Time evolution of an unstable quantum system

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Outline

1. General discussion of the decay law
2. Non-exponential decay: experiments
3. Theory: from Lee Hamiltonian to QFT (*and my results*)
4. Decay of a moving particle. Is the usual Einstein-formula correct?

Part 1: General discussion

Exponential decay law

- N_0 : Number of unstable particles at the time t = 0.

$$\boxed{N(t) = N_0 e^{-\Gamma t}} \,, \quad \tau = 1/\Gamma \text{ mean lifetime}$$

Confirmend in countless cases!

- For a single unstable particle:

$$\boxed{p(t) = e^{-\Gamma t}}$$

is the survival probability for a single unstable particle created at t=0. (Intrinsic probabilty, see Schrödinger´s cat).

For small times: $p(t) = 1 - \Gamma t + \ldots$

Basic definitions

Let $|S\rangle$ be an unstable state prepared at $t = 0$.

Survival probabilty ammplitude at $t > 0$:

$$a(t) = \langle S | e^{-iHt} | S \rangle$$

Survival probability: $p(t) = |a(t)|^2$

Rep. Prog. Phys., Vol. 41, 1978. Printed in Great Britain

Decay theory of unstable quantum systems

L FONDA, G C GHIRARDI and A RIMINI

Deviations from the exp. law at short times

Taylor expansion of the amplitude:

$$a(t) = \langle S|e^{-iHt}|S\rangle = 1 - it\langle S|H|S\rangle - \frac{t^2}{2}\langle S|H^2|S\rangle + \ldots$$

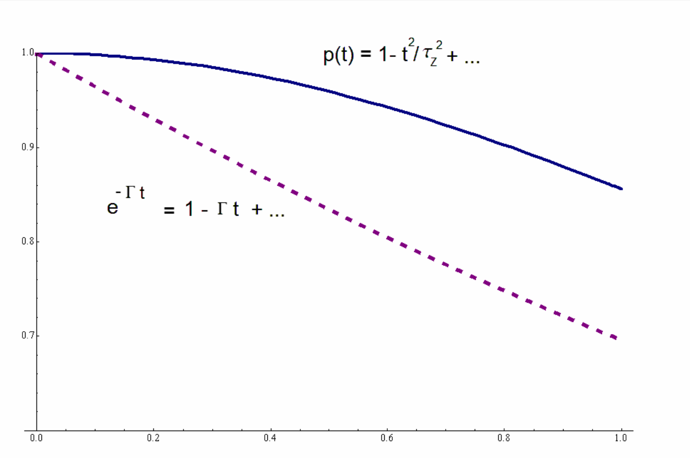
$$a^*(t) = \langle S|e^{-iHt}|S\rangle = 1 + it\langle S|H|S\rangle - \frac{t^2}{2}\langle S|H^2|S\rangle + \ldots$$

It follows:

$$\boxed{\mathrm{p(t)} = |\mathrm{a(t)}|^2 = \mathrm{a}^*(\mathrm{t})\mathrm{a(t)} = 1 - \mathrm{t}^2\left(\langle \mathrm{S}|\mathrm{H}^2|\mathrm{S}\rangle - \langle \mathrm{S}|\mathrm{H}|\mathrm{S}\rangle^2\right) + \ldots = 1 - \frac{\mathrm{t}^2}{\tau_Z^2} + \ldots}$$

p(t) decreases quadratically (not linearly);
no exp. decay for short times.
τ_Z is the `Zeno time´.

where $\tau_Z = \dfrac{1}{\sqrt{\langle \mathrm{S}|\mathrm{H}^2|\mathrm{S}\rangle - \langle \mathrm{S}|\mathrm{H}|\mathrm{S}\rangle^2}}$.



Note: the quadratic behavior holds
for any quantum transition, not only for decays.
It is an absolutely general property.

Time evolution and energy distribution (1)

The unstable state $|S\rangle$ is not an eigenstate of the Hamiltonian H.

Let $d_S(E)$ be the energy distribution of the unstable state $|S\rangle$.

Normalization holds: $\int_{-\infty}^{+\infty} d_S(E)dE = 1$

$$\boxed{a(t) = \int_{-\infty}^{+\infty} d_S(E) e^{-iEt} dE}$$

In stable limit : $d_S(E) = \delta(E-M) \rightarrow a(t) = e^{-iMt} \rightarrow p(t) = 1$

Time evoluition and energy distribution (2)

Breit-Wigner distribution:

$$d_S(E) = \frac{\Gamma}{2\pi} \frac{1}{(E-M)^2 + \Gamma^2/4} \rightarrow a(t) = e^{-iM_0 t - \Gamma t/2} \rightarrow p(t) = e^{-\Gamma t}.$$

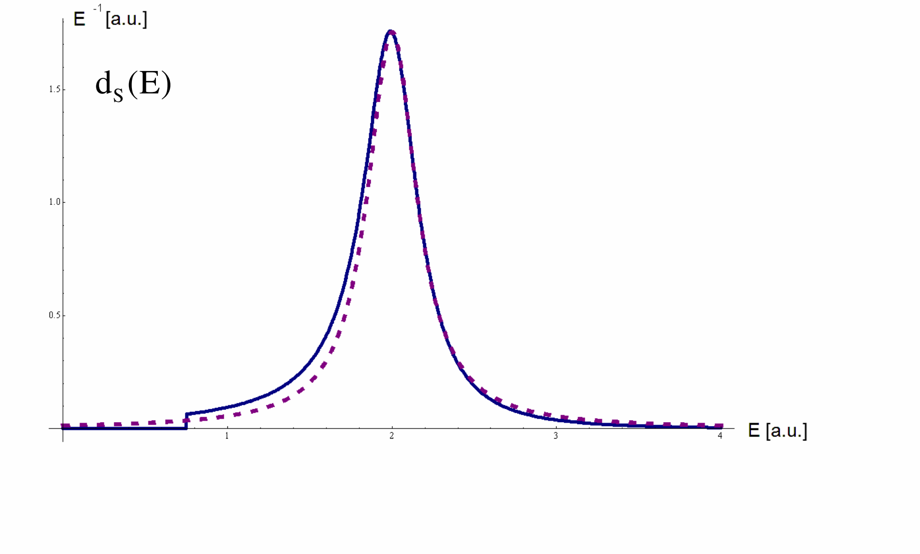
The Breit-Wigner energy distribution cannot be exact.

Two physical conditions for a realistic $d_S(E)$ are:

1) Minimal energy: $d_S(E) = 0 \text{ for } E < E_{\min}$

2) Mean energy finite: $\langle \mathrm{E} \rangle = \int_{-\infty}^{+\infty} d_S(E) E dE = \int_{E_{\min}}^{+\infty} d_S(E) E dE < \infty$

A very simple numerical example

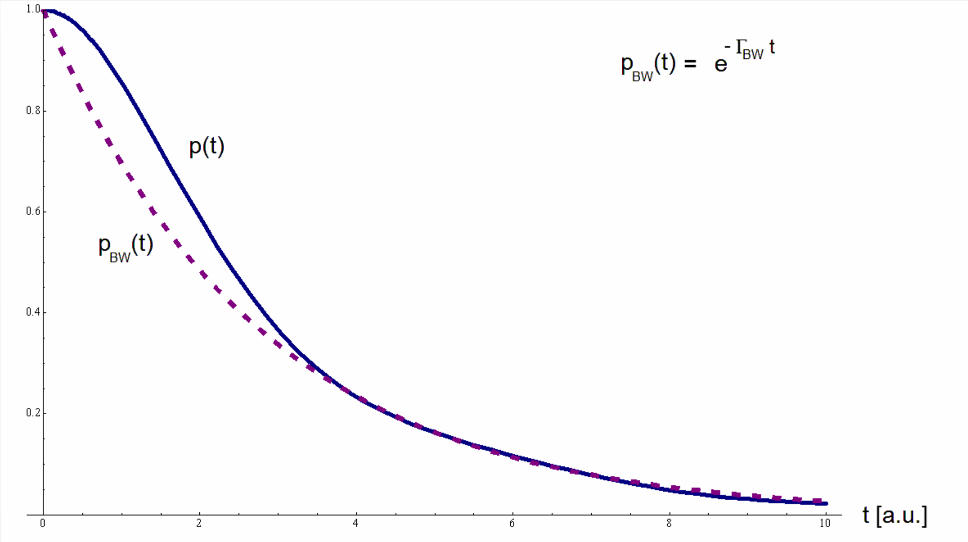


$$M_0 = 2;\ E_{\min} = 0.75;\ \ \Gamma = 0.4; \Lambda = 3$$

$$d_S(E) = N_0 \frac{\Gamma}{2\pi} \frac{e^{-(E^2 - E_0^2)/\Lambda^2}\theta(E - E_{\min})}{(E - M_0)^2 + \Gamma^2/4}$$

$$d_{BW}(E) = \frac{\Gamma_{BW}}{2\pi} \frac{1}{(E - M_0)^2 + \Gamma_{BW}{}^2/4}$$

Γ_{BW}, such that $d_{BW}(M_0) = d_S(M_0)$



$$a(t) = \int_{-\infty}^{+\infty} d_S(E) e^{-iEt} dE\ ; \quad p(t) = |a(t)|^2$$

$$p_{BW}(t) = e^{-\Gamma_{BW} t}$$

The quantum Zeno effect

We perform N inst. measurements:

the first one at time $t = t_0$, the second at time $t = 2t_0$, ..., the N-th at time $T = Nt_0$.

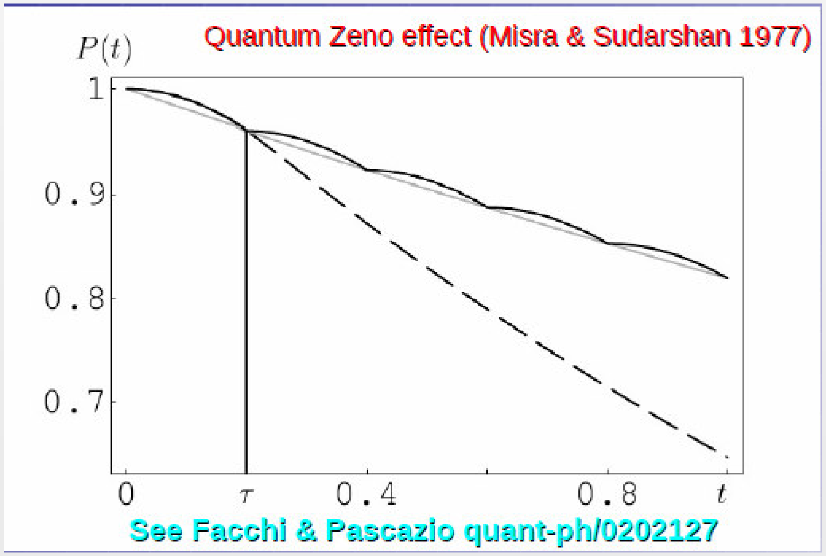
$$p_{\text{after N measurements}} = p(t_0)^N \approx \left(1 - \frac{t_0^2}{\tau_Z^2}\right)^N = \left(1 - \frac{T^2}{N^2\tau_Z^2}\right)^N$$

under the assumption that t_0 is small enough.

