Dipole-dipole interactions between neutrons

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Colaboration with
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Outline

- Motivation
- Static and frequency-dependent dipole polarizabilities
- neutron-neutron Casimir-Polder interaction
- neutron-Wall and Wall-neutron-Wall interactions
- Summary and outlook

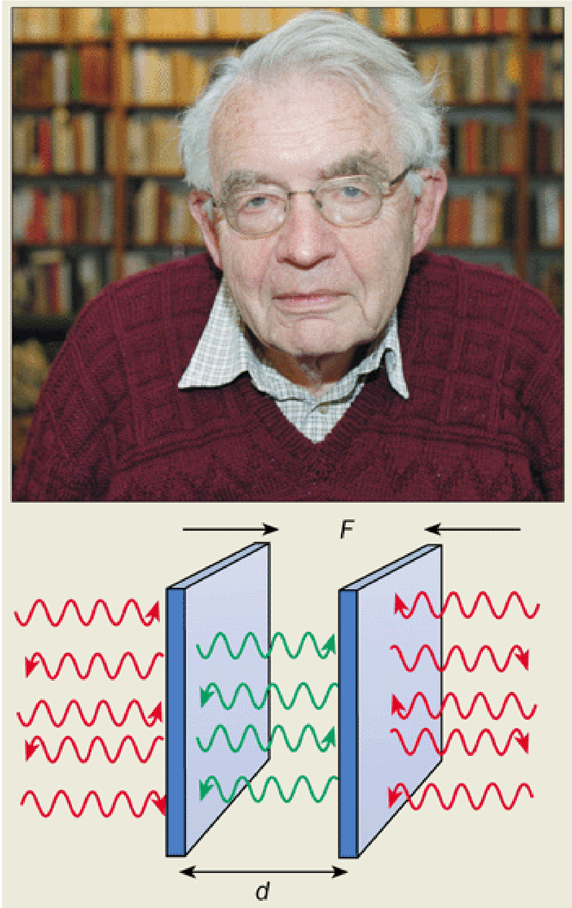
Casimir-Polder interaction between two neutral objects

Phillips Lab., Netherlands

- Overbeek, 1940: suspensions of quartz powder $\Rightarrow$ no van der Waals tail
- Casimir & Polder: $r^{-6} \to r^{-7}$

  (Retardation effects: c finite)
- Niels Bohr: "zero-point energy"

Casimir-Polder interaction between two neutral objects



Casimir

Welton & Weisskopf

Schwinger

. . .

Casimir-Polder interaction between two neutral objects

Feinberg & Sucher, PRA 2, 2395 (1970), Spruch & Kelsey, PRA 18, 845 (1978)

$$V_{CP;ij}(r) = -\frac{\alpha_0}{\pi r^6} I_{ij}(r)\,,$$

$$I_{ij}(r) = \int_0^\infty d\omega\, e^{-2\alpha_0\omega r}\Big\{\Big[\alpha_i(i\omega)\alpha_j(i\omega) + \beta_i(i\omega)\beta_j(i\omega)\Big]P_E(\alpha_0\omega r) + \Big[\alpha_i(i\omega)\beta_j(i\omega) + \beta_i(i\omega)\alpha_j(i\omega)\Big]P_M(\alpha_0\omega r)\Big\}\,,$$

$$P_E(x) = x^4 + 2x^3 + 5x^2 + 6x + 3\,, \quad P_M(x) = -(x^4 + 2x^3 + x^2)\,.$$

neutron-neutron:

$$V_{CP,nn}(r\to\infty) \sim -\frac{1}{4\pi r^7}\Big[23(\alpha_n^2 + \beta_n^2) - 14\alpha_n\beta_n\Big]$$

Detour: van der Waals of the nuclear force

1.B

Nuclear Physics **A294** (1978) 417–434;

LONG-RANGE NN INTERACTION AND AXIAL POLARIZABILITY

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CERN, Geneva

and

Department of Theoretical Physics, Barcelona

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Institut de Physique Nucléaire, Lyon

Detour: van der Waals of the nuclear force

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Detour: gravitational physics

Graviton physics

Barry R. Holstein

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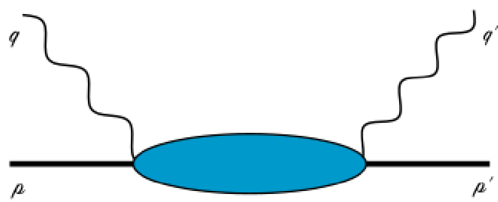
Quantum Gravitational Force Between Polarizable Objects

L. H. Ford, Mark P. Hertzberg, and J. Karouby
Phys. Rev. Lett. **116**, 151301 – Published 15 April 2016

Polarizabilities: induced response to an applied external field

$$\mathcal{H}_{\text{eff}} = -4\pi \left[\frac{1}{2}\alpha_E\, \boldsymbol{E}^2 + \frac{1}{2}\beta_M\, \boldsymbol{H}^2\right]$$

Nucleon/nuclear Compton scattering



$$= T_{\text{Born}} + T_{\text{non-Born}}$$

Hagelstein, Miskimen, Pascalutsa, Prog.Part.Nucl.Phys., 2016

$T_{\text{Born}} =$

Polarizabilities: induced response to an applied external field

Hagelstein, Miskimen, Pascalutsa, Prog.Part.Nucl.Phys., 2016

$$T_{\lambda'_\gamma \lambda'_N \lambda_\gamma \lambda_N} = \bar{u}_{\lambda'_N}(\boldsymbol{p}')\varepsilon^\star_{\lambda'_\gamma,\mu}\, T^{\mu\nu}(q', q, P)\, \varepsilon^\star_{\lambda_\gamma,\nu} u_{\lambda_N}(\boldsymbol{p})$$

$$(8\pi\sqrt{s})\phi_1 \equiv T_{-1/2-1/2} = T_{+1/2+1/2}\,,$$

$$(8\pi\sqrt{s})\phi_2 \equiv T_{-1/2+1/2} = T_{+1/2-1/2}\,,$$

$$(8\pi\sqrt{s})\phi_3 \equiv T_{-1/2+3/2} = T_{+1/2-3/2} = T_{+3/2-1/2} = T_{-3/2+1/2}\,,$$

$$(8\pi\sqrt{s})\phi_4 \equiv T_{-1/2-3/2} = T_{+1/2+3/2} = T_{+3/2+1/2} = T_{-3/2-1/2}\,,$$

$$(8\pi\sqrt{s})\phi_5 \equiv T_{+3/2+3/2} = T_{-3/2-3/2}\,,$$

$$(8\pi\sqrt{s})\phi_6 \equiv T_{-3/2+3/2} = T_{+3/2-3/2}\,.$$

$$T^J_{H'H}(\omega) = \frac{1}{2}\int_{-1}^{1} d(\cos\theta) T_{H'H}(\omega,\theta) d^J_{HH'}(\theta)\,,$$

Polarizabilities: induced response to an applied external field

Hagelstein, Miskimen, Pascalutsa, Prog.Part.Nucl.Phys., 2016

$$f^{1+}_{\substack{EE\\MM}} = \frac{1}{8}\left[\left(\phi_1^{3/2} \mp \phi_2^{3/2}\right) + 2\sqrt{3}\left(\pm\phi_3^{3/2} - \phi_4^{3/2}\right) + 3\left(\phi_5^{3/2} \mp \phi_6^{3/2}\right)\right]$$

$$f^{1-}_{\substack{EE\\MM}} = \frac{1}{2}\left(\phi_1^{1/2} \mp \phi_2^{1/2}\right)$$

$$\begin{bmatrix}\alpha_E(\omega)\\ \beta_M(\omega)\end{bmatrix} = \frac{1}{\omega^2}\left[2f^{1+}_{\substack{EE\\MM}}(\omega) + f^{1-}_{\substack{EE\\MM}}(\omega)\right]$$

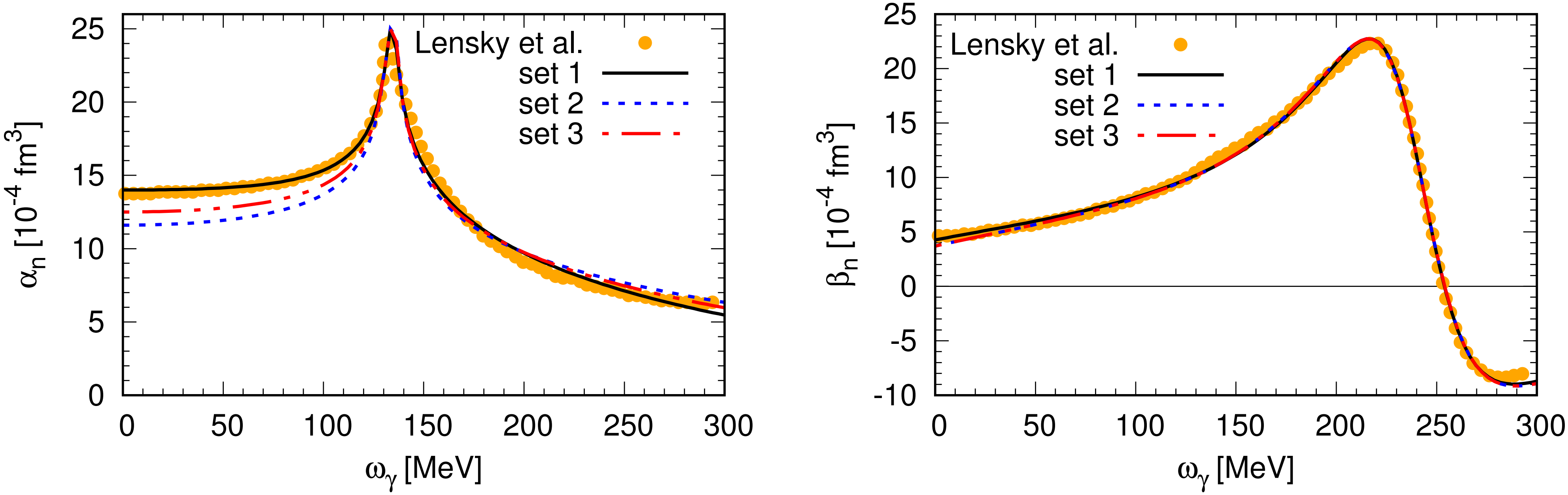
our fits:

$$\alpha_n(\omega) = \alpha_E(\omega) = \frac{\alpha_n(0)\sqrt{(m_\pi + a_1)(2M_n + a_2)}(0.2a_2)^2}{\sqrt{(\sqrt{m_\pi^2 - \omega^2} + a_1)(\sqrt{4M_n^2 - \omega^2} + a_2)}\left[|\omega|^2 + (0.2a_2)^2\right]}$$

$$\beta_n(\omega) = \beta_M(\omega) = \frac{\beta_n(0) - b_1^2\omega^2 + b_2^3\,\mathrm{Re}(\omega)}{(\omega^2 - \omega_\Delta^2)^2 + |\omega^2\Gamma_\Delta^2|}$$

