

# **Study of hadronic hydrogen-like atoms in DIRAC experiment at PS CERN**

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**Jagiellonian Symposium of Fundamental  
and Applied Subatomic Physics  
KRAKÓW, June 4 – 9 2017**

# ***DIRAC collaboration***

## ***DImeson Relativistic Atomic Complexes***



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JINR  
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SINP of Moscow State University  
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*Protvino, Russia*



Santiago de Compostela University  
*Santiago de Compostela, Spain*



Bern University  
*Bern, Switzerland*



Zurich University  
*Zurich, Switzerland*

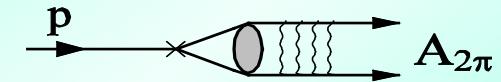
totally 68 physicists from 20 Institutes

# Production of pionium

**Atoms** are Coulomb bound state of two pions produced in one proton-nucleus collision

$$\frac{d\sigma_{nlm}^A}{d\vec{P}} = (2\pi)^3 \frac{E_A}{M_A} \left| \psi_{nlm}^{(C)}(0) \right|^2 \frac{d\sigma_s^0}{d\vec{p}_+ d\vec{p}_-} \Bigg|_{\vec{p}_+ = \vec{p}_-} \quad \sigma_A = k \sigma_C (Q < Q_0)$$

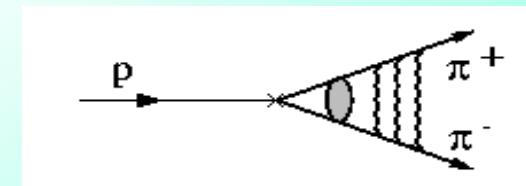
Nemenov 1985



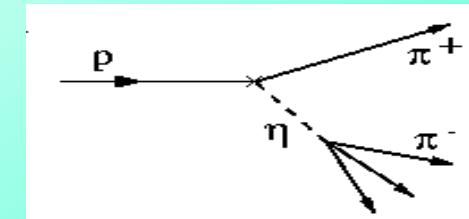
## Background processes:

**Coulomb pairs.** They are produced in one proton nucleus collision from fragmentation or short lived resonances and exhibit Coulomb interaction in the final state

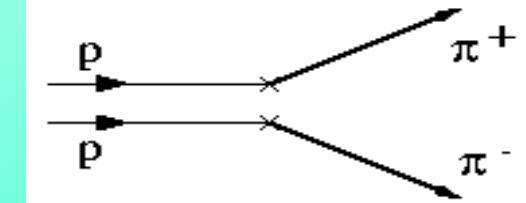
$$\frac{d^2\sigma_C}{d\vec{p}_+ d\vec{p}_-} = A_C(q) \frac{d\sigma_s^0}{d\vec{p}_+ d\vec{p}_-}, \quad A_C(q) = \frac{2\pi m_\pi \alpha / q}{1 - \exp(-2\pi m_\pi \alpha / q)}$$



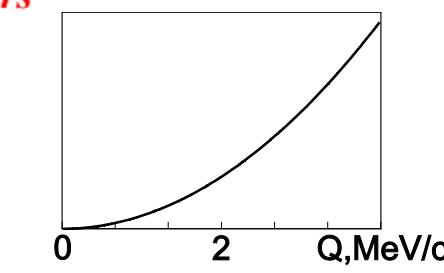
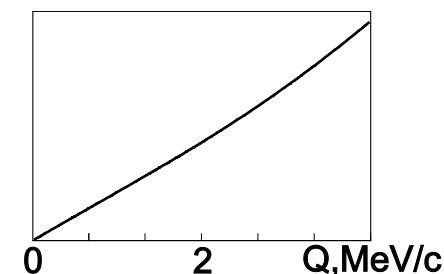
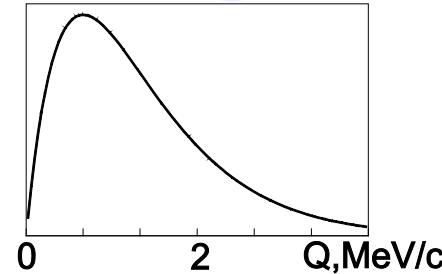
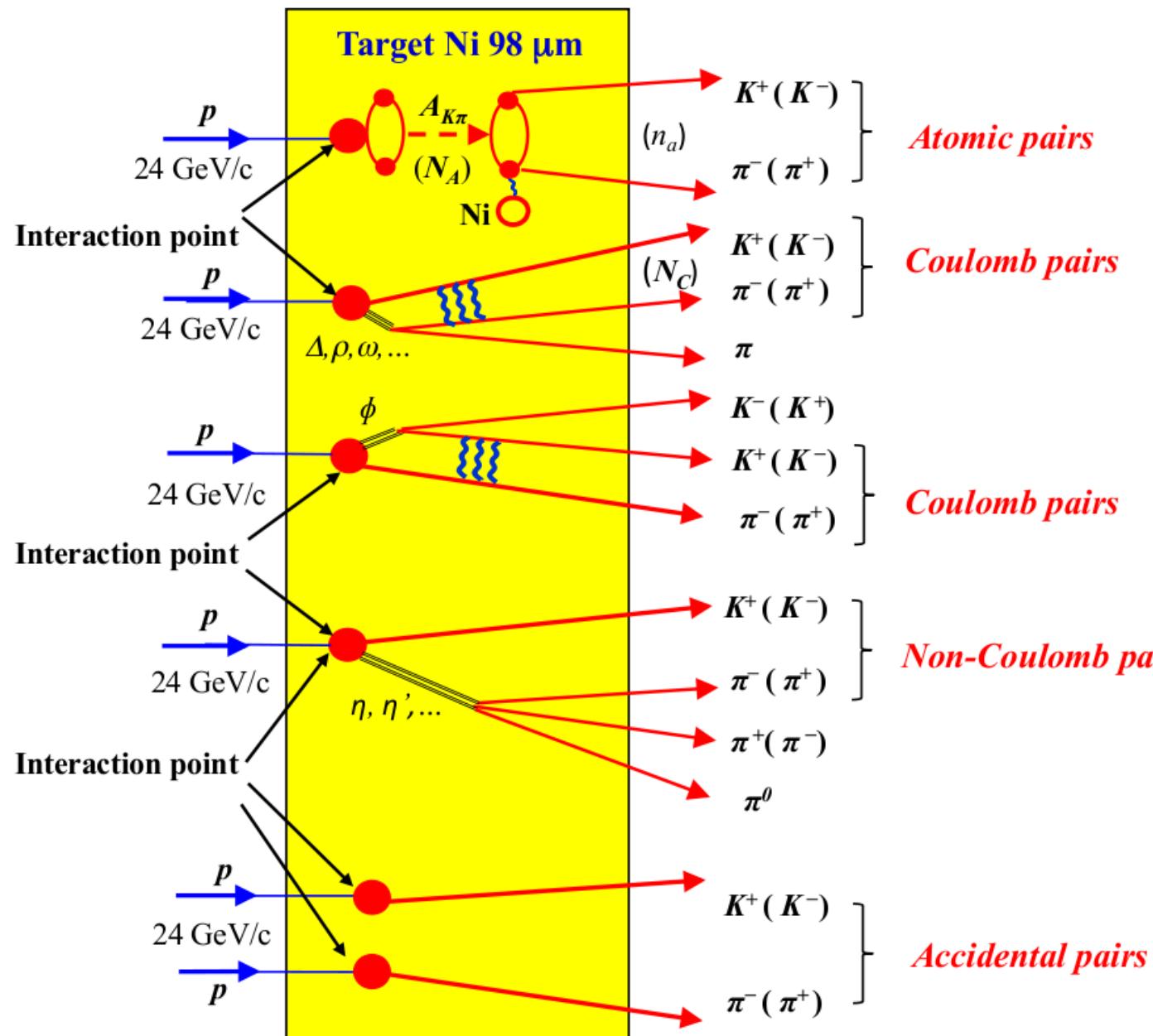
**Non-Coulomb pairs.** They are produced in one proton nucleus collision. At least one pion originates from a long lived resonance. No Coulomb interaction in the final state



