenboim
PRL 99, 203401 (2007)

Compare: decay rate of para-Ps ~8/ns. This confirms the picture of the ion as a Ps + a satellite electron. Probability that the core is pPs = 1/4.

Theory of the Ps ion and its magnetic moment



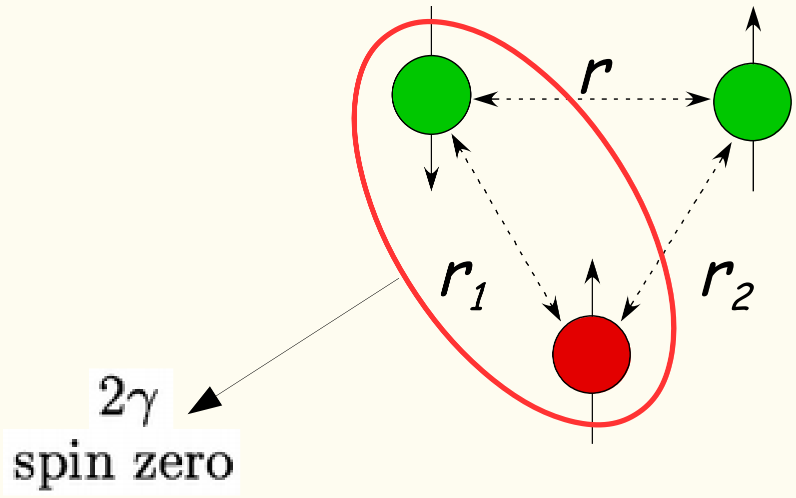
$$\psi(\vec{r}_1,\vec{r}_2) = \frac{\uparrow\downarrow - \downarrow\uparrow}{\sqrt{2}}\phi(r_1, r_2, r)$$

The two electrons form a spin singlet;
positron dominates the magnetic interaction

The wave function is not known analytically,
but can be found using the variational method.

The ion resembles a positronium "shell"
and a loosely-bound electron.

Magnetic moment of the Ps$^-$ ion



surviving electron remembers positron spin

$$g = 2\left[1 - \frac{11\alpha^2}{18}\langle \pi_{13}^2\rangle - \frac{\alpha^2}{6}\left(\left\langle\frac{\rho_{13}\cdot\rho_{12}}{\rho_{13}^3}\right\rangle + \left\langle\frac{\rho_{23}\cdot\rho_{21}}{\rho_{23}^3}\right\rangle\right)\right]$$

Yi Liang, P. McGrath, AC 2014

New J. Phys.

Numerical results:

$$\begin{aligned}
\mathrm{Ps}: \quad & g = 2 - \frac{5}{12}\alpha^2 \simeq 2 - 0.4\alpha^2 \\
\mathrm{Ps}^-: \quad & g \simeq 2 - 0.5\alpha^2 \\
\mathrm{H}: \quad & g = 2 - \frac{2}{3}\alpha^2 \simeq 2 - 0.67\alpha^2
\end{aligned}$$

More recent study

PHYSICAL REVIEW A **90**, 022508 (2014)

Electromagnetic moments of the bound system of charged particles

Albert Wienczek,[1] Mariusz Puchalski,[2] and Krzysztof Pachucki[1]

Quite a different result:

$$g_{\mathrm{Ps}^-} = g_e + \alpha^2\,\delta g,$$

$$\delta g = -\left\langle \vec{p}_3^{\,2}\left(\frac{2}{3} + \frac{g_e}{6}\right) + \frac{1}{r_{13}}\left(\frac{10}{27} + \frac{2}{9}\,g_e\right) + \frac{\vec{r}_{13}\cdot\vec{r}_{23}}{r_{13}^3}\left(\frac{10}{27} - \frac{4}{9}\,g_e\right)\right\rangle,$$

Reconciliation using virial relations

We
$$\frac{\Delta g_{\text{bound}}}{\alpha^2} = -\frac{11}{9}A - \frac{2}{3}(B-C),$$

They
$$\frac{\Delta g_{\text{bound}}}{\alpha^2} = -A - \frac{22}{27}B + \frac{14}{27}C.$$

$$A = \langle p_3^2 \rangle = -\langle \nabla_{13}^2 \rangle = 0.257\,532\,962$$

$$B = \left\langle \frac{1}{r_{13}} \right\rangle = 0.339\,821\,023$$

$$C = \left\langle \frac{\vec{r}_{13}\cdot\vec{r}_{23}}{r_{13}^3} \right\rangle = 0.046\,478\,421,$$

Many more digits known than in C

Virial relation (an example):

$$0 = \left\langle \left[\vec{r}_{23}\cdot\vec{\nabla}_{13}, H \right] \right\rangle = \left\langle \nabla_{13}^2 - \vec{\nabla}_{12}\cdot\vec{\nabla}_{13} + \frac{1}{r_{23}} + \frac{1}{r_{13}} - \frac{\vec{r}_{12}\cdot\vec{r}_{13}}{r_{13}^3} \right\rangle$$

inspired by V. Shabaev's work

symmetry of the two electrons

$$0 = \left\langle \frac{3}{2}\nabla_{13}^2 + 2\frac{1}{r_{13}} - \frac{\vec{r}_{12}\cdot\vec{r}_{13}}{r_{13}^3} \right\rangle = \left\langle \frac{3}{2}\nabla_{13}^2 + \frac{1}{r_{13}} + \frac{\vec{r}_{23}\cdot\vec{r}_{13}}{r_{13}^3} \right\rangle = -\frac{3}{2}A + B + C.$$

New expression for Δg

$$\frac{\Delta g_{\text{bound}}}{\alpha^2} = \left(\frac{g_{\text{free}}}{2} - \frac{11}{9}\right) A - \frac{2 g_{\text{free}}}{3} B,$$

