

$d + d$ η -nucleus interaction from the reaction around the η production threshold

Satoru Hirenzaki (Nara Womens University, Japan)



- Interests of Meson – Nucleus bound systems

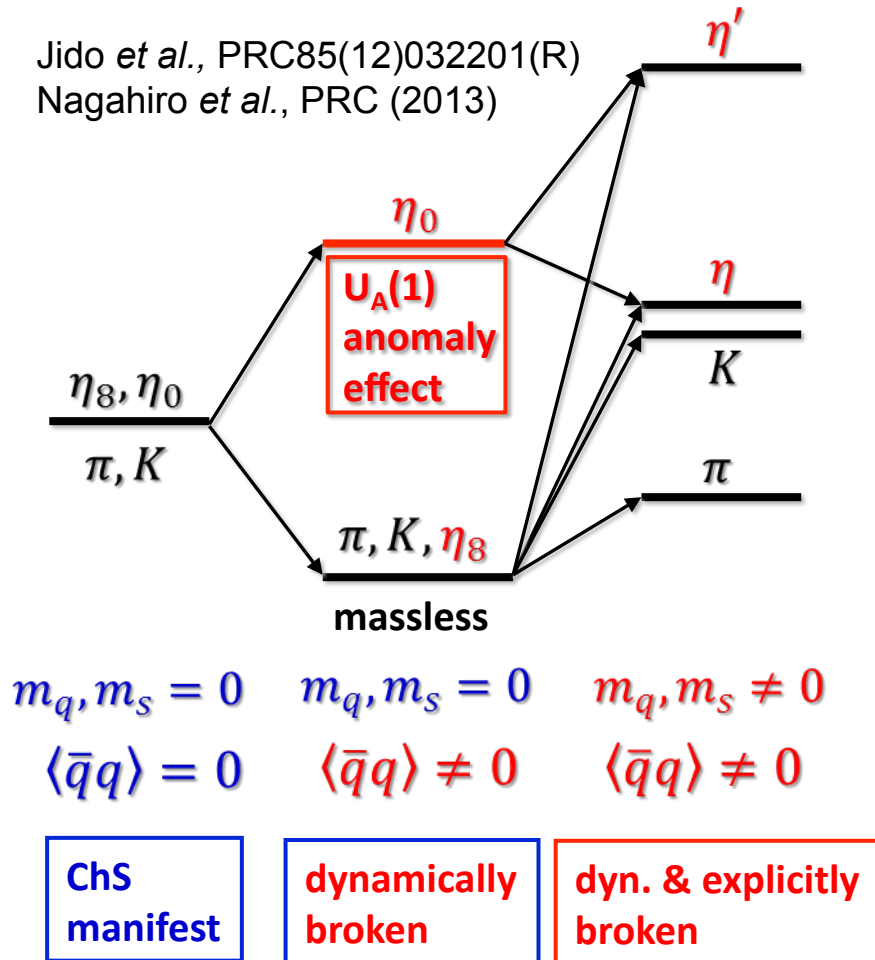
1. Hadron properties in Nucleus,

Aspects of the strong interaction symmetry at finite density

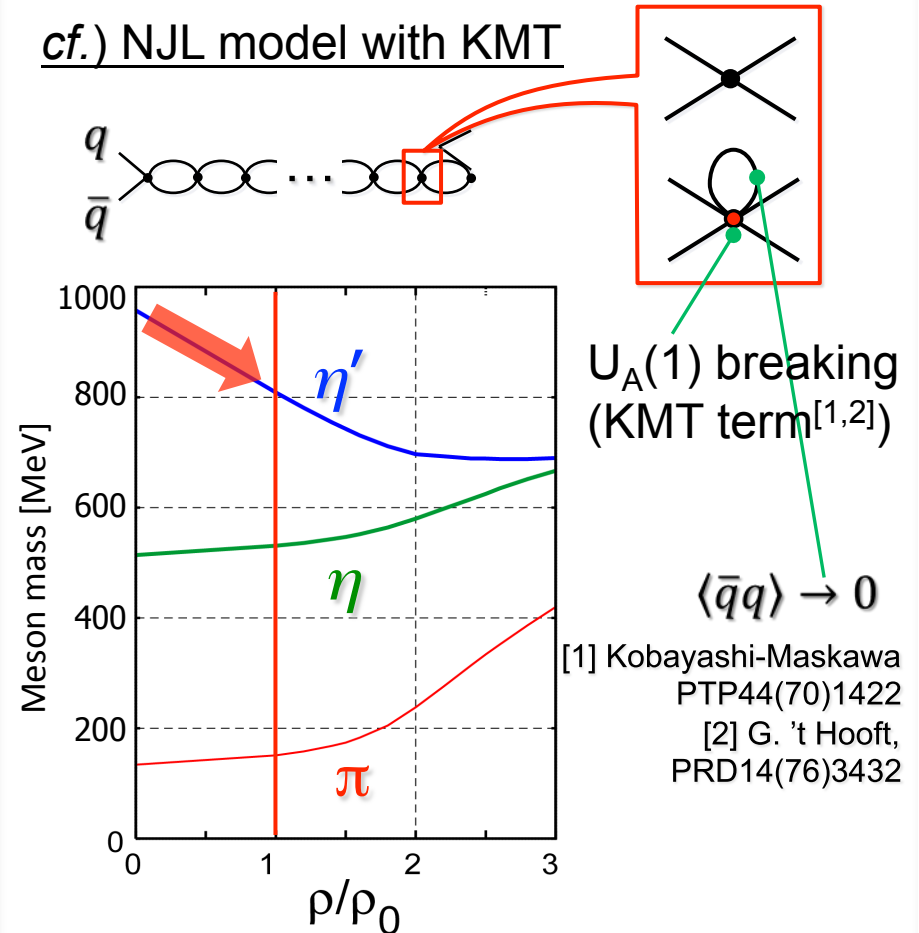
Meson mass spectrum and Symmetry Breaking Pattern (PS)

schematic view of the mass of π, K, η & η'

Jido *et al.*, PRC85(12)032201(R)
Nagahiro *et al.*, PRC (2013)



cf.) NJL model with KMT



$$\Delta m \sim -150 \text{ MeV} @ \rho_0$$

Costa *et al.*, PLB560(03)171,
Nagahiro-Takizawa-Hirenzaki, PRC74(06)045203

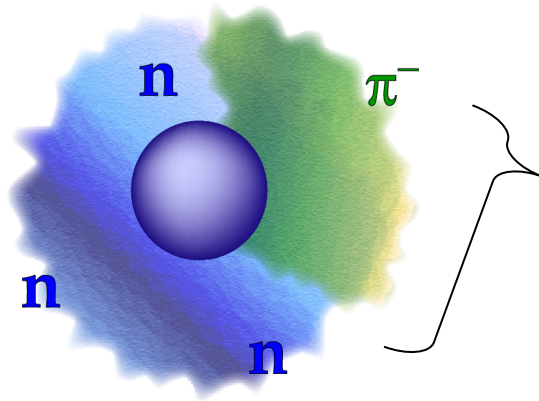
- Interests of Meson – Nucleus bound systems

1. Hadron properties in Nucleus,

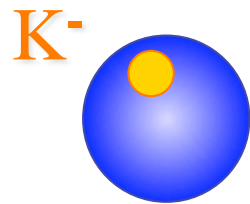
Aspects of the strong interaction symmetry at finite density

2. New exotic Hadron many body systems

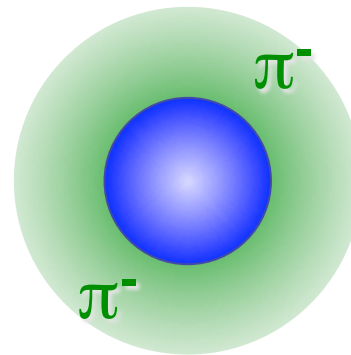
➤ Exotic Many Body Physics



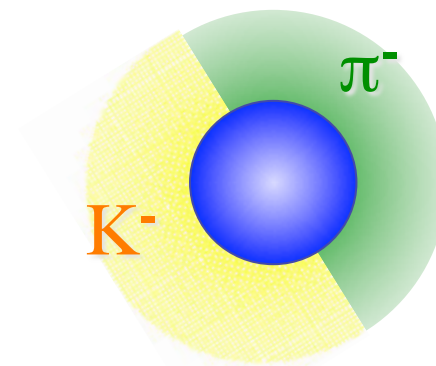
Core nucleus + Halo neutrons + pion



Kaonic nucleus



Double Pionic atom



Pionic & Kaonic atom

- Interests of Meson – Nucleus bound systems

1. Hadron properties in Nucleus,

Aspects of the strong interaction symmetry at finite density

2. New exotic Hadron many body systems

3. Baryon resonances at finite density

$$N^*(1535) - \eta + \text{Nucleus}$$

$$\Lambda(1405) - K^- + \text{Nucleus}$$

$d + d$ η -nucleus interaction from the reaction around the η production threshold

with

N. Ikeno (Tottori Univ.)

H. Nagahiro (Nara Women's Univ.)

D. Jido (Tokyo Metropolitan Univ)

Introduction of η -mesic nuclei

properties of eta meson

η meson

- » $m_\eta = 547.3$ [MeV] » $I = 0, J^P = 0^-$
- » $\Gamma = 1.18$ [keV] ($2\gamma, 3\pi^0, \pi^+\pi^-\pi^0, \dots$)

η -N system

■ Strong Coupling to $N^*(1535)$,

- » $\Gamma_{\pi N} \sim \Gamma_{\eta N} \sim 75$ [MeV]

eta-Nucleus system



Doorway to $N^*(1535)$

$$J^P = \frac{1}{2}^-$$

ηNN^* system

- No $I=3/2$ baryon contamination
- Large coupling constant
- no suppression at threshold
(s-wave coupling)

$$\mathcal{L}_{\eta NN^*} = g_\eta \eta \bar{N} N + h.c.$$

Motivation and our aim

- » η -N system ... strongly couples to the **$N^*(1535)$ resonance**
→ η -mesic nuclei ... doorway to **in-medium $N^*(1535)$**
- » **$N^*(1535)$** ... a candidate of the chiral partner of nucleon
→ **chiral symmetry for baryons**

η -Nucleus Interaction: general remark

~ N^* dominance model ~

optical potential

$$V_{\text{opt}} = \frac{g_\eta^2}{2\mu \omega + m_N(\rho) - m_{N^*}(\rho) + i\Gamma_{N^*}(s; \rho)/2} \rho$$

energy dependence

density-dependence

potential nature

In free space ($V \sim t\rho$)

$$\omega + m_N - m_{N^*} < 0 \quad \longrightarrow \quad \text{attractive}$$

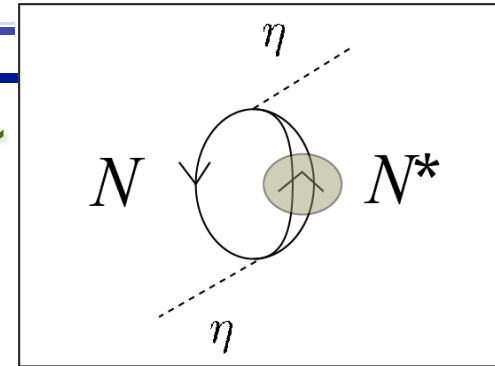
$$(m_\eta + m_N - m_{N^*} \sim -50\text{MeV})$$

medium effect

m_N & m_{N^*} change ??

$$\omega + m_N(\rho) - m_{N^*}(\rho) > 0 \quad \longrightarrow \quad \text{Repulsive ??}$$

N & N^* properties in medium evaluated by two kinds of **Chiral Models**



(Chiang, Oset, Liu PRC44(1991)738)

(D.Jido, H.N., S.Hirenzaki, PRC66(2002)045202)

$$g_\eta \simeq 2.0$$

to reproduce the partial width
 $\Gamma_{N^* \rightarrow \eta N} \simeq 75 \text{ MeV}$
 at tree level.