**Accidental pairs.** They are produced in two independent proton nucleus collision. They do not exhibit Coulomb interaction in the final state



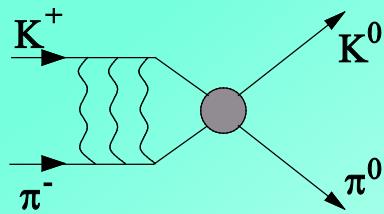
# Method of $\pi K$ ( $\pi\pi$ ) atom observation and investigation



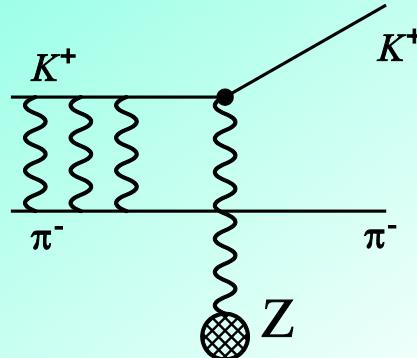
# Break-up probability

During propagation in matter atoms:

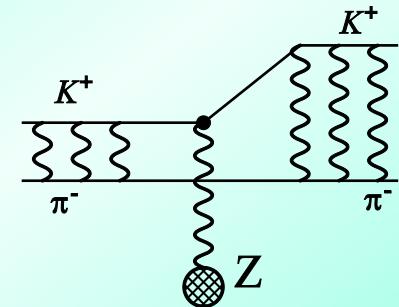
**annihilate**



**break up(ionized)**



**excite**



result in production of  $\pi K$  ( $\pi^+\pi^-$ ) atomic pairs  $n_A$

$$\frac{\sigma_A}{\sigma_C} = \frac{|\psi_{nlm}^{(C)}|^2}{|\psi_{\vec{q}}^{(C)}|^2}$$

$$N_A = K(Q_0)N_C(Q \leq Q_0), \frac{\delta K(Q_0)}{K(Q_0)} \leq 10^{-2}$$

$$n_A - \text{atomic pairs number}, \quad P_{br} = \frac{n_A}{N_A}$$

# Lifetime and breakup probability

The  $P_{br}$  value depends on the lifetime value,  $\tau$ . To obtain the precise  $P_{br}(\tau)$  curve a large differential equation system must be solved:

$$\frac{dp_{nlm}(s)}{ds} = \sum_{n'l'm'} a_{nlm}^{n'l'm'} p_{n'l'm'}(s)$$

where  $s$  is the position in the target,  $p_{nlm}$  is the population of a definite hydrogen-like state of pionium. The  $a_{nlm}^{n'l'm'}$  coefficients are given by:

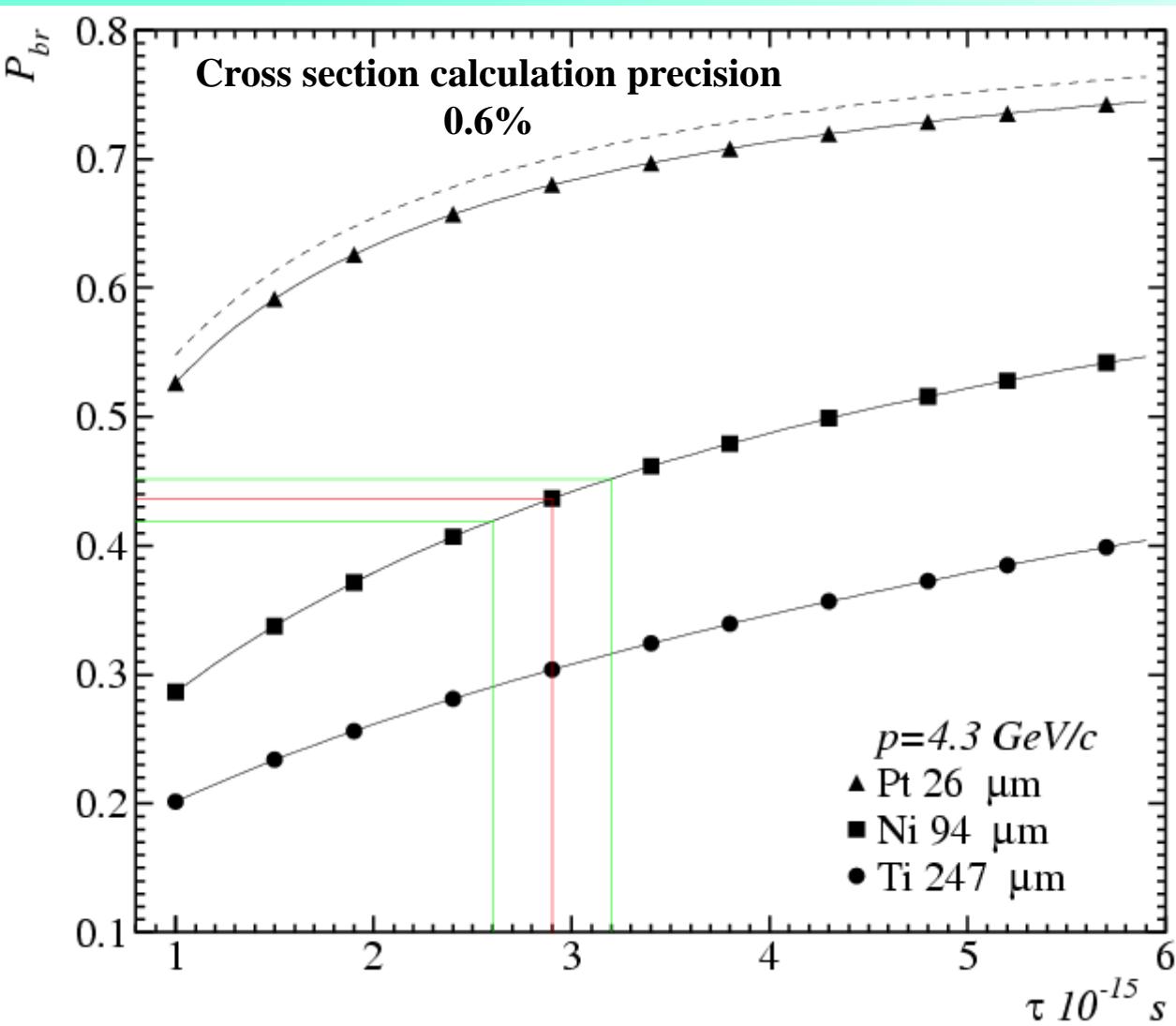
$$a_{nlm}^{n'l'm'} = \frac{\sigma_{nlm}^{n'l'm'} \rho N_0}{A} \quad \text{if } nlm \neq n'l'm', \quad a_{nlm}^{nlm} = -\frac{\sigma_{nlm}^{tot} \rho N_0}{A} - \begin{cases} 2M_\pi / P c \tau_n & l=0, \\ 0 & l \neq 0. \end{cases}$$