Our fits:

	$\alpha_n(0)$ (10^{-4}fm^3)	$\beta_n(0)$ (10^{-4}fm^3)
Set 1 (fit parameter)	13.9968	4.2612
Set 2 (PDG)	11.6	3.7
Set 3 (Kossert *et al.*, 2003)	12.5	2.7



HB-χEFT: Hildebrandt *et al.*, EPJA 20, 290 (2004)

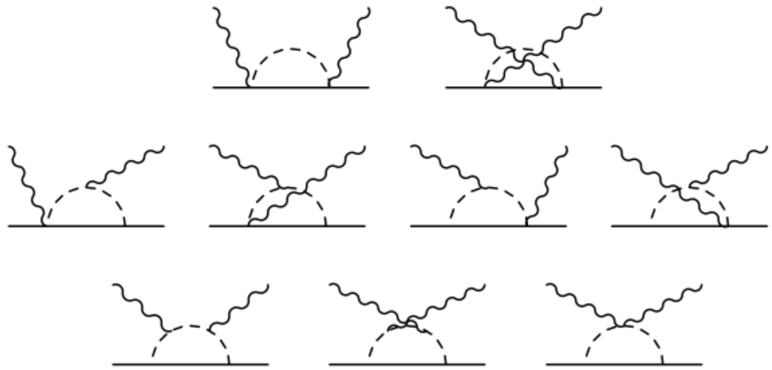


Fig. 2. Leading-one-loop $N\pi$ continuum contributions to nucleon polarizabilities.

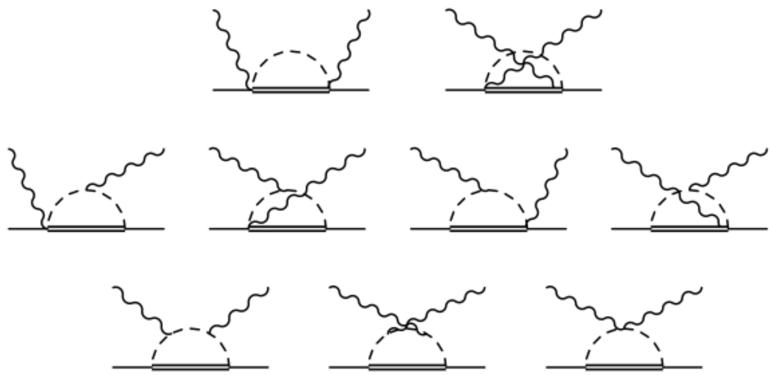


Fig. 3. Leading-one-loop $\Delta\pi$ continuum contributions to nucleon polarizabilities.

Fig. 4. Δ-pole and short-distance contributions to nucleon polarizabilities.

one-pion production threshold

$$\omega_\pi = \frac{m_\pi^2 + 2m_\pi M}{2(m_\pi + M)} \approx 131 \text{ MeV} \qquad (3.7)$$

is therefore not at the correct location. We correct for

HB-χEFT: Hildebrandt *et al.*, EPJA 20, 290 (2004)

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Appendix A. Projection formulae in Dispersion Theory

In this appendix, we give the relevant formulae to calculate the multipole amplitudes for Compton scattering from the invariant amplitudes A_i^L. Following the notation of ref. [5], we introduce the following six independent helicity amplitudes $\phi_{\Lambda'\Lambda}$, with $\Lambda = \lambda_\gamma - \lambda_N$ ($\Lambda' = \lambda'_\gamma - \lambda'_N$) related to the helicities of the initial (final) photon and nucleon, λ_γ (λ'_γ) and λ_N (λ'_N), respectively,

$$\begin{aligned} \phi_1 &\equiv \phi_{1/2\,1/2}\,, \\ \phi_2 &\equiv \phi_{1/2\,-1/2}\,, \\ \phi_3 &\equiv \phi_{1/2\,-3/2}\,, \\ \phi_4 &\equiv \phi_{1/2\,3/2}\,, \\ \phi_5 &\equiv \phi_{3/2\,3/2}\,, \\ \phi_6 &\equiv \phi_{3/2\,-3/2}\,. \end{aligned} \qquad \text{(A.1)}$$

The invariant amplitudes A_i^L are connected to the helicity amplitudes ϕ_i by the relations

$$\begin{aligned} \phi_1 &= \frac{\sqrt{(1-\sigma)}}{8\pi\sqrt{s}}\frac{(s-M^2)[2(s-M^2)+t]}{2M^3[M^2\sigma - s(\sigma-2)]} \\ &\quad\times\{(\sigma-1)s[2M^2A_3^L - (s-M^2)A_4^L] \\ &\quad+2M^2A_6^L(\sigma M^2 - s)\}, \\ \phi_2 &= -\frac{\sqrt{\sigma}}{8\pi\sqrt{s}}\frac{(s-M^2)^2}{4M^2s^{3/2}} \\ &\quad\times\{-2M^2\sigma[A_1^L(s+M^2)+A_2^L(s-M^2)] \\ &\quad+sA_5^L(\sigma-2)[2(s-M^2)+t]\}, \\ \phi_3 &= -\frac{\sigma\sqrt{1-\sigma}}{8\pi\sqrt{s}}\frac{(s-M^2)^2}{4Ms} \\ &\quad\times\{4M^2A_1^L - A_5^L[2(s-M^2)+t]\}, \\ \phi_4 &= \frac{\sqrt{\sigma}(1-\sigma)}{8\pi\sqrt{s}}\frac{\sqrt{s}(s-M^2)[2(s-M^2)+t]}{2M^2[M^2\sigma - s(\sigma-2)]} \\ &\quad\times[2M^2A_6^L + A_3^L(s+M^2)], \\ \phi_5 &= -\frac{(1-\sigma)\sqrt{(1-\sigma)}}{8\pi\sqrt{s}}\frac{s(s-M^2)[2(s-M^2)+t]}{M[M^2\sigma - s(\sigma-2)]} \\ &\quad\times\left[A_3^L + A_6^L + A_4^L\frac{(s-M^2)}{2M^2}\right], \\ \phi_6 &= \frac{\sigma\sqrt{\sigma}}{8\pi\sqrt{s}}\frac{(s-M^2)^2}{4s\sqrt{s}}\{2(s-M^2)A_2^L \\ &\quad-2A_1^L(s+M^2)+A_5^L[2(s-M^2)+t]\}, \end{aligned} \qquad \text{(A.2)}$$

where $\sigma = -s\,t/(s-M^2)^2 = \sin^2(\theta/2)$.

The helicity amplitudes have the following standard partial-wave decomposition in terms of the reduced matrices $d^J_{\Lambda\Lambda'}$:

$$\phi_{\Lambda'\Lambda} = \sum_J (2J+1)\phi^J_{\Lambda'\Lambda}\, d^J_{\Lambda\Lambda'}(\theta), \qquad \text{(A.3)}$$

which, by inversion, gives

$$\phi^J_{\Lambda'\Lambda} = \frac{1}{2}\int_{-1}^{+1} \mathrm{d}\cos\theta\ \phi_{\Lambda'\Lambda}(\cos\theta)d^J_{\Lambda\Lambda'}(\theta). \qquad \text{(A.4)}$$

With the partial-wave decomposition of eq. (A.3), we finally obtain the relations between the multipole amplitudes of Compton scattering and the helicity partial waves:

$$\begin{aligned} f_{EE}^{l+} &= \frac{1}{(l+1)^2}\left[\frac{1}{2}\left(\phi_1^{l+1/2} - \phi_2^{l+1/2}\right)\right. \\ &\quad\left.+\sqrt{\frac{l+2}{l}}\left(\phi_3^{l+1/2}-\phi_4^{l+1/2}\right)+\frac{l+2}{2l}\left(\phi_5^{l+1/2}-\phi_6^{l+1/2}\right)\right], \\ f_{MM}^{l+} &= \frac{1}{(l+1)^2}\left[\frac{1}{2}\left(\phi_1^{l+1/2} + \phi_2^{l+1/2}\right)\right. \\ &\quad\left.-\sqrt{\frac{l+2}{l}}\left(\phi_3^{l+1/2}+\phi_4^{l+1/2}\right)+\frac{l+2}{2l}\left(\phi_5^{l+1/2}+\phi_6^{l+1/2}\right)\right], \\ f_{EE}^{l-} &= \frac{1}{l^2}\left[\frac{1}{2}\left(\phi_1^{l-1/2}+\phi_2^{l-1/2}\right)+\sqrt{\frac{l-1}{l+1}}\left(\phi_3^{l-1/2}+\phi_4^{l-1/2}\right)\right. \\ &\quad\left.+\frac{l-1}{2(l+1)}\left(\phi_5^{l-1/2}+\phi_6^{l-1/2}\right)\right], \\ f_{MM}^{l-} &= \frac{1}{l^2}\left[\frac{1}{2}\left(\phi_1^{l-1/2}-\phi_2^{l-1/2}\right)-\sqrt{\frac{l-1}{l+1}}\left(\phi_3^{l-1/2}-\phi_4^{l-1/2}\right)\right. \\ &\quad\left.+\frac{l-1}{2(l+1)}\left(\phi_5^{l-1/2}-\phi_6^{l-1/2}\right)\right], \\ f_{EM}^{l+} &= \frac{1}{(l+1)^2}\left[-\frac{1}{2}\left(\phi_1^{l+1/2}-\phi_2^{l+1/2}\right)\right. \\ &\quad\left.-\frac{1}{\sqrt{l(l+2)}}\left(\phi_3^{l+1/2}-\phi_4^{l+1/2}\right)+\frac{1}{2}\left(\phi_5^{l+1/2}-\phi_6^{l+1/2}\right)\right], \\ f_{ME}^{l+} &= \frac{1}{(l+1)^2}\left[-\frac{1}{2}\left(\phi_1^{l+1/2}+\phi_2^{l+1/2}\right)\right. \\ &\quad\left.+\frac{1}{\sqrt{l(l+2)}}\left(\phi_3^{l+1/2}+\phi_4^{l+1/2}\right)+\frac{1}{2}\left(\phi_5^{l+1/2}+\phi_6^{l+1/2}\right)\right]. \end{aligned} \qquad \text{(A.5)}$$