General feature

Chiral models for N and N* and η-nucleus interaction

Chiral doublet model

DeTar, Kunihiro, PRD39 (89)2805
 Jido, Nemoto, Oka, Hosaka NPA671(00)471
 Jido, Oka, Hosaka, PTP106(01)873
 Jido, Hatsuda, Kunihiro, PRL84(00)3252
 etc

Extended SU(2) Linear Sigma Model
 for N and N*

Lagrangian

$$\mathcal{L} = \sum_{j=1,2} [\bar{N}_j i \not{\partial} N_j - g_j \bar{N}_j (\sigma + (-)^{j-1} i \gamma_5 \vec{\tau} \cdot \vec{\pi}) N_j] - m_0 (\bar{N}_1 \gamma_5 N_2 - \bar{N}_2 \gamma_5 N_1)$$

Physical fields

$$\begin{pmatrix} N \\ N^* \end{pmatrix} = \begin{pmatrix} \cos \theta & \gamma_5 \sin \theta \\ -\gamma_5 \sin \theta & \cos \theta \end{pmatrix} \begin{pmatrix} N_1 \\ N_2 \end{pmatrix}$$

N* : chiral partner of nucleon

Mass difference

$$m_N^*(\rho) - m_N^*(\rho) = (1 - C \frac{\rho}{\rho_0})(m_N - m_{N^*})$$

* C~0.2 :the strength of the Chiral restoration at the nuclear saturation density

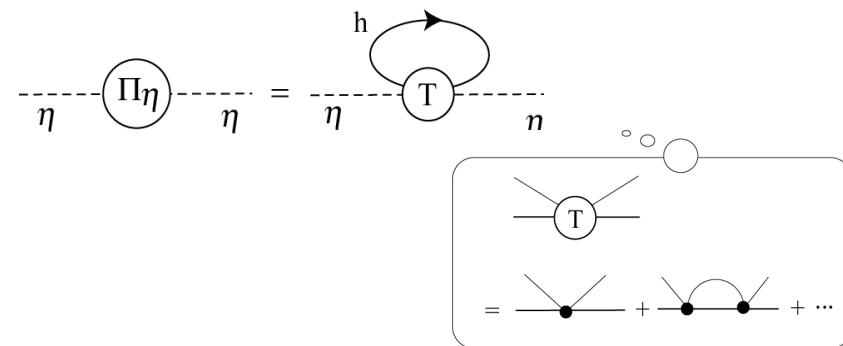
*** reduction of mass difference**

Chiral unitary model

Kaiser, Siegel, Weise, PLB362(95)23
 Waas, Weise, NPA625(97)287
 Garcia-Recio, Nieves, Inoue, Oset, PLB550(02)47
 Inoue, Oset, NPA710(02) 354

A coupled channel Bethe-Salpeter eq.

$$\{\pi^- p, \pi^0 n, \eta n, K^0 \Lambda, K^+ \Sigma^-, K^0 \Sigma^0, \pi^0 \pi^- p, \pi^+ \pi^- n\}$$

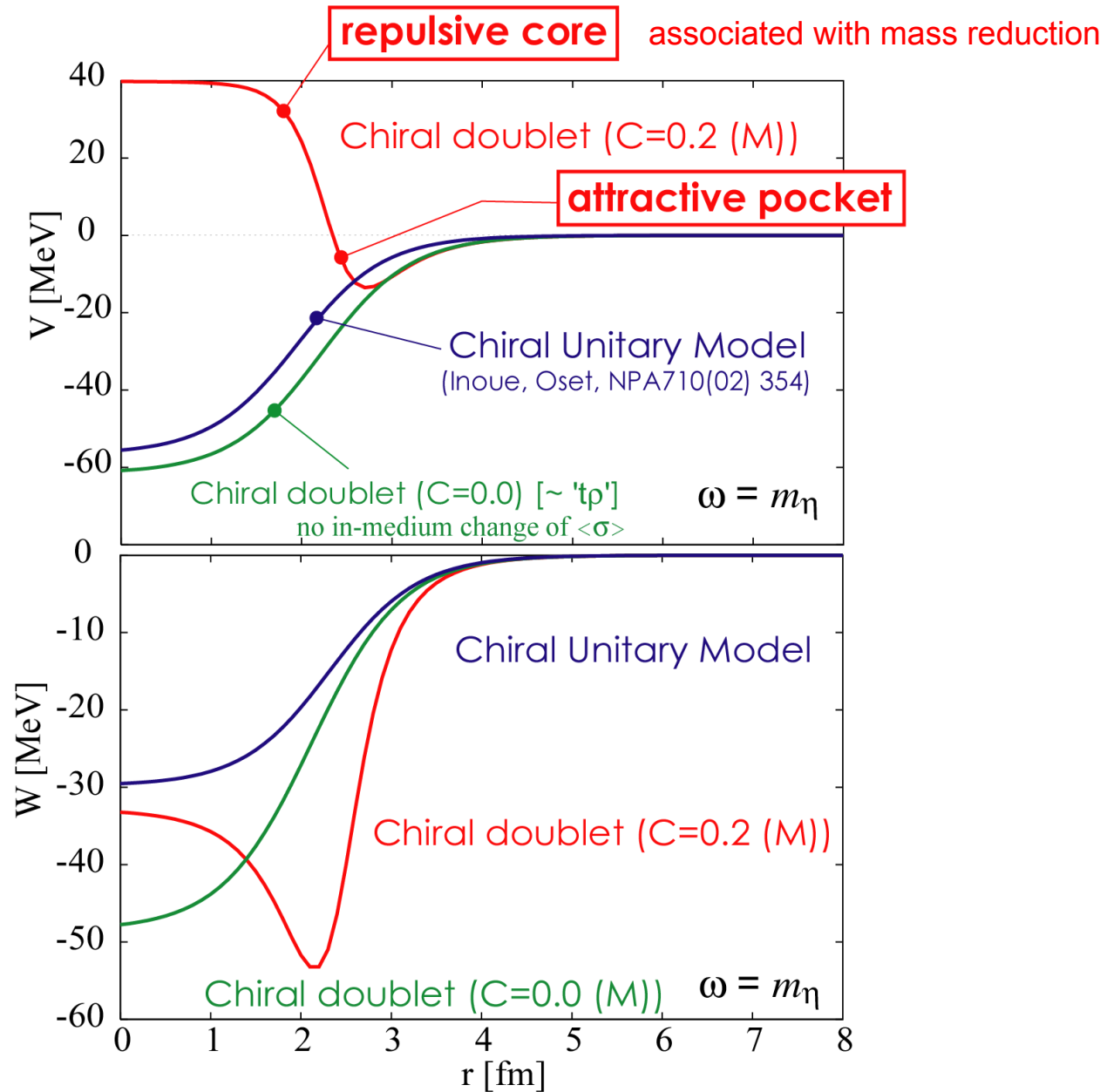


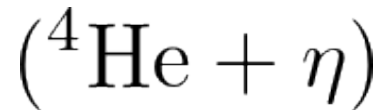
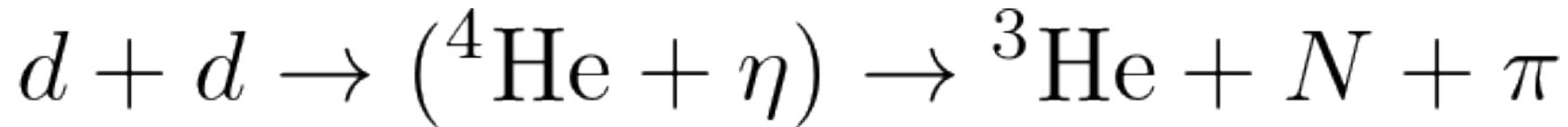
* the N* is introduced as **a resonance generated dynamically** from meson-baryon scattering.

*** No mass shift of N* is expected in the nuclear medium.**

* In this study, we directly take the eta-self-energy in the ref.NPA710(02)354

η -Nucleus optical potential



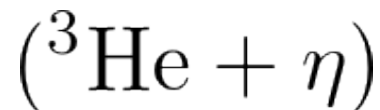


- 2008: ($^3\text{He} + p + \pi^-$) final state (W. Krzemien, P. Moskal)

- 2010: ($^3\text{He} + n + \pi^0$) and ($^3\text{He} + p + \pi^-$) states
(M. Skurzok and W. Krzemien, P. Moskal)

Search for η -mesic ^4He in the $dd \rightarrow ^3\text{He}n\pi^0$ and
 $dd \rightarrow ^3\text{He}p\pi^-$ reactions with the WASA-at-COSY
facility

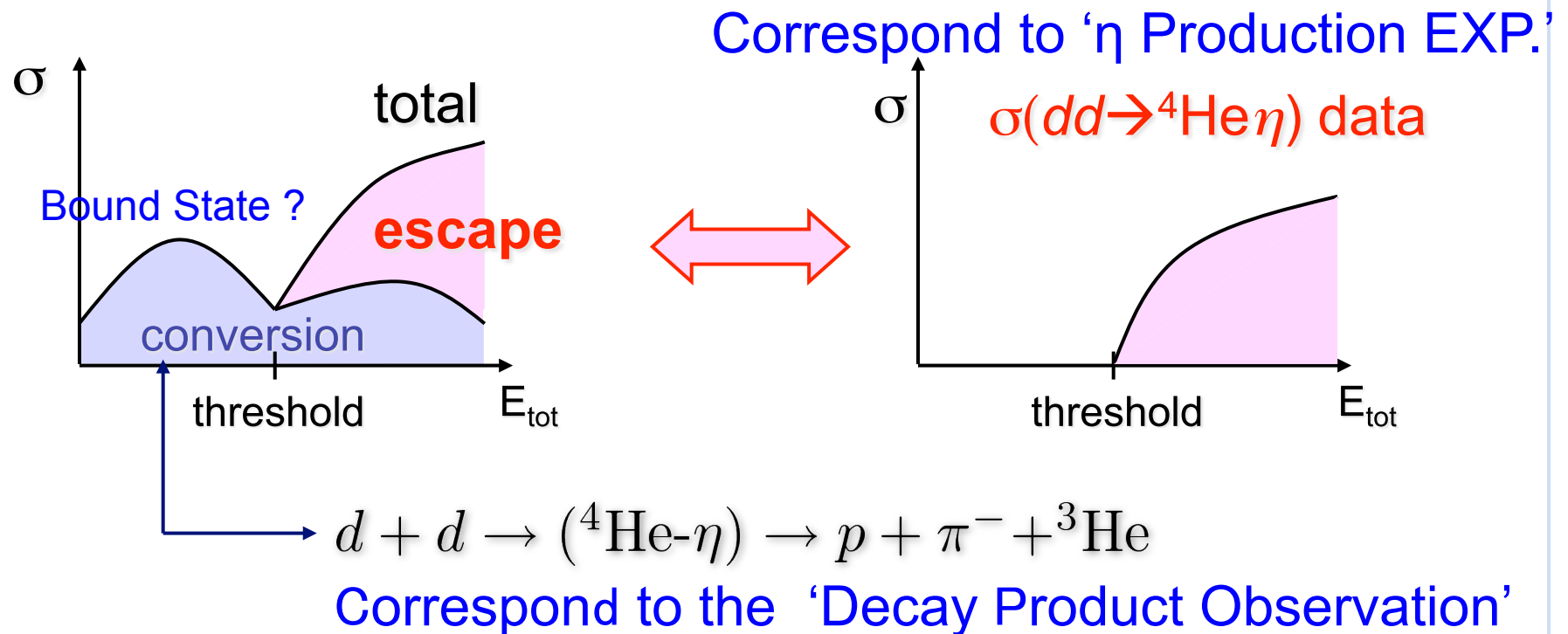
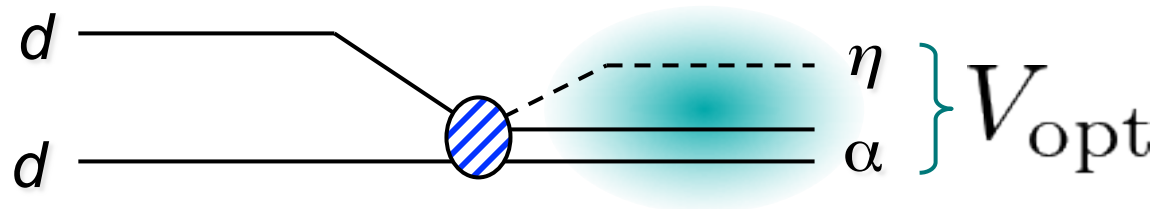
[Nuclear Physics A 959 \(2017\) 102–115](#)