$\sigma_{nlm}^{n'l'm'}$  being the pionium-target atom cross section,  $N_0$  the Avogadro Number,  $\rho$  the material density and  $A$  its atomic weight.

The detailed knowledge of the cross sections (Afanashev&Tarasov, Trautmann et al) (Born and Glauber approach) together with the accurate solution of the differential equation system permits us to know the curves within 1%.

# Break-up probability

Solution of the transport equations provides one-to-one dependence of the measured break-up probability ( $P_{br}$ ) on pionium lifetime  $\tau$



$$\delta\tau=10\% \rightarrow \delta P_{br}=4\%$$

All targets have the same thickness in radiation lengths  $6.7 \times 10^{-3} X_0$

There is an optimal target material for a given lifetime

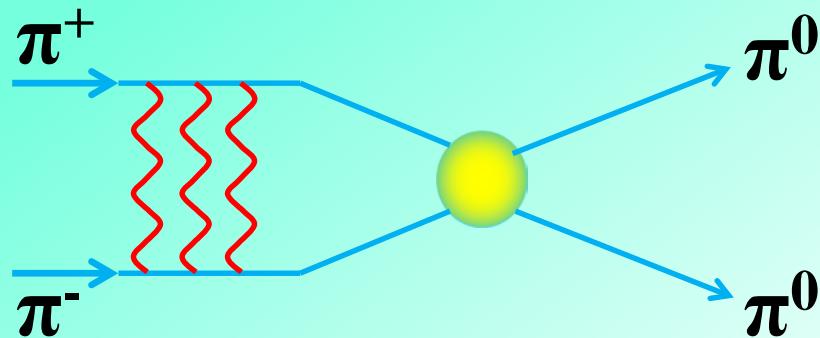
# $\pi^+\pi^-$ atom lifetime

$\pi^+\pi^-$  atom (pionium) is a hydrogen-like atom consisting of  $\pi^+$  and  $\pi^-$  mesons:

$$E_B = -1.86 \text{ keV},$$

$$r_B = 387 \text{ fm},$$

$$p_B \approx 0.5 \text{ MeV/c}$$



The lifetime of  $\pi^+\pi^-$  atom is dominated by the decay into  $\pi^0\pi^0$  mesons:

$$\Gamma = \frac{1}{\tau} = \Gamma_{2\pi^0} + \Gamma_{2\gamma} \quad \frac{\Gamma_{2\gamma}}{\Gamma_{2\pi^0}} \approx 4 \times 10^{-3}$$

$$\Gamma_{ns \rightarrow 2\pi^0} = R |\psi_{ns}(0)|^2 |a_0 - a_2|^2$$

$$\tau_{1s} = (2.9 \pm 0.1) \times 10^{-15} \text{ s}$$

$a_0$  and  $a_2$  are the  $\pi\pi$  s-wave scattering lengths for isospin  $I=0$  and

$I=2$ .

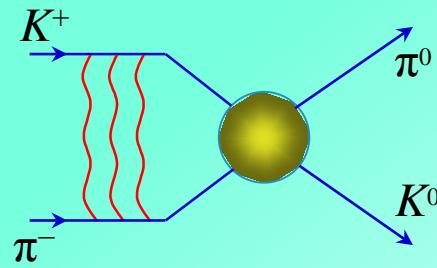
$$\psi_{nl} \begin{cases} \neq 0 \text{ for } l=0 & A_{2\pi}(1s, 2s, \dots, (n-1)s) \xrightarrow{\text{ }} \pi^0\pi^0 \\ = 0 \text{ for } l \neq 0 & A_{2\pi}(np) \xrightarrow{\gamma} A_{2\pi}(1s, 2s, \dots, (n-1)s) \xrightarrow{\text{ }} \pi^0\pi^0 \end{cases}$$

The lifetime of  $np$  states depends on transition  $np \xrightarrow{\text{ }} 1s, 2s, \dots, (n-1)s$  probability  
 This probability is about three orders less than  $ns \xrightarrow{\text{ }} \pi^0\pi^0$  decay into  $\pi^0\pi^0$

# $K^{\dagger}\pi$ and $K\pi^{\dagger}$ atoms lifetime

$K\pi$ -atom ( $A_{K\pi}$ ) is a hydrogen-like atom consisting of  $K^{\pm}$  and  $\pi^{\mp}$  mesons:

$$E_B = -2.9 \text{ keV} \quad r_B = 249 \text{ fm} \quad p_B = 0.79 \text{ MeV}$$



$$\frac{1}{\tau} = \frac{8}{9} \alpha^3 \mu^2 p^* (a_{1/2} - a_{3/2})^2 (1 + \delta_K)$$

The  $K\pi$ -atom lifetime ground state  $1S$ ,  $\tau = 1/\Gamma$  is dominated by the annihilation process into  $K^0\pi^0$ :

$$A_{K^+\pi^-} \rightarrow \pi^0 K^0$$

$$A_{\pi^+K^-} \rightarrow \pi^0 \bar{K}^0$$

$$\mu = 109 \text{ MeV}/c^2$$

$$p^* = 11.8 \text{ MeV}/c$$

$$\delta_k = 0.040 \pm 0.022$$

[S.Bilenky et al., Sov. J. Nucl. Phys. 10 (1969) 469]

[J. Schweizer, Phys. Lett. B 587 (2004) 33]