Appendix B. Compton amplitudes to leading-one-loop order in χEFT

The formulae which connect the amplitudes R_i discussed in the text to the A_i^H basis usually used in χEFT calculations of nucleon Compton scattering read [7]

$$\begin{aligned} A_1^H &= 4\pi\frac{W}{M}(R_1 + zR_2)\,, \\ A_2^H &= -4\pi\frac{W}{M}R_2\,, \\ A_3^H &= 4\pi\frac{W}{M}(R_3 + zR_4 + 2zR_5 + 2R_6)\,, \\ A_4^H &= 4\pi\frac{W}{M}R_4\,, \\ A_5^H &= -4\pi\frac{W}{M}(R_4 + R_5)\,, \\ A_6^H &= -4\pi\frac{W}{M}R_6\,. \end{aligned} \qquad \text{(B.1)}$$

As discussed in sect. 2.1 we need to know both the pole as well as the structure-dependent contributions to A_i^H.

The cm pole contributions to the Compton amplitudes A_1^H to A_6^H for the case of a proton target have been calculated up to leading-one-loop order in ref. [27]. For completeness, we list them here again ($\kappa = \frac{1}{2}(\kappa_v + \kappa_s)$):

$$\begin{aligned} A_1^{\text{pole}}(\omega, z) &= -\frac{e^2}{M} + \mathcal{O}(\epsilon^4)\,, \\ A_2^{\text{pole}}(\omega, z) &= \frac{e^2\omega}{M^2} + \mathcal{O}(\epsilon^4)\,, \\ A_3^{\text{pole}}(\omega, z) &= \frac{e^2\omega\,\left(1 + 2\kappa - (1+\kappa)^2 z\right)}{2M^2} \\ &\quad-\frac{e^2 g_A}{4\pi^2 f_\pi^2}\frac{\omega^3(1-z)}{m_\pi^2 + 2\omega^2(1-z)} + \mathcal{O}(\epsilon^4)\,, \\ A_4^{\text{pole}}(\omega, z) &= -\frac{e^2\omega\,(1+\kappa)^2}{2M^2} + \mathcal{O}(\epsilon^4)\,, \\ A_5^{\text{pole}}(\omega, z) &= \frac{e^2\omega\,(1+\kappa)^2}{2M^2} \\ &\quad-\frac{e^2 g_A}{8\pi^2 f_\pi^2}\frac{\omega^3}{m_\pi^2 + 2\omega^2(1-z)} + \mathcal{O}(\epsilon^4)\,, \\ A_6^{\text{pole}}(\omega, z) &= -\frac{e^2\omega\,(1+\kappa)}{2M^2} \\ &\quad+\frac{e^2 g_A}{8\pi^2 f_\pi^2}\frac{\omega^3}{m_\pi^2 + 2\omega^2(1-z)} + \mathcal{O}(\epsilon^4). \end{aligned} \qquad \text{(B.2)}$$

Finally, we present explicit expressions for the leading-one-loop order structure-dependent SSE Compton amplitudes including the kinematical as well as the short-distance corrections discussed in sect. 3.2. The threshold correction was done as follows for each diagram in fig. 2: If the pion propagator in a loop integral exhibits a cut at $\omega = m_\pi$, one replaces ω in that propagator by eq. (3.8) in order to obtain the physically correct s-channel cut position at $\omega = \omega_\pi$. The u-channel contribution is unchanged. We are aware, that this procedure violates crossing symmetry, but the crossing violating effects in the u-channel are quite small. Formally, the terms correcting for the exact location of the pion threshold start to appear at $\mathcal{O}(p^4)$.

$$\begin{aligned} \bar{A}_1^H(\omega,z) &= \frac{b_1^2 e^2\omega^2 z}{9M^2}\left(-\frac{1}{\omega_s - \Delta_0} + \frac{1}{\omega_u + \Delta_0}\right) + \frac{\alpha\,(g_{118}t - g_{117}\omega^2)}{2\pi f_\pi^2 M} \\ &+\frac{\alpha}{18\pi f_\pi^2}\int_0^1 \mathrm{d}x\int_0^1 \mathrm{d}y\left\{9g_A^2\left[m_\pi\pi + \frac{\pi\,(2m_\pi{}^2 - t)}{2\sqrt{-t}}\arctan\left(\frac{\sqrt{-t}}{2m_\pi}\right) + \frac{\omega_s - \omega}{8\omega_s\omega}(m_\pi{}^2\pi^2 - 4\omega_s\omega)\right.\right. \\ &+\frac{m_\pi{}^2}{2\omega_s\omega}\left(\omega\arccos^2\left(-\frac{\omega_s}{m_\pi}\right) - \omega_s\arccos^2\left(\frac{\omega}{m_\pi}\right)\right) - (1-y)\left(\frac{1}{c_u}\left[5c_u^2 - (1-y)\left(\omega^2x^2(1-y)\right.\right.\right. \\ &\left.\left.+t\left(\frac{x}{2} + (1-x)\,y\right)\right)\right]\arccos\left(\frac{\omega x\,(1-y)}{d}\right) + \frac{1}{c_s}\left[5c_s^2 - (1-y)\left(\omega^2x^2(1-y) + t\left(\frac{x}{2} + (1-x)\,y\right)\right)\right] \\ &\left.\left.\times\arccos\left(\frac{\omega_s x\,(-1+y)}{d}\right)\right)\right] + 16g_{\pi N\Delta}^2\left[-2\Delta_0\ln m_\pi - 3\Delta_0\ln\sqrt{m_\pi^2 - t\,(1-x)\,x}\right. \\ &+\sqrt{-m_\pi^2 + (\Delta_0 - \omega)^2}\ln R(\Delta_0 - \omega) + \sqrt{-m_\pi^2 + (\Delta_0 + \omega)^2}\ln R(\Delta_0 + \omega) - 2\sqrt{-m_\pi^2 + (\Delta_0 - \omega x)^2}\ln R(\Delta_0 - \omega x) \\ &-2\sqrt{-m_\pi^2 + (\Delta_0 + \omega x)^2}\ln R(\Delta_0 + \omega x) - \frac{(3\Delta_0{}^2 - 3m_\pi^2 + 4t\,(1-x)\,x)}{\sqrt{\Delta_0{}^2 - m_\pi^2 + t\,(1-x)\,x}}\ln\left(\frac{\Delta_0 + \sqrt{\Delta_0{}^2 - m_\pi^2 + t\,(1-x)\,x}}{\sqrt{m_\pi^2 - t\,(1-x)\,x}}\right) \\ &+\left(\frac{1}{C_s}\left(5C_s^2 + \omega^2x^2(1-y)^2 + \frac{1}{2}tx\,(1-y) + t\,(1-x)\,(1-y)\,y\right)\ln\bar{R}(\Delta_0 - \omega x\,(1-y)) + 10\Delta_0\ln d\right. \\ &\left.\left.\left.+\frac{1}{C_u}\left(5C_u^2 + \omega^2x^2(1-y)^2 + \frac{1}{2}tx\,(1-y) + t\,(1-x)\,(1-y)\,y\right)\ln\bar{R}(\Delta_0 + \omega x(1-y))\right)(1-y)\right]\right\} + \mathcal{O}(\epsilon^4)\,, \end{aligned} \qquad \text{(B.3)}$$

See equations (B.3) *above and* (B.4)-(B.8)

on the following pages

HB-χEFT: Hildebrandt *et al.*, EPJA 20, 290 (2004)

$$
\begin{aligned}
\bar{A}_2^H(\omega,z) = {}& \frac{b_1^2 e^2 \omega^2}{9 M^2}\left(\frac{1}{\omega_s-\Delta_0}-\frac{1}{\omega_u+\Delta_0}\right)-\frac{\alpha\, g_{118}}{\pi f_\pi^2 M}\omega^2+\frac{\alpha}{18\pi f_\pi^2}\int_0^1 \mathrm{d}x \int_0^1 \mathrm{d}y\, \omega^2 (1-y)\left\{9 g_A^2\left[(1-x)\, x\right.\right.\\
&\times\left(\frac{\omega_s}{c_s^2 d^2}-\frac{\omega}{c_u^2 d^2}\right)(1-y)^3 y\left(\omega^2 x^2 (1-y)+t\left(\frac{x}{2}+(1-x)\, y\right)\right)-\frac{1}{c_s^3}\Big((-1+x)\,(1-y)^2 y\Big(\omega^2 x^2 (1-y)\\
&+t\left(\frac{x}{2}+(1-x)\, y\right)\Big)+c_s^2\,(x y+(1-x)\,(1-7y+7y^2))\Big)\arccos\left(\frac{\omega_s x\,(-1+y)}{d}\right)-\frac{1}{c_u^3}\\
&\times\left((-1+x)\,(1-y)^2 y\left(\omega^2 x^2\,(1-y)+t\left(\frac{x}{2}+(1-x)\, y\right)\right)+c_u^2\,(x y+(1-x)\,(1-7y+7y^2))\right)\arccos\left(\frac{\omega x\,(1-y)}{d}\right)\Big]\\
&-16\, g_{\pi N\Delta}^2\left[(1-x)\left(\frac{-\Delta_0+\omega x\,(1-y)}{C_s^2 d^2}-\frac{\Delta_0+\omega x\,(1-y)}{C_u^2 d^2}\right)(1-y)^2 y\left(\omega^2 x^2\,(1-y)+\frac{1}{2} t x+t\,(1-x)\, y\right)\right.\\
&+\frac{1}{C_s^3}\left(C_s^2\,((1-x)\,(1-7y)\,(1-y)+y)+(1-x)\,(1-y)^2 y\left(\omega^2 x^2\,(1-y)+\frac{1}{2} t x+t\,(1-x)\, y\right)\right)\\
&\times\ln\bar{R}\,(\Delta_0-\omega x\,(1-y))+\frac{1}{C_u^3}\Big(C_u^2\,((1-x)\,(1-7y)\,(1-y)+y)+(1-x)\,(1-y)^2 y\\
&\times\left.\left.\left(\omega^2 x^2\,(1-y)+\frac{1}{2} t x+t\,(1-x)\, y\right)\Big)\ln\bar{R}\,(\Delta_0+\omega x(1-y))\right]\right\}+\mathcal{O}\left(\epsilon^4\right), \qquad \text{(B.4)}
\end{aligned}
$$

