Theoretical and experimental approaches to the alpha clustering in nuclei

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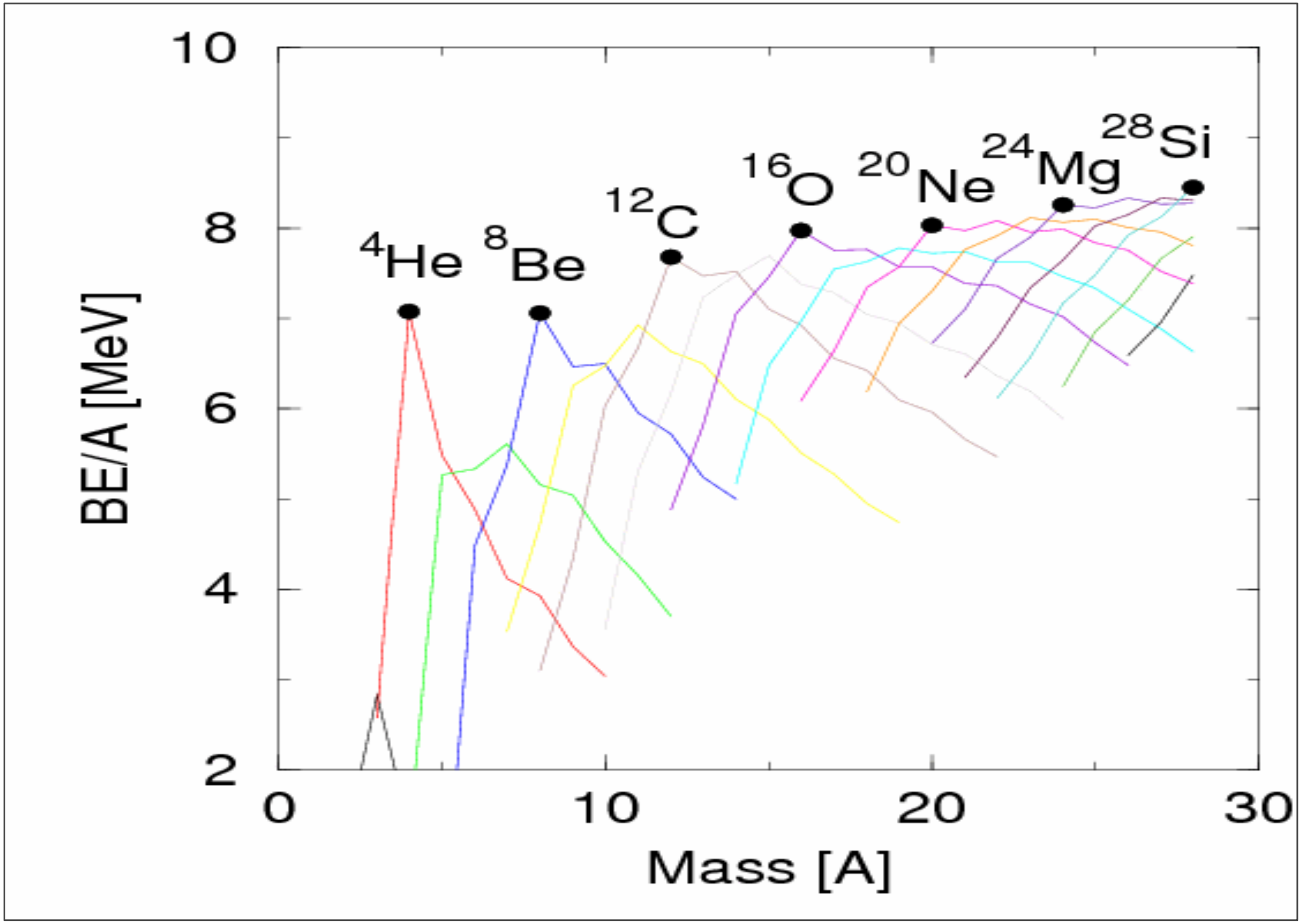
Outline