If $N \gg 1$ (at fixed T): $p_{\text{after N measurements}} \approx e^{-\frac{T^2}{N\tau_Z^2}} \approx 1.$

For large but finite N :

$\rightarrow$ slowing down of the decay.



Part 2: Experimental evidence of non-exponential decay

Experimental confirmation of non-exponential decays (1)

NATURE | VOL 387 | 5 JUNE 1997

Experimental evidence for non-exponential decay in quantum tunnelling

Steven R. Wilkinson, Cyrus F. Bharucha, Martin C. Fischer, Kirk W. Madison, Patrick R. Morrow, Qian Niu, Bala Sundaram* & Mark G. Raizen

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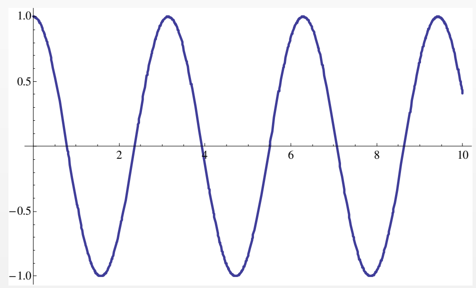
An exponential decay law is the universal hallmark of unstable systems and is observed in all fields of science. This law is not, however, fully consistent with quantum mechanics and deviations from exponential decay have been predicted for short as well as long times[1–8]. Such deviations have not hitherto been observed experimentally. Here we present experimental evidence for short-time deviation from exponential decay in a quantum tunnelling experiment. Our system consists of ultra-cold sodium atoms that are trapped in an accelerating periodic optical potential created by a standing wave of light. Atoms can escape the wells by quantum tunnelling, and the number that remain can be measured as a function of interaction time for a fixed value of the well depth and acceleration. We observe that for short times the survival probability is initially constant before developing the characteristics of exponential decay. The conceptual simplicity of the experiment enables a detailed comparison with theoretical predictions.

Cold Na atoms in a optical potential

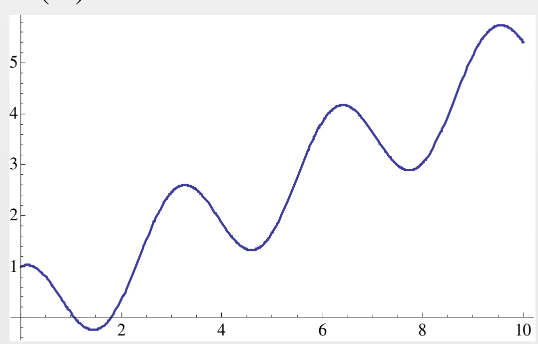
$V(x,0)$



x [a.u.]

$$V(x,t) = V_0 \cos\left(2k_L x - k_L a t^2\right)$$

$U(x')$



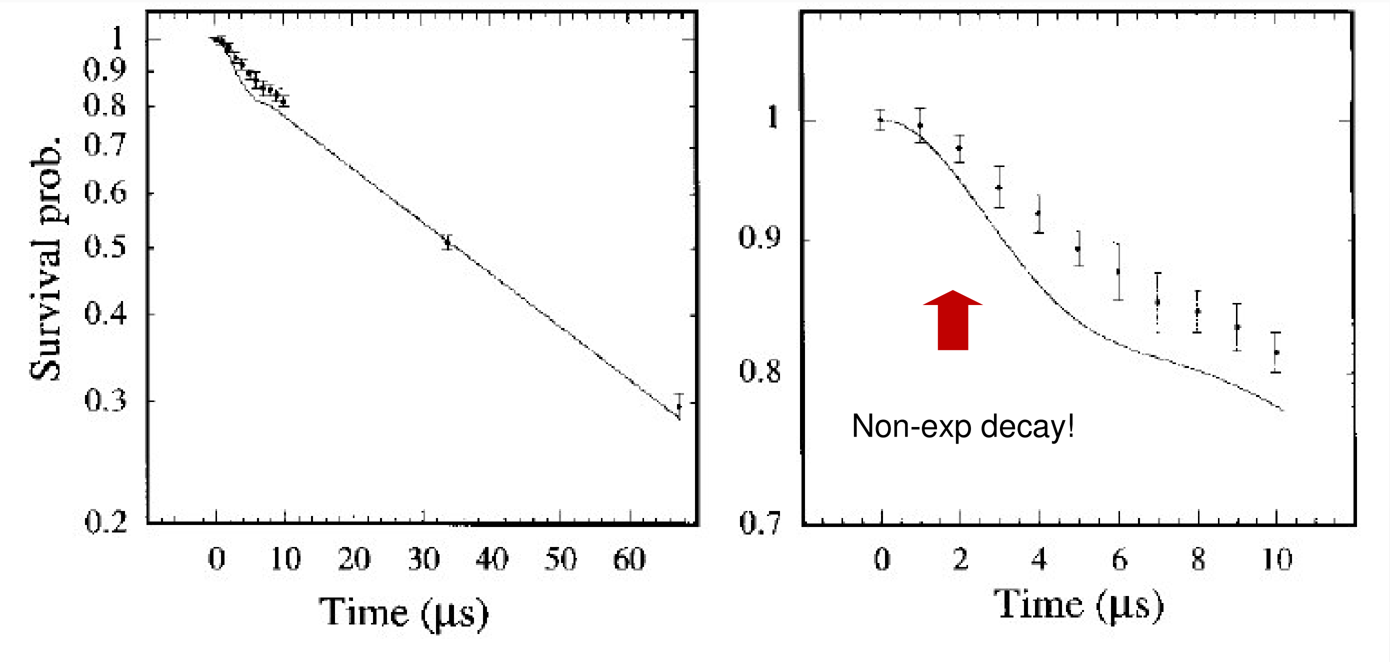
x' [a.u.]

$$x' = x - \frac{1}{2} a t^2$$

$$U(x') = V_0 \cos\left(2k_L x'\right) + Max'$$

Experimental confirmation of non-exponential decays (2)

Measured survival probabilty p(t)



Experimental confirmation of non-exponential decays and Zeno /Anti-Zeno effects

VOLUME 87, NUMBER 4 PHYSICAL REVIEW LETTERS 23 JULY 2001

Observation of the Quantum Zeno and Anti-Zeno Effects in an Unstable System

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(Received 30 March 2001; published 10 July 2001)

We report the first observation of the quantum Zeno and anti-Zeno effects in an unstable system. Cold sodium atoms are trapped in a far-detuned standing wave of light that is accelerated for a controlled duration. For a large acceleration the atoms can escape the trapping potential via tunneling. Initially the number of trapped atoms shows strong nonexponential decay features, evolving into the characteristic exponential decay behavior. We repeatedly measure the number of atoms remaining trapped during the initial period of nonexponential decay. Depending on the frequency of measurements we observe a decay that is suppressed or enhanced as compared to the unperturbed system.

Same exp. setup,
but with measurements in between

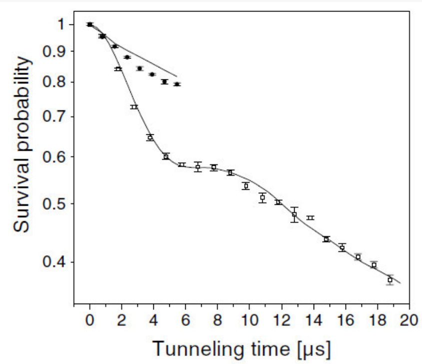


FIG. 3. Probability of survival in the accelerated potential as a function of duration of the tunneling acceleration. The hollow squares show the noninterrupted sequence, and the solid circles show the sequence with interruptions of 50 μs duration every 1 μs. The error bars denote the error of the mean. The data have been normalized to unity at $t_{\rm tunnel} = 0$ in order to compare with the simulations. The solid lines are quantum mechanical simulations of the experimental sequence with no adjustable parameters. For these data the parameters were $a_{\rm tunnel} = 15\,000$ m/s^2, $a_{\rm interr} = 2000$ m/s^2, $t_{\rm interr} = 50$ μs, and $V_0/h = 91$ kHz, where h is Planck's constant.

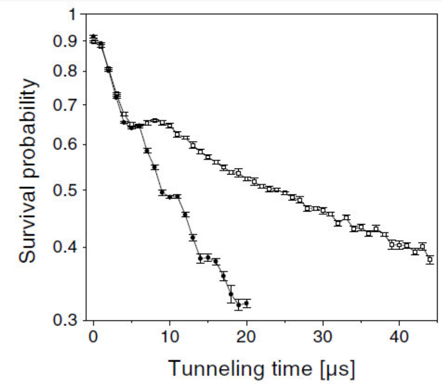


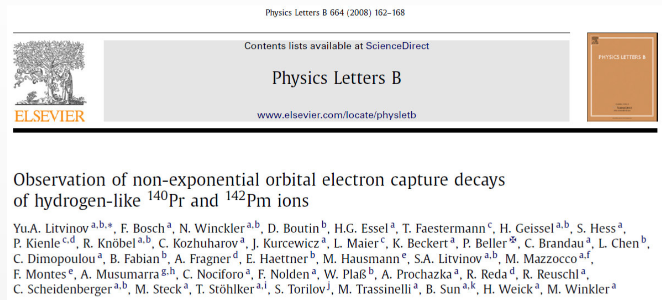
FIG. 4. Survival probability as a function of duration of the tunneling acceleration. The hollow squares show the noninterrupted sequence, and the solid circles show the sequence with interruptions of 40 μs duration every 5 μs. The error bars denote the error of the mean. The experimental data points have been connected by solid lines for clarity. For these data the parameters were: $a_{\rm tunnel} = 15\,000$ m/s^2, $a_{\rm interr} = 2800$ m/s^2, $t_{\rm interr} = 40$ μs, and $V_0/h = 116$ kHz.

Zeno effekt

Anti-Zeno effect

GSI oscillations

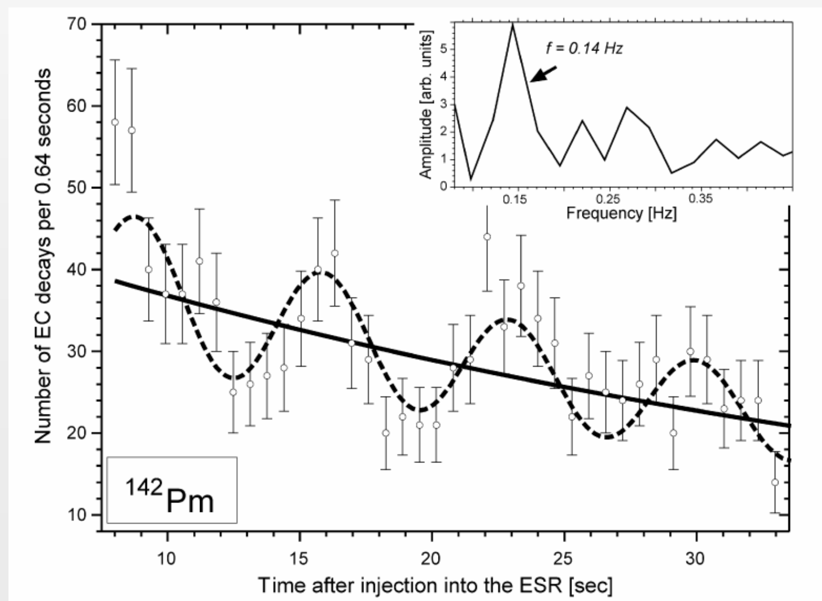
Measurement of weak decays of ions.