- 2014: From proton + deuteron

Expected spectra for $d + d \rightarrow (^4\text{He} + \eta) \rightarrow ^3\text{He} + N + \pi$

Schematic picture



Theoretical Model for $d + d \rightarrow ({}^4\text{He}-\eta) \rightarrow p + \pi^- + {}^3\text{He}$

Some remarks

- Momentum transfer – Large
= $p_d = 1.025 \text{ GeV}/c$, $p_\alpha = p_\eta = 0$ at threshold in C.M.
- Energy range – Narrow (around threshold)
= Calculated range is;
1004 – 1041 MeV/c for p_d
-20 – +15 MeV for energy around the threshold
- Momentum transfer – Range is about 100MeV/c
= from 920 – 1024MeV/c

Conjecture, Assumption (should be checked numerically)

Energy dependence of the dd distortion and eta-alpha potential
→ Effects to the 'spectrum SHAPE' could be small

Momentum transfer varies 100MeV/c → Include 'Form factor'

Theoretical Model for $d + d \rightarrow ({}^4\text{He}-\eta) \rightarrow p + \pi^- + {}^3\text{He}$

Additional remarks

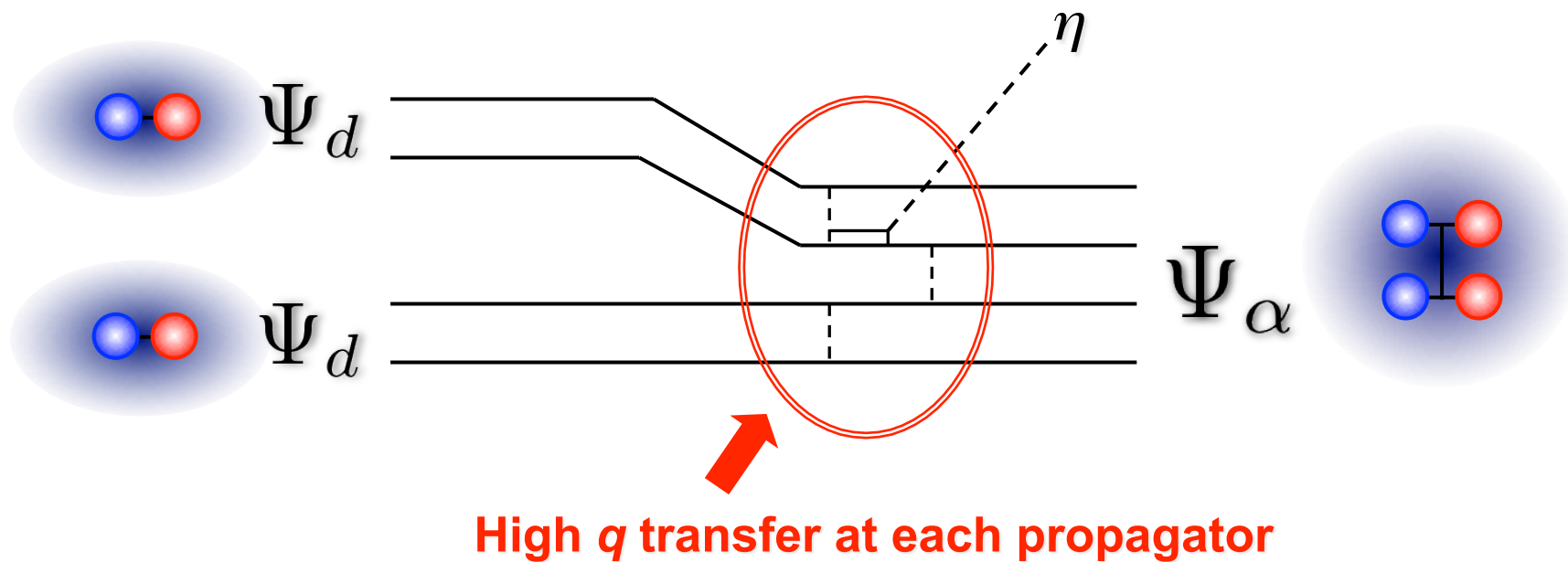
- System consists of
2 Nucleon + 2 Nucleon \rightarrow 4 Nucleon + 1 meson
= Simple spectral structure is expected for light systems
(.... 1 or 0 bound state with small binding energy ?)
- Data of $d d \rightarrow {}^4\text{He} \eta$ above threshold
= Information on the strength of the nuclear fusion,
eta production, dd distortion etc at this energy region

Conjecture, Assumption

= Absolute value of the spectra could be determined
based on the data

Theoretical Model for $d + d \rightarrow ({}^4\text{He}-\eta) \rightarrow p + \pi^- + {}^3\text{He}$

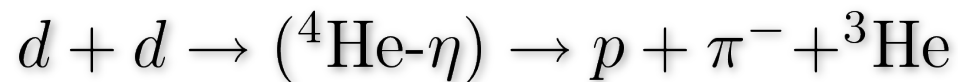
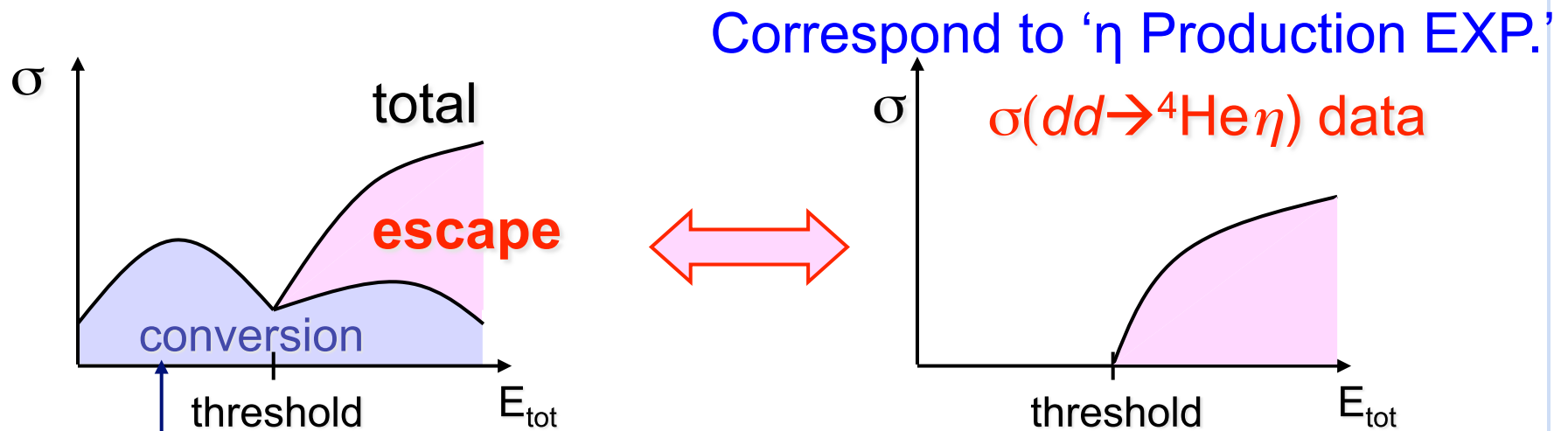
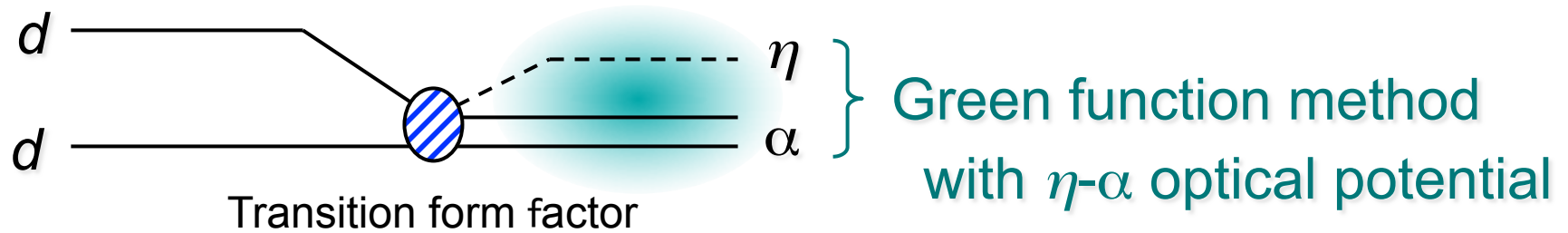
- Transition (η -production) part



→ Parameterize this part. Fix by η production data

Theoretical Model for $d + d \rightarrow ({}^4\text{He}-\eta) \rightarrow p + \pi^- + {}^3\text{He}$

Schematic picture



Correspond to the 'Decay Product Observation'

Theoretical Model for $d + d \rightarrow ({}^4\text{He}-\eta) \rightarrow p + \pi^- + {}^3\text{He}$

$$\epsilon^{\mu\nu\rho\sigma} \partial_\mu A_\nu \partial_\rho A_\sigma P$$

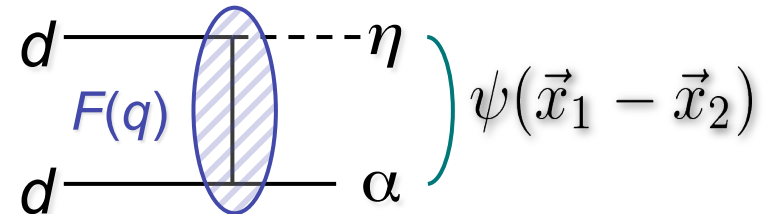
Coupling:

Axial vector -- Axial vector -- Pseudoscalar – Scalar

$$\mathcal{H}_{\text{int}} = -ic\epsilon^{ijk} ((\partial_{x_2^0} \nabla_{x_1}^i - \partial_{x_1^0} \nabla_{x_2}^i) \hat{\phi}_d^j(x_1) \hat{\phi}_d^k(x_2)) \hat{\phi}_\eta^\dagger(x_1) \hat{\phi}_\alpha^\dagger(x_2) \mathcal{F}(x_1, x_2)$$

Interaction Hamiltonian

$$\mathcal{F}(x_1, x_2) = \int \frac{d^4q}{(2\pi)^4} F(\mathbf{q}) e^{iq \cdot (x_1 - x_2)}$$



Transition Form Factor

$$S = -i\mathcal{N}_{d_1}\mathcal{N}_{d_2}\mathcal{N}_\eta\mathcal{N}_\alpha c \int d^4x_1 d^4x_2 \epsilon^{ijk} (\partial_{x_2^0} \nabla_{x_1}^i - \partial_{x_1^0} \nabla_{x_2}^i)$$

$$\left[\chi_{d_1}^j \chi_{d_2}^k e^{-ip_1 \cdot x_1} e^{-ip_2 \cdot x_2} + \chi_{d_2}^j \chi_{d_1}^k e^{-ip_2 \cdot x_1} e^{-ip_1 \cdot x_2} \right]$$