1. Historical development
2. Model calculations and results
3. Systematic study
4. Conclusions

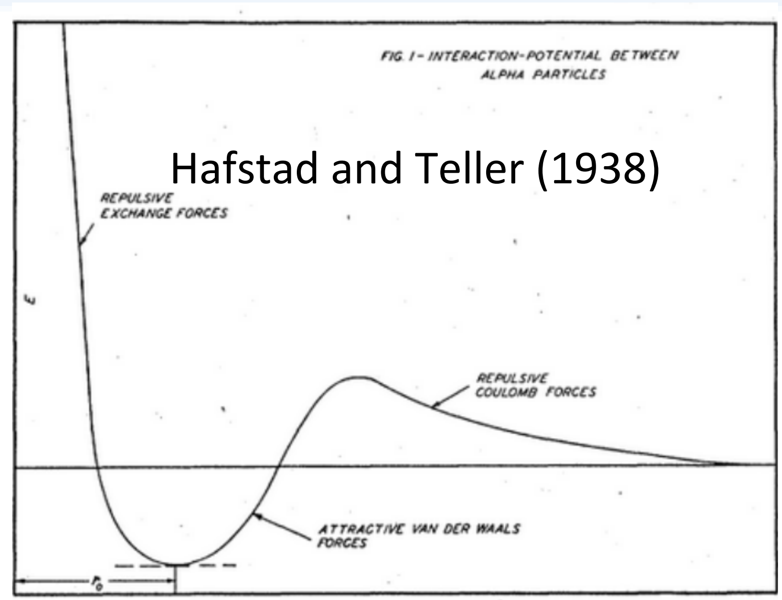
Experimental binding energy curve

Same color, same Z

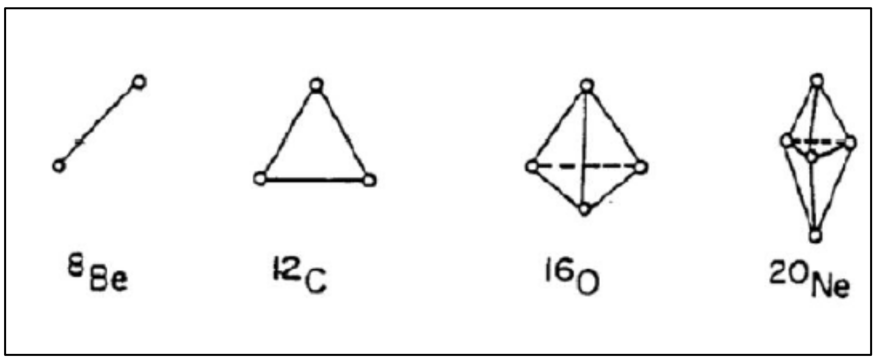


History

Molecular viewpoint of alpha clustered nuclei - Bethe and Bacher (1936)



Hafstad and Teller (1938)



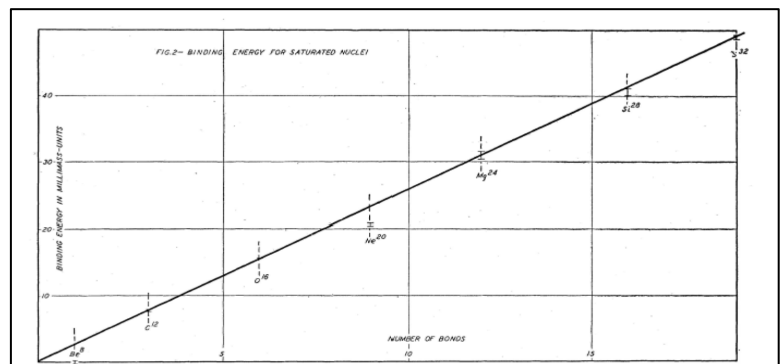


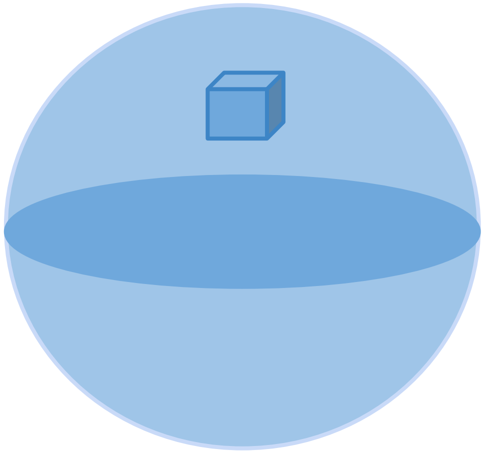
Fig. 2. Binding energy for saturated nuclei.

Hafstad and Teller (1938)

Equation of State (EOS) of nuclear matter

Nuclear matter

infinite nucleons of density $\rho(\vec{r},t)$ temperature $T(\vec{r},t)$



$$E_V = \int_V \rho e(\rho,T)dV$$

$$\frac{E}{A} = e(\rho,T)$$

Asymmetric nuclear matter at low temperature

$$e(\rho,\delta) = T_F(\rho,\delta) + V_0(\rho) + \delta^2 V_2(\rho)$$

Isospin dependent

$$e(\rho,\delta) = e(\rho,\delta=0) + \delta^2 e_{sym}(\rho)$$

$$\xi = \frac{\rho-\rho_0}{\rho_0}$$

$$\delta = \frac{\rho_n-\rho_p}{\rho}$$

$$e = e_{00} + \frac{K_0}{18}\xi^2 + \delta^2\left(e_{I0} + \frac{L_I}{3}\xi + \frac{K_I}{18}\xi^2\right)$$

New form of EOS

$$\eta_n = \frac{\rho_{n\uparrow}-\rho_{n\downarrow}}{\rho}$$

$$\eta_p = \frac{\rho_{p\uparrow}-\rho_{p\downarrow}}{\rho}$$

Spin and Isospin dependent

$$e = e_{00} + \frac{K_0}{18}\xi^2 + \delta^2\left(e_{I0} + \frac{L_I}{3}\xi + \frac{K_I}{18}\xi^2\right) + (\eta_n^2+\eta_p^2)\left(e_{ii0} + \frac{L_{ii}}{3}\xi + \frac{K_{ii}}{18}\xi^2\right) + 2\eta_n\eta_p\left(e_{ij0} + \frac{L_{ij}}{3} + \frac{K_{ij}}{18}\xi^2\right)$$