Physics Letters B 664 (2008) 162–168
Contents lists available at ScienceDirect
Physics Letters B
www.elsevier.com/locate/physletb

Observation of non-exponential orbital electron capture decays of hydrogen-like ^{140}Pr and ^{142}Pm ions

Yu.A. Litvinov[a,b,*], F. Bosch[a], N. Winckler[a,b], D. Boutin[b], H.G. Essel[a], T. Faestermann[c], H. Geissel[a,b], S. Hess[a], P. Kienle[c,d], R. Knöbel[a,b], C. Kozhuharov[a], J. Kurcewicz[a], L. Maier[c], K. Beckert[a], P. Beller[✠], C. Brandau[a], L. Chen[b], C. Dimopoulou[a], B. Fabian[b], A. Fragner[d], E. Haettner[b], M. Hausmann[e], S.A. Litvinov[a,b], M. Mazzocco[a,f], F. Montes[e], A. Musumarra[g,h], C. Nociforo[a], F. Nolden[a], W. Plaß[b], A. Prochazka[a], R. Reda[d], R. Reuschl[a], C. Scheidenberger[a,b], M. Steck[a], T. Stöhlker[a,i], S. Torilov[j], M. Trassinelli[a], B. Sun[a,k], H. Weick[a], M. Winkler[a]

Decay of H-like Pm into: neutrino + Nd



Measurement was:

$$\frac{dN_{decays}}{dt} \propto -\frac{dp(t)}{dt}$$

Oscillations later confirmed.

arXiv:1309.7294 [nucl-ex]. Explanation still missing!

Late-time deviations

PRL **96,** 163601 (2006) PHYSICAL REVIEW LETTERS week ending 28 APRIL 2006

Violation of the Exponential-Decay Law at Long Times

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(Received 4 July 2005; published 26 April 2006)

First-principles quantum mechanical calculations show that the exponential-decay law for any metastable state is only an approximation and predict an asymptotically algebraic contribution to the decay for sufficiently long times. In this Letter, we measure the luminescence decays of many dissolved organic materials after pulsed laser excitation over more than 20 lifetimes and obtain the first experimental proof of the turnover into the nonexponential decay regime. As theoretically expected, the strength of the nonexponential contributions scales with the energetic width of the excited state density distribution whereas the slope indicates the broadening mechanism.

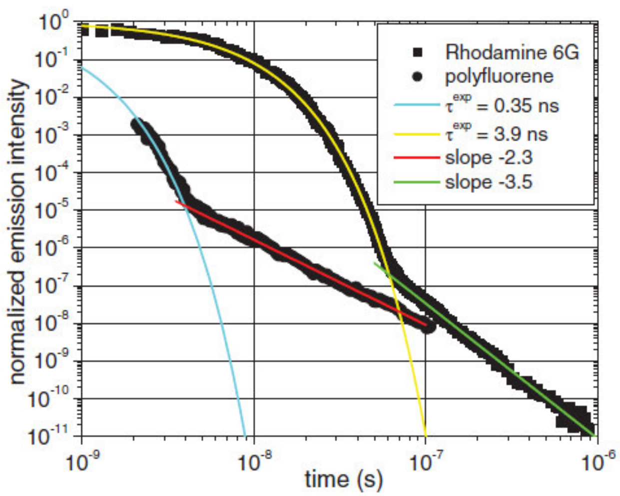


FIG. 2 (color). Corresponding double logarithmic fluorescence decays of the emissions shown in Fig. 1. Exponential and power law regions are indicated by solid lines and the emission intensity at time zero has been normalized.

Confirmation of: L. A. Khalfin. 1957. 1957 (Engl. trans. Zh.Eksp.Teor.Fiz.,33,1371)

Considerations

- No other short- or long-time deviation from the exp. law was seen in unstable states.
- Verification of the two aforementioned works (Reizen + Rothe) would be needed.
- The measurement of deviations in simple natural systems (elementary particles, nuclei, atoms) would be a great achievement.

Part 3: from the Lee Hamiltonian to Quantum Field Theory

Lee Hamiltonian

$$H = H_0 + H_1$$

$$H_0 = M_0 \left| S \right\rangle \left\langle S \right| + \int_{-\infty}^{+\infty} dk\, \omega(k) \left| k \right\rangle \left\langle k \right|$$

$$H_1 = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} dk \left(g \cdot f(k) \right) \left(\left| S \right\rangle \left\langle k \right| + \left| k \right\rangle \left\langle S \right| \right)$$

|S> is the initial unstable state, coupled to an infinity of final states |k>. (Poincare-time is infinite. Irreversible decay). General approach, similar Hamiltonians used in many areas of Physics.

(Ex: Jaynes-Cummings approach)

Example/1: spontaneous emission. |S> represents an atom in the excited state, |k> is the ground-state plus photon.

Example/2: pion decay. |S> represents a neutral pion, |k> represents two photons (flying back-to-back)

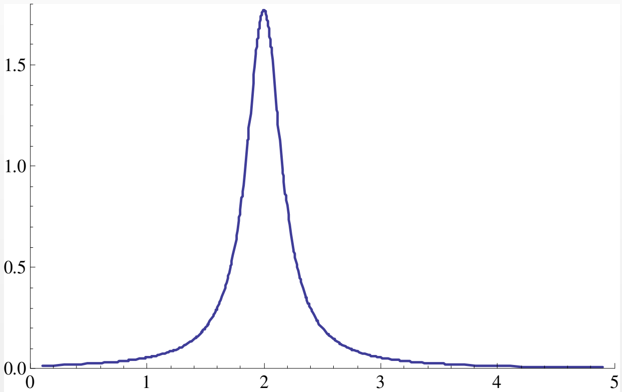
Exponential limit

$$H = H_0 + H_1 \;;\; H_0 = M_0 |S\rangle\langle S| + \int_{-\infty}^{+\infty} dk\,\omega(k) |k\rangle\langle k| \;;\; H_1 = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} dk (g \cdot f(k))(|S\rangle\langle k| + |k\rangle\langle S|)$$

$$\omega(k) = k \;;\quad f(k) = 1 \quad \Rightarrow \quad \Pi(E) = ig^2/2 \;;\; \Gamma = g^2$$

$$d_S(E) = \frac{\Gamma}{2\pi} \frac{1}{(E - M_0)^2 + \Gamma^2/4}$$

$$\Rightarrow a(t) = e^{-i(M_0 - i\Gamma/2)t} \Rightarrow p(t) = e^{-\Gamma t}$$



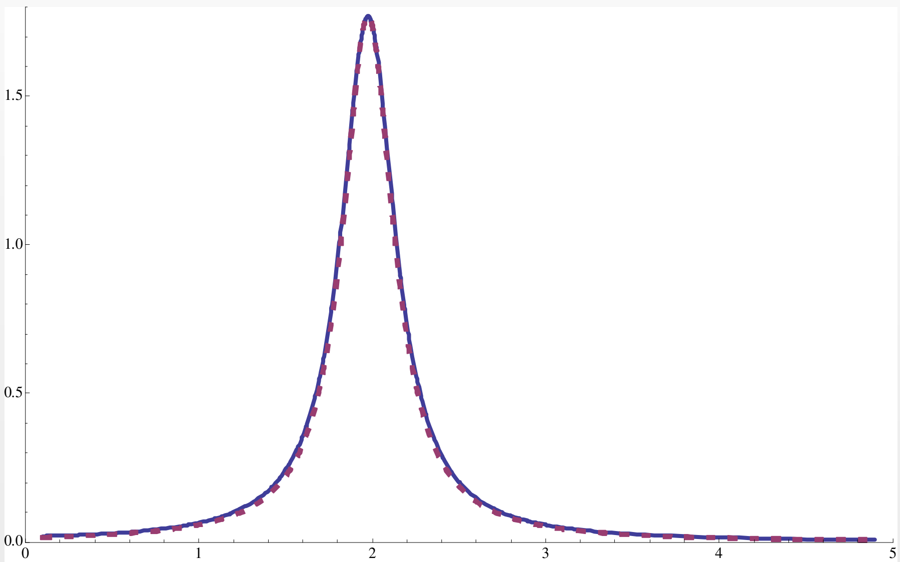
The exponential limit is obtained when the unstable state couples to all the states of the continuum with the same strength

Non-exponential case (1)

$$H_1 = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} dk (g \cdot f(k)) (|S\rangle\langle k| + |k\rangle\langle S|)$$

$$f(k) = \begin{cases} 0 \text{ for } k < E_{min} \\ 1 \text{ for } E_{min} \le k \le E_{max} \\ 0 \text{ for } k > E_{max} \end{cases}$$

$d_S(E)$

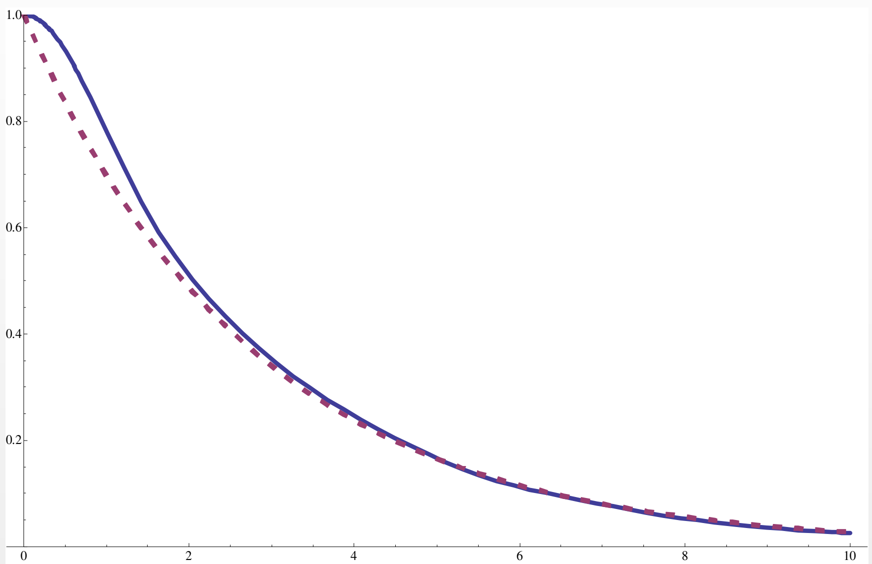


$M_0 = 2;\ E_{min} = 0;\ E_{max} = 5;\ g^2 = 0.36$ (all in a.u. of energy) E

This is what I have said at the beginning of the talk, but now “well done”

Non-exponential case (2)