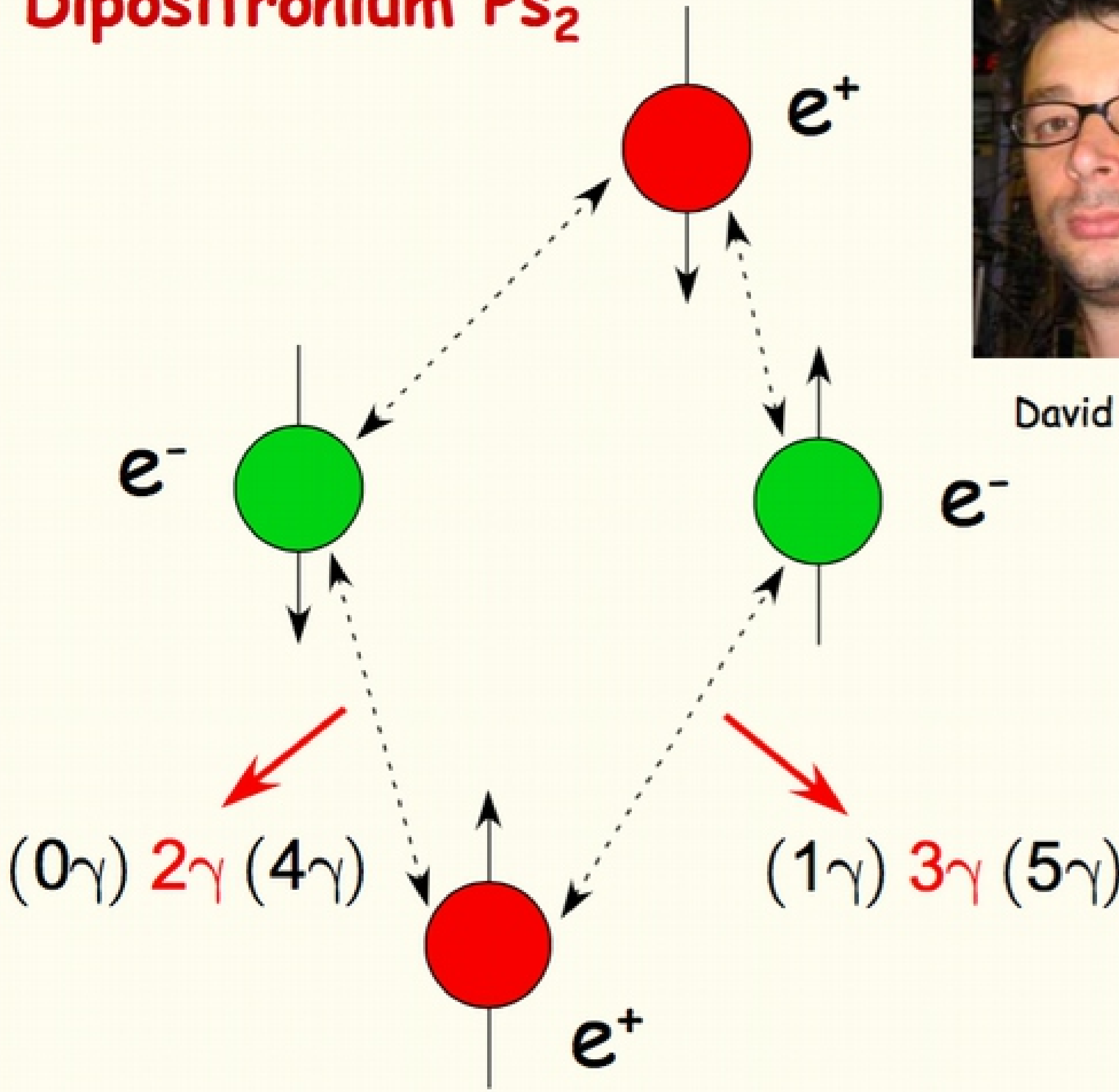
Note: eliminated the less-well known operator *C*:
very high precision possible

$$\Delta g_{\text{bound}} = -0.510\,551\,028\,187\,6(6)\alpha^2 + \mathcal{O}\left(\alpha^4\right)$$