S-matrix

$$\times \int \frac{d^4q}{(2\pi)^4} F(\mathbf{q}) e^{iq \cdot (x_1 - x_2)} \phi_\eta^*(\mathbf{x}_1) e^{iE_\eta x_1^0} \phi_\alpha^*(\mathbf{x}_2) e^{iE_\alpha x_2^0}$$

Theoretical Model for $d + d \rightarrow ({}^4\text{He}-\eta) \rightarrow p + \pi^- + {}^3\text{He}$

$$\begin{aligned}
 S = & -i2\pi\delta(E_1 + E_2 - E_f)\mathcal{N}_{d_1}\mathcal{N}_{d_2}\mathcal{N}_f \\
 & \times c\epsilon^{ijk}2(E_2p_1^i - E_1p_2^i)\chi_{d_1}^j\chi_{d_2}^k \\
 & \times \int d\mathbf{R}d\mathbf{r} e^{i(\mathbf{p}_1+\mathbf{p}_2)\cdot\mathbf{R}} e^{i(\frac{M_\alpha}{m_\eta+M_\alpha}\mathbf{p}_1 - \frac{m_\eta}{m_\eta+M_\alpha}\mathbf{p}_2)\cdot\mathbf{r}} \\
 & \times \int \frac{d\mathbf{q}}{(2\pi)^3} F(\mathbf{q})e^{-i\mathbf{q}\cdot\mathbf{r}} \phi_f^*(\mathbf{r})e^{-i\mathbf{p}_f\cdot\mathbf{R}}.
 \end{aligned}$$

(Center of Mass and Relative coordinates)

Obtain T by $S = 1 - i(2\pi)^4 T \delta^4(p)$

$$d\sigma = \frac{1}{9} \sum_{\chi_{d_1}, \chi_{d_2}, f} \frac{|T|^2}{8p_{\text{c.m.}} E_d} (2\pi)^4 \delta^{(4)}(p_i - p_f) \frac{d\mathbf{p}_f}{(2\pi)^3 2E_f},$$

Theoretical Model for $d + d \rightarrow ({}^4\text{He}-\eta) \rightarrow p + \pi^- + {}^3\text{He}$

$$\sigma = -\frac{8\pi}{9}c^2 p_{cm} \text{Im} \int r_1^2 dr_1 r_2^2 dr_2 f(r_1) f^*(r_2) \\ \times \sum_{\ell} (2\ell + 1) j_{\ell}(pr_1) G^{\ell}(E_i; r_1, r_2) j_{\ell}(pr_2),$$

: Total Cross section

$$\sigma_{\text{conv}} = -\frac{8\pi}{9}c^2 p_{cm} \int r_1^2 dr_1 r_2^2 dr_2 r_3^2 dr_3 f(r_1) f^*(r_3) \\ \times \text{Im} U_{\text{opt}}(r_2) \sum_{\ell} (2\ell + 1) j_{\ell}(pr_1) \\ \times G^{\ell*}(E_i; r_1, r_2) G^{\ell}(E_i; r_2, r_3) j_{\ell}(pr_3),$$

: Conversion part

$$\sum_f \delta(E_f - E_i) \phi_f^*(\mathbf{r}_1) \phi_f(\mathbf{r}_2) \\ = -\frac{1}{\pi} \text{Im} \sum_f \phi_f^*(\mathbf{r}_1) \frac{1}{E_f - E_i + i\epsilon} \phi_f(\mathbf{r}_2) \\ = -\frac{1}{\pi} \text{Im} \sum_f \phi_f^*(\mathbf{r}_1) \frac{1}{\hat{H} - E_i + i\epsilon} \phi_f(\mathbf{r}_2),$$

: Green's function

Theoretical Model for $d + d \rightarrow (^4\text{He}-\eta) \rightarrow p + \pi^- + ^3\text{He}$

4 parameters in this model

- η - α optical potential (2 parameters)

$$V_{\text{opt}} = (V_0 + iW_0) \frac{\rho_\alpha(r)}{\rho_\alpha(0)}$$

- Transition form factor
(η production and nuclear fusion parts $dd \rightarrow ^4\text{He}\eta$)

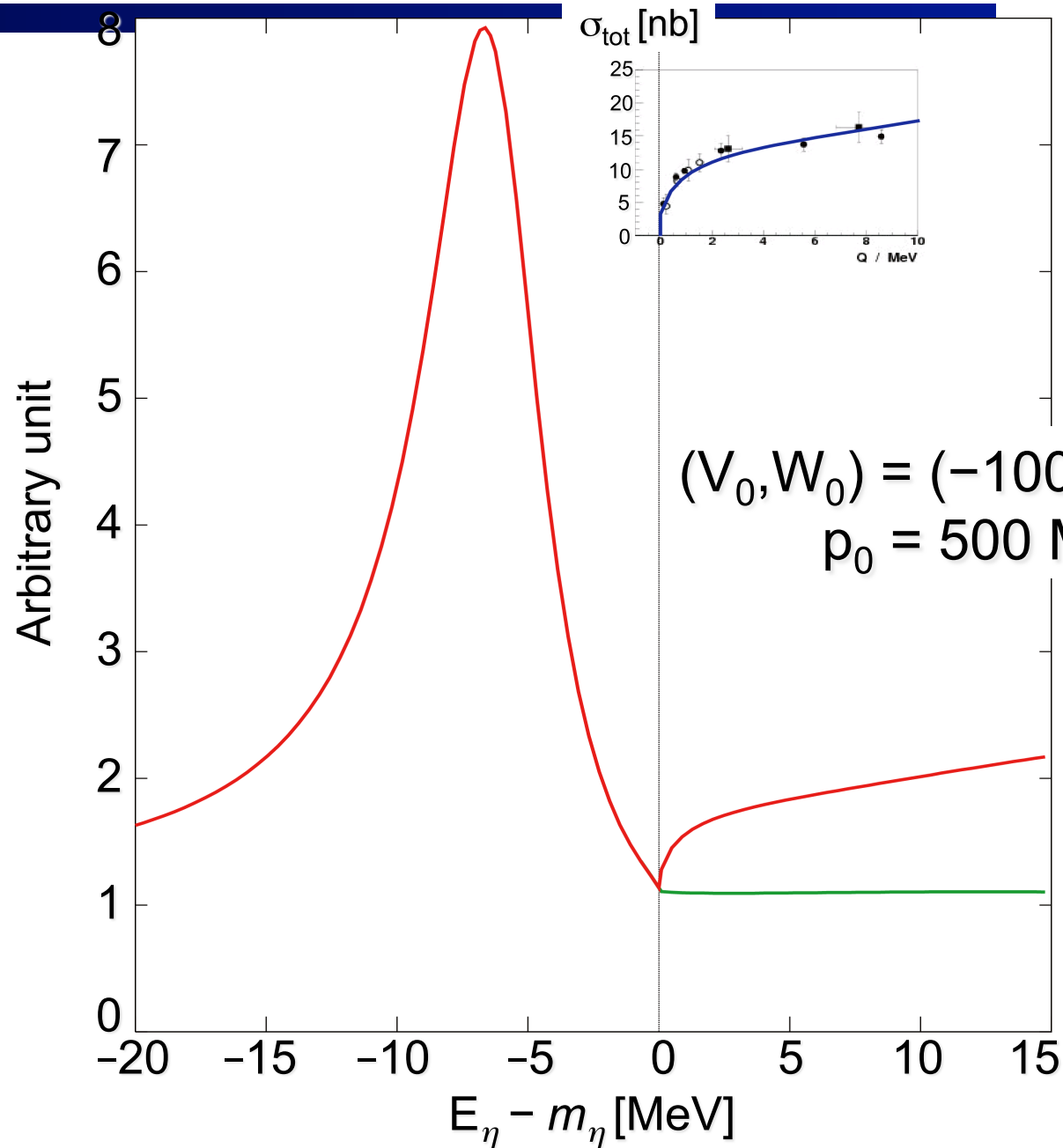
$$F(q) = N \exp \left[-\frac{p^2}{p_0^2} \right]$$

* Possibilities of other functional form

- Coupling strength C

Numerical Results, An example

(Ideal case)



Eta-production data and Calculated escape part

Exp. Data of $dd \rightarrow {}^4\text{He}\eta$

R.Frascaria et al., Rhys.Rev.C50 (1994) 573,
N. Wills et al., Phys. Lett. B 406 (1997) 14,
A.Wronska et al., Eur. Phys. J. A 26, 421-428 (2005).

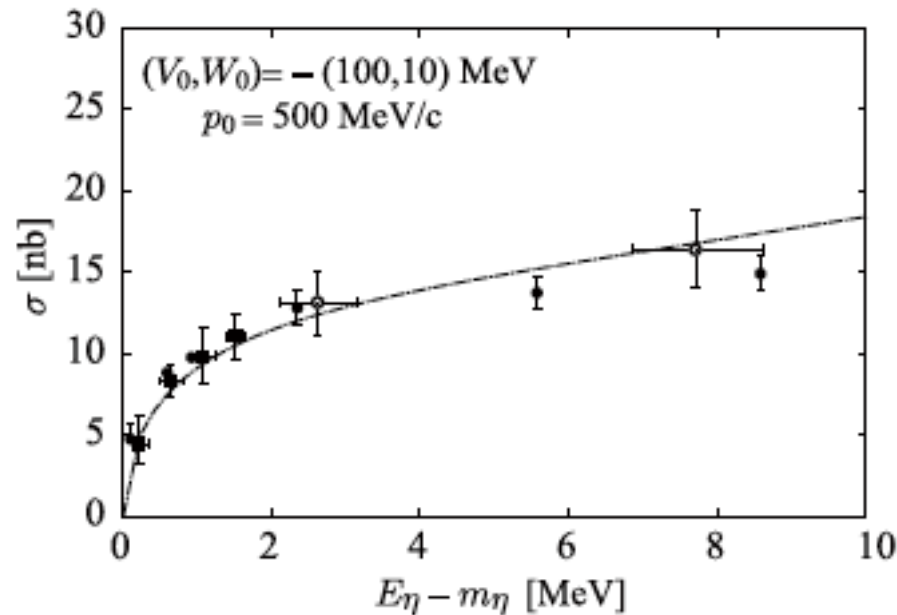
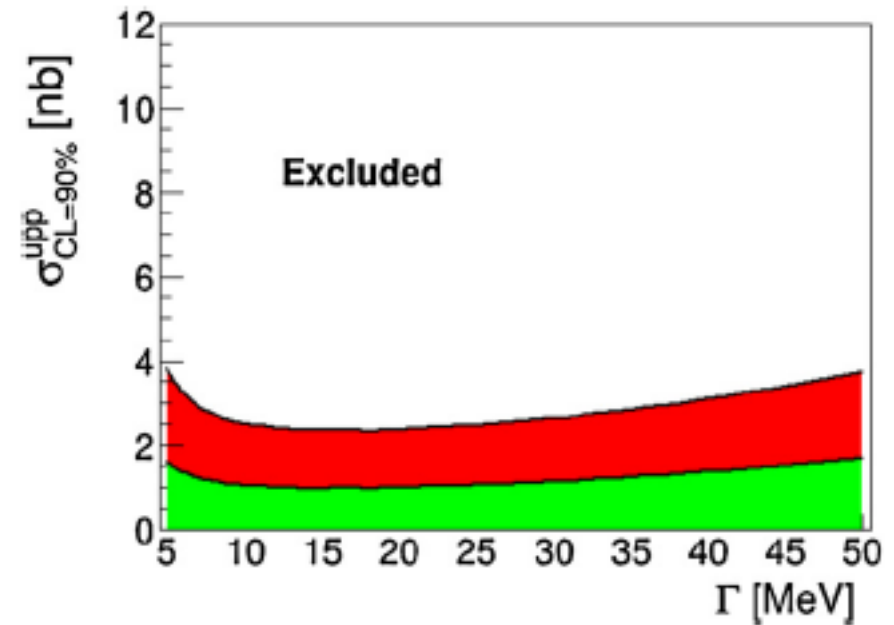
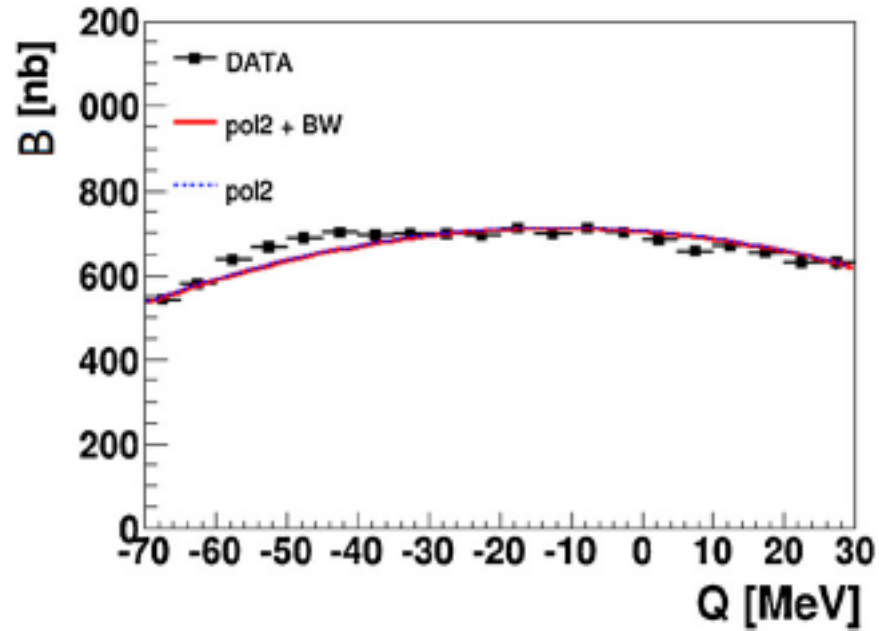
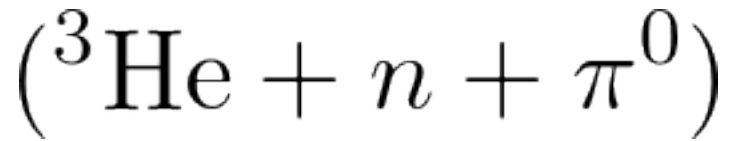