Draw back:
Valid only near to the $\rho=\rho_0$
when density $\rho = 0$, $e \neq 0$

$$e(\rho,0) = \alpha_0\rho + \beta_0\rho^2 + \gamma_0\rho^3$$

Model assumptions

$$\Phi = \prod_{k=1}^{A} {}^{k}\phi_{I_k S_k}$$

$${}^{k}\phi_{I_k S_k} = \frac{1}{\left(2\pi\sigma_k^2(r)\right)^{3/4}} \exp\left(\frac{-\left(\mathbf{r}_k - \langle \mathbf{r}_k \rangle\right)^2}{4\sigma_k^2(r)} + \frac{i}{\hbar}\mathbf{r_k}\langle \mathbf{p}_k \rangle \right)$$

$\langle \mathbf{r}_k \rangle$, $\langle \mathbf{p}_k \rangle$ and $\sigma_k^2(r)$ are time dependent parameters

Time evolution - self-consistent variational principle

Nuclear matter - four component fluid $\rho_{p\uparrow}$ $\rho_{p\downarrow}$ $\rho_{n\uparrow}$ $\rho_{n\downarrow}$, energy density $\varepsilon\left(\rho_{p\uparrow}, \rho_{p\downarrow}, \rho_{n\uparrow}, \rho_{n\downarrow}\right)$

$$P_k(\mathbf{r}) = \left|{}^{k}\phi_{I_k S_k}\right|^2$$

$$e_k = \langle \varepsilon \rangle_k + \lambda \sigma_k(\varepsilon)$$

$$\langle \varepsilon \rangle_k = \int P_k(\mathbf{r}) \varepsilon\left(\rho_{p\uparrow}, \rho_{p\downarrow}, \rho_{n\uparrow}, \rho_{n\downarrow}\right) d^3\mathbf{r}$$

$$\sigma_k^2(\varepsilon) = \int P_k(\mathbf{r}) \left(\varepsilon\left(\rho_{p\uparrow}, \rho_{p\downarrow}, \rho_{n\uparrow}, \rho_{n\downarrow}\right) - \langle \varepsilon \rangle_k\right)^2 d^3\mathbf{r}$$

λ is a parameter associated with the surface energy

For nuclear matter $\sigma_k(\varepsilon)$'s zero

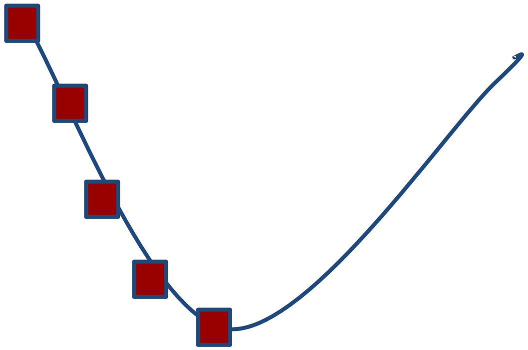
$\varepsilon\left(\rho_{p\uparrow}, \rho_{p\downarrow}, \rho_{n\uparrow}, \rho_{n\downarrow}\right)$ is EOS

Microscopic approach to Liquid Drop Model

For finite nuclei

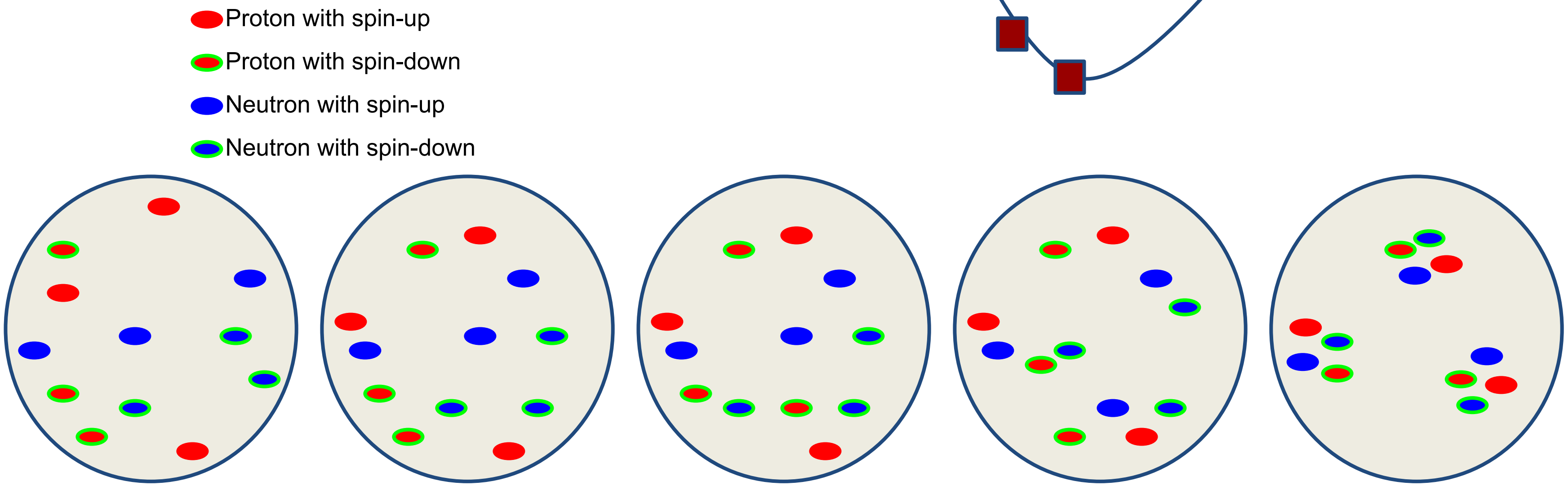
$$\langle \Phi | H | \Phi \rangle = \int \varepsilon \left(\rho_{p\uparrow}, \rho_{p\downarrow}, \rho_{n\uparrow}, \rho_{n\downarrow} \right) \rho \left(\mathbf{r} \right) d^3\mathbf{r} + \lambda \sum_{k=1}^{k=A} \sigma_k(\varepsilon) + \langle \Phi | V_C | \Phi \rangle$$