$p(t)$

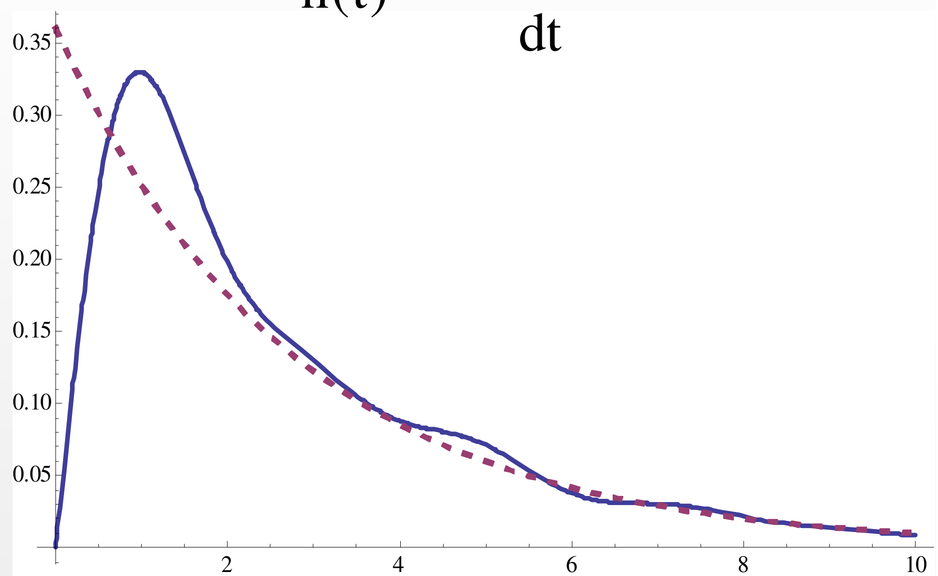


t

Dashed: $p_{BW}(t) = e^{-\Gamma t}$ with $\Gamma = \mathrm{Im}[\Pi(M)]/2$

$h(t)$

$$h(t) = -\frac{dp(t)}{dt}$$



t

Dashed: $h_{BW}(t) = \Gamma e^{-\Gamma t}$ with $\Gamma = \mathrm{Im}[\Pi(M)]/2$

$$\int_0^t h(u)du = 1 - p(t)$$

Namley: $h(t)dt = p(t) - p(t+dt)$ is the probability that the particles decays between t and t+dt

Presentation of 3 results

obtained within this theoretical framework.

Prediction 1 / Two-channel case (a)

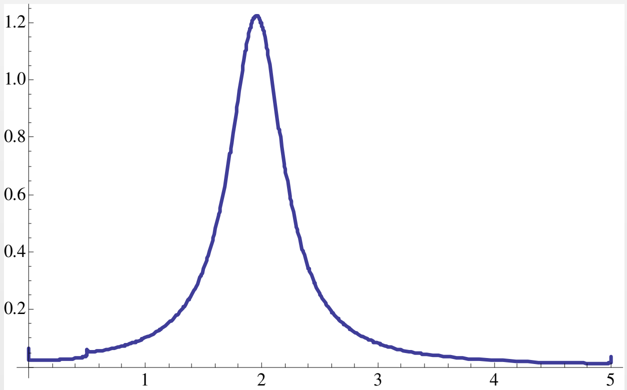
Found Phys (2012) 42:1262–1299
DOI 10.1007/s10701-012-9667-3

Non-exponential Decay in Quantum Field Theory and in Quantum Mechanics: The Case of Two (or More) Decay Channels

Francesco Giacosa

$$H_1 = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty} dk(g_1 \cdot f_1(k))(|S\rangle\langle k,1| + |k,1\rangle\langle S|) + \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty} dk(g_2 \cdot f_2(k))(|S\rangle\langle k,2| + |k,2\rangle\langle S|)$$

$$f_i(k) = \begin{cases} 0 \text{ for } k < E_{i,min} \\ 1 \text{ for } E_{i,min} \leq k \leq E_{i,max} \\ 0 \text{ for } k > E_{i,max} \end{cases}$$



$$M_0 = 2;\ E_{1,min} = 0;\ E_{2,min} = 0\ ;\ E_{1,max} = E_{2,max} = 5;$$

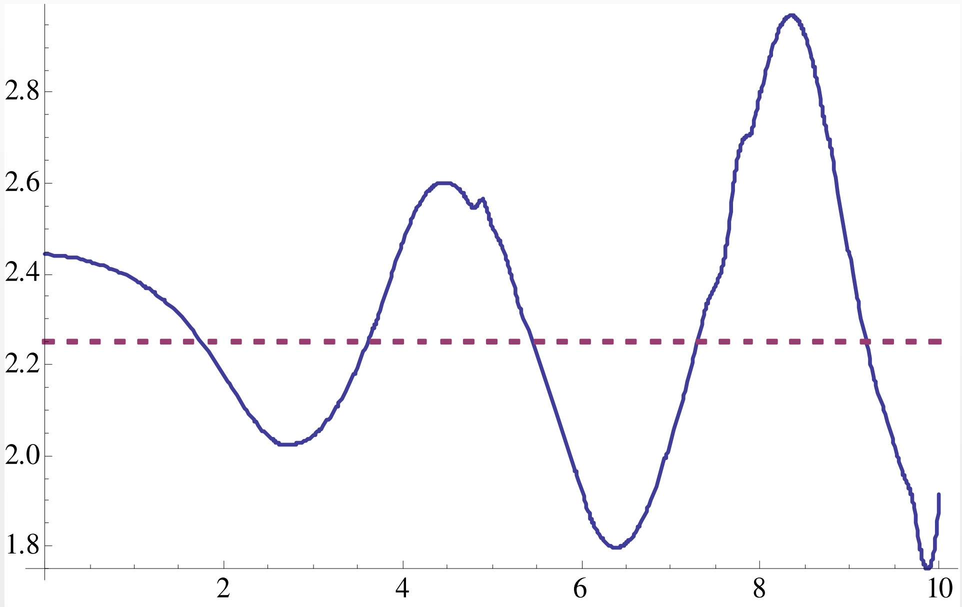
$$g_1^2 = 0.36\ ;\ \ g_2^2 = 0.16 \quad \text{(all in a.u. of energy)}$$

Prediction 1/ Two-channel case (b)

$h_1(t)dt =$ probabilty that the state $|S\rangle$ decays in the first channel between (t,t+dt)

$h_2(t)dt =$ probabilty that the state $|S\rangle$ decays in the second channel between (t,t+dt)

$$\frac{h_1(t)}{h_2(t)}$$



t

Dashed: $\frac{h_{1,BW}(t)}{h_{2,BW}(t)} = \frac{\Gamma_1}{\Gamma_2} = \text{const}$

Measurable effect???

Details in:

F. G., Non-exponential decay in quantum field theory and in quantum mechanics: the case of two (or more) decay channels, Found. Phys. 42 (2012) 1262 [arXiv:1110.5923].

Prediction 2 /
Exponential limit and final state spectrum (a)

PHYSICAL REVIEW A 88, 052131 (2013)

Energy uncertainty of the final state of a decay process

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(Received 22 May 2013; revised manuscript received 17 July 2013; published 27 November 2013)

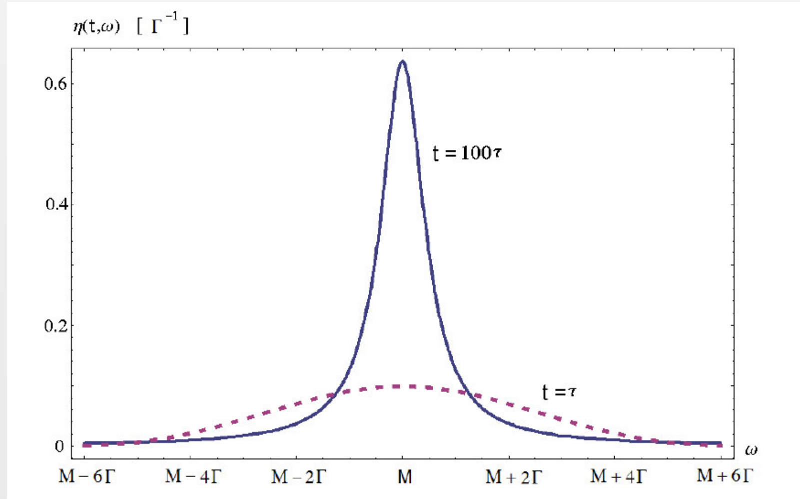
$\left|\left\langle k\right|e^{-iHt}\left|S\right\rangle\right|^2$ is the prob. that $\left|S\right\rangle$ transforms into $\left|k\right\rangle$

Translating into energy:

$$\eta(t,\omega)=\frac{\Gamma}{2\pi}\left|\frac{e^{-i\omega t}-e^{-i(M_0-i\Gamma/2)t}}{E-M_0+i\Gamma/2}\right|^2 \quad ;$$

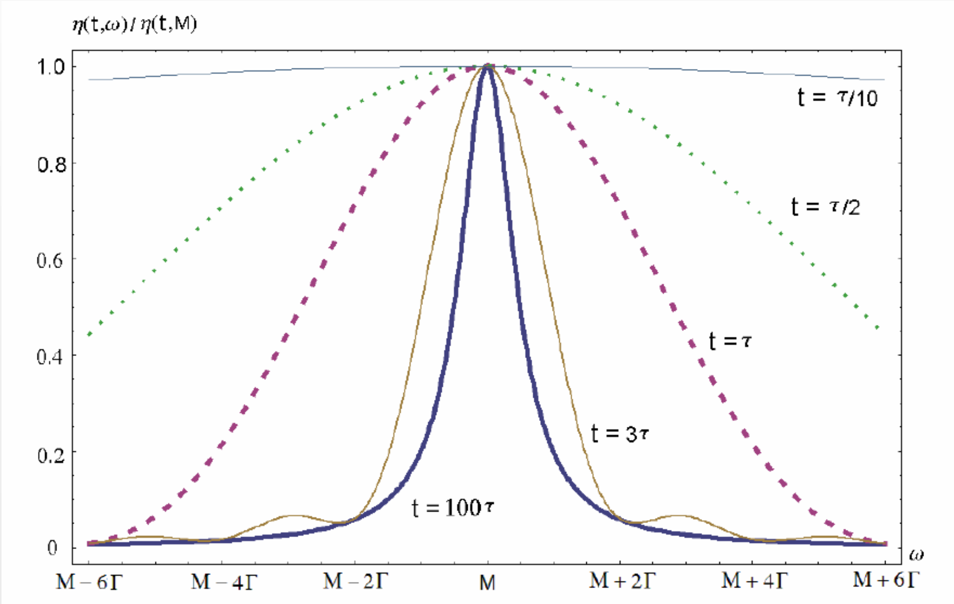
In spont. emission:

$\eta(t,\omega)d\omega$ is the prob. that the outgoing photon has an energy between ω and $\omega+d\omega$



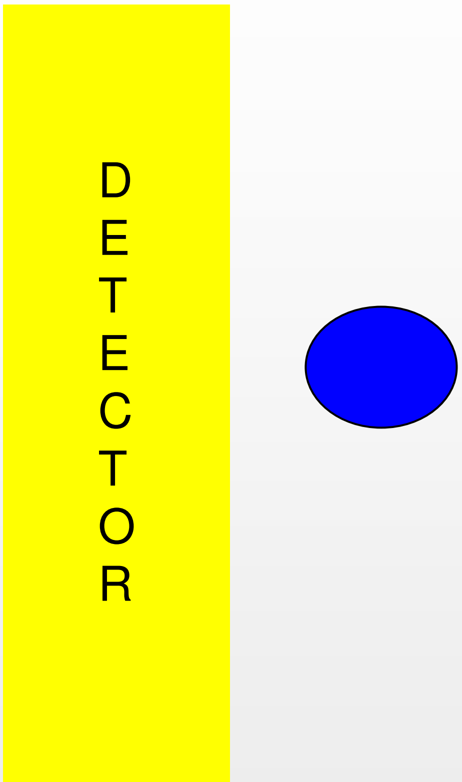
Prediction 2/
Exponential limit and final state spectrum (b)

$$\eta(t,\omega) = \frac{\Gamma}{2\pi}\left|\frac{e^{-i\omega t} - e^{-i(M_0 - i\Gamma/2)t}}{E - M_0 + i\Gamma/2}\right|^2$$



Details in: F. G., arXiv:1305.4467 [quant-ph].