with Y. Liang, PRD 2015

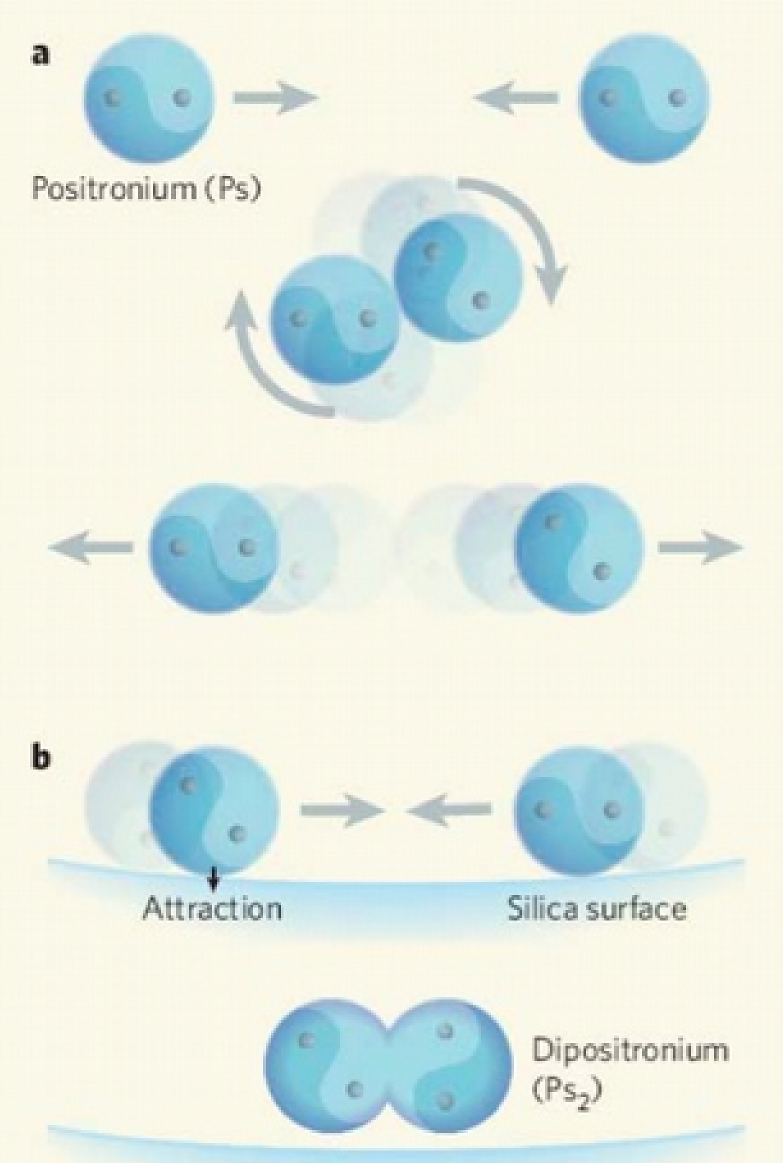
Di-positronium molecule

Dipositronium Ps_2



David Cassidy   Allen Mills

Discovery of dipositronium 2007



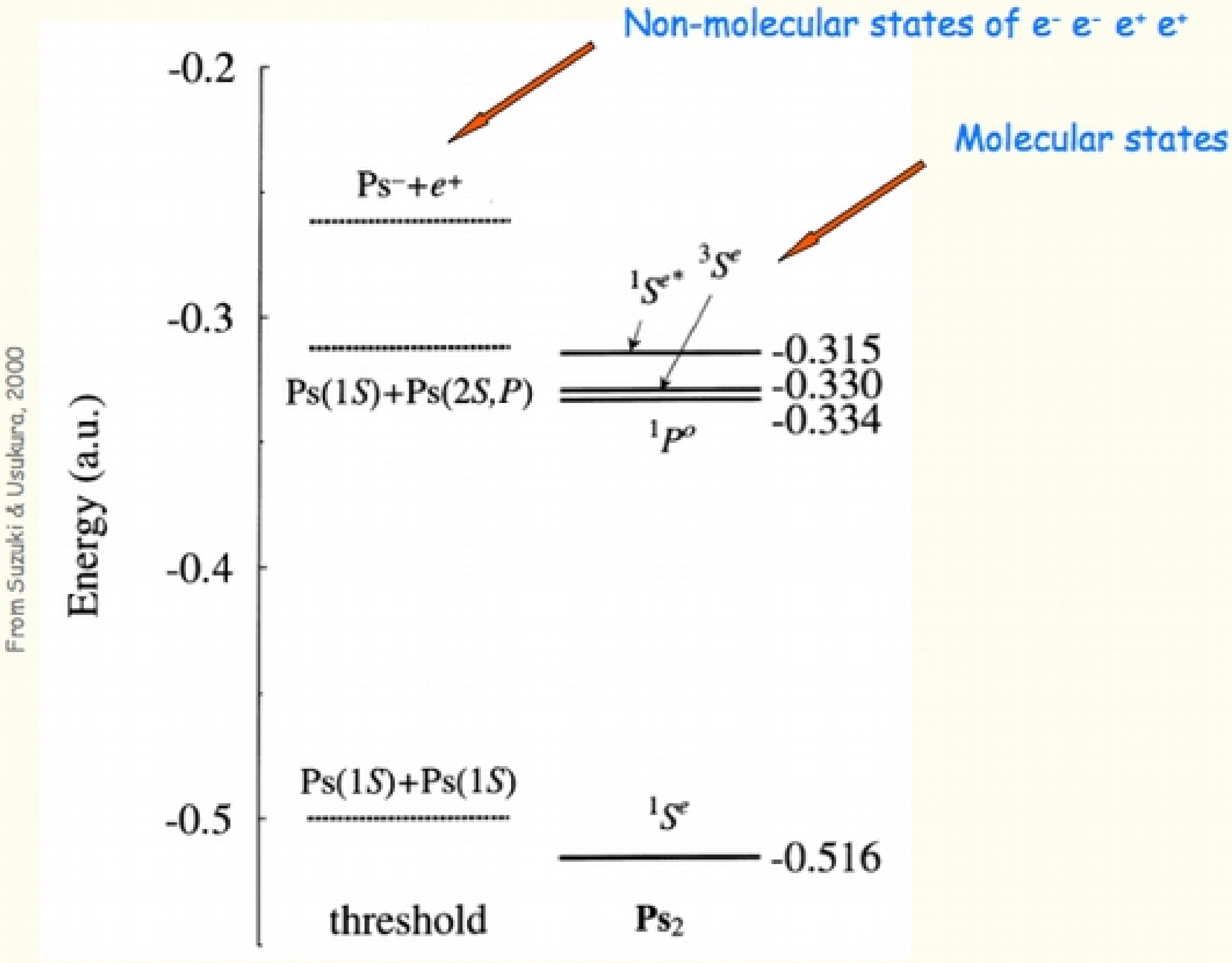
Molecule formation kills long-lived positronia.

At higher temperature, fewer atoms on the surface, fewer molecules formed.