Fig. 4. Calculated escape part σ_{esc} in Fig. 3 plotted with the experimental data of $d + d \rightarrow \eta + \alpha$ reaction indicated by black squares [32], black circles [33], and open circles [34]. The parameters of the $\eta - \alpha$ optical potential are $(V_0, W_0) = -(100, 10)$ MeV and the p_0 parameter is fixed to be $p_0 = 500$ MeV/c. The height of the calculated spectrum is adjusted so as to reproduce the data by changing the interaction strength c given in Eq. (1).

Search for η -mesic ${}^4\text{He}$ in the $dd \rightarrow {}^3\text{He}n\pi^0$ and $dd \rightarrow {}^3\text{He}p\pi^-$ reactions with the WASA-at-COSY facility

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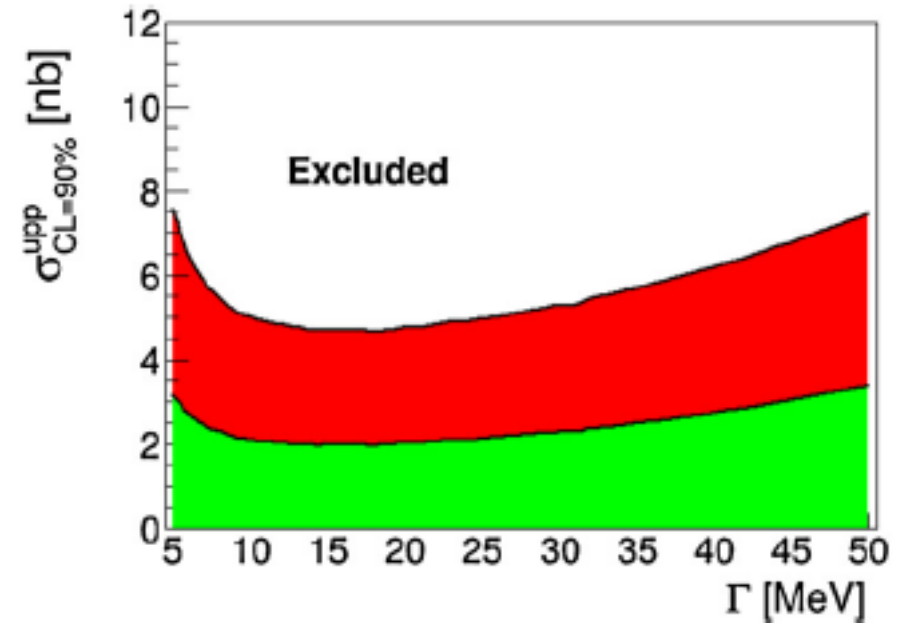
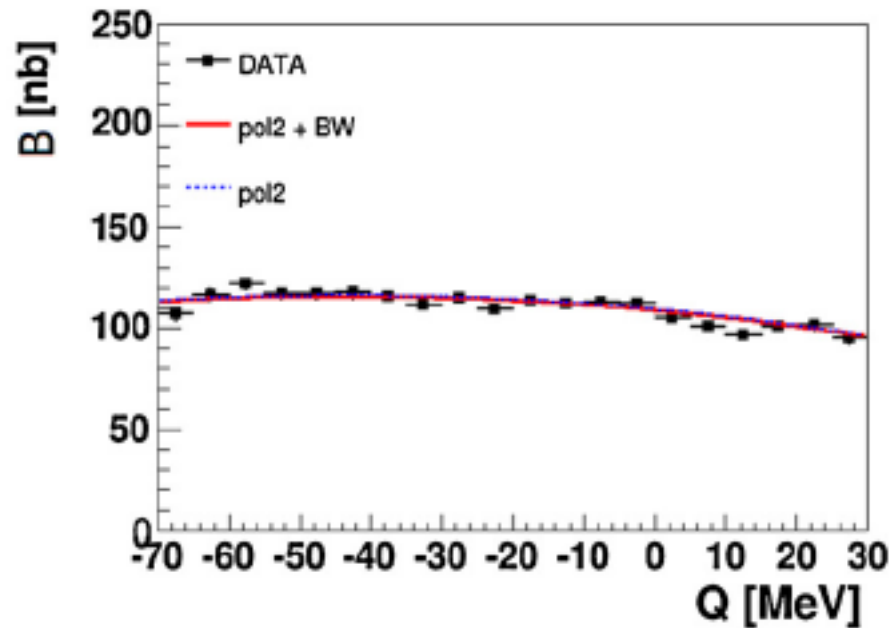


Excitation function and Upper limit of total cross section

Search for η -mesic ${}^4\text{He}$ in the $dd \rightarrow {}^3\text{He}n\pi^0$ and $dd \rightarrow {}^3\text{He}p\pi^-$ reactions with the WASA-at-COSY facility

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$({}^3\text{He} + p + \pi^-)$



Excitation function and Upper limit of total cross section

Table 1

The upper limit of the total cross-section for the $dd \rightarrow (^4\text{He}-\eta)_{\text{bound}} \rightarrow ^3\text{He}n\pi^0$ process determined at CL=90% for different values of binding energy B_s and width Γ . The upper limit of the total cross-section for the $dd \rightarrow (^4\text{He}-\eta)_{\text{bound}} \rightarrow ^3\text{He}p\pi^-$ process according to isospin relation is two times larger.

| B_s [MeV] | Γ [MeV] | $\sigma_{90\%}^{\text{upp}}$ [nb] | B_s [MeV] | Γ [MeV] | $\sigma_{90\%}^{\text{upp}}$ [nb] |
|-------------|----------------|-----------------------------------|-------------|----------------|-----------------------------------|
| 10 | 5 | 3.8 | 30 | 5 | 3.8 |
| 10 | 10 | 2.6 | 30 | 10 | 2.5 |
| 10 | 20 | 2.6 | 30 | 20 | 2.4 |
| 10 | 30 | 3.1 | 30 | 30 | 2.6 |
| 10 | 40 | 3.8 | 30 | 40 | 3.1 |
| 10 | 50 | 4.8 | 30 | 50 | 3.7 |
| 20 | 5 | 3.9 | 40 | 5 | 3.9 |
| 20 | 10 | 2.6 | 40 | 10 | 2.6 |
| 20 | 20 | 2.6 | 40 | 20 | 2.4 |
| 20 | 30 | 3.0 | 40 | 30 | 2.7 |
| 20 | 40 | 3.7 | 40 | 40 | 3.1 |
| 20 | 50 | 4.7 | 40 | 50 | 3.7 |

- These numbers correspond to the calculated **CONVERSION** parts.

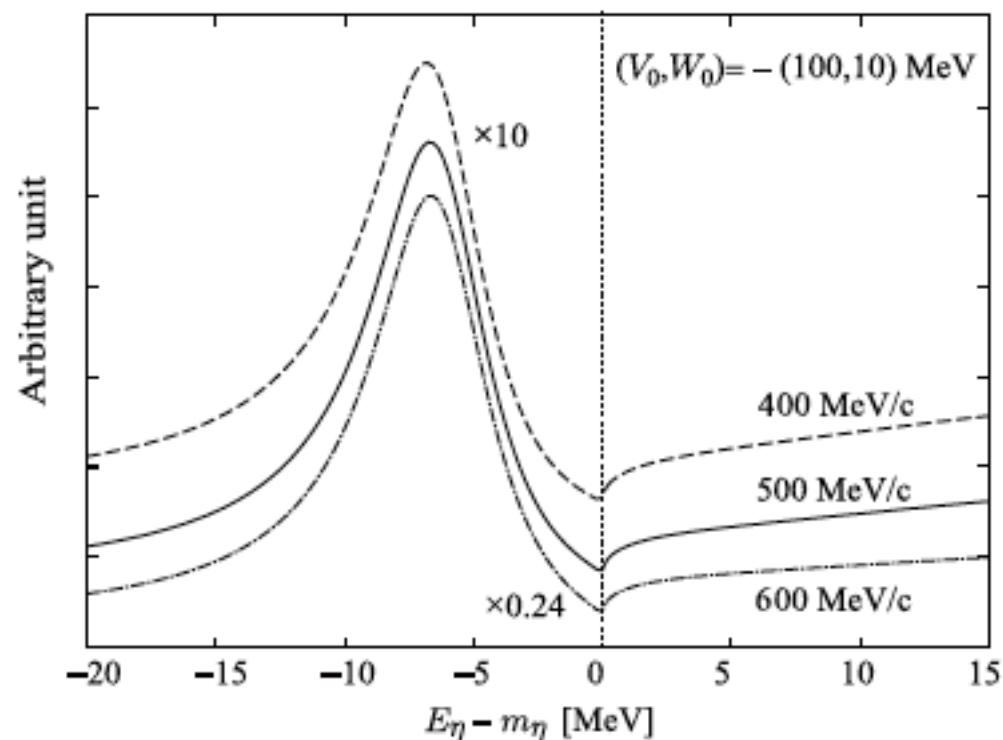


Fig. 2. Calculated total cross sections of the $d+d \rightarrow (\eta+\alpha) \rightarrow X$ reaction for the formation of the $\eta - \alpha$ bound system with $p_0 = 400, 500, 600$ MeV/c cases plotted as functions of the η excited energy $E_\eta - m_\eta$. The parameters of the $\eta - \alpha$ optical potential are fixed to be $(V_0, W_0) = -(100, 10)$ MeV. It is noted that the result with $p_0 = 600$ MeV/c is scaled by factor 0.24 and that with $p_0 = 400$ MeV/c scaled by factor 10.