EOS Parameters from BE and r.m.s radius of light nuclei

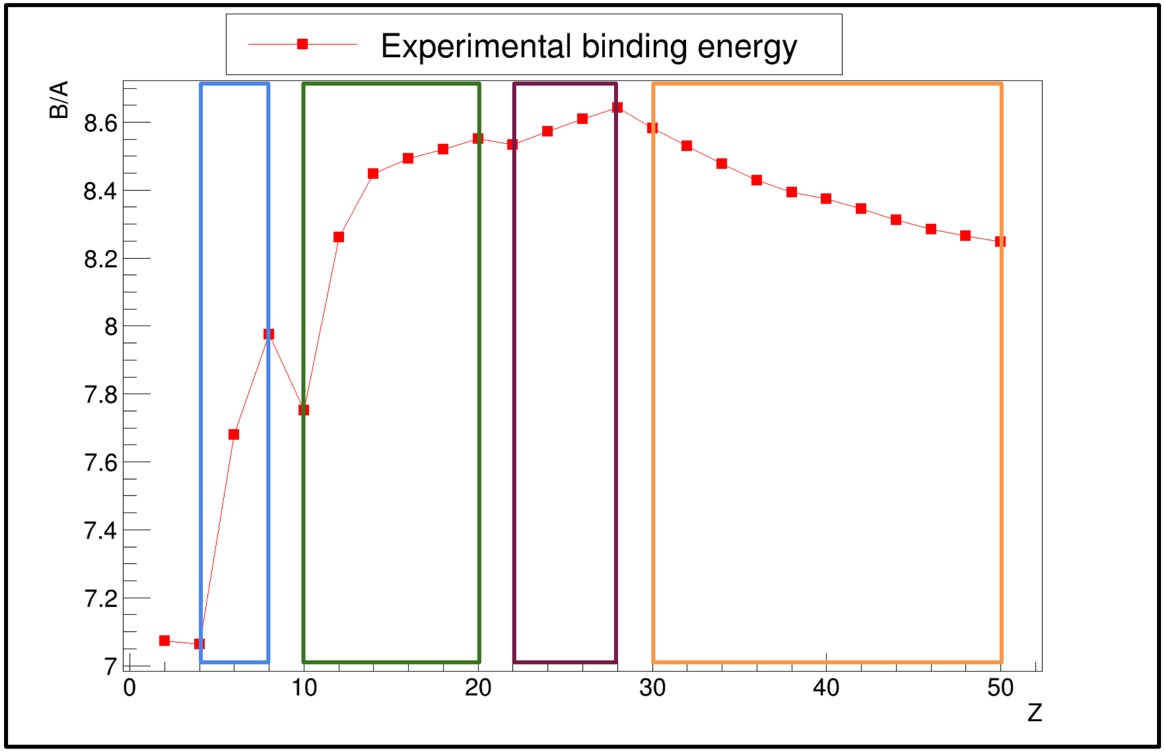


Oversimplified simulation

Center positions of nucleon Gaussian wave packet



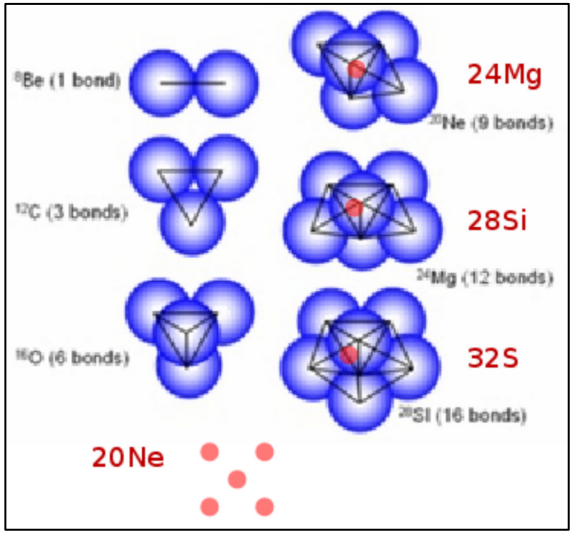
Assumptions in the calculation