SU(3) ChPT predictions [J. Bijnens et al. JHEP 0405 (2004) 036]

$$\frac{1}{3} M_\pi (a_{1/2} - a_{1/3}) = M_\pi a_0^- = 0.071(CA) \rightarrow 0.079(1l) \rightarrow 0.89(2l) \quad [\text{P.Buttiker et al., Eur. Phys. J. C33 (2004) 409}]$$

$$\rightarrow 0.090 \pm 0.005(\text{dispersion}) \rightarrow \tau = (3.5 \pm 0.4) \times 10^{-15} \text{ s}$$

Lattice QCD calculations of ChPT low energy constant

[NPLQCD, Phys. Rev. D74 (2006) 114503]

[Z.Fu, Phys. Rev. D85 (2012) 074501]

[C.B. Lang et al., Phys. Rev. D86 (2012) 054508]

$$M_\pi a_0^- = 0.077 \pm 0.001^{+0.002}_{-0.005}$$

$$M_\pi a_0^- = 0.0777 \pm 0.0013 \pm ?$$

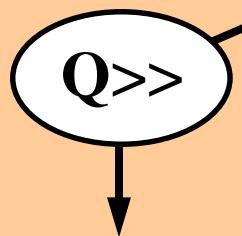
$$M_\pi a_0^- = 0.0811 \pm 0.0143$$

# Theoretical motivation



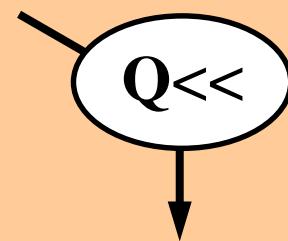
Strong interaction:  $L_{QCD} = L_{sym} + L_{sym-break}$  ( $m_q \neq 0$ )

HIGH energy  
(small distance)



(chiral symmetry)

LOW energy  
(large distance)



**perturbative QCD:**

$$L_{QCD}(q,g)$$

Interaction → „weak“ (asympt. freedom)  
Method: expansion in coupling

Checks only  $L_{sym}$  ( $m_q \ll 0$ ) !

**non-perturbative QCD:**

$$L_{eff}(\text{GB: } \pi, K, \eta); L_{lattice}(q,g)$$

Interaction → „strong“ (confinement)  
Methods: 1) Chiral Perturbation Theory  
2) Lattice Gauge Theory

Checks  $L_{sym}$  as well as  $L_{sym-break}$  !

spontaneously broken symmetry

quark-condensate

# Theoretical motivation

## $\pi\pi$ scattering length

In ChPT the effective Lagrangian, which describes the  $\pi\pi$  interaction, is an expansion in terms:

$$L_{\text{eff}} = \underset{\text{(tree)}}{L^{(2)}} + \underset{\text{(1-loop)}}{L^{(4)}} + \underset{\text{(2-loop)}}{L^{(6)}} + \square$$

G. Colangelo, J. Gasser and H. Leutwyler, Nucl. Phys. B603 (2001) 125,  
using ChPT (2-loop) & Roy equations:

$$\left. \begin{array}{l} a_0 = 0.220 \pm 2.3\% \\ a_2 = -0.0444 \pm 2.3\% \end{array} \right\} a_0 - a_2 = 0.265 \pm 1.5\%$$

These results (precision) depend on the low-energy constants (LEC)  $\bar{l}_3$  and  $\bar{l}_4$ :  
Lattice gauge calculations from 2006 provided values for these  $\bar{l}_3$  and  $\bar{l}_4$ .

# Theoretical motivation

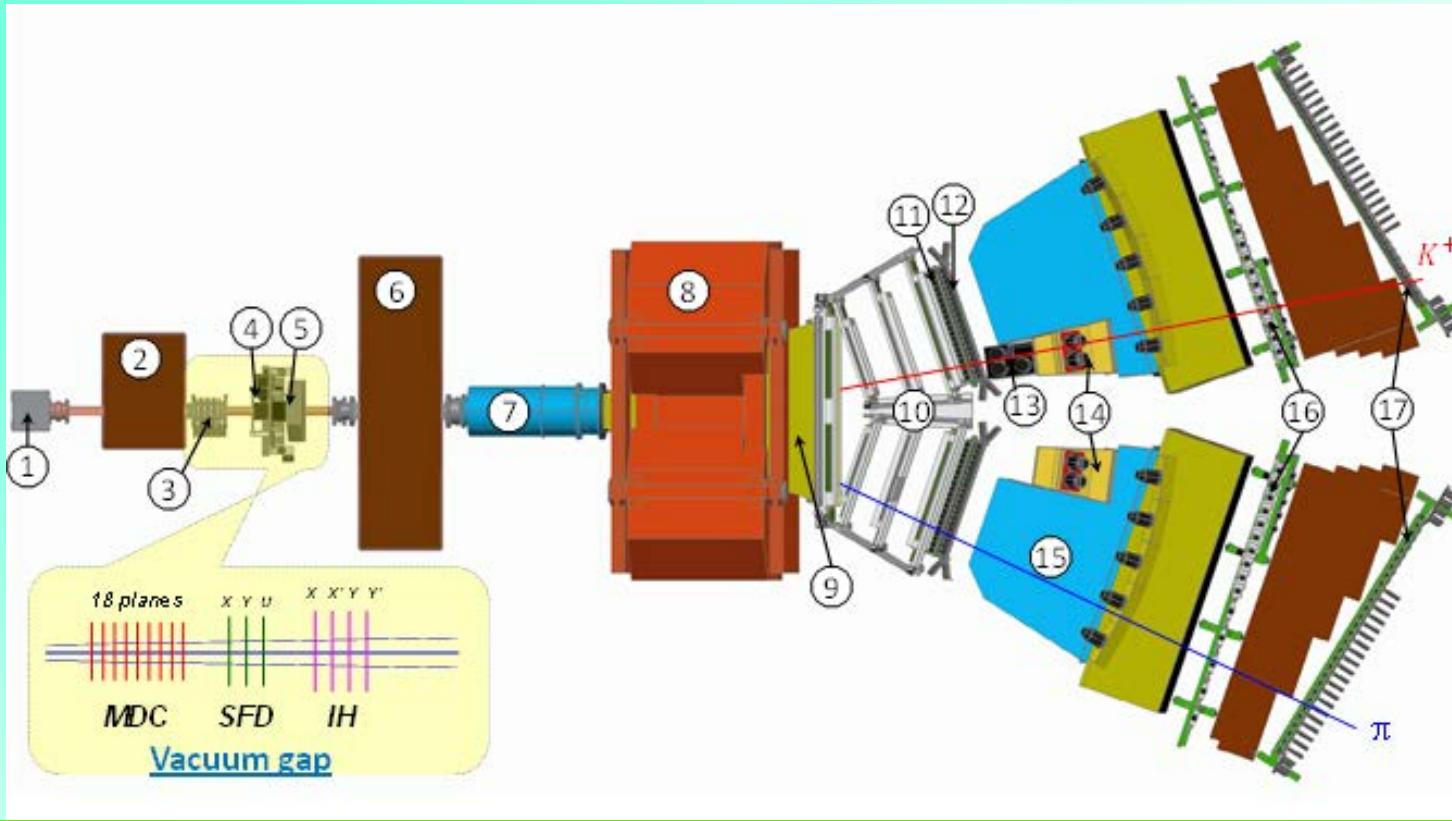
## Lattice calculations of $\bar{l}_3, \bar{l}_4$

- 2006:  $\bar{l}_3, \bar{l}_4$  ... first lattice calculations
- 2012: 10 collaborations: 3 in USA, 5 in Europe and 2 in Japan
- J. Gasser, H. Leutwyler: model calculation (1985)  
 $\bar{l}_3 = 2.9 \pm 2.4, \quad \bar{l}_4 = 4.3 \pm 0.9$
- Lattice calculations of these constants have been done in 20 works.  
Best result:  $l_3 = 2.6 \pm 0.5 \text{st} \pm 0.4 \text{sy}, \quad l_4 = 3.8 \pm 0.4 \text{st} \pm 0.2 \text{sy}$

Therefore, the theoretical pion-pion scattering length precision can be improved.