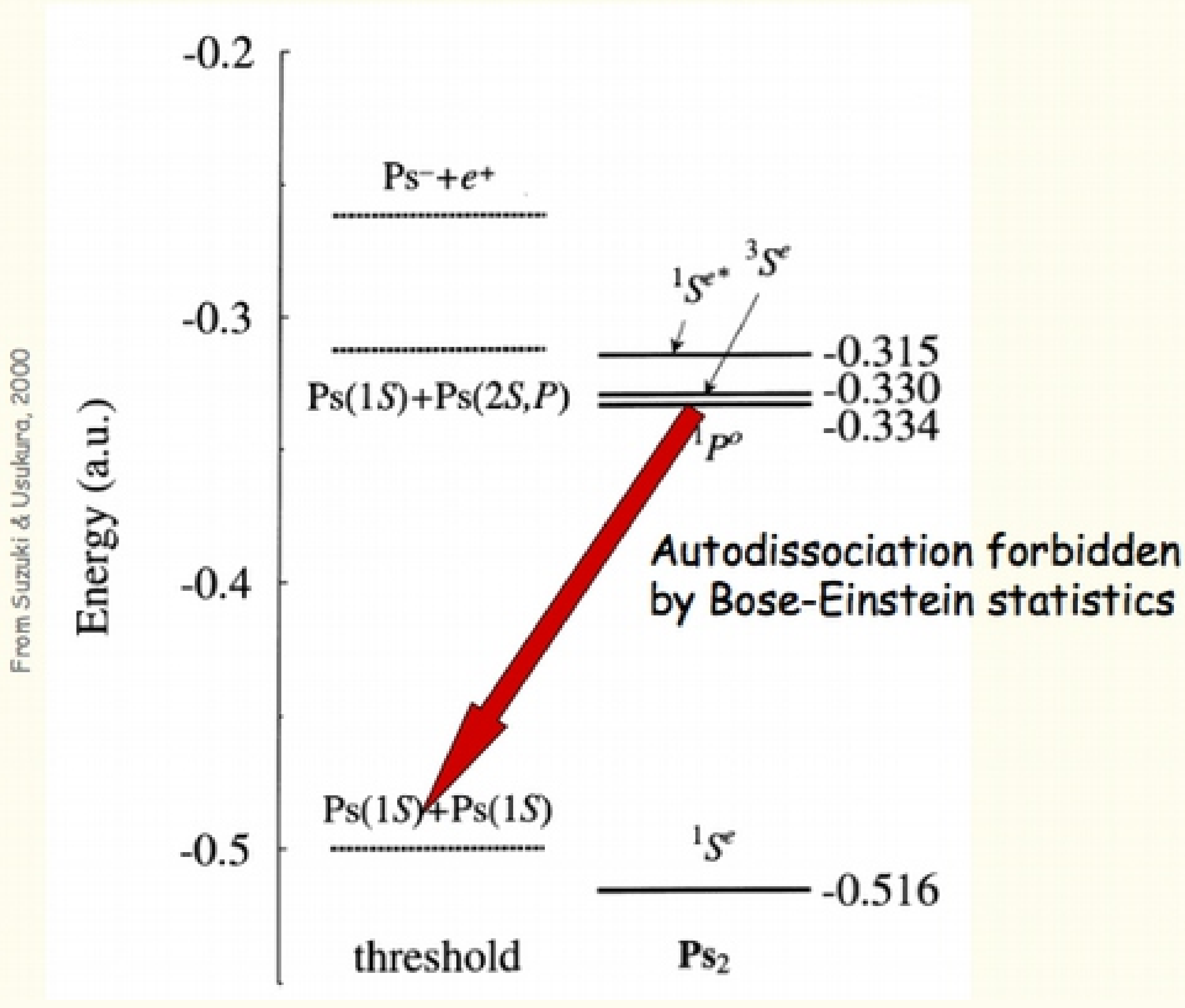
Indeed: at high-T, more long-lived positronia observed.

Cassidy & Mills, Nature 2007

Spectrum of the molecule Ps_2



A direct signal of the molecule: transition line.



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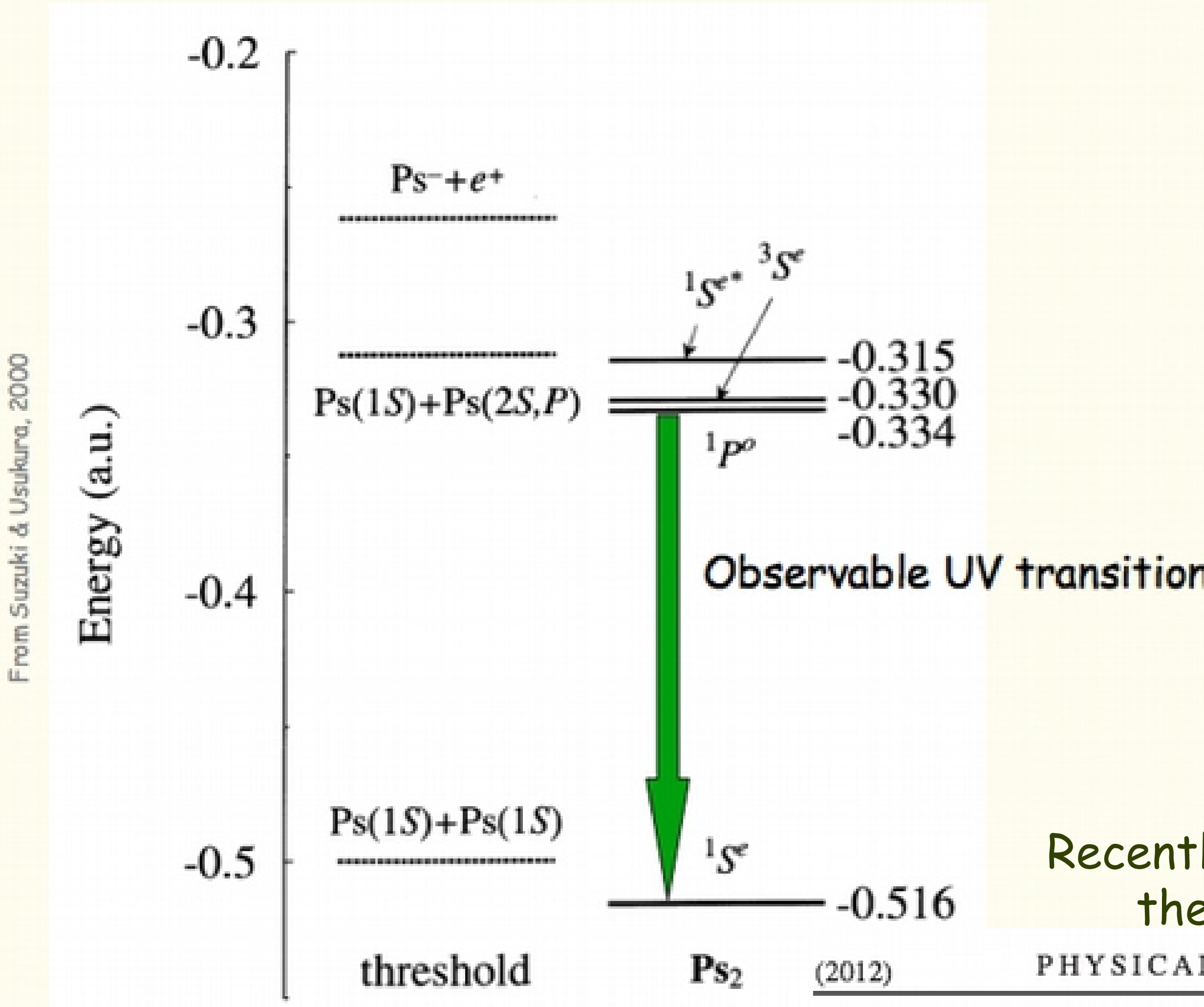