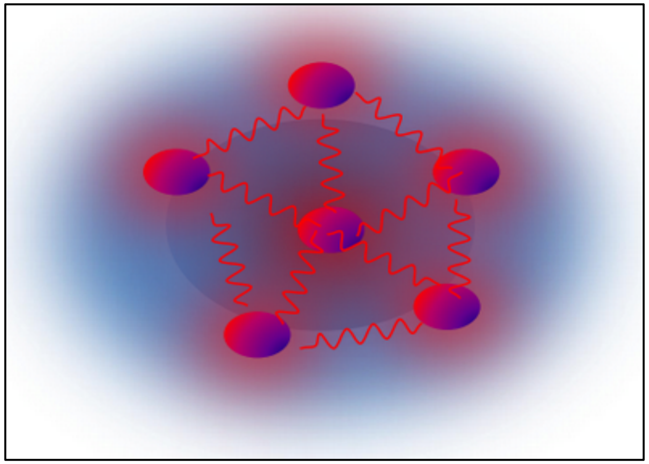
For Z≤8

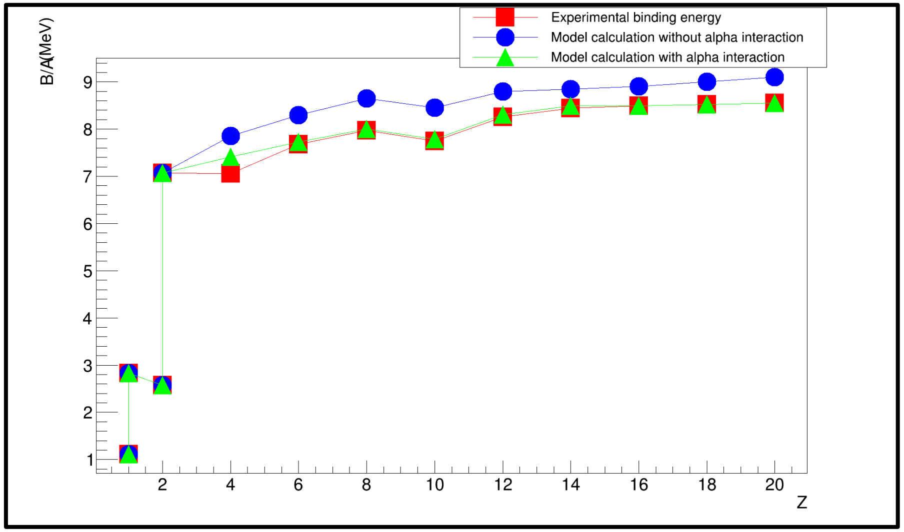
8 < Z ≤ 20 single alpha core

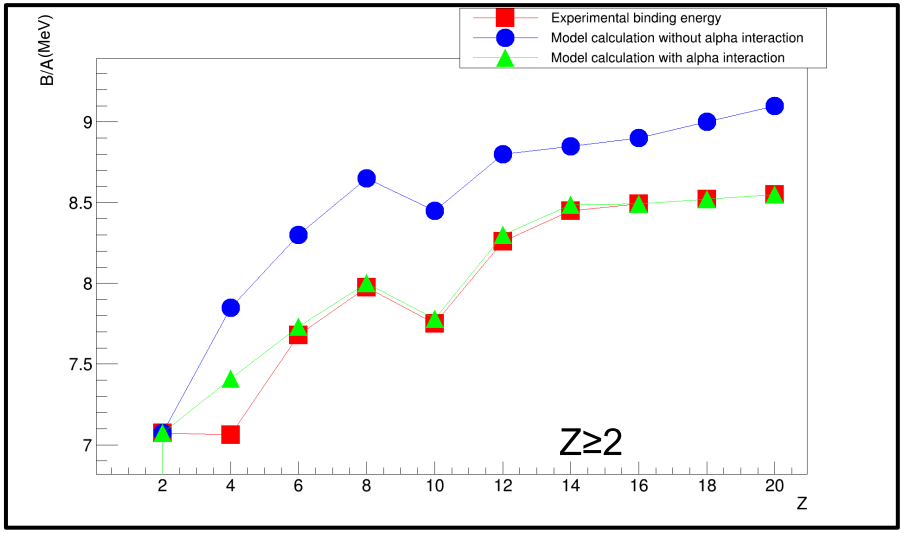
20 < Z ≤ 28 double alpha core



Matter without and with alpha cluster interaction