$$
\begin{aligned}
\bar{A}_3^H(\omega,z) = {}& \frac{b_1^2 e^2 \omega^3 z}{18 M^2 \Delta_0}\left(\frac{1}{\omega_s-\Delta_0}-\frac{1}{\omega_u+\Delta_0}\right)+\frac{\alpha}{\pi f_\pi^2}\int_0^1 \mathrm{d}x\int_0^1 \mathrm{d}y\left\{\frac{g_A^2}{2}\left[-\frac{\omega_s+\omega}{8\,\omega_s\,\omega}\,\left(m_\pi{}^2\pi^2+4\,\omega_s\,\omega\right)\right.\right.\\
&+\frac{m_\pi{}^2}{2\,\omega_s\,\omega}\left(\omega\arccos^2\left(-\frac{\omega_s}{m_\pi}\right)+\omega_s\arccos^2\left(\frac{\omega}{m_\pi}\right)\right)+\omega^4\,(1-x)\, x\,(1-y)^3 y\,(1-z^2)\\
&\times\left(\left(\frac{\omega_s}{c_s^2 d^2}+\frac{\omega}{c_u^2 d^2}\right)x\,(1-y)-\frac{1}{c_u^3}\arccos\left(\frac{\omega x\,(1-y)}{d}\right)+\frac{1}{c_s^3}\arccos\left(\frac{\omega_s x\,(-1+y)}{d}\right)\right)\Big]\\
&+\frac{4\, g_{\pi N\Delta}^2}{9}\Big[-\sqrt{-m_\pi^2+(\Delta_0-\omega)^2}\,\ln R(\Delta_0-\omega)+\sqrt{-m_\pi^2+(\Delta_0+\omega)^2}\,\ln R(\Delta_0+\omega)\\
&+2\sqrt{-m_\pi^2+(\Delta_0-\omega x)^2}\,\ln R(\Delta_0-\omega x)-2\sqrt{-m_\pi^2+(\Delta_0+\omega x)^2}\,\ln R(\Delta_0+\omega x)-\omega^4\,(1-x)\, x(1-y)^3 y\,(1-z^2)\\
&\times\left.\left.\left(\frac{\Delta_0-\omega x\,(1-y)}{C_s^2 d^2}-\frac{\Delta_0+\omega x\,(1-y)}{C_u^2 d^2}-\frac{1}{C_s^3}\ln\bar{R}\,(\Delta_0-\omega x(1-y))+\frac{1}{C_u^3}\ln\bar{R}\,(\Delta_0+\omega x\,(1-y))\right)\right]\right\}+\mathcal{O}(\epsilon^4), \qquad \text{(B.5)}
\end{aligned}
$$

$$
\begin{aligned}
\bar{A}_4^H(\omega,z) = {}& \frac{b_1^2 e^2 \omega^3}{18 M^2 \Delta_0}\left(\frac{1}{\omega_s-\Delta_0}-\frac{1}{\omega_u+\Delta_0}\right)+\frac{\alpha}{\pi f_\pi^2}\int_0^1 \mathrm{d}x\int_0^1 \mathrm{d}y\,\omega^2 x(1-y)^2\left\{\frac{g_A^2}{2}\left[-\frac{1}{c_u}\arccos\left(\frac{\omega x\,(1-y)}{d}\right)\right.\right.\\
&\left.+\frac{1}{c_s}\arccos\left(\frac{\omega_s x\,(-1+y)}{d}\right)\right]+\frac{4\, g_{\pi N\Delta}^2}{9}\left.\left[-\frac{1}{C_s}\ln\bar{R}\,(\Delta_0-\omega x(1-y))+\frac{1}{C_u}\ln\bar{R}\,(\Delta_0+\omega x\,(1-y))\right]\right\}+\mathcal{O}(\epsilon^4), \qquad \text{(B.6)}
\end{aligned}
$$

$$
\begin{aligned}
\bar{A}_5^H(\omega,z) = {}& \frac{b_1^2 e^2 \omega^3}{18 M^2 \Delta_0}\left(-\frac{1}{\omega_s-\Delta_0}+\frac{1}{\omega_u+\Delta_0}\right)+\frac{\alpha}{\pi f_\pi^2}\int_0^1 \mathrm{d}x\int_0^1 \mathrm{d}y\,\omega^2\,(1-y)y\left\{\frac{g_A^2}{2}\left[\omega^2\left(\frac{\omega_s}{c_s^2 d^2}+\frac{\omega}{c_u^2 d^2}\right)\right.\right.\\
&\times\,(1-x)\, x^2\,(1-y)^3 z-\frac{1}{c_u^3}\left(-c_u^2+\omega^2\,(1-x)\, x\,(1-y)^2 z\right)\arccos\left(\frac{\omega x\,(1-y)}{d}\right)\\
&\left.+\frac{1}{c_s^3}\left(-c_s^2+\omega^2\,(1-x)\, x\,(1-y)^2 z\right)\arccos\left(\frac{\omega_s x\,(-1+y)}{d}\right)\right]\\
&+\frac{4\, g_{\pi N\Delta}^2}{9}\Big[\frac{1}{C_s}\ln\bar{R}\,(\Delta_0-\omega x(1-y))-\frac{1}{C_u}\ln\bar{R}\,(\Delta_0+\omega x\,(1-y))-\omega^2\,(1-x)\, x\,(1-y)^2 z\\
&\times\left.\left.\left(\frac{\Delta_0-\omega x\,(1-y)}{C_s^2 d^2}-\frac{\Delta_0+\omega x\,(1-y)}{C_u^2 d^2}-\frac{1}{C_s^3}\ln\bar{R}\,(\Delta_0-\omega x(1-y))+\frac{1}{C_u^3}\ln\bar{R}\,(\Delta_0+\omega x(1-y))\right)\right]\right\}+\mathcal{O}(\epsilon^4), \qquad \text{(B.7)}
\end{aligned}
$$

$$
\begin{aligned}
\bar{A}_6^H(\omega,z) = {}& \frac{\alpha}{\pi f_\pi^2}\int_0^1 \mathrm{d}x\int_0^1 \mathrm{d}y\,\omega^2\,(1-y)\,y\left\{\frac{g_A^2}{2}\left[-\omega^2\left(\frac{\omega_s}{c_s^2 d^2}+\frac{\omega}{c_u^2 d^2}\right)(1-x)\, x^2\,(1-y)^3+\frac{1}{c_u^3}\Big(-c_u^2\right.\right.\\
&\left.+\omega^2\,(1-x)\, x(1-y)^2\Big)\arccos\left(\frac{\omega x\,(1-y)}{d}\right)-\frac{1}{c_s^3}\left(-c_s^2+\omega^2\,(1-x)\, x(1-y)^2\right)\arccos\left(\frac{\omega_s x\,(-1+y)}{d}\right)\right]\\
&+\frac{4 g_{\pi N\Delta}^2}{9}\Big[-\frac{1}{C_s}\ln\bar{R}\,(\Delta_0-\omega x\,(1-y))+\frac{1}{C_u}\ln\bar{R}\,(\Delta_0+\omega x\,(1-y))+\omega^2\,(1-x)\, x\,(1-y)^2\\
&\times\left.\left.\left(\frac{\Delta_0-\omega x\,(1-y)}{C_s^2 d^2}-\frac{\Delta_0+\omega x\,(1-y)}{C_u^2 d^2}-\frac{1}{C_s^3}\ln\bar{R}\,(\Delta_0-\omega x\,(1-y))+\frac{1}{C_u^3}\ln\bar{R}\,(\Delta_0+\omega x\,(1-y))\right)\right]\right\}+\mathcal{O}(\epsilon^4). \qquad \text{(B.8)}
\end{aligned}
$$

In eqs. (B.3)-(B.8) we have used the following abbreviations:

$$
\begin{aligned}
d^2 &= m_\pi^2-t\,(1-x)\,(1-y)\, y\,,\\
c_s^2 &= d^2-\omega_s^2 x^2\,(1-y)^2\,,\\
c_u^2 &= d^2-\omega^2 x^2\,(1-y)^2\,,\\
C_s^2 &= (\Delta_0-\omega x\,(1-y))^2-d^2\,,\\
C_u^2 &= (\Delta_0+\omega x\,(1-y))^2-d^2\,;\\
\omega_s &= \sqrt{s}-M\,,\\
\omega_u &= M-\sqrt{u}\,,\\
s &= (p+k)^2=\left(\omega+\sqrt{M^2+\omega^2}\right)^2,\\
t &= (k-k')^2=2\omega^2\,(z-1)\,,\\
u &= (p-k')^2=M^2-2\,\omega\sqrt{M^2+\omega^2}-2\,\omega^2 z\,;
\end{aligned}
$$

$$
R(X)=\frac{X}{m_\pi}+\sqrt{\frac{X^2}{m_\pi^2}-1},\quad \bar{R}(X)=\frac{X}{d}+\sqrt{\frac{X^2}{d^2}-1}\,.
$$

For the *isovector* Compton structure amplitudes, one finds a null result to leading-one-loop order:

$$
\bar{A}_i^{H\,(v)}=0+\mathcal{O}(\epsilon^4)\,, \qquad \text{(B.5)}
$$

with $i=1,\ldots,6$.