Recently measured for the first time:

(2012) PHYSICAL REVIEW LETTERS

Optical Spectroscopy of Molecular Positronium

D. B. Cassidy, T. H. Hisakado, H. W. K. Tom, and A. P. Mills, Jr.

with Puchalski,
PRL 101, 183001 (2008)

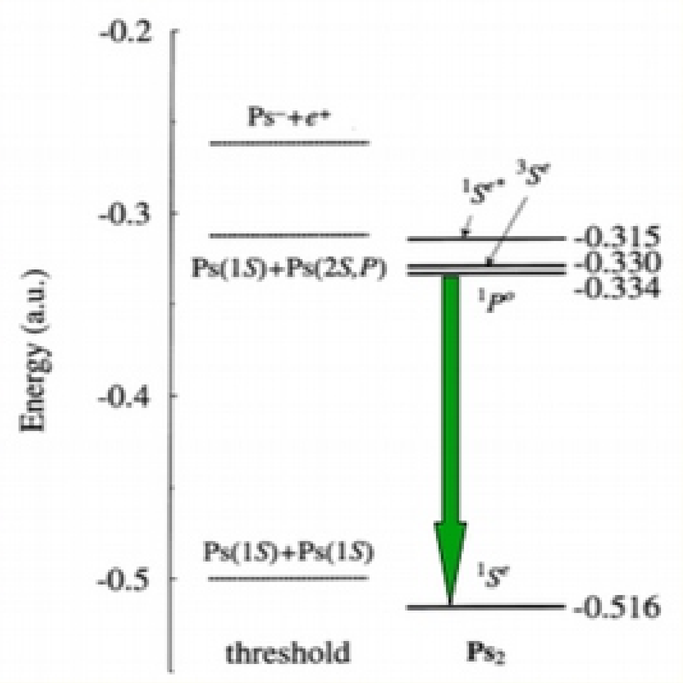
Questions about this transition:

What is its accurate energy?

$$\Delta E = E_P - E_S = 0.1815867(8)\,\text{a.u.}$$
$$\simeq 4.9\,\text{eV}$$

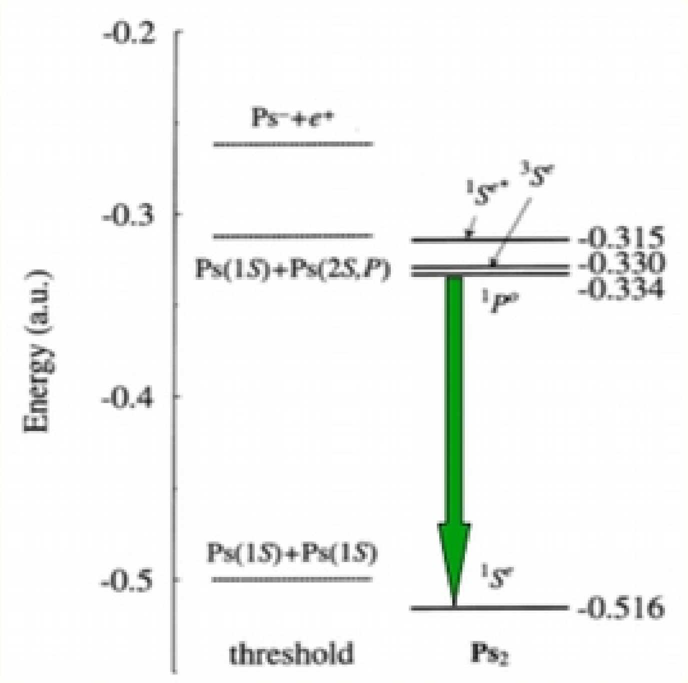
Similar to atomic positronium, but softer (dielectric effect?):

$$E_P - E_S = \frac{3}{4} \times \frac{1}{4}\,\text{a.u.} = 0.1875\,\text{a.u.}$$



with Puchalski,
PRL 101, 183001 (2008)

Questions about this transition:

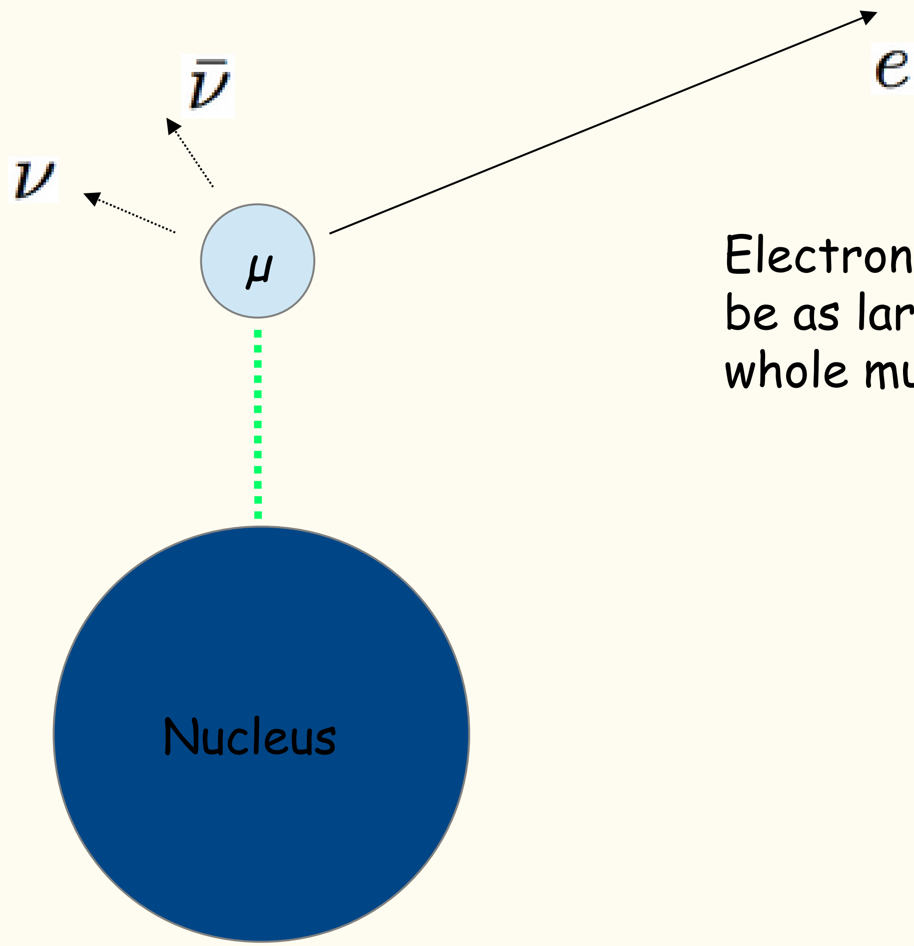


How often does
radiative transition appear
(before annihilation)?

$$\mathrm{BR}(P \to S) = \frac{\Gamma_{\mathrm{dip}}(P \to S)}{\Gamma_{\mathrm{annih}}(P) + \Gamma_{\mathrm{dip}}(P \to S)} = 0.191(2)$$

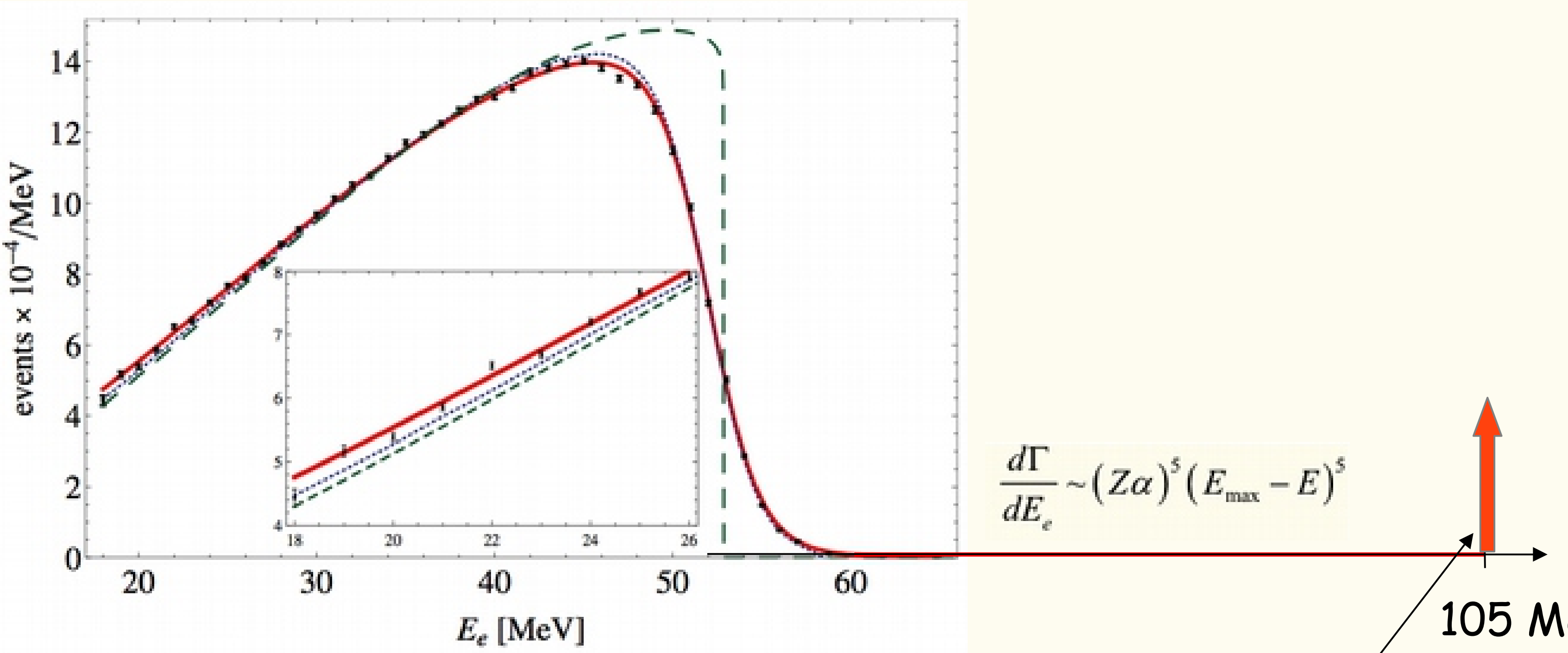
Muonic atoms

Electron spectrum in a mu-decay near nucleus



Electron energy can be as large as the whole muon mass

Spectrum of the bound muon decay

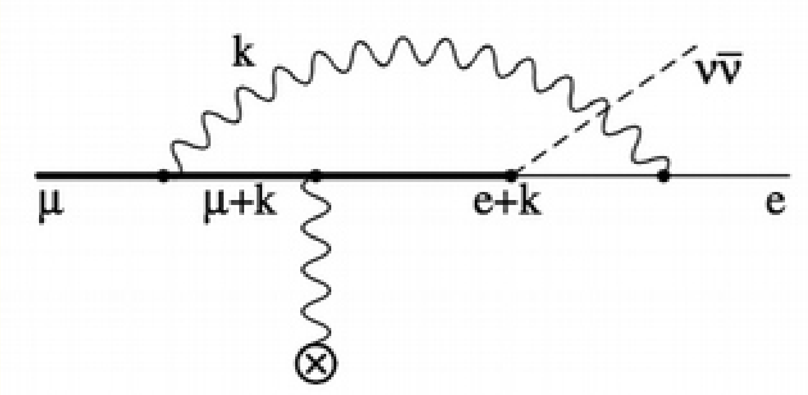


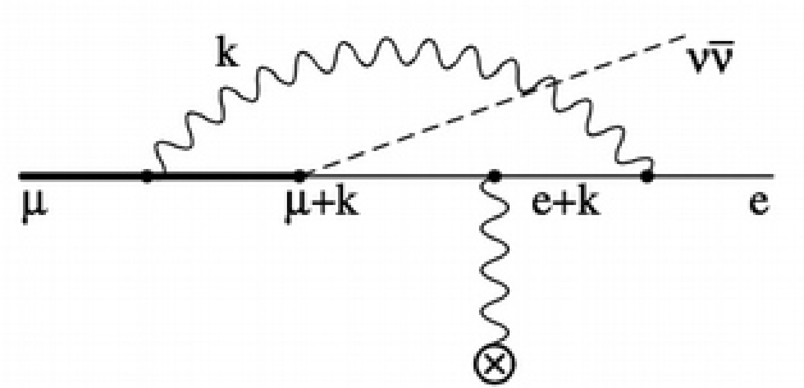
$$\frac{d\Gamma}{dE_e} \sim (Z\alpha)^5 (E_{max} - E)^5$$