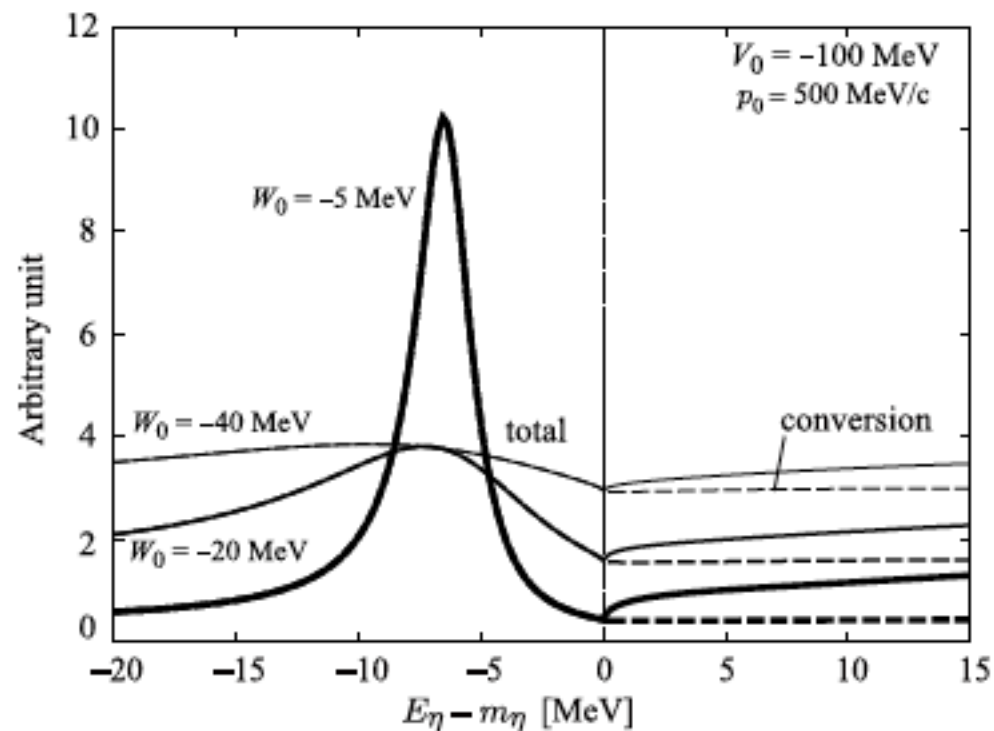


Fig. 5. Calculated cross sections of the $d + d \rightarrow (\eta + \alpha) \rightarrow X$ reaction for the formation of the $\eta - \alpha$ bound system plotted as functions of the η excited energy $E_\eta - m_\eta$. The parameters of the $\eta - \alpha$ optical potential are $(V_0, W_0) = -(100, 5)$, $-(100, 20)$, and $-(100, 40)$ MeV, and the p_0 parameter is fixed to be $p_0 = 500$ MeV/ c . The solid lines indicate the total cross sections σ and the dashed lines the conversion parts σ_{conv} .

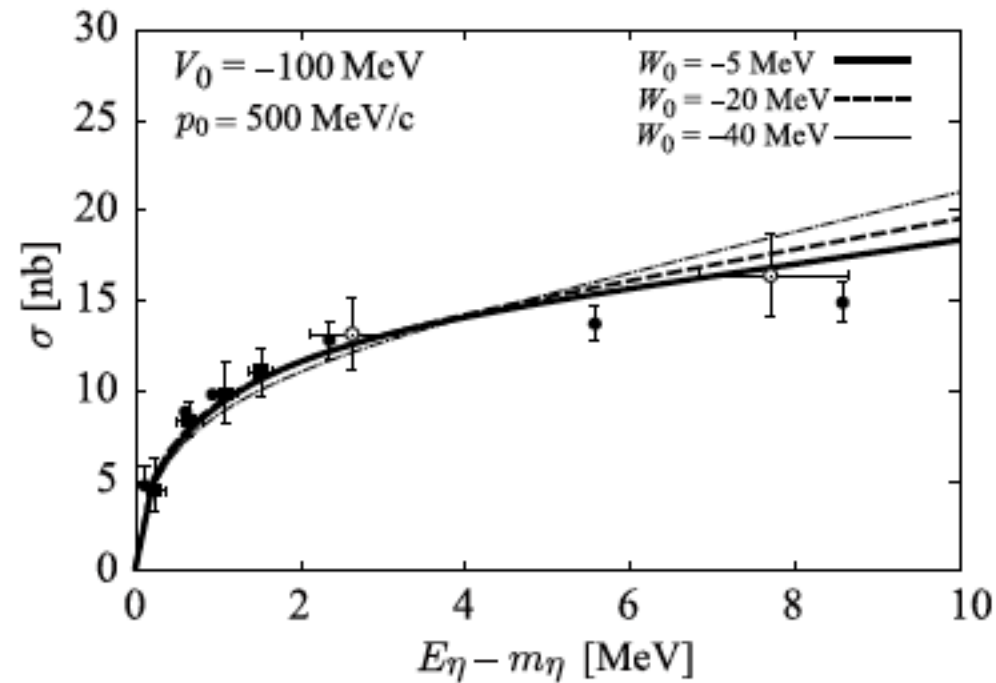


Fig. 9. Calculated escape part σ_{esc} plotted with the experimental data of $d + d \rightarrow \eta + \alpha$ reaction indicated by black squares [32], black circles [33], and open circles [34]. The parameters of the $\eta - \alpha$ optical potential are $(V_0, W_0) = -(100, 5), -(100, 20),$ and $-(100, 40)$ MeV as shown in the figure. The p_0 parameter is fixed to be $p_0 = 500$ MeV/ c .

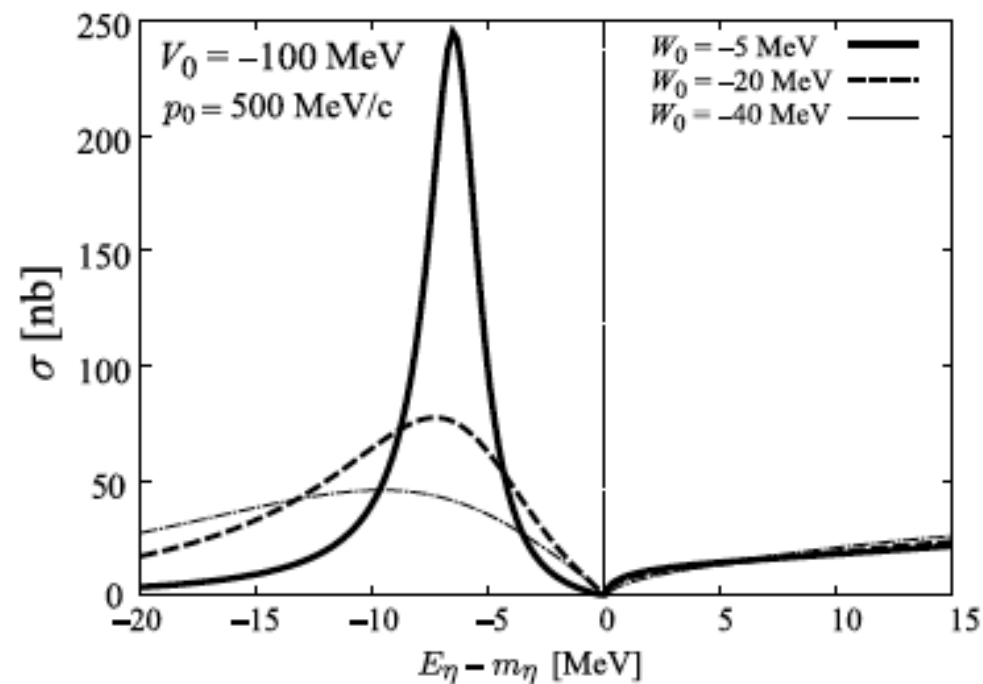


Fig. 13. Calculated total cross sections of $d+d \rightarrow (\eta+\alpha) \rightarrow X$ reaction scaled by the same factor used in Fig. 9 plotted as functions of the η excited energy $E_\eta - m_\eta$. The flat contributions are subtracted. The parameters of the $\eta - \alpha$ optical potential are $(V_0, W_0) = -(100, 5)$, $-(100, 20)$, and $-(100, 40)$ MeV, and the p_0 parameter is fixed to be $p_0 = 500$ MeV/ c .