The best experimental results on the scattering length have a precision not better than 4%.

# Experimental setup



1 Target station with Ni foil; 2 First shielding; 3 Micro Drift Chambers;  
4 Scintillating Fiber Detector; 5 Ionization Hodoscope; 6 Second Shielding;  
7 Vacuum Tube; 8 Spectrometer Magnet; 9 Vacuum Chamber; 10 Drift  
Chambers; 11 Vertical Hodoscope; 12 Horizontal Hodoscope; 13 Aerogel  
Čerenkov; 14 Heavy Gas Čerenkov; 15 Nitrogen Čerenkov; 16 Preshower;  
17 Muon Detector

# Spectrometer resolutions

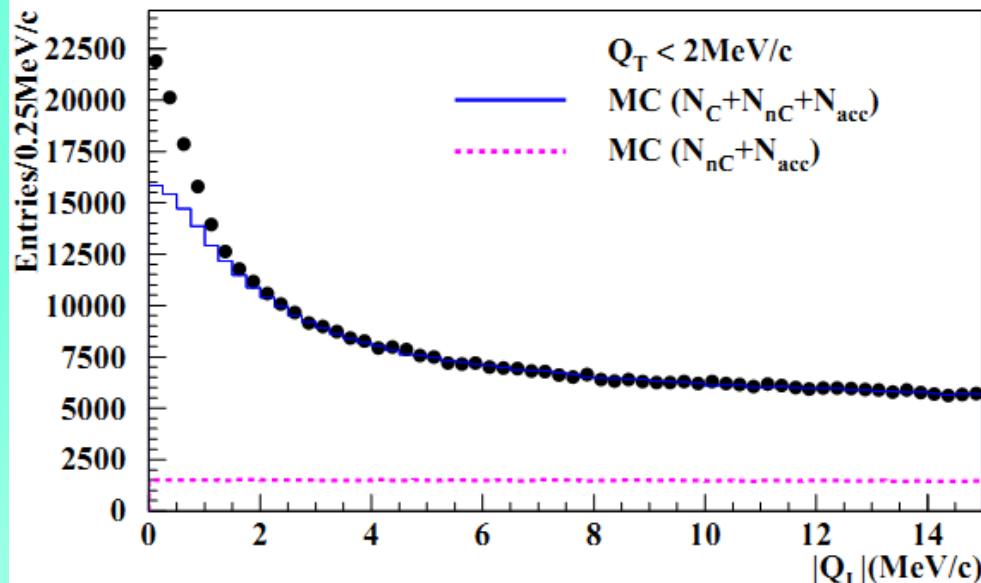
SFD			
Coordinate precision	$\sigma_X = 60 \mu\text{m}$	$\sigma_Y = 60 \mu\text{m}$	$\sigma_W = 120 \mu\text{m}$
Time precision	$\sigma^t_X = 380 \text{ ps}$	$\sigma^t_Y = 512 \text{ ps}$	$\sigma^t_W = 522 \text{ ps}$

DC	VH
Coordinate precision $\sigma = 85 \mu\text{m}$	Time precision $\sigma = 100 \text{ ps}$

Spectrometer		
Relative resolution on the particle momentum in L.S.		$3 \cdot 10^{-3}$
Precision on Q-projections	$\sigma_{QX} = \sigma_{QY} = 0.5 \text{ MeV/c}$	$\sigma_{QL} = 0.5 \text{ MeV/c } (\pi\pi)$ $\sigma_{QL} = 0.9 \text{ MeV/c } (\pi K)$

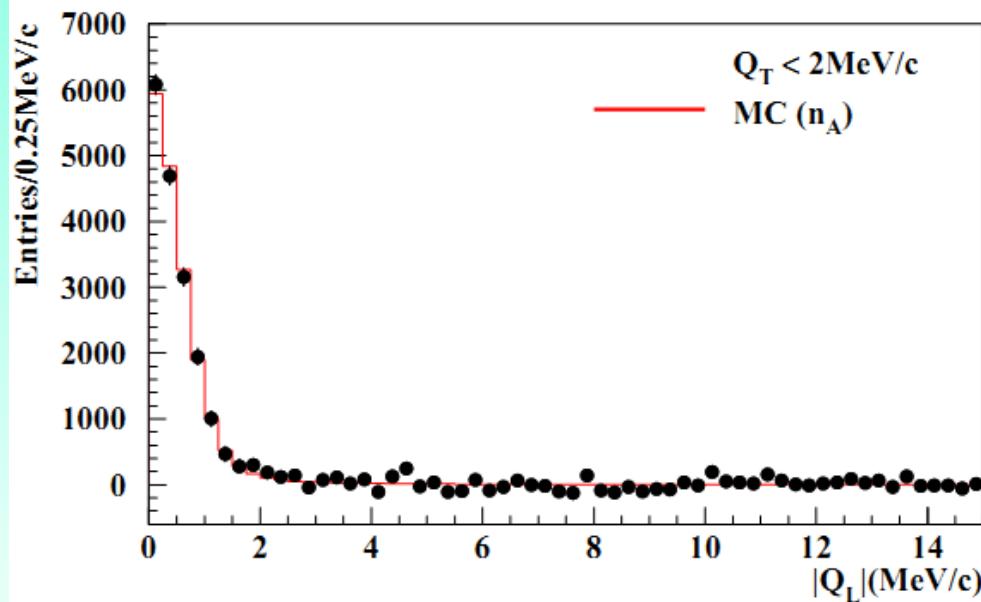
Trigger efficiency 98 % for pairs with  $Q_L < 28 \text{ MeV/c}$   
 $Q_X < 6 \text{ MeV/c}$   
 $Q_Y < 4 \text{ MeV/c}$