Prediction 3/ The role of an imperfect detector (a)



PHYSICAL REVIEW A **90**, 052107 (2014)

Pulsed and continuous measurements of exponentially decaying systems

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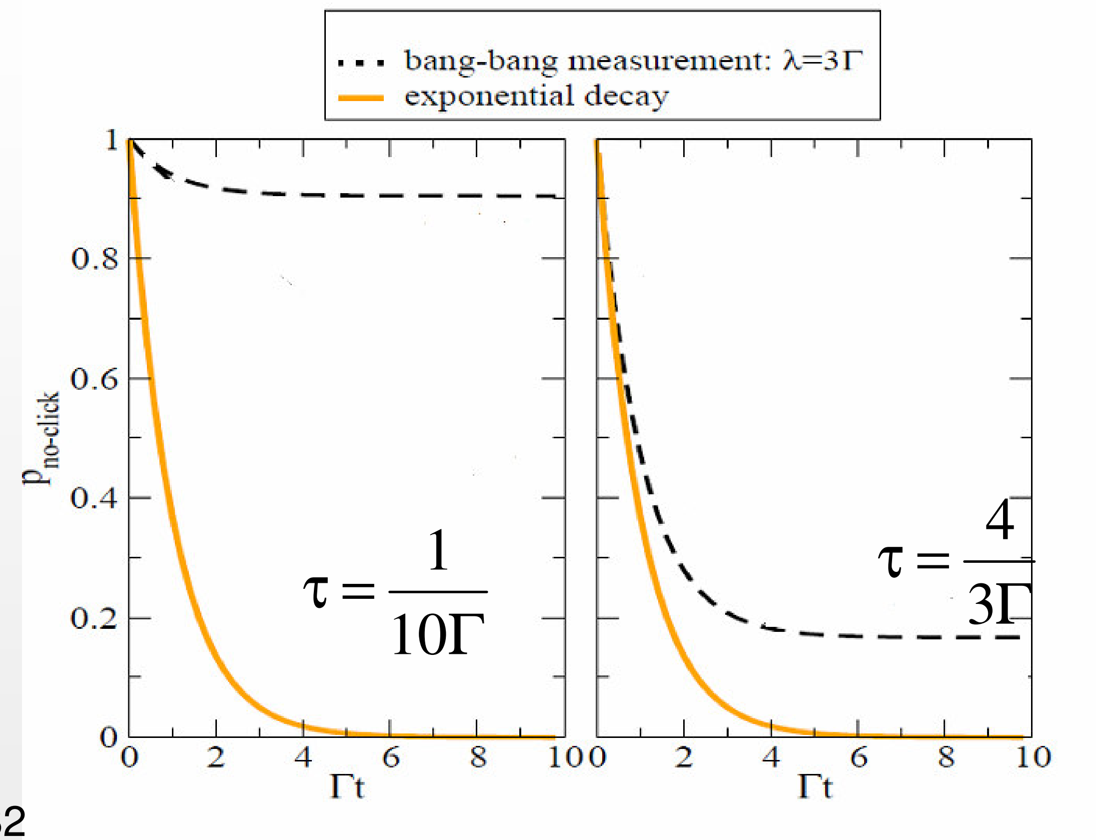
(Received 3 June 2014; published 11 November 2014)

arXiv:1405.6882 [quant-ph].

Detector: It is not perfect. It can measures the emitted photon only in a certain energy range.

Prediction 3/ The role of an imperfect detector (b)



arXiv:1405.6882

Even in the exponential limit for the decay, a QZE can take place due to the interaction with an imperfect detector. However, for that to happen, the 'bang-bang' measurements have to take place very often.

Prediction 3/ The role of an imperfect detector (c)

Let us now assume no collapse, but continuous evolution.
In this case the whole ket is a superposition of all possible outcomes,
In which the detector is now part of the game.

$$e^{-iH_{full}t}\left|S\right\rangle\left|D_0\right\rangle = \left(e^{-iH^C t}\left|S\right\rangle\right)\left|D_0\right\rangle + \int_{-\infty}^{+\infty} \mathrm{dk} \sum_{\alpha} q(k,t)\left|k\right\rangle\left|D_k^{\alpha}\right\rangle$$

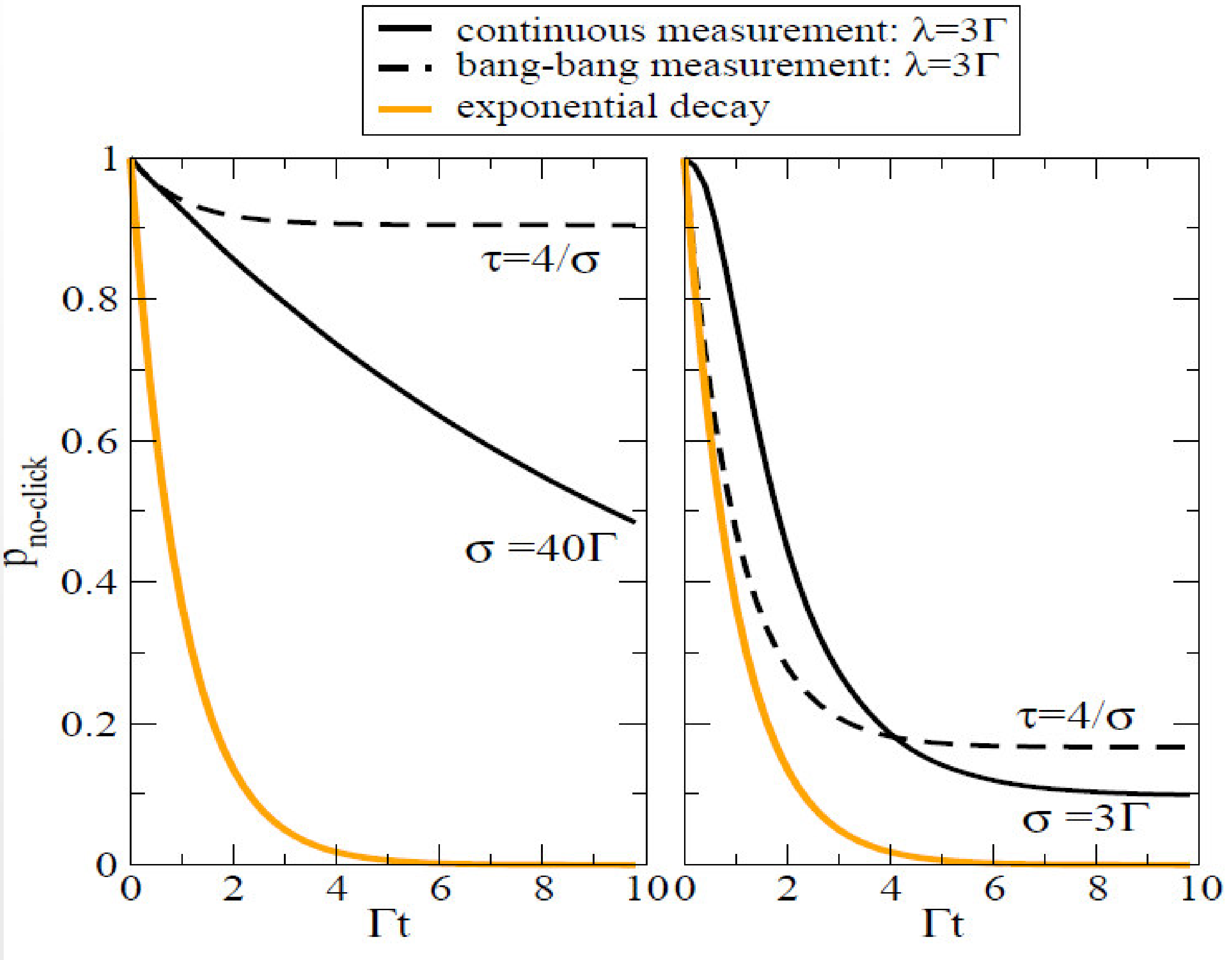
The norm of the ket proportional to D_0 gives us the no-click probability.
At a practical level, the Hamiltonian H^C is non-Hermitian with:

$$\omega(k) = k \rightarrow k - i\sigma(k)/2$$

$$\sigma(k) = \sigma \text{ for } k \subset (-\lambda, \lambda), \text{ 0 otherwise.}$$

Details in: arXiv:1405.6882 [quant-ph].
See alsothe review K. Koshino and A. Shimizu, Phys. Rept. 412 (2005) 191.

Prediction 3: The role of imperfect detector (d)



arXiv:1405.6882

Considerations

For nonexponential decay the QZE applies, but in the exponential limit it doesn‘t.

However, we can have the QZE even in the exponential limit and
pulsed measurements if the detector is not perfect
(i.e., it measures the final state only in a certain range).

A continuous measurement generates also a QZE! But in a different way than the bang-bang case.

What about QFT?