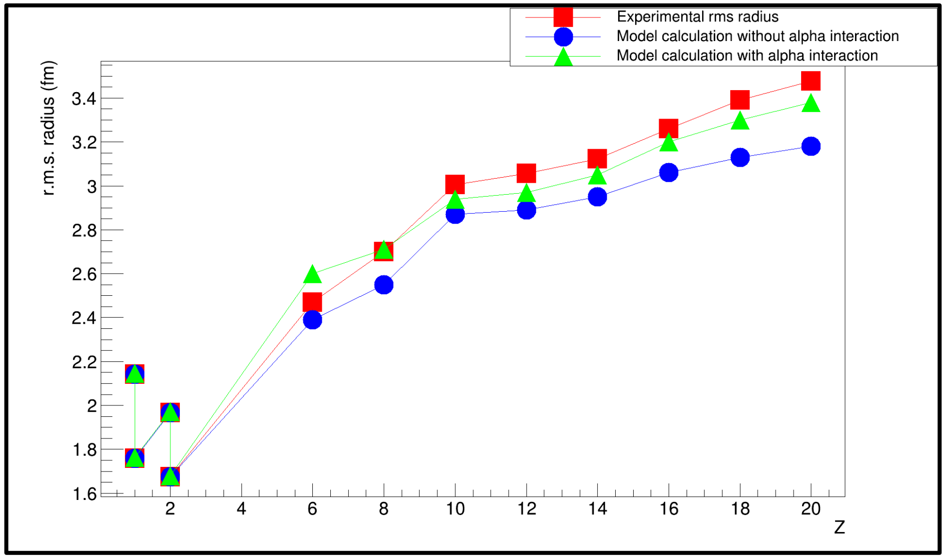


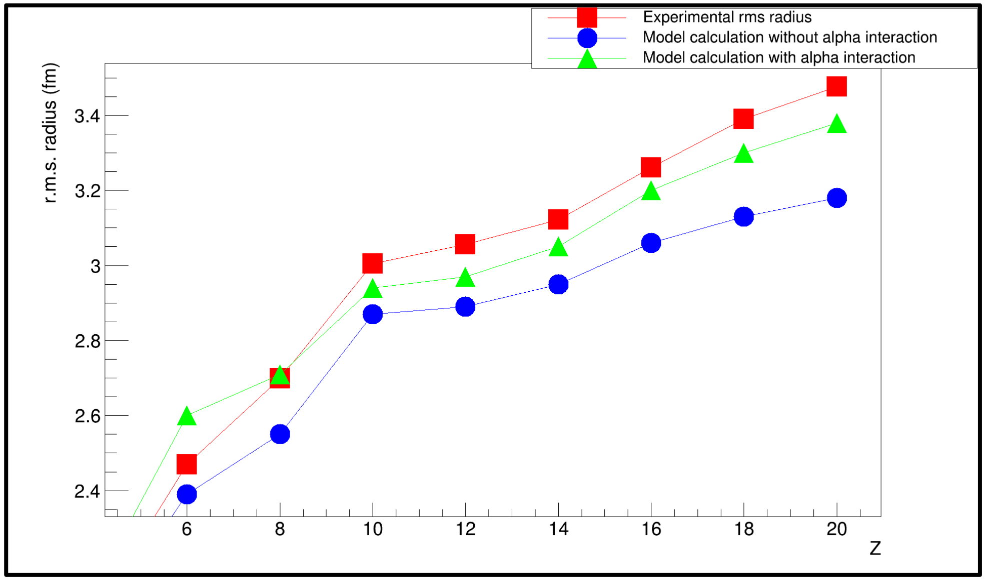




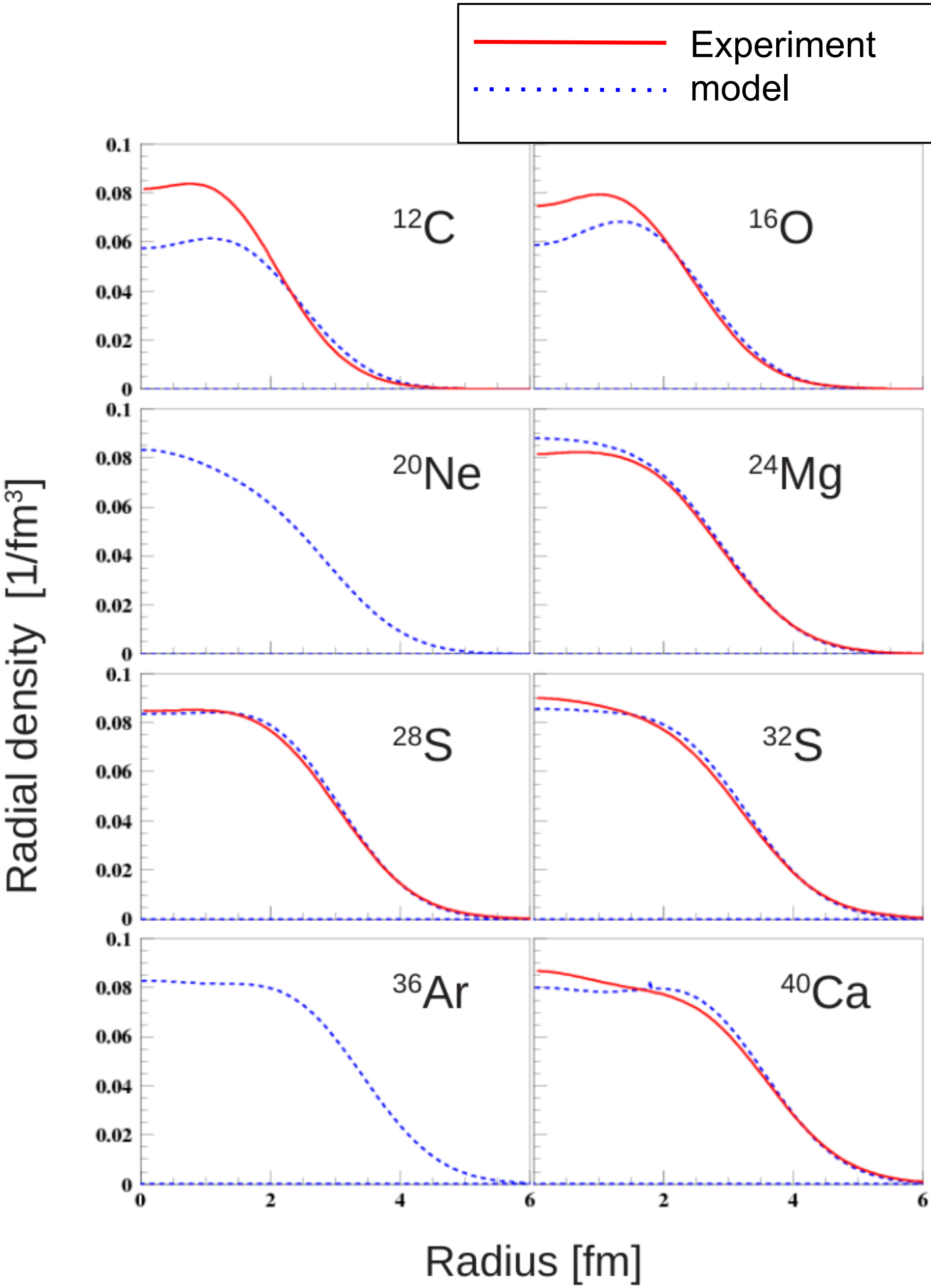
$$\langle \Phi | \Delta H_1 | \Phi \rangle = \sum_{i \neq j} P_\alpha(i) P_\alpha(j) V_{\alpha\alpha}(d_{ij})$$