# $\pi^+\pi^-$ atoms 2001-2003



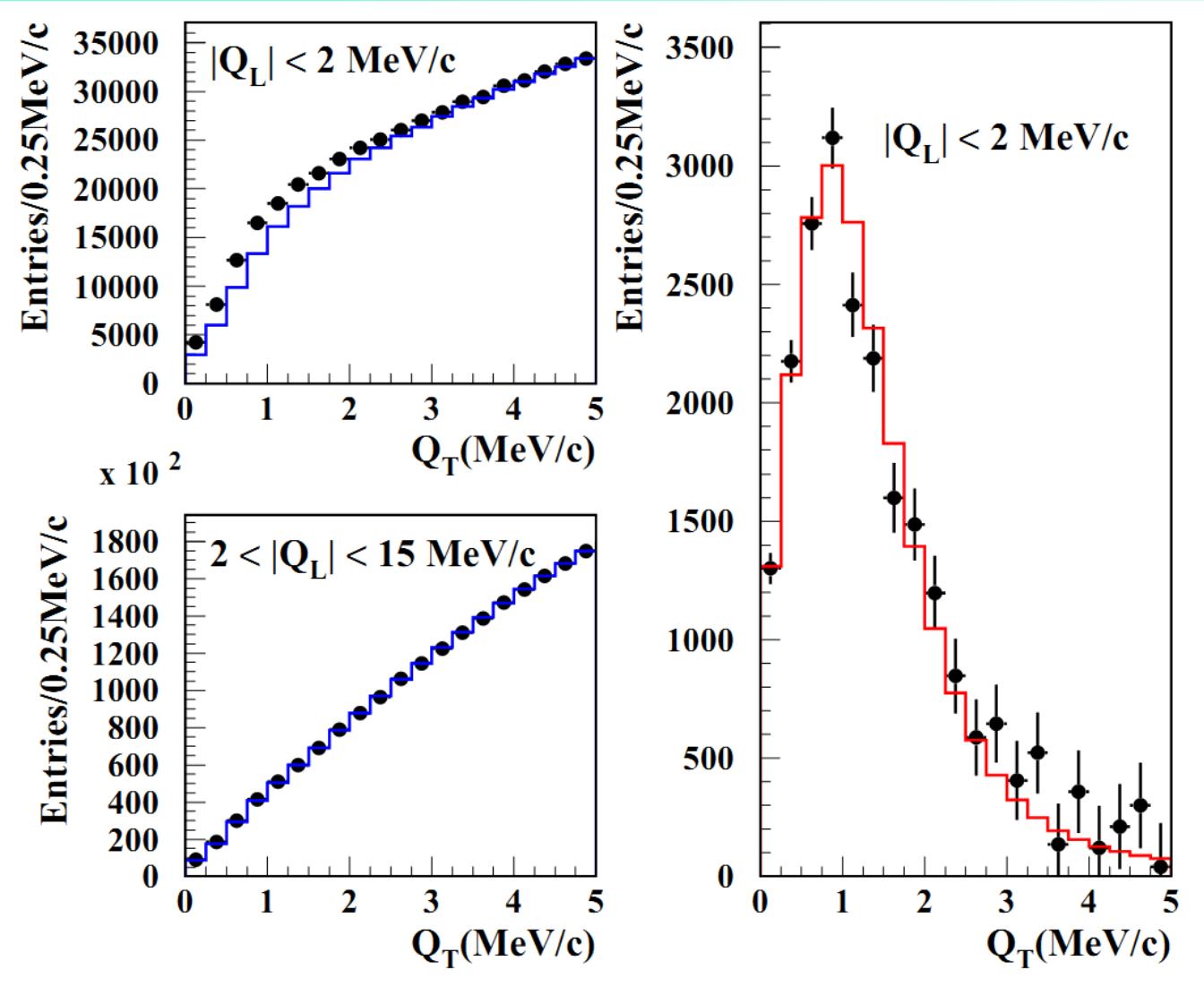
$Q_L$  distribution

← All events



← After background subtraction

# $\pi^+\pi^-$ atoms 2001-2003



$Q_T$  distribution

← After  
background  
subtraction  
for  
 $Q_L < 2 \text{ MeV}/c$

# $\pi^+\pi^-$ atoms 2001-2003

Ni, $p_{\text{beam}}$	$\chi^2/\text{ndf}$	$n_A$	$N_C$	$N_{nC}$	$N_{\text{acc}}$	$P_{\text{br}}$
94 $\mu\text{m}$ , 24 GeV/c	2127/2079	$6020 \pm 216$	$546003 \pm 4549$	$45624 \pm 4501$	$63212 \pm 208$	$0.441 \pm 0.018$
98 $\mu\text{m}$ , 24 GeV/c	4288/4149	$9321 \pm 274$	$828554 \pm 5811$	$93148 \pm 5754$	$98499 \pm 255$	$0.452 \pm 0.015$
98 $\mu\text{m}$ , 20 GeV/c	4257/4144	$5886 \pm 210$	$496820 \pm 4441$	$60867 \pm 4397$	$59392 \pm 144$	$0.472 \pm 0.020$
combined samples		$21227 \pm 407$	$1871377 \pm 8613$	$199639 \pm 8526$	$221103 \pm 359$	

DIRAC data	$\tau_{1s} (10^{-15} \text{ s})$					$ a_0 - a_2 $					Reference
	value	stat	syst	<i>theo</i> *	tot	value	stat	syst	<i>theo</i> *	tot	
2001	<b>2.91</b> $^{+0.45}_{-0.38}$ $^{+0.19}_{-0.49}$ $\left[ ^{+0.49}_{-0.62} \right]$					<b>0.264</b> $^{+0.017}_{-0.020}$ $^{+0.022}_{-0.009}$ $\left[ ^{+0.033}_{-0.020} \right]$					PL B 619 (2005) 50
2001-03	<b>3.15</b> $^{+0.20}_{-0.19}$ $^{+0.20}_{-0.18}$ $\left[ ^{+0.28}_{-0.26} \right]$					<b>0.2533</b> $^{+0.0078}_{-0.0080}$ $^{+0.0072}_{-0.0077}$ $\left[ ^{+0.0106}_{-0.0111} \right]$					PL B 704 (2011) 24