What about QFT? /1
Quantum field theory: textbook treatment

$$d\Gamma = \frac{(2\pi)^4}{2M} |\mathcal{M}|^2 \, \delta(p - k_1 - k_2) \frac{d^3 k_1}{(2\pi)^3 2E_1} \frac{d^3 k_2}{(2\pi)^3 2E_2}$$

see e.g. Peskin-Schroeder ord PDG

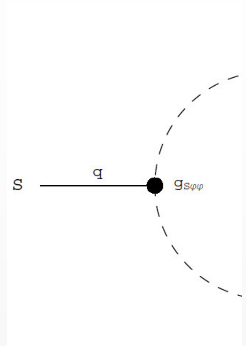
Care is needed:

- An unstable state is not an asymptotic state
- The formula is valid only for $\Gamma \ll M$
- Within this treatment the decay is purely exponential
- One needs to go beyond to study non-exp. decays

What about QFT/2

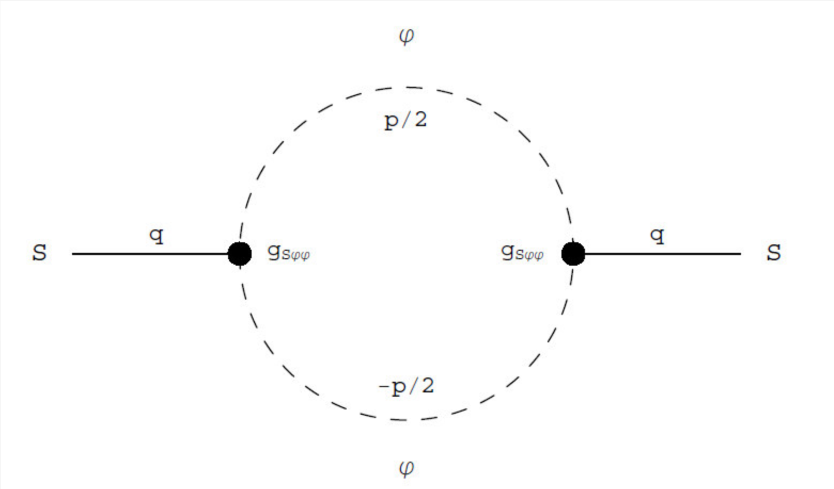
$$L_{int} = gS\varphi^2$$

[g] =[Energy]; QFT super-renorm.



Propagator:

$$\Delta_S(p^2) = \frac{1}{p^2 - M_0^2 + \Pi(p^2) + i\varepsilon}$$



Spectral function (or energy distribution):

$$d_S(m) = \frac{2m}{\pi} \mathrm{Im}[\Delta_S(p^2 = m^2)]$$

Normalization follows authomatically:

$$\int_0^\infty dm d_S(m) = 1$$

F.G. and G. Pagliara, *On the spectral functions of scalar mesons*, Phys. Rev. C 76 (2007) 065204 [arXiv:0707.3594].

What about QFT/3
Survival amplitude and example

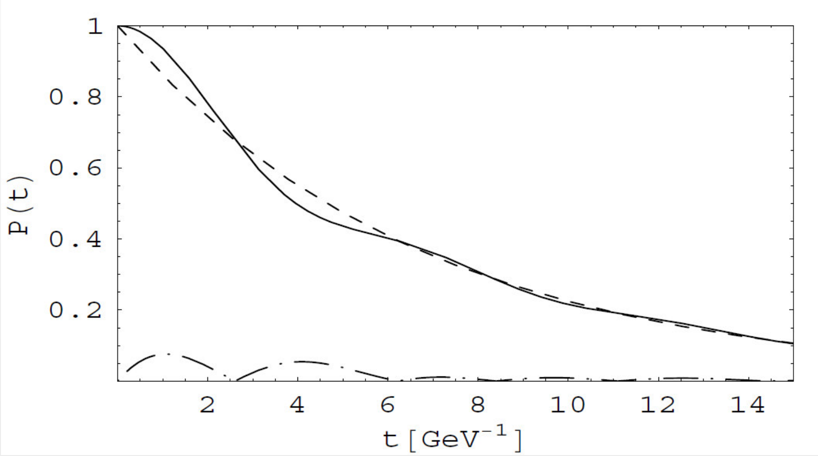
Survival probability amplitude:

$$a(t) = \int_0^\infty dm\, d_S(m) e^{-imt}$$

Just as in QM: non-trivial result!

No dep. on cutoff for a superrenormalizable field theory

Example: p(t) for the ρ meson



[arXiv:1005.4817 [hep-ph]]

In order to show it:
(i) Analogy to Lee models
(ii) Work in the Schroedinger picture in QFT

What about QFT/4
Is there a “maximal energy scale“?

PHYSICAL REVIEW D **88**, 025010 (2013)

Spectral function of a scalar boson coupled to fermions

Francesco Giacosa[1] and Giuseppe Pagliara[2]

[1]*Institut für Theoretische Physik Johann Wolfgang Goethe - Universität, Max von Laue–Strasse 1, D-60438 Frankfurt, Germany*
[2]*Dipartimento di Fisica e Scienze della Terra dell'Università di Ferrara and INFN Sezione di Ferrara, Via Saragat 1, I-44100 Ferrara, Italy*

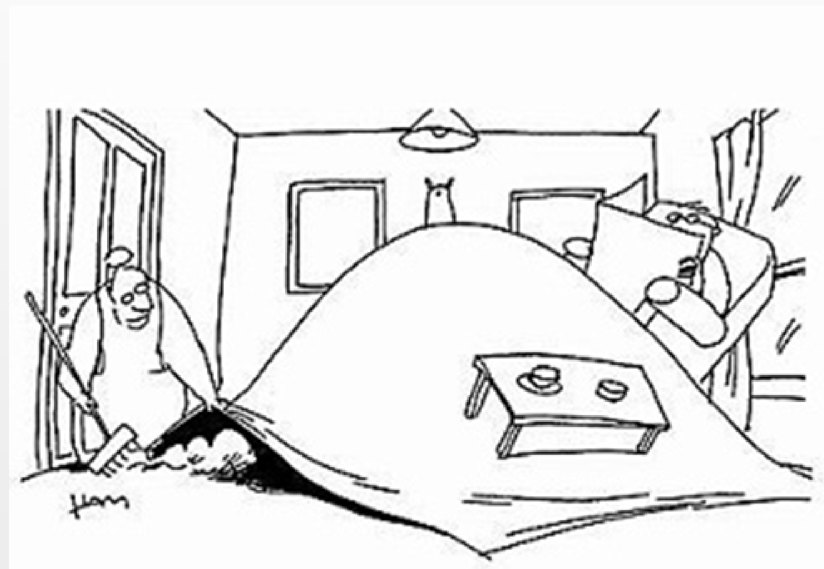
$$\int_0^{\Lambda} d_H(m)dm = 1$$

no matter how large is Λ...

but if one tries to do $\Lambda \to \infty$ one encounters problems: normalization, etc.

$$d_H(m) \propto 1/(m \cdot \ln^2 m) \qquad \text{for large m}$$

Finite outcome: even for a renorm. QFT the existence of a maximal energy scale (i.e., a minimal length) is needed.



Renormalization: sweep dirt under the carpet?

Part 4: Decay of a moving particle

Unstable particle with momentum p

Up to now: in rest frame of the decaying particle. But what if it moves?
Let us consider a momentum translation.

$$|S,p\rangle = U_p \; |S,0\rangle$$

We **expect** in the exponential limit:

$$P_{nd}(t) = e^{-\frac{\Gamma}{\gamma}t} \;,\; \tau = \gamma\Gamma^{-1} \text{ `dilated lifetime'.}$$

Reduction of the decay width

$$\frac{\Gamma}{\gamma} \equiv \frac{\Gamma M}{\sqrt{p^2+M^2}} = \tilde{\Gamma}_p$$

Finite momentum vs finite velocity

- Subtle but important point: in the long-life limit, a particle with definite momentum has also definite velocity.
- $$p = \frac{M}{\sqrt{1-v^2}} v$$
- In general, however, there is a difference! For an unstable state a boost is not equivalent to a momentum translation.
- Here, we consider a definite momentum

$$|S, p\rangle = \int_0^\infty \mathrm{d}m a_S(m) \, |m, p\rangle$$

Unstable particle with momentum p: previous works

L. A. Khalfin, Theory of unstable particles and relativity, PDMI Preprint/1997

M. I. Shirokov, Int. J. Theor. Phys. 43 (2004) 1541.

E. V. Stefanovich, Int. Jour. Theor. Phys, 35 12 (1996)

K. Urbanowski, Phys. Lett. B 737 (2014) 346.

S. A. Alavi and C. Giunti, Europhys. Lett. 109 (2015) 6, 6001

My recent paper:

Vol. 47 (2016) *ACTA PHYSICA POLONICA B* No 9

DECAY LAW AND TIME DILATATION*

Francesco Giacosa

Institute of Physics, Jan Kochanowski University
Świętokrzyska 15, 25-406, Kielce, Poland

Unstable particle with momentum p: unexpected result for the nondecay probability

$$|S,p\rangle = U_p \; |S,0\rangle$$

$$|S,p\rangle = \int_0^\infty dm a_S(m) \, |m,p\rangle$$

The non-decay probability: $P_{nd}(t) = e^{-\Gamma_p t}$

$$\boxed{\Gamma_p = \sqrt{2}\sqrt{\left[\left(M^2 - \frac{\Gamma^2}{4} + p^2\right)^2 + M^2\Gamma^2\right]^{1/2} - \left(M^2 - \frac{\Gamma^2}{4} + p^2\right)}}$$

F. G. arXiv:1512.00232 [hep-ph]

$$\Gamma_p \neq \tilde{\Gamma}_p = \Gamma M / \sqrt{p^2 + M^2}.$$

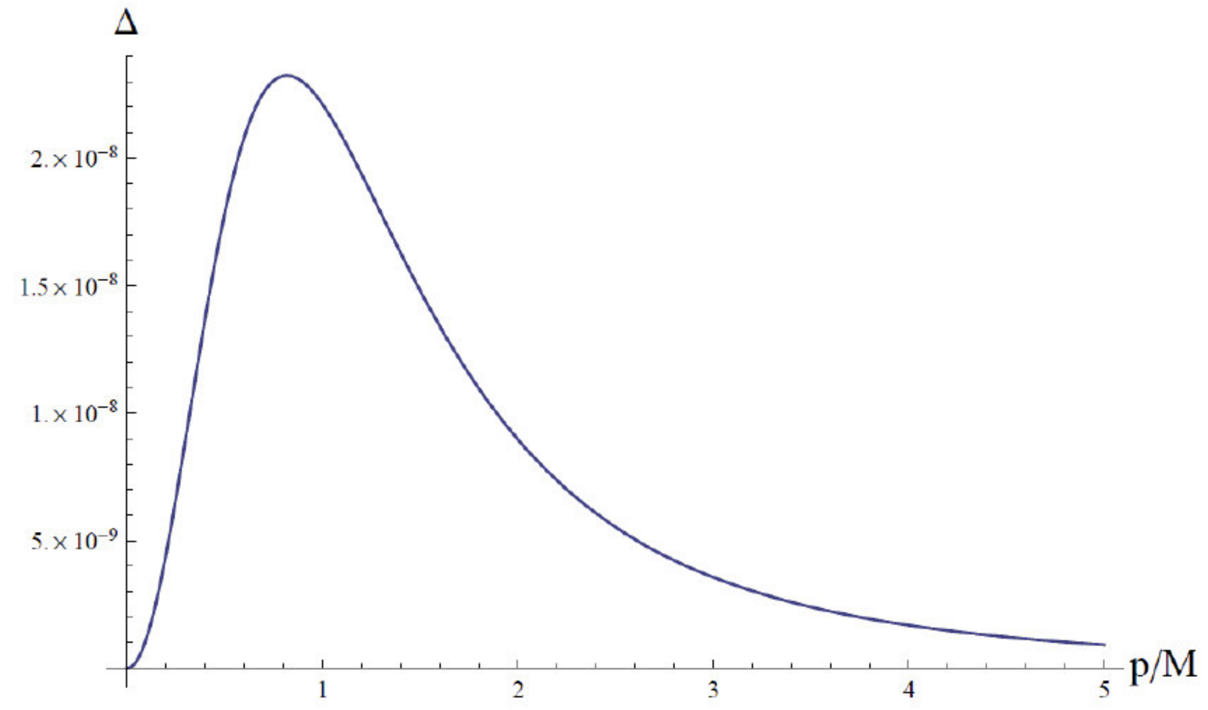
But this is not a breaking of relativity!
It is a different setup.