It is the main background for the expected conversion signal

AC, M. Dowling, X. Garcia i Tormo, W. Marciano, R. Szafron
R. Szafron, AC

Radiative corrections to the electron spectrum





Competing effects:
- vacuum polarization in the hard photon; and
- self-energy and real radiation

$$\frac{1}{\Gamma_0}\frac{\mathrm{d}\Gamma}{\mathrm{d}E_e}\bigg|_{E_e\to\tilde{m}} = B\Delta^5 + \mathcal{O}\left(\Delta^6\right)$$

$$B|_{(Z\alpha)^5} = \frac{1024}{5\pi m_\mu^6}(Z\alpha)^5\left(\frac{\Delta}{m_\mu}\right)^{\frac{\alpha}{\pi}\delta_S}(1+\delta_{VP}+\delta_F)$$

$$\delta_S = 10.1$$

Szafron, AC 2015

Summary

We have a good theoretical understanding
of the three- and four-body systems involving positrons.

The ion, since recently, can be routinely produced.
Precise measurements are possible.

The molecule has been discovered more recently.
First spectroscopic observations have been made.

Understanding of bound states recently extended to
decays of bound muons, including radiative corrections.

Extra slides

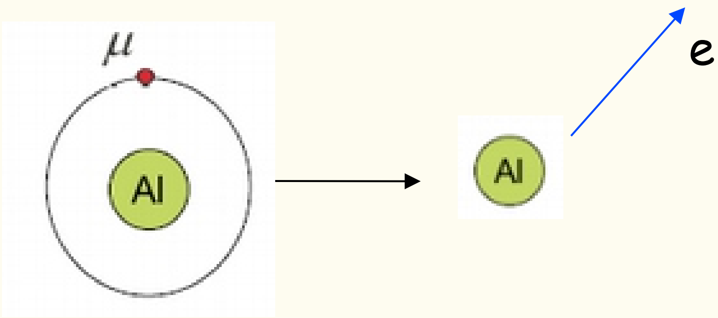
Conversion: probes dipole and non-dipole interactions

So far, we have only talked about dipole interactions.
There are also vectors and scalars.

They are not (directly) probed by processes with external photons, by gauge invariance requirements.

New process: muon-electron conversion

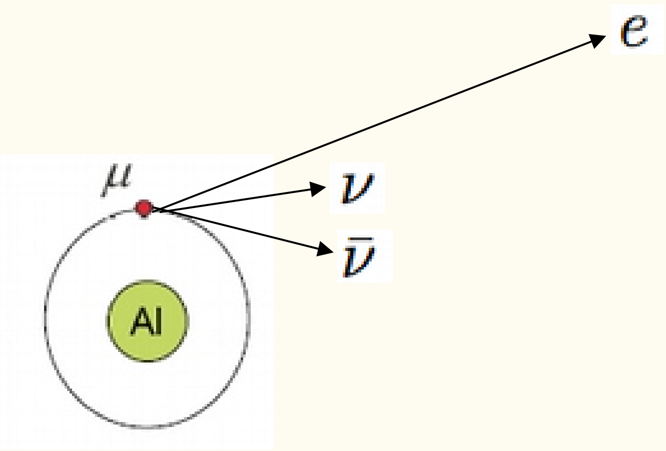
(as well as mu --> eee)



Variety of mechanisms:

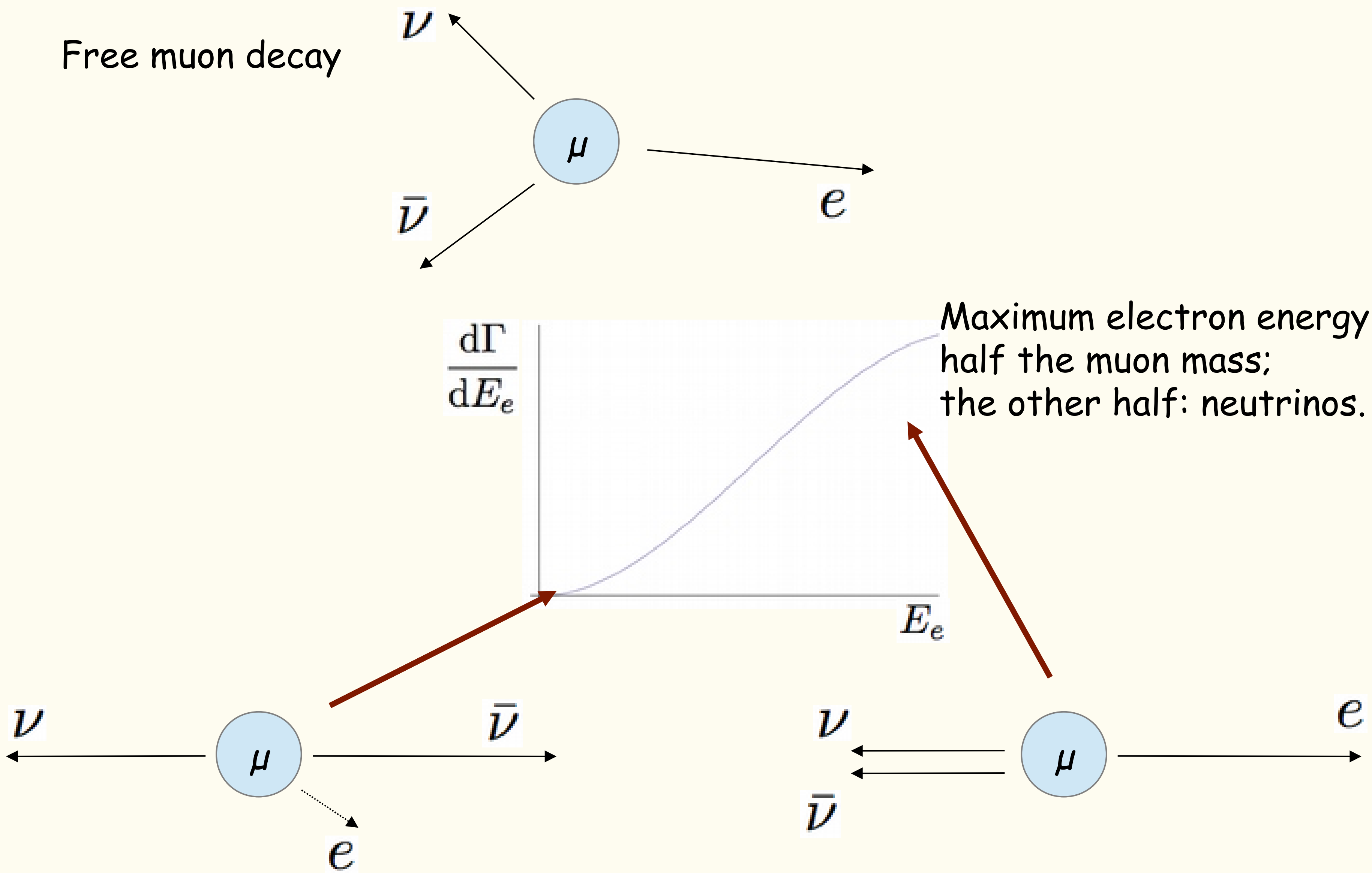
Background for the conversion search

Normal decay of the muon bound in the atom
can produce high-energy electron,

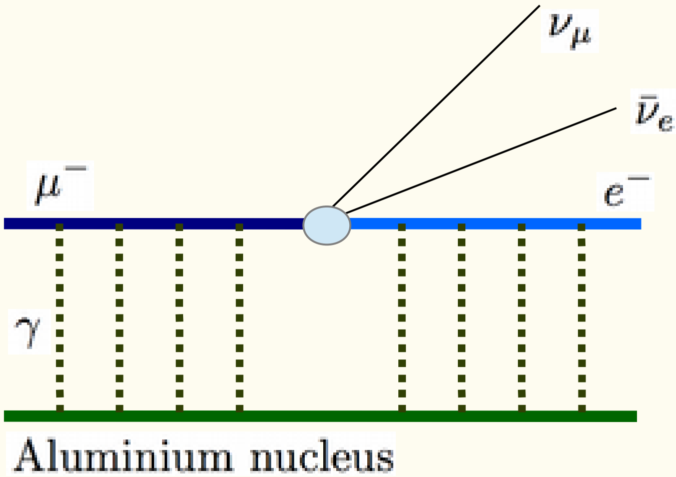


Spectrum has to be well understood.

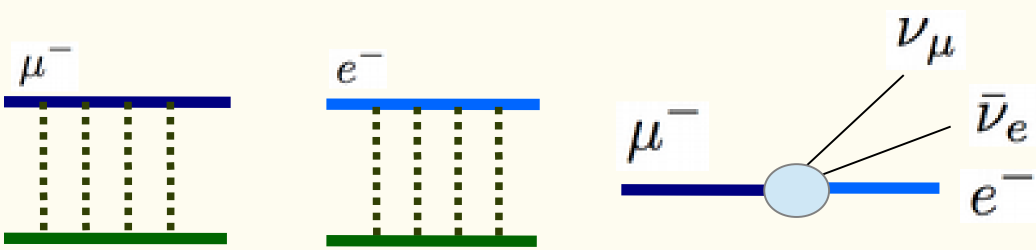
Muon decay: electron energy spectrum



Decay of a muon bound in aluminium



factorization



Free muon decay rate,
with all corrections!

$$\frac{\mathrm{d}\Gamma}{\mathrm{d}E_e} = \int \mathrm{d}\lambda\, s(\lambda) \frac{\mathrm{d}\Gamma_{\mathrm{free}}}{\mathrm{d}z} \frac{\mathrm{d}z}{\mathrm{d}E_e}\bigg|_{z\to z(\lambda)}$$

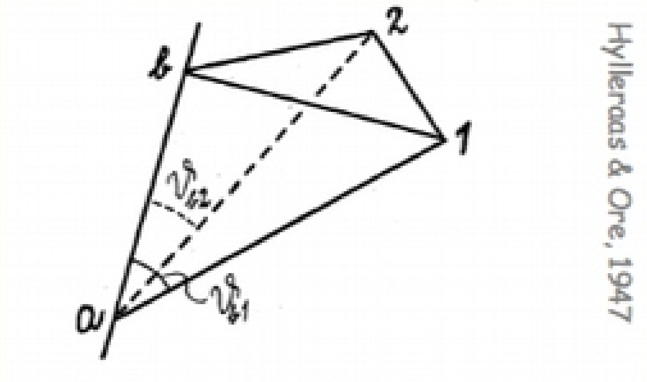
$$z(\lambda) = \frac{2(E_e+\lambda) + (Z\alpha)^2 m_\mu}{m_\mu + \lambda}$$

AC, M. Dowling, X. Garcia i Tormo, W. Marciano, R. Szafron

with Puchalski,
PRL 101, 183001 (2008)

Variational determination of the Ps_2 wave fnc.

Gaussian basis $$\exp\left(-\sum_{i=1}^{6} a_i r_i^2\right)$$



Coordinate system for the positronium molecule

Relatively small basis ~ 2000
QR decomposition of the eigenvalue equation
Optimization of individual basis elements with Powell's method
→nonrelativistic energies ~1ppb
(test: Lithium)

Relativistic corrections dominated by annihilation: repulsive,
more effective in the ground state → decreases the SP interval.