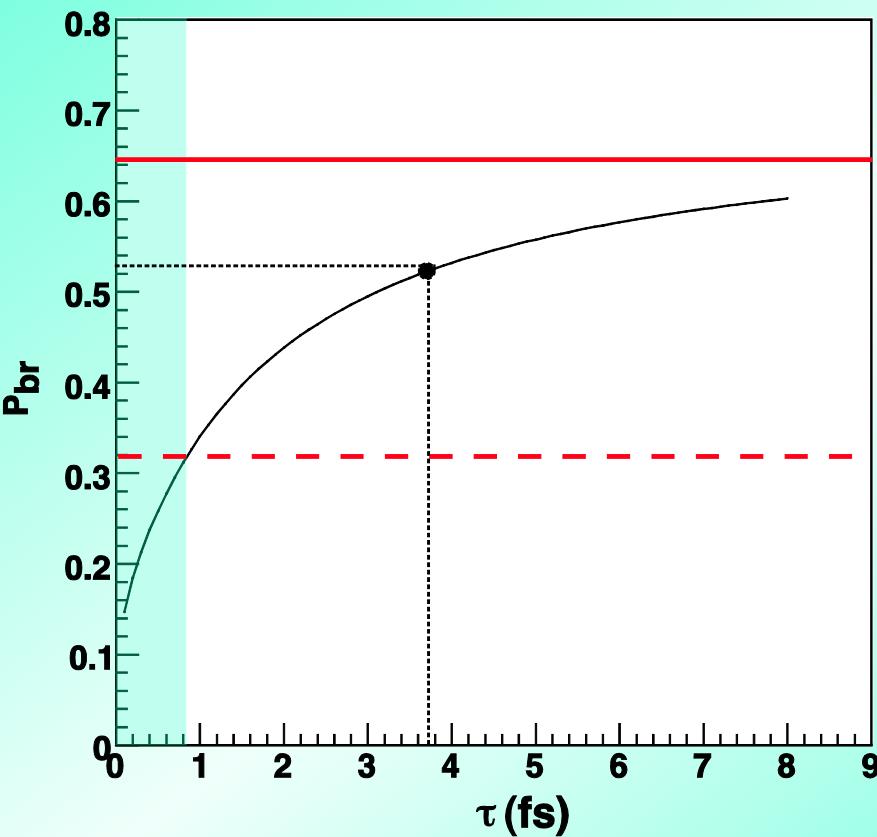
# *First evidence for $\pi K$ atoms*

2007, Platinum target 28 $\mu\text{m}$ :

$$n_A(\pi^- K^+) = 143 \pm 53, \quad n_A(\pi^+ K^-) = 29 \pm 15$$

**Evidence for  $\pi K$  -atoms observation with DIRAC**

[Adeva et al. (DIRAC Collaboration) Phys. Lett. B674 (2009) 11]



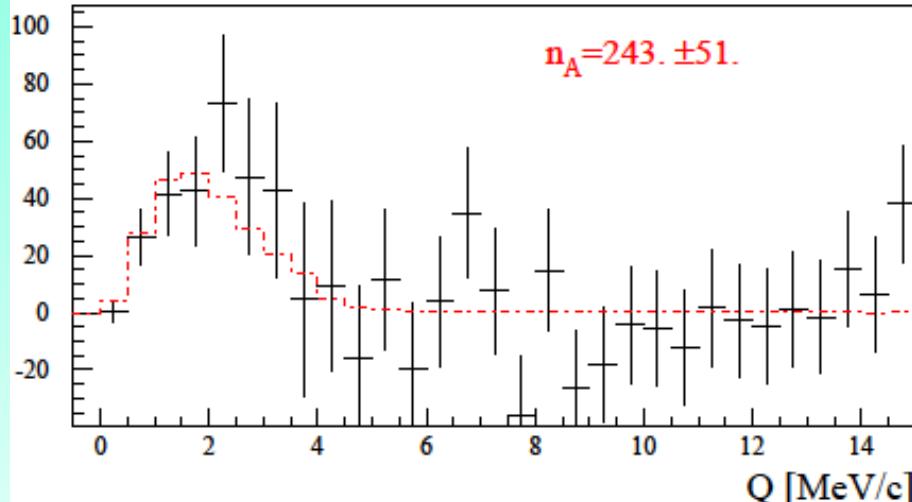
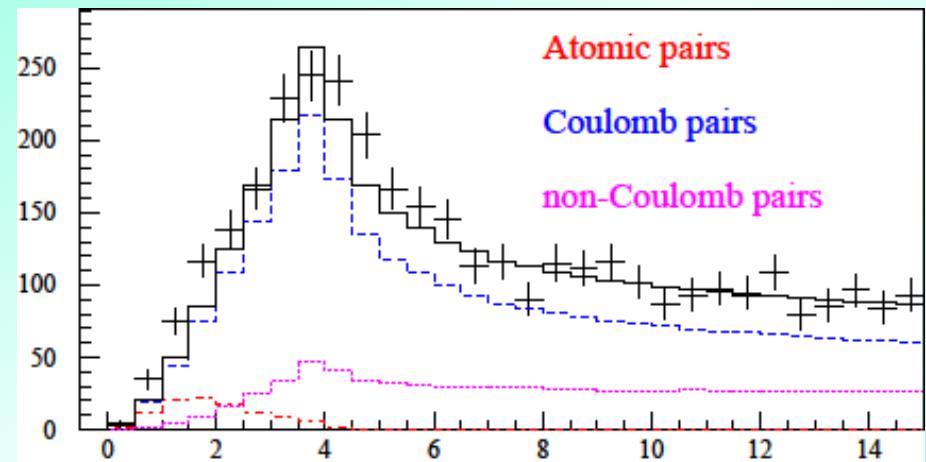
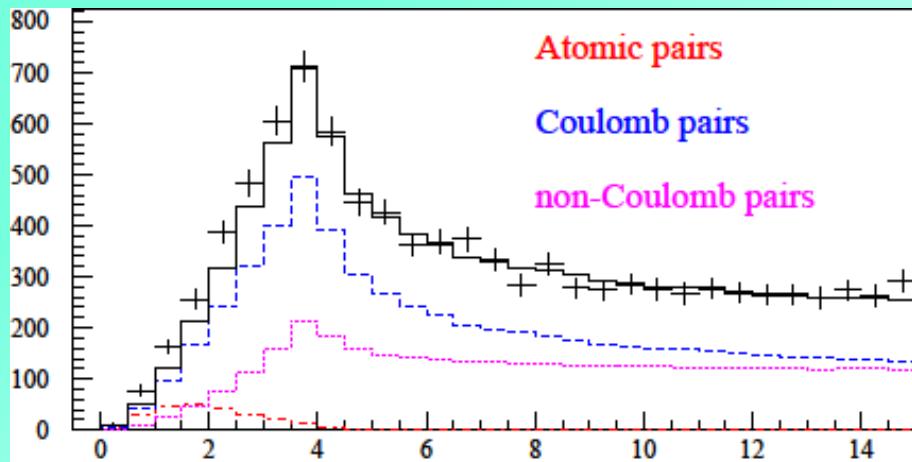
$$n_A(\pi^+ K^- + \pi^- K^+) = 173 \pm 54 \text{ (} 3.2\sigma \text{)}$$