Unstable particle with momentum p:deviation

$$\Delta = \frac{\Gamma_p - \tilde{\Gamma}_p}{M}$$

$$\Gamma_p = \sqrt{2}\sqrt{\left[\left(M^2 - \frac{\Gamma^2}{4} + p^2\right)^2 + M^2\Gamma^2\right]^{1/2} - \left(M^2 - \frac{\Gamma^2}{4} + p^2\right)}\,.$$

$$\tilde{\Gamma}_p = \frac{\Gamma}{\gamma} \equiv \frac{\Gamma M}{\sqrt{p^2 + M^2}}$$



arXiv:1512.00232 [hep-ph]

$$\frac{p_{\max}}{M} = \sqrt{\frac{2}{3}} \simeq 0.816$$

$$\Delta_{\max} = \frac{\Gamma_{p_{\max}} - \tilde{\Gamma}_{p_{\max}}}{M} \simeq \frac{3}{100}\sqrt{\frac{3}{5}}\left(\frac{\Gamma}{M}\right)^3$$

QFT text-book

Back to QFT. The S-matrix approach

$$d\Gamma = \frac{(2\pi)^4}{2M} \left|\mathcal{M}\right|^2 \delta(p - k_1 - k_2) \frac{d^3k_1}{(2\pi)^3 2E_1} \frac{d^3k_2}{(2\pi)^3 2E_2}$$

is again justified for very small decay width!
Here, the time-dilatation formula holds exactly.

The full QFT proof of the deviation is strictly speaking still missing.
(Technically, the formalism used above is based on so-called Lee Hamiltonians, which are QFT-like, but care is needed).

Unstable particle with momentum p: some examples of deviations

Muon
M = 105.65 MeV
Γ = $2.99 \cdot 10^{-16}$ MeV

$$\Gamma_{p_{\max}} - \tilde{\Gamma}_{p_{\max}} \simeq 5.598 \cdot 10^{-53} \text{ MeV}$$

Neutral pion
M = 134.98 MeV
Γ = $7.72 \cdot 10^{-6}$ MeV

$$\Gamma_{p_{\max}} - \tilde{\Gamma}_{p_{\max}} \simeq 5.81 \cdot 10^{-22} \text{ MeV}$$

Rho meson
M = 775.26 MeV
Γ = 147.8 MeV

$$\Gamma_{p_{\max}} - \tilde{\Gamma}_{p_{\max}} \simeq 0.125 \text{ MeV}$$

Very small deviations!

Boost: state with definite velocity revisited

$$U_v \left| S, 0 \right\rangle \equiv \left| S, v \right\rangle$$

$$\left| S, v \right\rangle = \int_0^\infty dm a_S(m) \sqrt{m} \gamma^{3/2} \left| m, m\gamma v \right\rangle$$

The survival (or better, non-decay) probability vanishes at all time!

$$P_{nd}(t) = 0$$

A boosted muon consists of an electron and two neutrinos!

In reality, wave packets smear the effect.

Details in arXiv:1512.00232 [hep-ph]

Summary...

- The decay is never exponential! This is a fact. This is so both in QM and QFT. Experiments exist, but new ones would be welcome.
- Experimentally seen, but new experiments would be needed
- New interesting effects still to be measured (two-channel case, short-time measurement,role of detector)
- Decay of a moving particle: interesting link between relativity and QM and QFT.
- For a particle with definite momentum p (for the measuring observer) there is a different formula. Numerically, the Einstein expression is very good but is not exact.
- A boost is a very subtle operation in QM and QFT.

…and outlook

Need of a ab initio Quantum Field Theoretical calculation… this is ongoing now. By using the standard technique (interaction picture, …).

Connection to the most fundamental environment for the study of decays.

Thank You

- **When Physicists Attack: Homeless Man Attacks Fellow Transient in Disagreement Over Quantum Physics**
- 1, June 25, 2009 jonathanturley Bizarre, Criminal law, Society
- This week a homeless man in California hit a fellow transient in the face with a skateboard over a disagreement about quantum physics. In San Francisco, Jason Everett Keller, 40, allegedly attacked, Stephan Fava, over a disputed physics question.
- At the time of the attack, Fava was discussing quantum physics with a third homeless man.
- I have been warning for years about the danger of "fighting words" in quantum physics discussions. I confess that I have come close to blows when I hear someone disparage Planck's Action Constant in a bar.

The Zeno's paradox in quantum theory

B. Misra and E. C. G. Sudarshan*

Center for Particle Theory, University of Texas at Austin, Austin, Texas 78712
(Received 24 February 1976)

Analogy to the arrow

Today we speak of Quantum Zeno effect (and not paradox)

Can the cat be saved? Can the cat save its own life?

There is a difference between an infinitely frequent ideal observations and a continuous observation (that was not yet clear in 1976/1977)

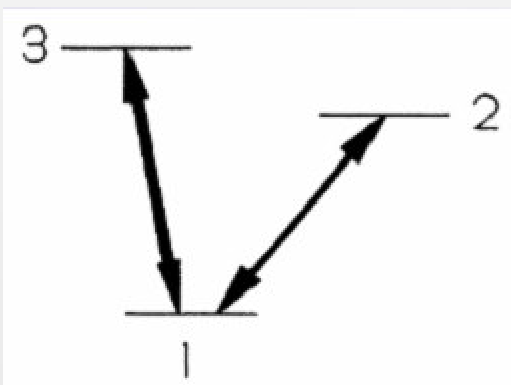
Experimental confirmation of the quantum Zeno effect - Itano et al (1)

PHYSICAL REVIEW A VOLUME 41, NUMBER 5 1 MARCH 1990

Quantum Zeno effect

Wayne M. Itano, D. J. Heinzen, J. J. Bollinger, and D. J. Wineland
Time and Frequency Division, National Institute of Standards and Technology, Boulder, Colorado 80303
(Received 12 October 1989)

The quantum Zeno effect is the inhibition of transitions between quantum states by frequent measurements of the state. The inhibition arises because the measurement causes a collapse (reduction) of the wave function. If the time between measurements is short enough, the wave function usually collapses back to the initial state. We have observed this effect in an rf transition between two $^9Be^+$ ground-state hyperfine levels. The ions were confined in a Penning trap and laser cooled. Short pulses of light, applied at the same time as the rf field, made the measurements. If an ion was in one state, it scattered a few photons; if it was in the other, it scattered no photons. In the latter case the wave-function collapse was due to a null measurement. Good agreement was found with calculations.



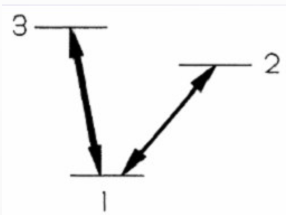
(Undisturbed) survival probability

At $t = 0$, the electron is in $|1\rangle$.

$$p(t) = \cos^2\left(\frac{\Omega t}{2}\right) = 1 - \frac{\Omega^2 t^2}{4} + \ldots$$

$$p(T) = 0 \text{ für } T = \pi/\Omega$$

Experimental confirmation of the quantum Zeno effect - Itano et al (2)



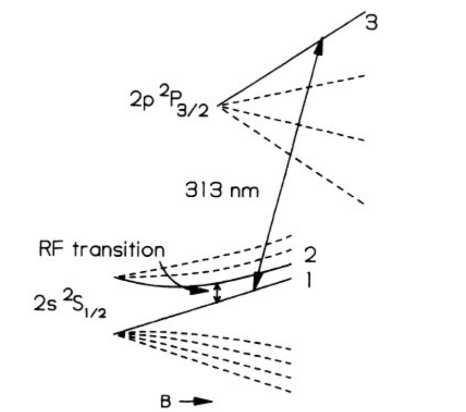


FIG. 2. Diagram of the energy levels of $^9Be^+$ in a magnetic field B. The states labeled 1, 2, and 3 correspond to those in Fig. 1 .

5000 Ions in a Penning trap

Short laser pulses 1-3 work as measurements.

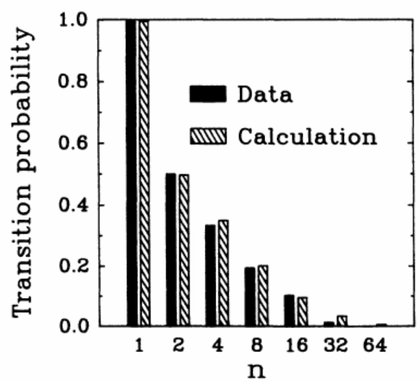


FIG. 3. Graph of the experimental and calculated $1 \to 2$ transition probabilities as a function of the number of measurement pulses n. The decrease of the transition probabilities with increasing n demonstrates the quantum Zeno effect.

$$p(t) = \cos^2(\Omega t/2) = 1 - \frac{\Omega^2 t^2}{4} + \ldots; \quad p(T) = 0 \text{ für } T = \pi/\Omega$$

(Transition probability (without measuring) at time T) : $1 - p(T) = 1$.

Witn n measurements in between the transition probabilty decreases!

The electron stays in state 1.

Other experiments about Zeno/Streed et al

PRL **97,** 260402 (2006) PHYSICAL REVIEW LETTERS week ending 31 DECEMBER 2006

Continuous and Pulsed Quantum Zeno Effect

Erik W. Streed,[1,2] Jongchul Mun,[1] Micah Boyd,[1] Gretchen K. Campbell,[1] Patrick Medley,[1] Wolfgang Ketterle,[1] and David E. Pritchard[1]

[1]*Department of Physics, MIT-Harvard Center for Ultracold Atoms, and Research Laboratory of Electronics, MIT, Cambridge, Massachusetts 02139, USA*

[2]*Centre for Quantum Dynamics, Griffith University, Nathan, QLD 4111, Australia*

(Received 14 June 2006; published 27 December 2006)

Use of BEC (with Rb). QZE confirmed.

The intensity of a continuous observation of a quantum state is equivalent to a certain t0 (Shulman, PRA 57, 1509 (1997)).

Other experiments about Zeno/Haroche

PRL **101,** 180402 (2008) PHYSICAL REVIEW LETTERS week ending 31 OCTOBER 2008

Freezing Coherent Field Growth in a Cavity by the Quantum Zeno Effect

J. Bernu,[1] S. Deléglise,[1] C. Sayrin,[1] S. Kuhr,[1,*] I. Dotsenko,[1,2] M. Brune,[1,+] J. M. Raimond,[1] and S. Haroche[1,2]

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(Received 21 July 2008; published 28 October 2008)

Cavity QED: the nr of photons is frozen.