Appendix C. Projection formulae for χEFT

The connection between the Compton structure amplitudes $\bar{A}_i^H(\omega,z)$, $i=1,\ldots,6$ given in the previous section and the cm Compton multipoles $f_{XX'}^{l\pm}(\omega)$, $X,X'=E,M$, introduced in sect. 2.1, reads

$$
\begin{aligned}
f_{EE}^{1+}(\omega) = {}& \int_{-1}^{1}\frac{M}{16\cdot 4\pi W}\Big[\bar{A}_3^H(\omega,z)\,(-3+z^2)\\
&+4\bar{A}_6^H(\omega,z)\,(-1+z^2)+\left(2\bar{A}_2^H(\omega,z)+\bar{A}_4^H(\omega,z)\right.\\
&\left.+2\,\bar{A}_5^H(\omega,z)\right) z\,(-1+z^2)+2\bar{A}_1^H(\omega,z)\,(1+z^2)\Big]\mathrm{d}z\,,
\end{aligned}
$$

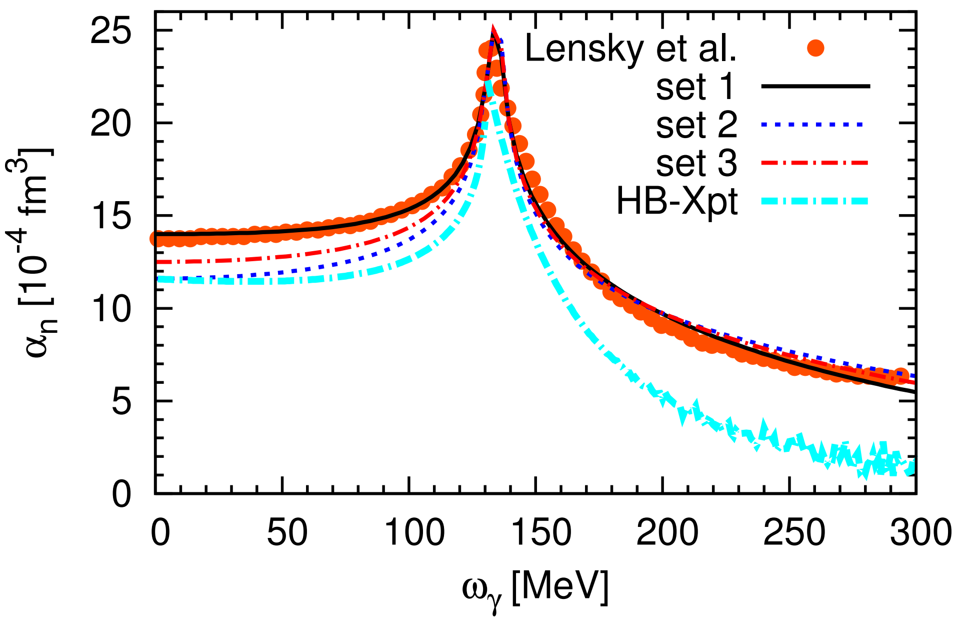
$$
\begin{aligned}
f_{EE}^{1-}(\omega) = {}& \int_{-1}^{1}\frac{M}{8\cdot 4\pi W}\Big[-\bar{A}_3^H(\omega,z)\,(-3+z^2)\\
&-4\bar{A}_6^H(\omega,z)\,(-1+z^2)-\left(-\bar{A}_2^H(\omega,z)+\bar{A}_4^H(\omega,z)\right.\\
&\left.+2\,\bar{A}_5^H(\omega,z)\right) z\,(-1+z^2)+\bar{A}_1^H(\omega,z)\,(1+z^2)\Big]\mathrm{d}z\,,
\end{aligned}
$$

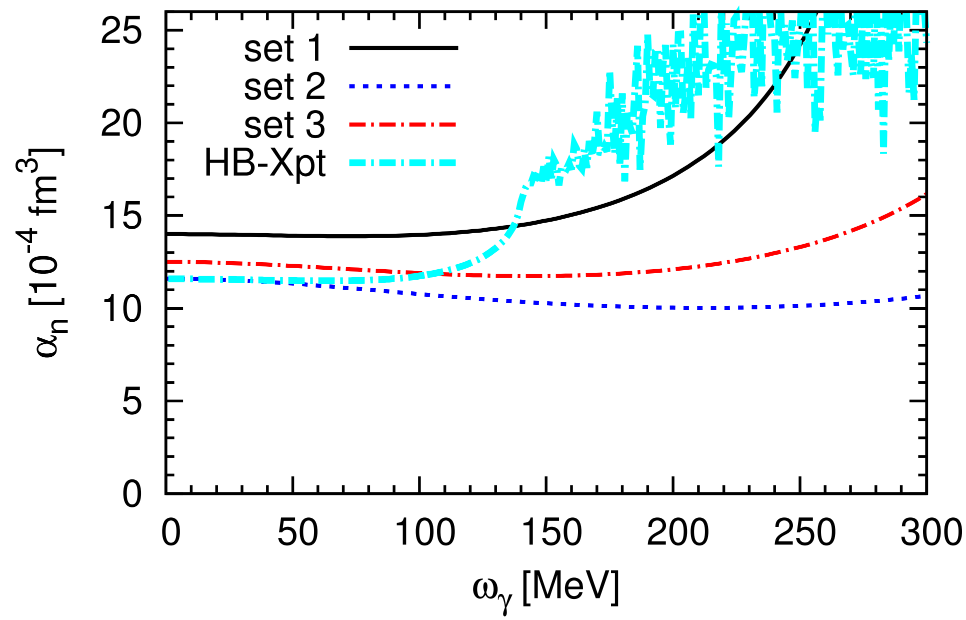
$$
\begin{aligned}
f_{MM}^{1+}(\omega) = {}& \int_{-1}^{1}\frac{M}{16\cdot 4\pi W}\Big[2\bar{A}_2^H(\omega,z)\,(-1+z^2)\\
&+\bar{A}_4^H(\omega,z)\,(-1+z^2)+2\Big(\bar{A}_5^H(\omega,z)\,(1-z^2)\\
&+\bar{A}_1^H(\omega,z)\,2z-\bar{A}_3^H(\omega,z)\, z\Big)\Big]\mathrm{d}z\,,
\end{aligned}
$$

$$
\begin{aligned}
f_{MM}^{1-}(\omega) = {}& \int_{-1}^{1}\frac{M}{8\cdot 4\pi W}\Big[\bar{A}_4^H(\omega,z)\,(1-z^2)\\
&+\bar{A}_2^H(\omega,z)\,(-1+z^2)+2\Big(\bar{A}_5^H(\omega,z)\,(-1+z^2)\\
&+\bar{A}_1^H(\omega,z)\, z+\bar{A}_3^H(\omega,z)\, z\Big)\Big]\mathrm{d}z\,,
\end{aligned}
$$

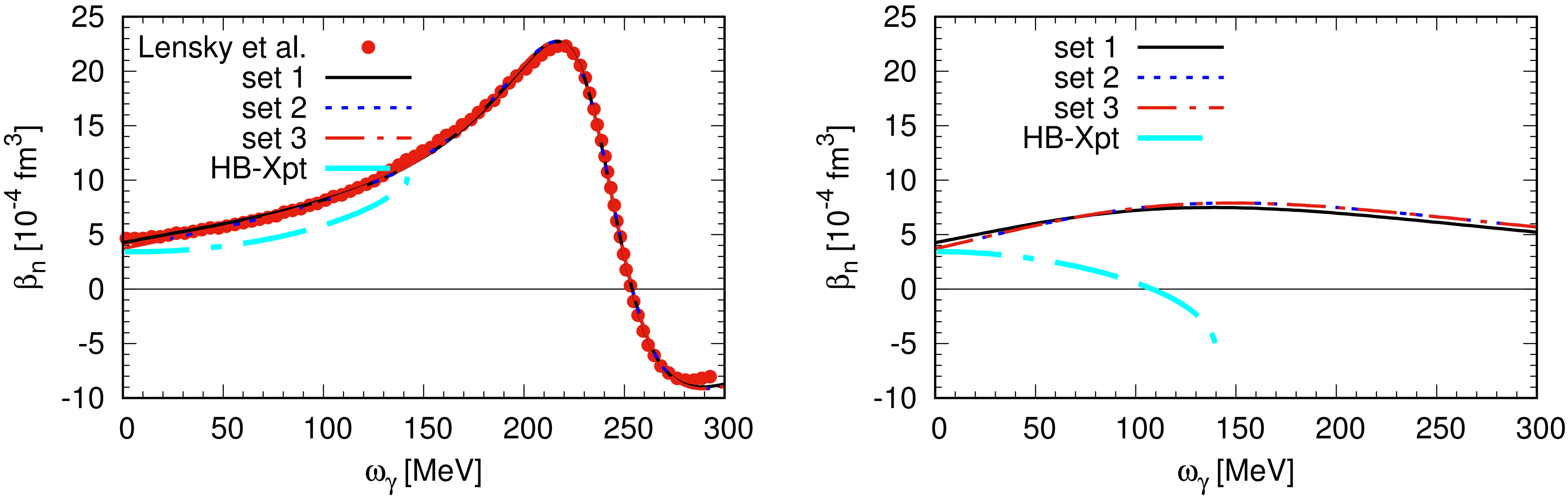
$$
\begin{aligned}
f_{EE}^{2+}(\omega) = {}& \int_{-1}^{1}\frac{M}{72\cdot 4\pi W}\Big[\bar{A}_4^H(\omega,z)\,(-1-3z^2+4z^4)\\
&+\bar{A}_2^H(\omega,z)\,(3-9z^2+6z^4)+2\Big(\bar{A}_5^H(\omega,z)\\
&\times(-1-3z^2+4z^4)+\bar{A}_1^H(\omega,z)\,3z^3\\
&+\bar{A}_3^H(\omega,z)\,(2z^3-3z)+\bar{A}_6^H(\omega,z)\,(6z^3-6z)\Big)\Big]\mathrm{d}z\,,
\end{aligned}
$$

HB-χEFT: Hildebrandt *et al.*, EPJA 20, 290 (2004)





HB-χEFT: Hildebrandt *et al.*, EPJA 20, 290 (2004)