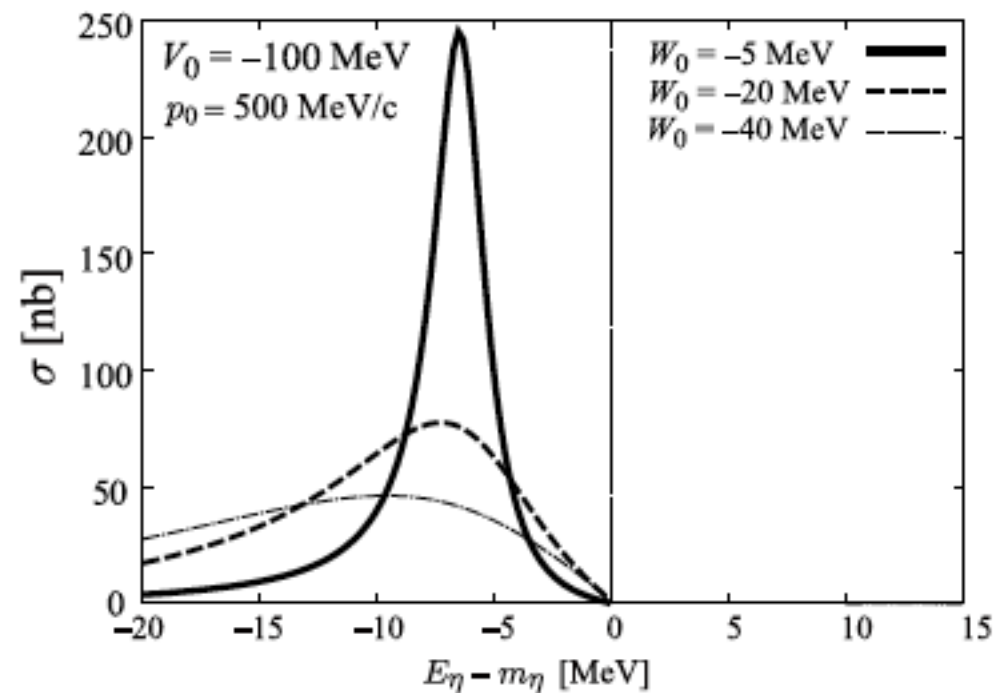
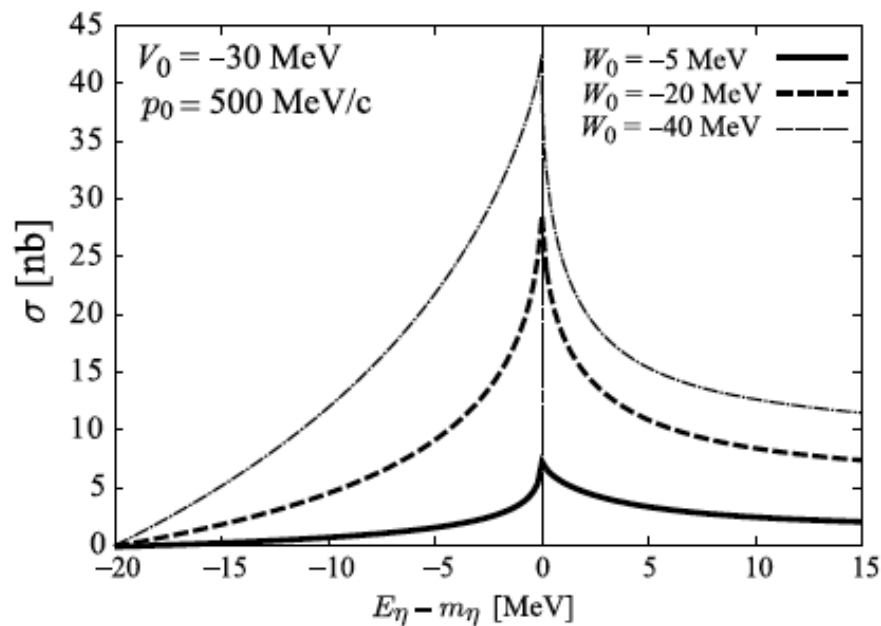
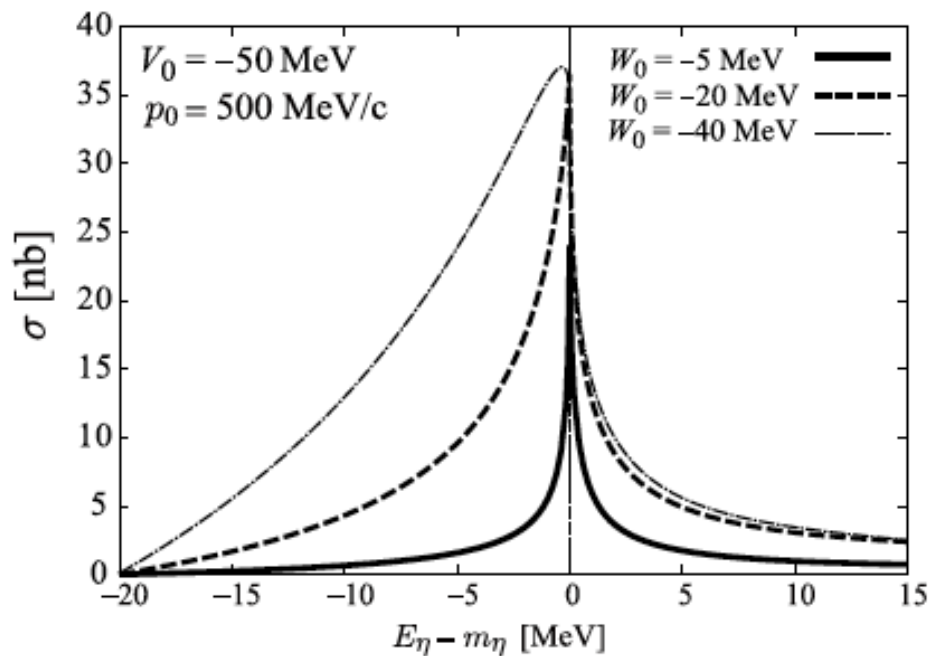
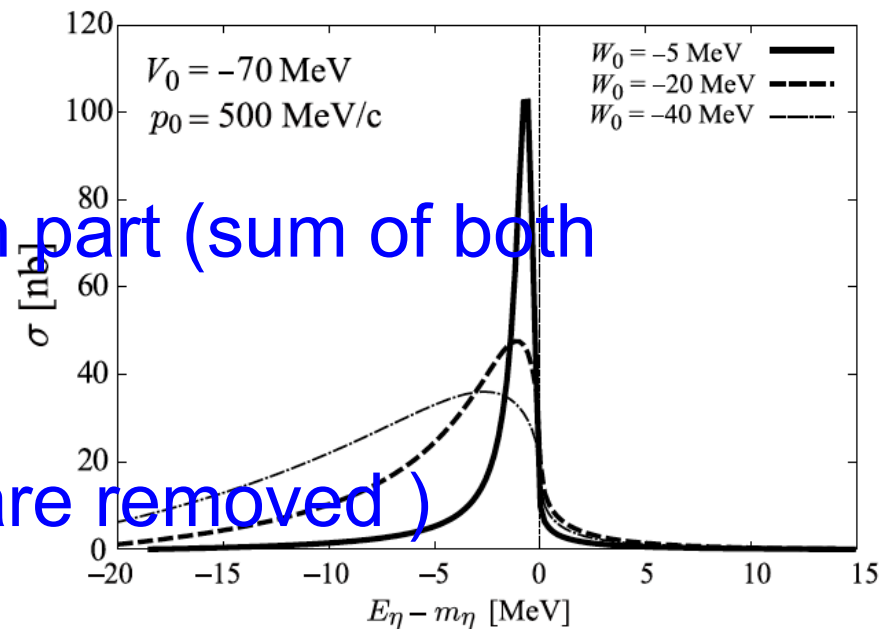
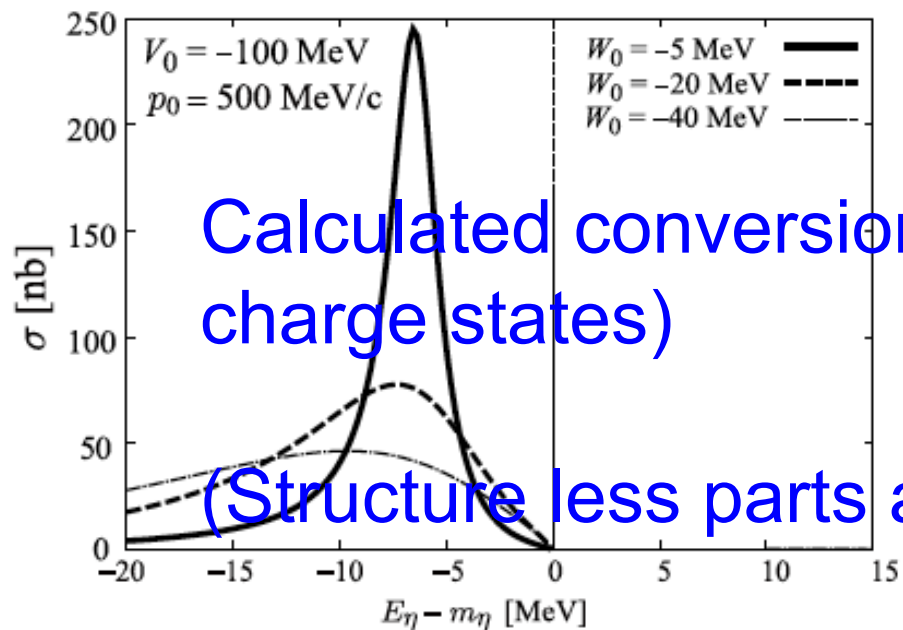
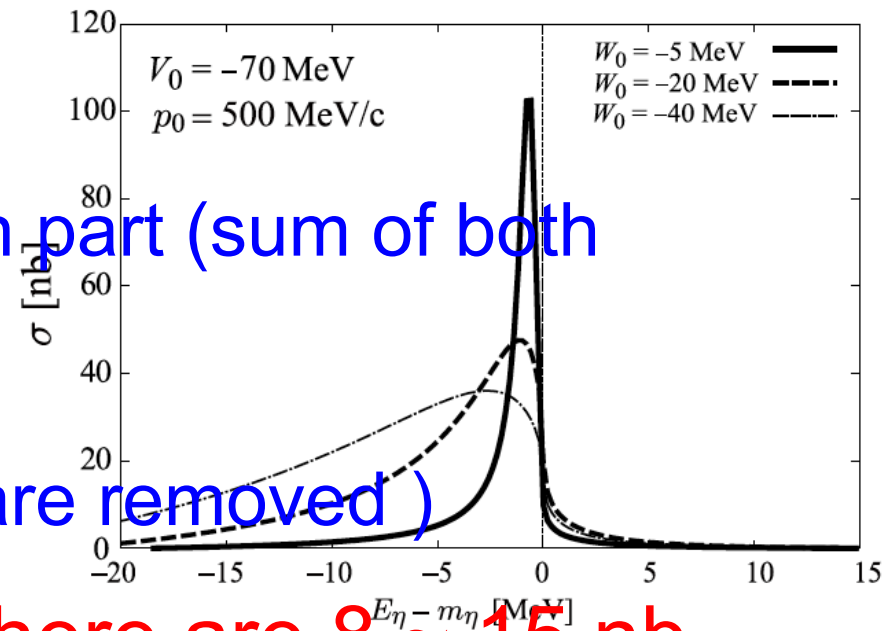
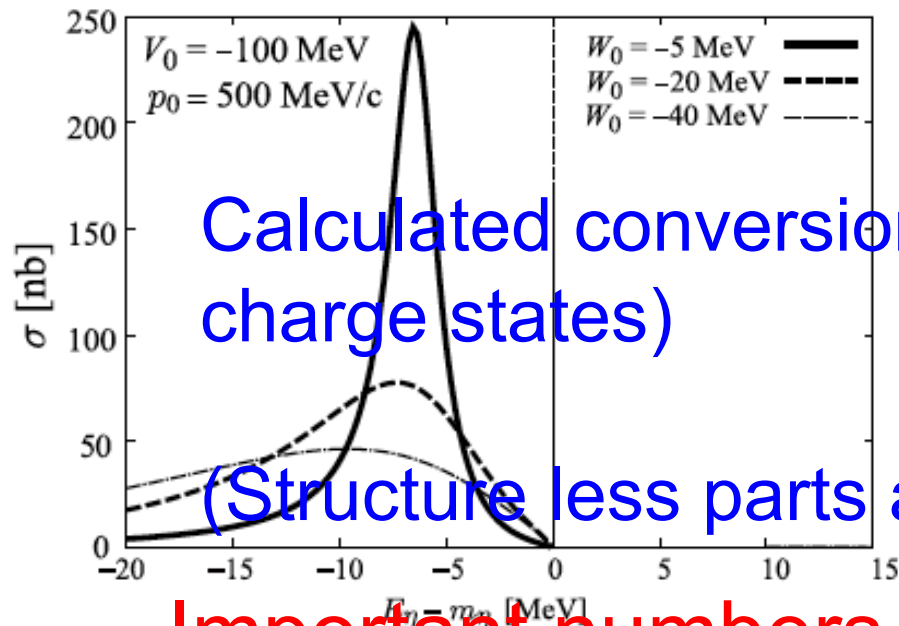


Fig. 17. Calculated conversion part of cross sections of $d + d \rightarrow (\eta + \alpha) \rightarrow X$ reaction scaled by the same factor used in Fig. 9 plotted as functions of the η excited energy $E_\eta - m_\eta$. The flat contributions are subtracted. The parameters of the η - α optical potential are $(V_0, W_0) = -(100, 5)$, $-(100, 20)$, and $-(100, 40)$ MeV, and the p_0 parameter is fixed to be $p_0 = 500$ MeV/ c .