Another verification of QZE.

Direction QFT.

Quantum Zeno dynamics, Quantum computations, ...

Sudarshan: Seven Science Quests IOP Publishing
Journal of Physics: Conference Series **196** (2009) 012017 doi:10.1088/1742-6596/196/1/012017

Quantum Zeno dynamics and quantum Zeno subspaces

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[2] Dipartimento di Scienze Fisiche, Università di Napoli "Federico II"
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[3] Dipartimento di Fisica, Università di Bari
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PRL **108,** 080501 (2012) PHYSICAL REVIEW LETTERS week ending 24 FEBRUARY 2012

Zeno Effect for Quantum Computation and Control

Gerardo A. Paz-Silva,[1,4] A. T. Rezakhani,[1,4,5] Jason M. Dominy,[1,4] and D. A. Lidar[1,2,3,4]

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[2] *Department of Electrical Engineering, University of Southern California, Los Angeles, California 90089, USA*
[3] *Department of Physics, University of Southern California, Los Angeles, California 90089, USA*
[4] *Center for Quantum Information Science and Technology, University of Southern California, Los Angeles, California 90089, USA*
[5] *Department of Physics, Sharif University of Technology, Tehran, Iran*

(Received 28 April 2011; revised manuscript received 9 February 2012; published 22 February 2012)

C. R. Physique 17 (2016) 685–692

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ELSEVIER

Quantum microwaves / Micro-ondes quantiques

Quantum Zeno dynamics in atoms and cavities

CrossMark

Dynamique de Zénon quantique avec des atomes et des cavités

Sébastien Gleyzes *, Jean-Michel Raimond

Laboratoire Kastler Brossel, Collège de France, CNRS, ENS–PSL Research University, UPMC–Sorbonne Universités, 11, place Marcelin-Berthelot, 75005 Paris, France

Other experiments about Zeno/Balzer

ELSEVIER

1 June 2000

OPTICS COMMUNICATIONS

Optics Communications 180 (2000) 115–120

www.elsevier.com/locate/optcom

The quantum Zeno effect – evolution of an atom impeded by measurement

Chr. Balzer, R. Huesmann, W. Neuhauser, P.E. Toschek *

Institut für Laser-Physik, Universität Hamburg, Jungiusstrasse 9, D-20355 Hamburg, Germany

Received 11 April 2000; accepted 14 April 2000

ELSEVIER

1 October 2002

OPTICS COMMUNICATIONS

Optics Communications 211 (2002) 235–241

www.elsevier.com/locate/optcom

A relaxationless demonstration of the Quantum Zeno paradox on an individual atom

Chr. Balzer, Th. Hannemann, D. Reiß, Chr. Wunderlich, W. Neuhauser, P.E. Toschek *

Institut für Laser-Physik, Universität Hamburg, D-20355 Hamburg, Germany

Received 3 April 2002; received in revised form 5 July 2002; accepted 2 August 2002

Same setup as Itano et al.(different ions are used, YB instead of Be),

But now the measurement takes place between 3 and 2.

Results in agreement with Itano, but here the QZE is associated by a seires of null-measurements.

Propagator and spectral function

$$H = H_0 + H_1 \; ; \; H_0 = M_0 |S\rangle\langle S| + \int_{-\infty}^{+\infty} dk\,\omega(k) |k\rangle\langle k| \; ; \; H_1 = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} dk (g \cdot f(k)) (|S\rangle\langle k| + |k\rangle\langle S|)$$

$$G_S(E) = \left\langle S \left| (E - H + i\varepsilon)^{-1} \right| S \right\rangle = (E - M_0 + \Pi(E) + i\varepsilon)^{-1}$$

$$\Pi(E) = -\int_{-\infty}^{+\infty} \frac{dk}{2\pi} \frac{g^2 f(k)^2}{E - \omega(k) + i\varepsilon}$$

$$d_S(E) = \tfrac{1}{\pi} \operatorname{Im} G_S(E) \; ;$$

$$\boxed{a(t) = \left\langle S \left| e^{-iHt} \right| S \right\rangle = \int_{-\infty}^{+\infty} dE\, d_S(E) e^{-iEt}}$$

It follows:

$$\boxed{\int_{-\infty}^{+\infty} dE\, d_S(E) = 1}$$

Fermi golden rule: $\Gamma = \operatorname{Im}[\Pi(M)]\,/2$.

Unstable particle with momentum p

We work in the exp. limit

M = rest mass; Γ= decay width in the rest frame.

An unstable particle moves with definite momentum p.

Which is its decay width? The stanard expression is:

$$\tilde{\Gamma}_p = \frac{\Gamma}{\gamma} \equiv \frac{\Gamma M}{\sqrt{p^2 + M^2}}$$

Important but sublte point:
in QM and QFT a state with definite momentum has not definite velocity.

Non-decay probability

Straightforward calculation

$$a(t,p)=\frac{1}{\delta(p=0)}\left\langle S,p\left|e^{-iHt}\right|S,p\right\rangle=\int_{-\infty}^{\infty}\mathrm{dm}d_S(m)e^{-i\sqrt{m^2+p^2}t}$$
$$\simeq\int_{-\infty}^{\infty}\mathrm{dm}d_S^{BW}(m)e^{-i\sqrt{m^2+p^2}t}=e^{-i\sqrt{(M-i\Gamma/2)^2+p^2}t}\ .$$

One obtains:

$$P_{nd}(t)=\left|a(t,p)\right|^2=e^{-\Gamma_p t}$$

$$\Gamma_p=2\,\mathrm{Im}\left[\sqrt{(M-i\Gamma/2)^2+p^2}\right]\ .$$

This expression does **not** coincide with the usual Einstein expression!

Decay width of a general state

$$|\Psi\rangle = \int_{-\infty}^{+\infty} dp B(p) \, |S,p\rangle$$

the quantity $\langle \Psi \, | e^{-iHt} | \, \Psi \rangle$ is *not* what we are looking for.

$$P_{nd}(t) = \int_{-\infty}^{+\infty} dp \left| \langle S,p \, | e^{-iHt} | \, \Psi \rangle \right|^2$$

$$P_{nd}(t) = \int_{-\infty}^{+\infty} dp \, |B(p)|^2 \, e^{-\Gamma_p t}$$

Inclusion of spatial wave function is simple. Generalization straightforward.
Details in arXiv:1512.00232.

Boost: state with definite velocity

Point: a velocity translation (i.e. a boost) is not a momentum translation!!!!

$$U_v \left|S,0\right\rangle \equiv \left|S,v\right\rangle$$

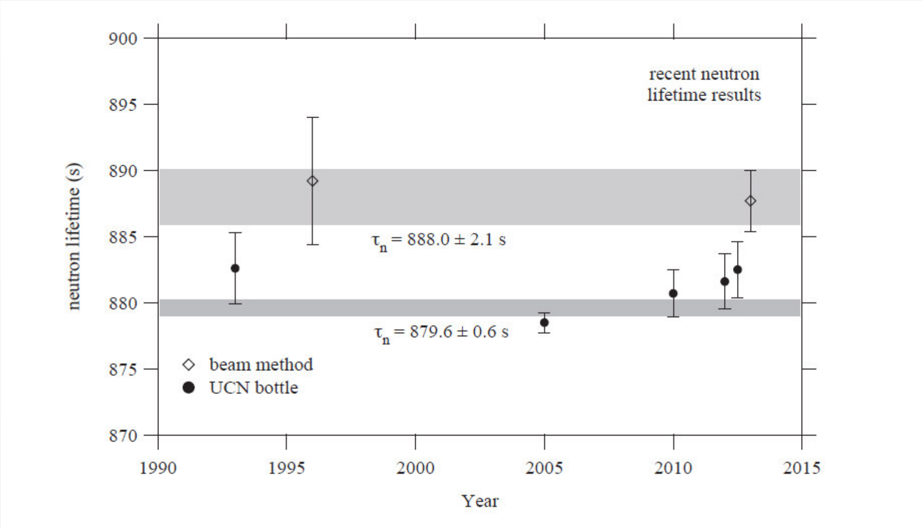
$$\left|\left\langle S,v\left|e^{-iHt}\right|S,v\right\rangle\right|^2 = e^{-i\gamma\Gamma t}$$

The survival probability shows here an absurd Lorentz contraction!

What about QFT? /5
Ongoing work in Kielce

Can we modify the QFT textbook approach (interaction picture) by keeping the time finite? Yes.
This is subject of an ongoing work with S. Mrówczyński.

Goal: neutron.



F. E. Wietfeldt, The neutron lifetime, arXiv: 1411.3687 [nucl-ex]

Prediction 3/ Details

Let us go back to the pure exponential case. So, no QZE should appear.

$$e^{-iHt}\left|S\right\rangle = a(t)\left|S\right\rangle + \int_{-\infty}^{+\infty} dk b(k,t)\left|k\right\rangle$$

$$a(t) = e^{-\Gamma t/2} \text{ , } \Gamma = g^2 \text{ , } b(k,t) = \sqrt{\frac{\Gamma}{2\pi}}\frac{e^{-ikt} - e^{-\Gamma t/2}}{k + i\Gamma/2} \text{ .}$$

$$\boxed{p(t) = \left|a(t)\right|^2 = e^{-\Gamma t}}$$

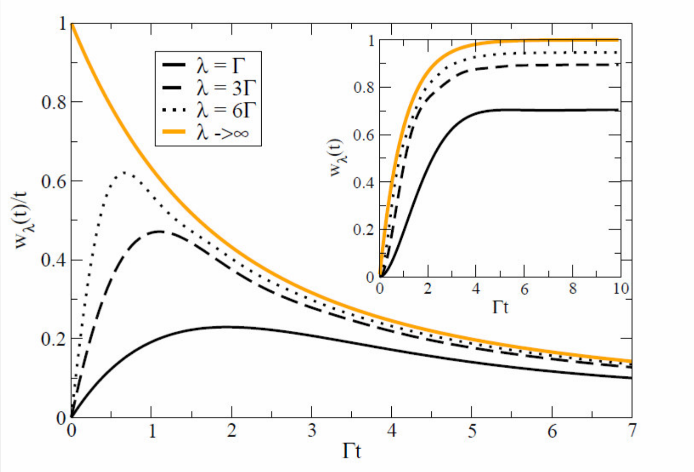
But: our detector can dected only final states in a certain energy range $(-\lambda, \lambda)$.

Following discussion is based on:
F.Giacosa and G. Pagliara,
Pulsed and continuous measurements of exponentially decaying systems,
arXiv:1405.6882 [quant-ph].

Prediction 3/ Details

Suppose that we perform at the time t a measurement if the state is decayed. The probability to hear click is given by:

$$w_\lambda(t) = \int_{-\lambda}^{+\lambda} \mathrm{dk}\, |b(k,t)|^2$$



If, now, we perform N measurements at τ, 2τ, ...,

$$p^{BB}_{\text{no-click}}(t) = 1 - w_\lambda(\tau)\frac{1-e^{-\Gamma t}}{1-e^{-\Gamma\tau}}$$