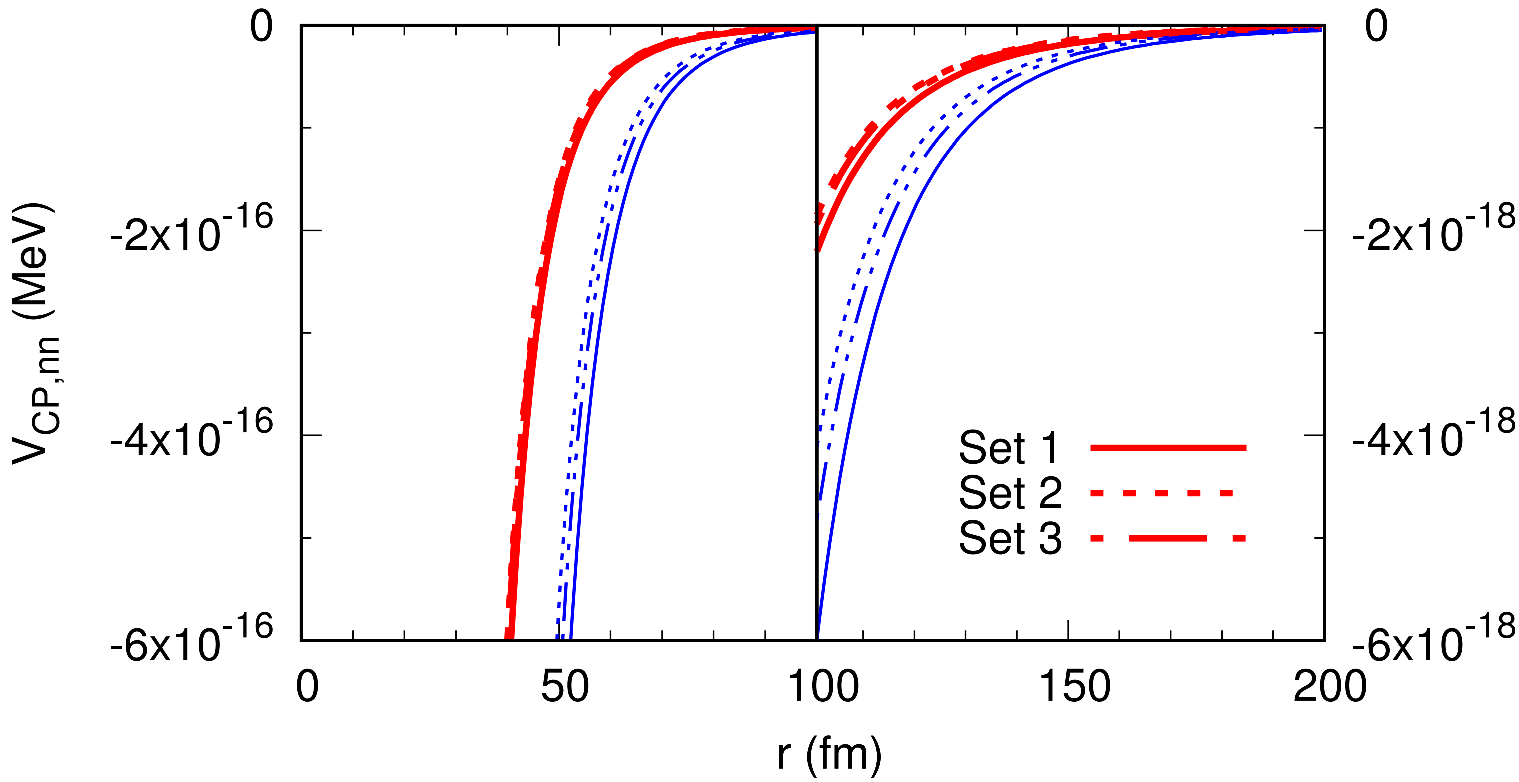


neutron-neutron Casimir-Polder interaction

$$I_{ij}(r) = \int_0^\infty d\omega\, e^{-2\alpha_0 \omega r} \Big\{ \Big[\alpha_i(i\omega)\alpha_j(i\omega) + \beta_i(i\omega)\beta_j(i\omega) \Big] P_E(\alpha_0 \omega r) + \Big[\alpha_i(i\omega)\beta_j(i\omega) + \beta_i(i\omega)\alpha_j(i\omega) \Big] P_M(\alpha_0 \omega r) \Big\} ,$$

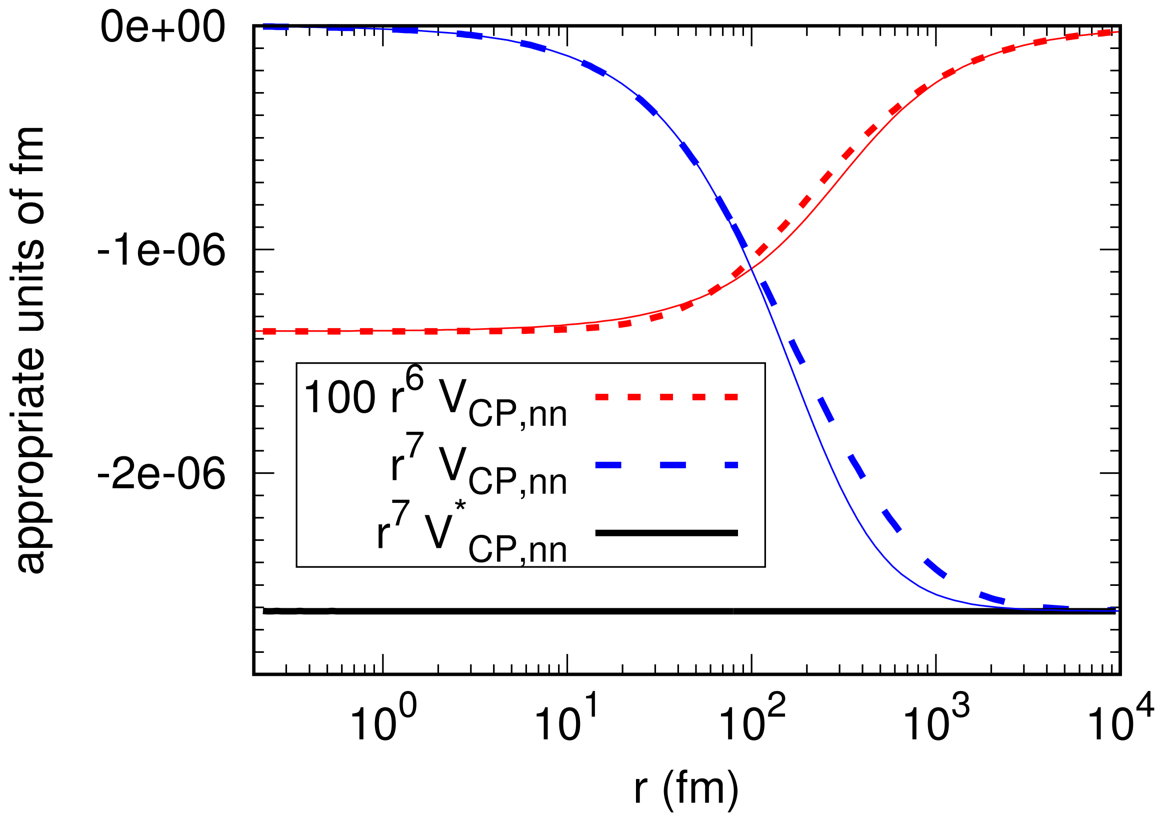
- $r \lesssim 20$ fm $\Rightarrow$ $(2\alpha_0 \times 20\,\mathrm{fm})^{-1} \sim 670$ MeV
- $r \sim 50$ fm $\Rightarrow$ $(2\alpha_0 \times 50\,\mathrm{fm})^{-1} \sim 270$ MeV $\sim \omega_\Delta$
- $r \gtrsim 100$ fm $\Rightarrow$ $(2\alpha_0 \times 100\,\mathrm{fm})^{-1} \sim 135$ MeV $\sim m_\pi$

neutron-neutron Casimir-Polder interaction



Thin blue curve: static limit of the polarizabilities ($V^{\star}_{CP,nn}$)

neutron-neutron Casimir-Polder interaction



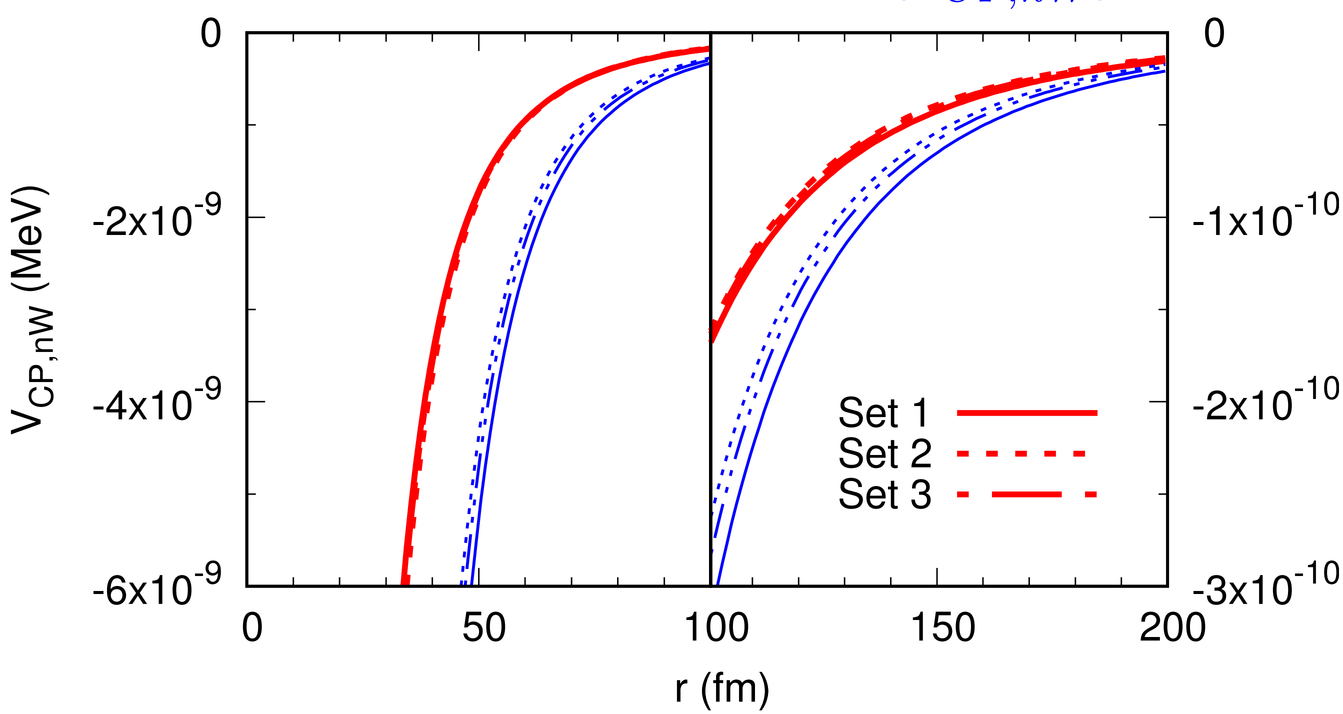
Thin continuous lines: arctan parametrization (O'Carroll & Sucher 69, Arnold 73)