Calculated conversion part (sum of both charge states)

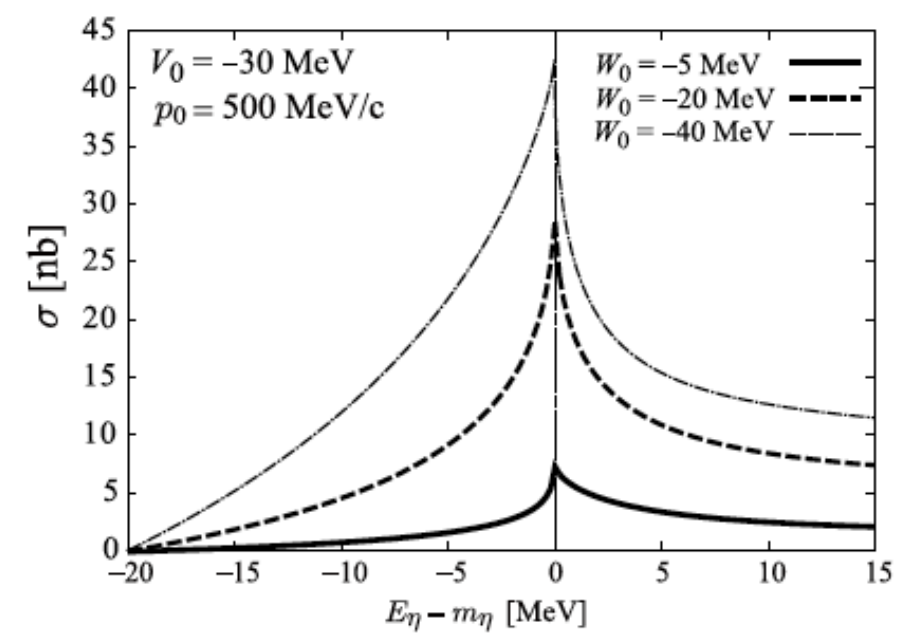
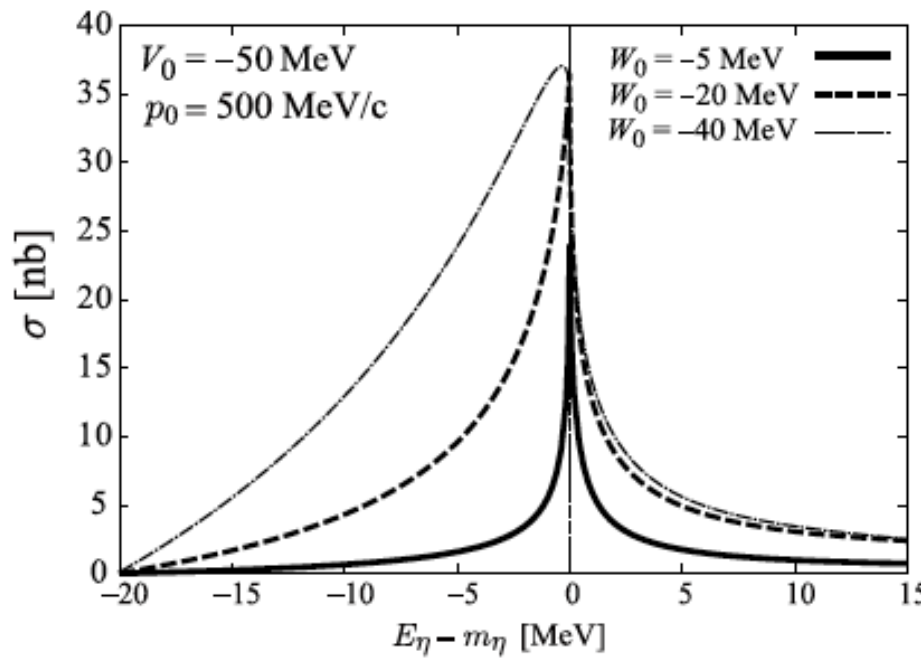
(Structure less parts are removed)



Calculated conversion part (sum of both charge states)

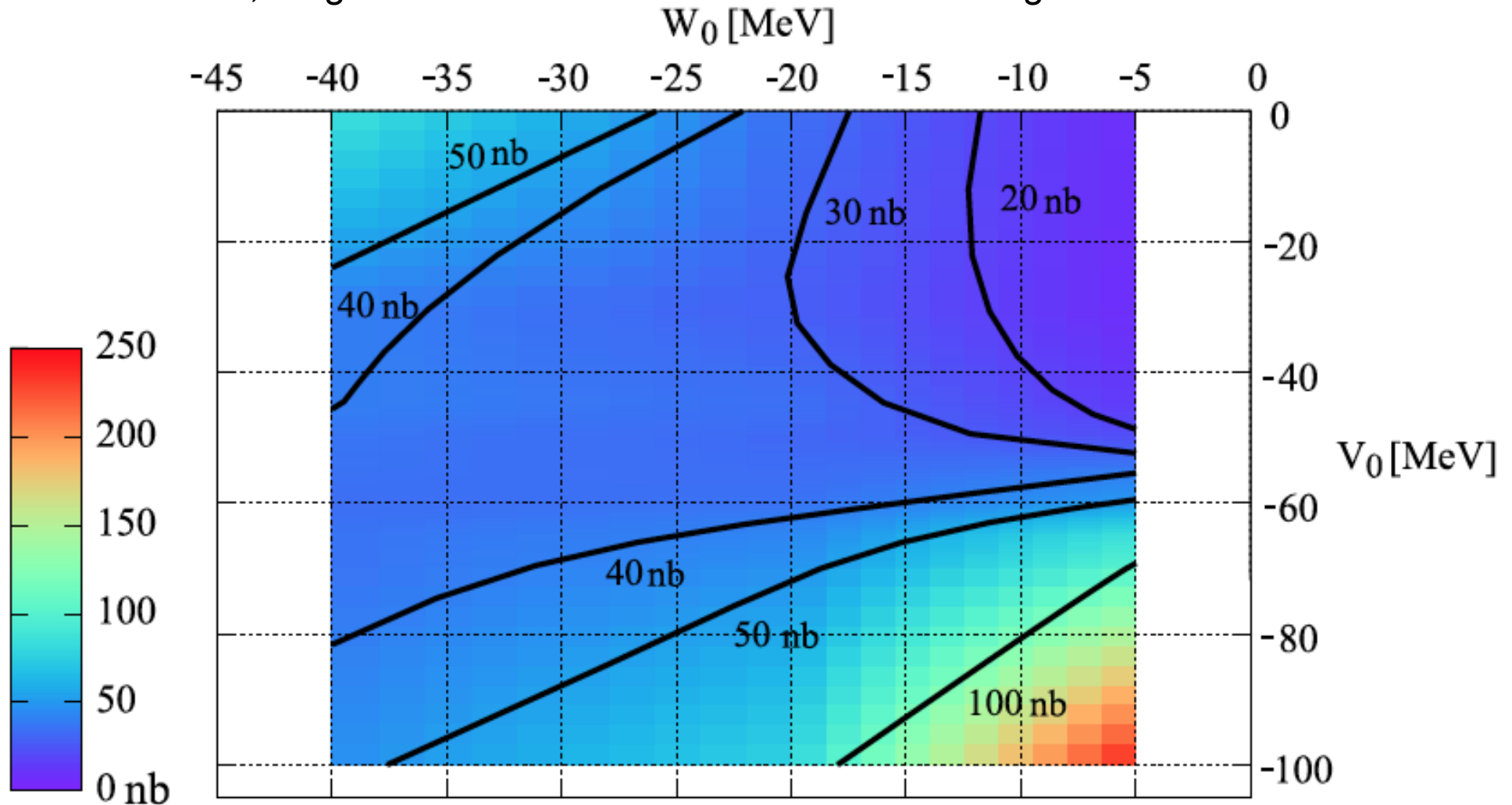
(Structure less parts are removed)

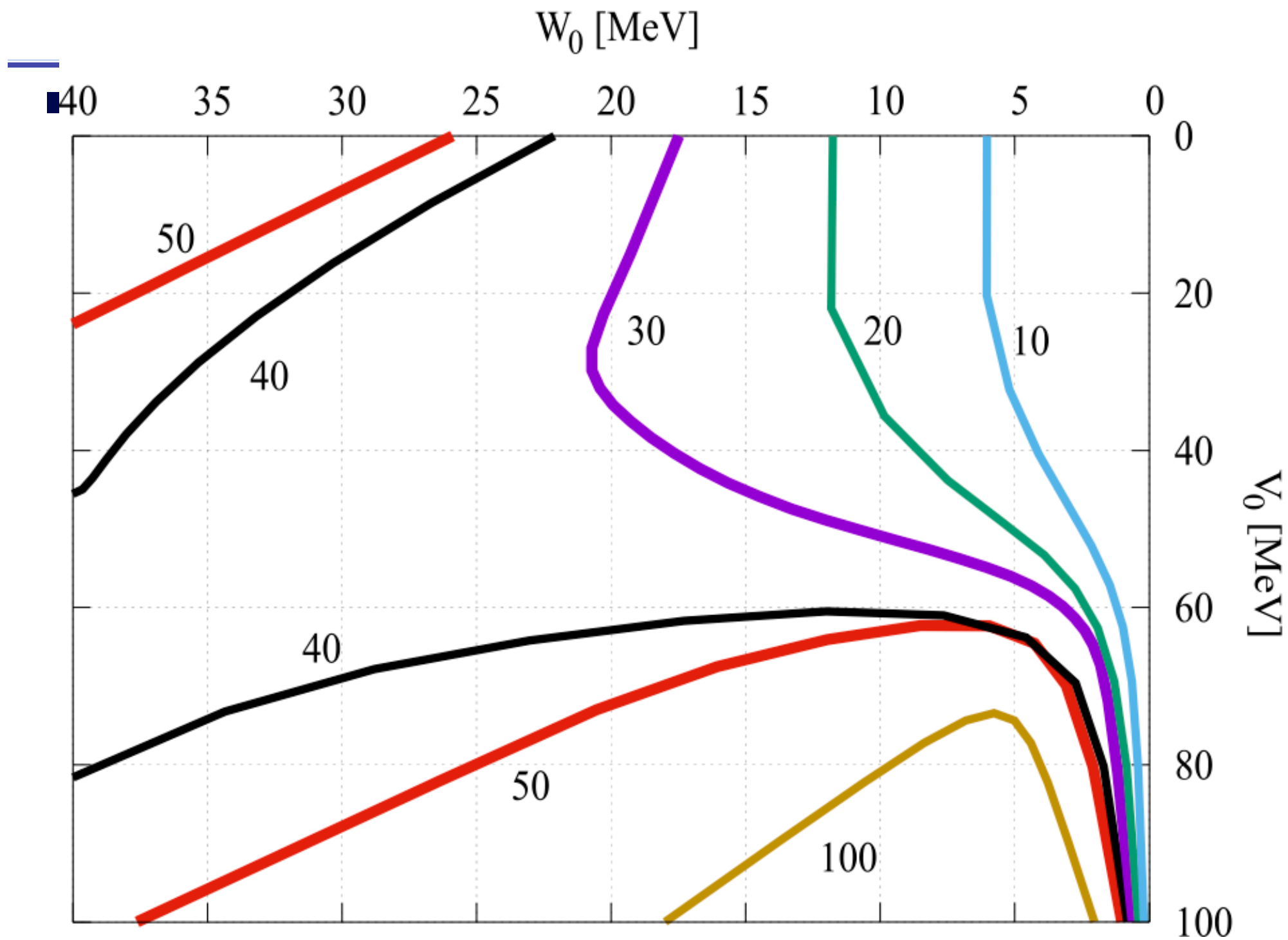
Important numbers here are $8 \sim 15 \text{ nb}$



•Contour plot of 'Peak Hight' of Conversion spectra

- Height=0 on the 'W0=0' axis because of No-conversion
- There are Two Mountains in this contour.
 - (1) Large V0, Small W0 area => High peak due to bound states
 - (2) Small V0, Large W0 area => Some structure due to large conversion effect





Summary for d+d reaction for eta threshold region

- Formation of η mesic nucleus
 - » $d + d \rightarrow ({}^4\text{He}-\eta) \rightarrow N + \pi + 3\text{He}$ reaction
 - » High momentum transfer ($\sim 1\text{GeV}/c$)
 - » η production data above threshold
 - » Simple spectra are expected
- A model with Green's function
 - Provide useful estimation and interpretation of data.

Fit to the real experimental spectra could be necessary.