Matter without and with alpha cluster interaction

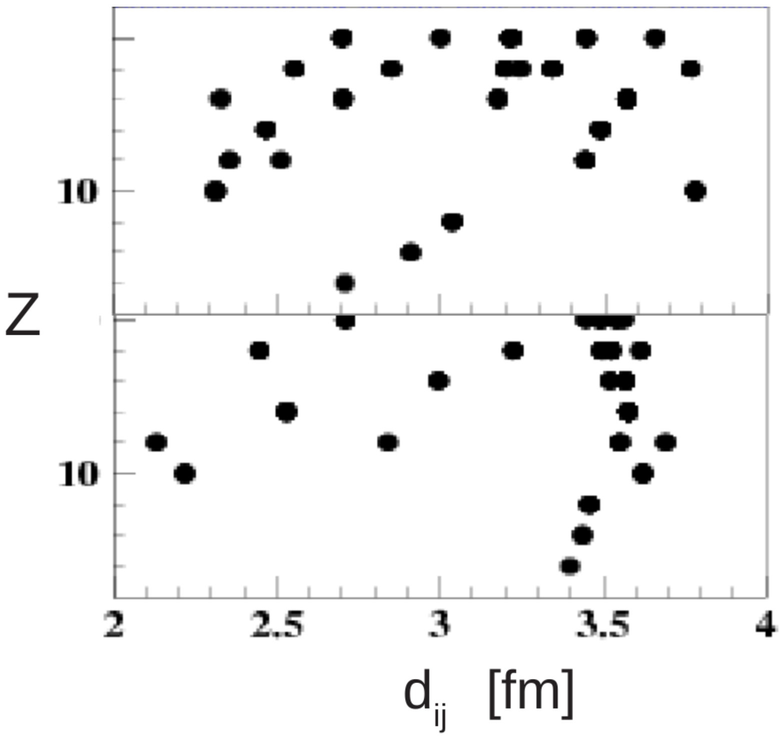




Radial density distribution

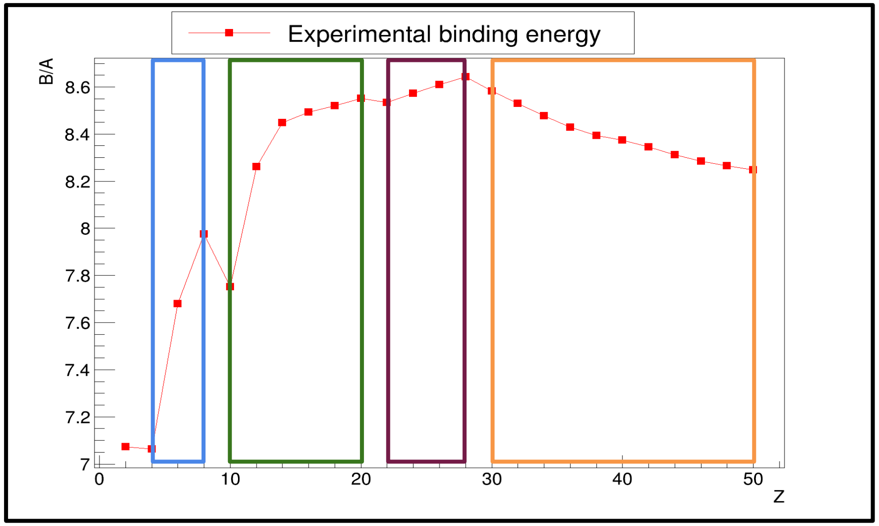


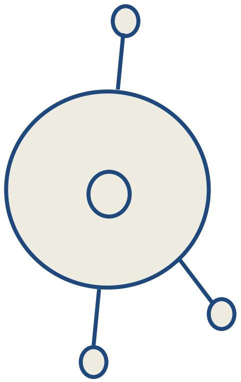
cluster center- center distance distribution

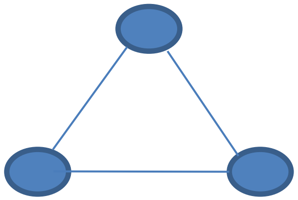


Article is under preparation

Systematic study

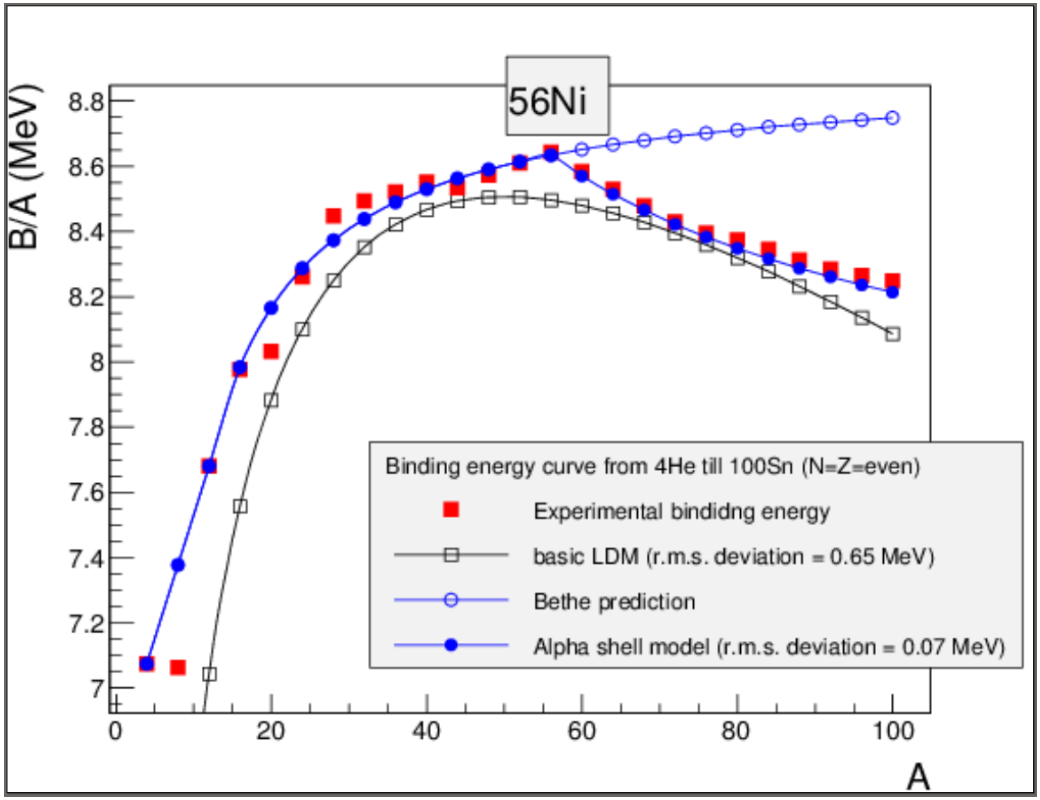






$$B = n_\alpha \times b_\alpha + n_b \times b_{\alpha-\alpha}$$

$$b_{\alpha-\alpha} = 2.425 MeV$$



Resulting structures

http://nz21-33.ifj.edu.pl/clusters/

Conclusions and perspectives

- Ground state structures are calculated
- Detector simulation and construction is finished. Undergoing tests

To be done

- Calculation of excited states and corresponding structure
- Comparison with experimental data
- Propose new experiments
- Analysis of results and comparison

Thank you…