neutron-Wall Casimir-Polder interaction

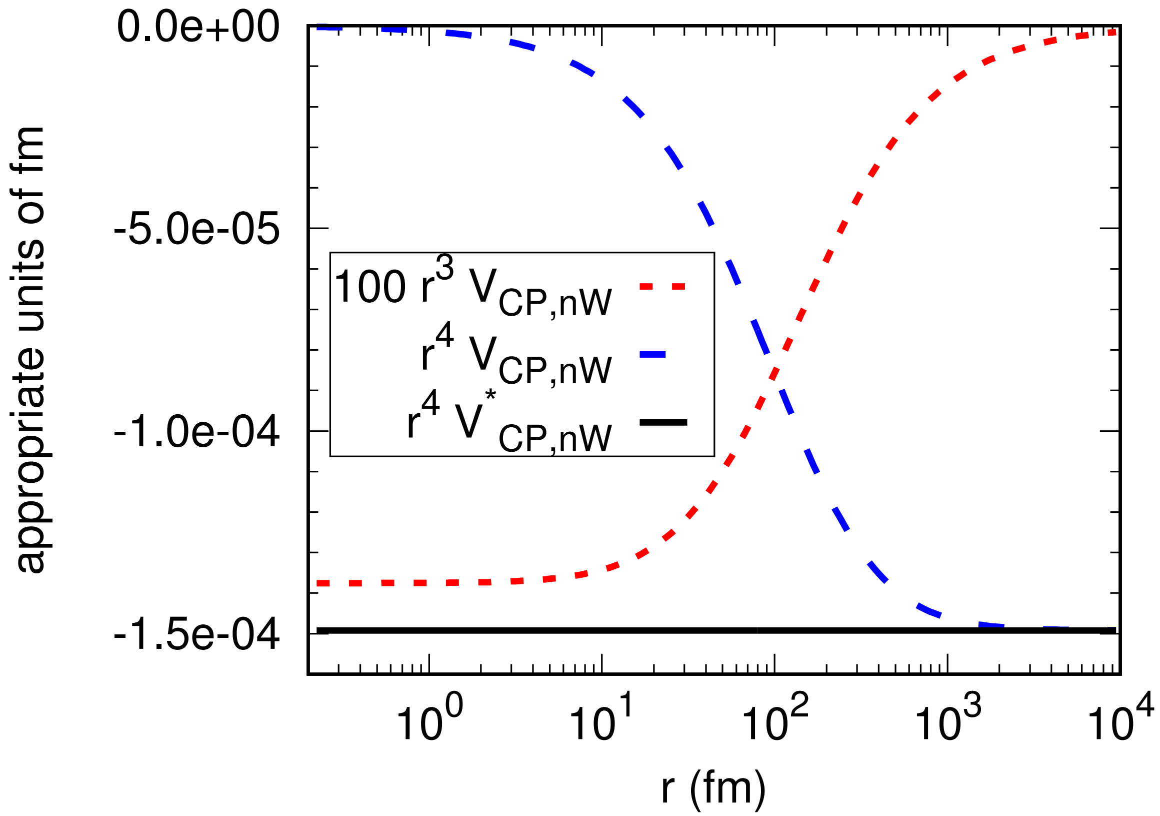
(Zhou & Spruch, PRA 52, 297 (95).)

$$V_{CP,nW}(r) = -\frac{\alpha_0}{4\pi r^3} J_{nW}(r), \quad J_{nW}(r) = \int_0^\infty d\omega\, e^{-2\alpha_0 \omega r} \alpha_n(i\omega) Q(\alpha_0 \omega r), \quad Q(x) = 2x^2 + 2x + 1$$

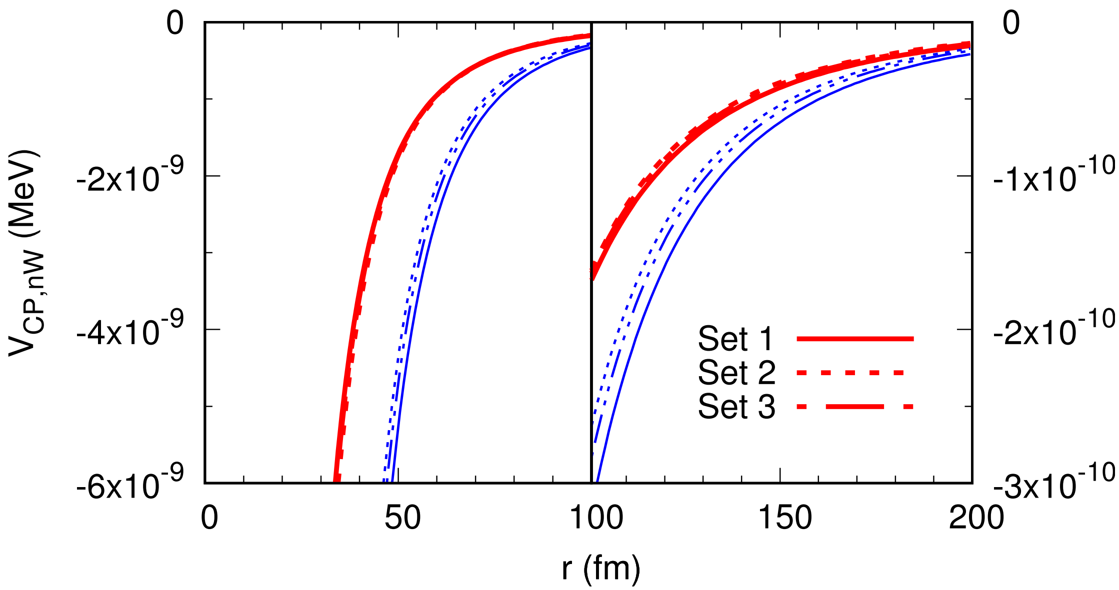
Thin blue curve: static limit of the polarizabilities ($V^{\star}_{CP,nW}$)



neutron-Wall Casimir-Polder interaction



neutron-Wall Casimir-Polder interaction



- UC neutrons: $v_n \sim$ 3-25 m/s
- Fermi pseudo-potential: $V_F = \rho a \left(2\pi\hbar^2/M_N\right)$ [Ni ≈ 252 neV, Al ≈ 54 neV]

Wall-neutron-Wall

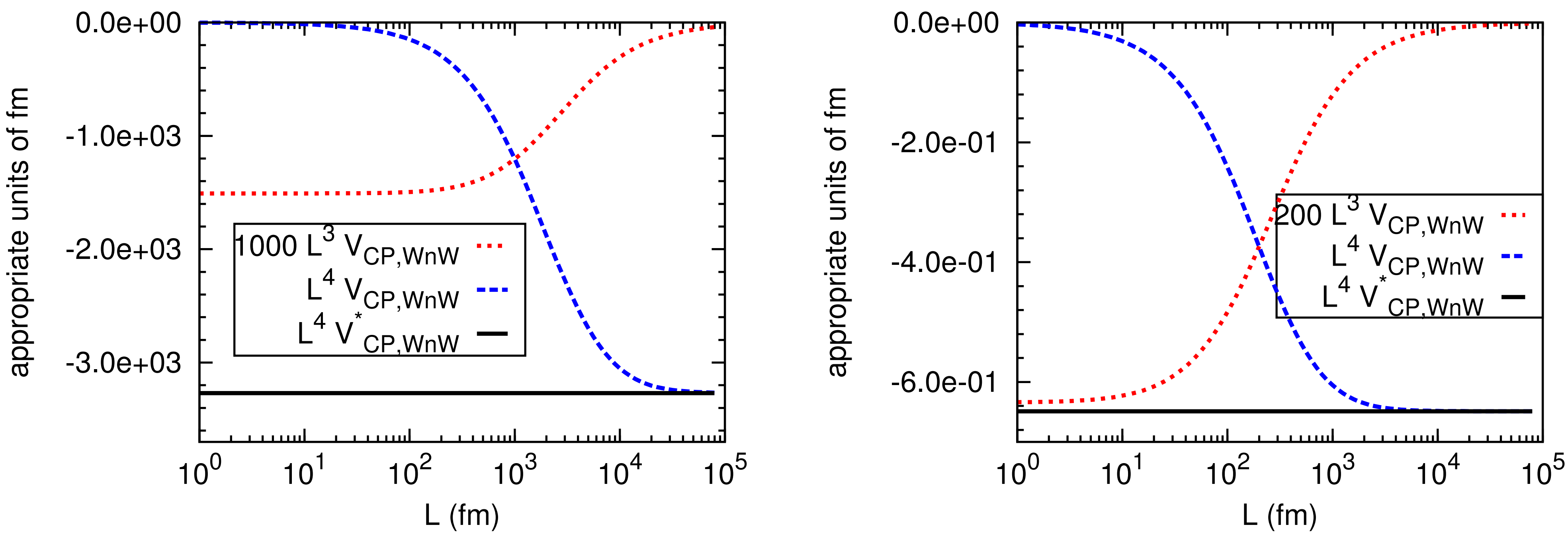
$$V_{CP,WnW}(z, L) = -\frac{1}{\pi L^3}\int_0^\infty dt \frac{t^2 \cosh\left(2tz/L\right)}{\sinh\left(t\right)} \int_0^{\frac{t}{\alpha_0 L}} d\omega\, \alpha(i\omega)$$
$$+ \frac{\alpha_0^2}{\pi L}\int_0^\infty d\omega\, \omega^2 \alpha(i\omega) \int_{\alpha_0 L\omega}^\infty dt \frac{e^{-t}}{\sinh\left(t\right)}$$
$$= -\frac{1}{\alpha_0 \pi L^4}\int_0^\infty u^3 du\, \alpha\left(i\frac{u}{\alpha_0 L}\right) \int_1^\infty \frac{dv}{\sinh(uv)} \left[v^2 \cosh\left(\frac{2z}{L}uv\right) - e^{-uv}\right]$$

$$-\frac{L}{2} \leq z \leq +\frac{L}{2}$$

Zhou & Spruch, PRA 52, 297 (95).

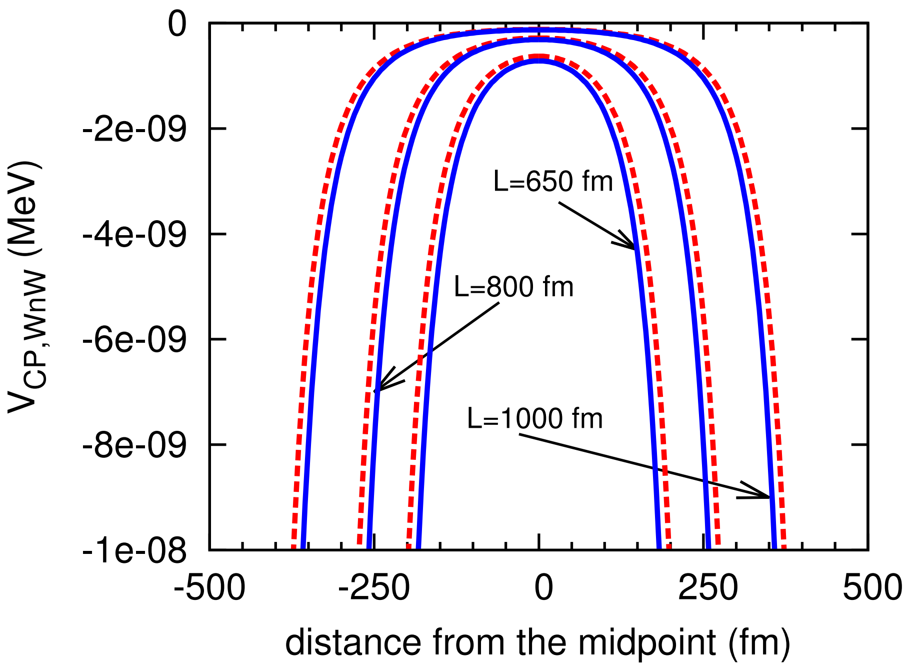
Kharchenko, Babb, Dalgarno, PRA 55, 3566 (97), for Na atoms.

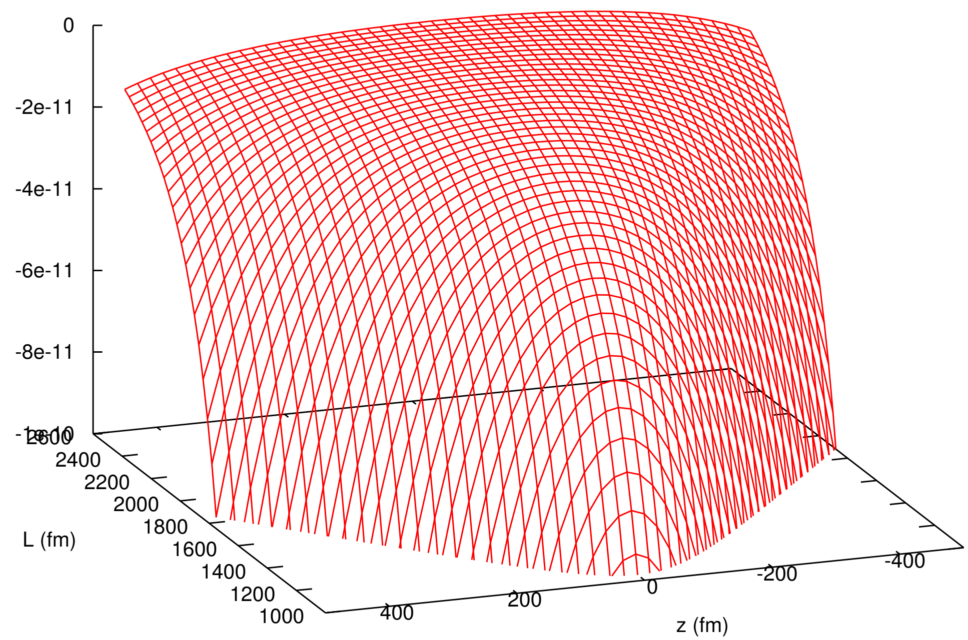
Wall-neutron-Wall



Left: $z = 0.45L$. Right: $z = 0.0L$.

Wall-neutron-Wall





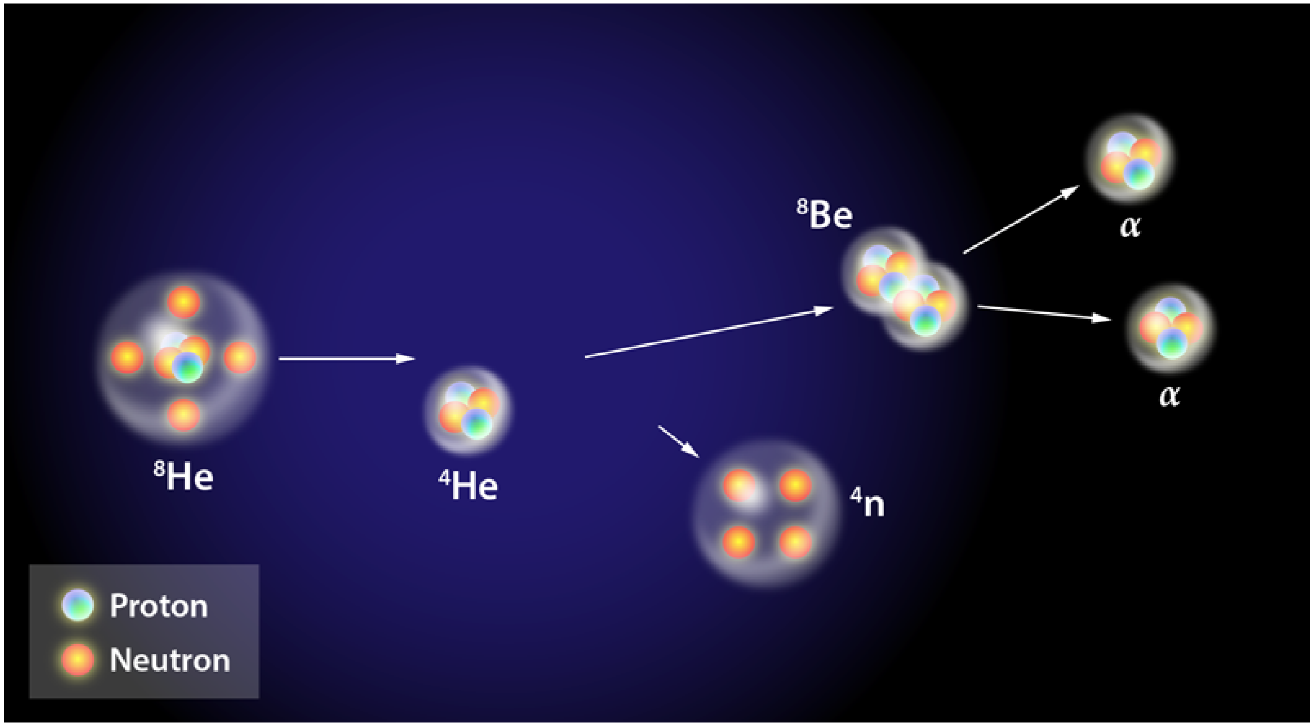
Tetraneutrons and nn VDW/CP interaction?

^{8}He + ^{4}He $\rightarrow 4n +$ ^{8}Be $\rightarrow 4n +$ ^{4}He + ^{4}He

K. Kisamori et al. Phys. Rev. Lett. **116**, 052501 (2016)
K. Hebeler et al., Constraints on Neutron Star Radii Based on Chiral Effective Field Theory Interactions, Phys. Rev. Lett. **105**, 161102 (2010).
J.P. Schiffer and R. Vandenbosch, Search for a Particle-Stable Tetra Neutron, Phys. Lett. **5**, 292 (1963).
F. M. Marqués et al., Detection of Neutron Clusters, Phys. Rev. C **65**, 044006 (2002).
Steven C. Pieper, Can Modern Nuclear Hamiltonians Tolerate a Bound Tetraneutron?, Phys. Rev. Lett. **90**, 252501 (2003),

Experiment indicates existence of 4n resonance cluster

Modern theory with usual inputs can not reproduce experiment



Would the addition of nn VDW/CP interactions in the theory alter the impasse?

Summary

- Casimir-Polder interactions: retardation effects, alters the van der Waals tail, zero-point energy
- Dipole polarizabilities: existence of a dipole-dipole dispersive interaction between neutrons [book of Rauch and Werner, *Neutron Interferometry* (Sec. 10.11)]
- Same book: neutron through a wire $\Rightarrow$ topological quantum phase
- Dipole polarizabilities: fit to RB-χEFT of Lensky *et al.*, up to the onset of Δ
- $\Rightarrow$ improvement over the arctan parametrization
- neutron-Wall and Wall-neutron-Wall: UCN, confinement in bottles, wires, etc.
- Perspectives: better modeling/extraction of dipole polarizabilities, magnetic moment interactions, quadrupole polarizabilities, Fermi pseudo-potential, etc.

Our fits:

	$\alpha_n(0)$ $(10^{-4}\mathrm{fm}^3)$	a_1 (MeV)	a_2 (MeV)	$\beta_n(0)$ $(10^{-4}\mathrm{fm}^3)$	b_1 (MeV)	b_2 (MeV)	ω_Δ (MeV)	Γ_Δ (MeV)
Set 1	13.9968	12.2648	1621.63	4.2612	8.33572	22.85	241.484	66.92 65
Set 2	11.6	2.2707	2721.47	3.7	8.67962	24.2003	241.593	68.3009
Set 3	12.5	5.91153	2118.79	2.7	9.27719	26.328	241.821	70.8674

Thank you