$$N_A(\pi^+ K^- + \pi^- K^+) = k N_c = 280 \pm 70$$

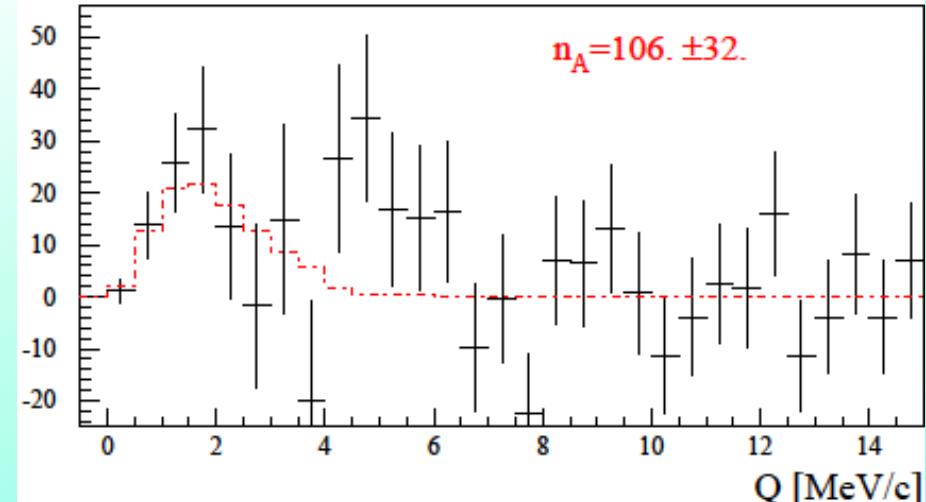
$$\tau > 0.8 \times 10^{-15} \text{ s (CL=0.9)}$$

# $\pi K$ atoms observation

All data Platinum and Nickel targets

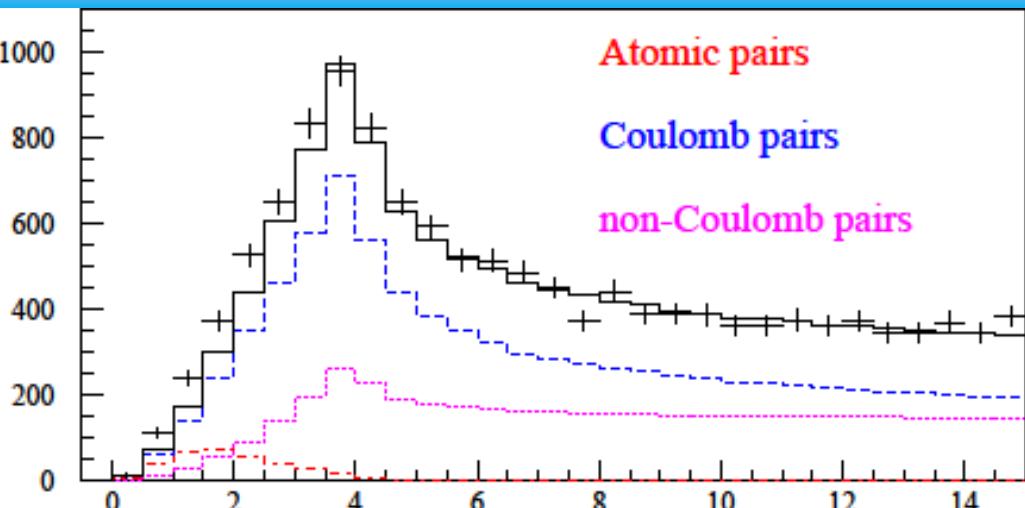


$K^+\pi^-$  atoms  
Q distribution for  $Q_T < 4$  MeV/c

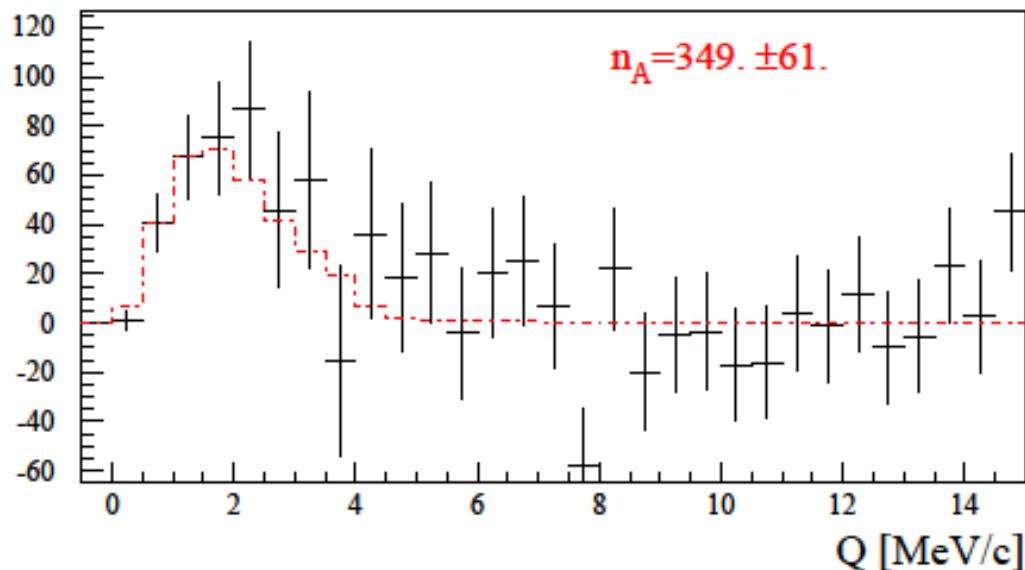


$K^-\pi^+$  atoms  
Q distribution for  $Q_T < 4$  MeV/c

# $\pi K$ atoms observation



All data Platinum and Nickel targets



$K^+\pi^-$  and  $K^-\pi^+$  atoms ,  $Q_L$  distribution for  $Q_T < 4$  MeV/c  $\chi^2 / \text{ndf} = 41/37$   
In absense of “atomic pairs”  $\chi^2 / \text{ndf} = 73/38$

# $K^+ \pi^-$ and $K^- \pi^+$ pairs analysis

Analysis	$\pi^- K^+$	$\pi^+ K^-$	$\pi^+ K^- + \pi^- K^+$
Q	$243 \pm 51$ ( $4.7\sigma$ )	$106 \pm 32$ ( $3.3\sigma$ )	$349 \pm 61$ ( $5.7\sigma$ )
$ Q_L $	$164 \pm 79$ ( $2.1\sigma$ )	$67 \pm 47$ ( $1.4\sigma$ )	$230 \pm 92$ ( $2.5\sigma$ )
$ Q_L , Q_T$	$237 \pm 50$ ( $4.7\sigma$ )	$78 \pm 32$ ( $2.5\sigma$ )	$314 \pm 59$ ( $5.3\sigma$ )

Analysis with  $|Q_L|, Q_T$

$$n_A = 314 \pm 59(\text{stat}) \pm 10(\text{syst}) = 314 \pm 60(\text{tot})$$

5.2 standard deviations

Analysis with  $Q$

$$n_A = 349 \pm 61(\text{stat}) \pm 9(\text{syst}) = 349 \pm 62(\text{tot})$$

5.6 standard deviations

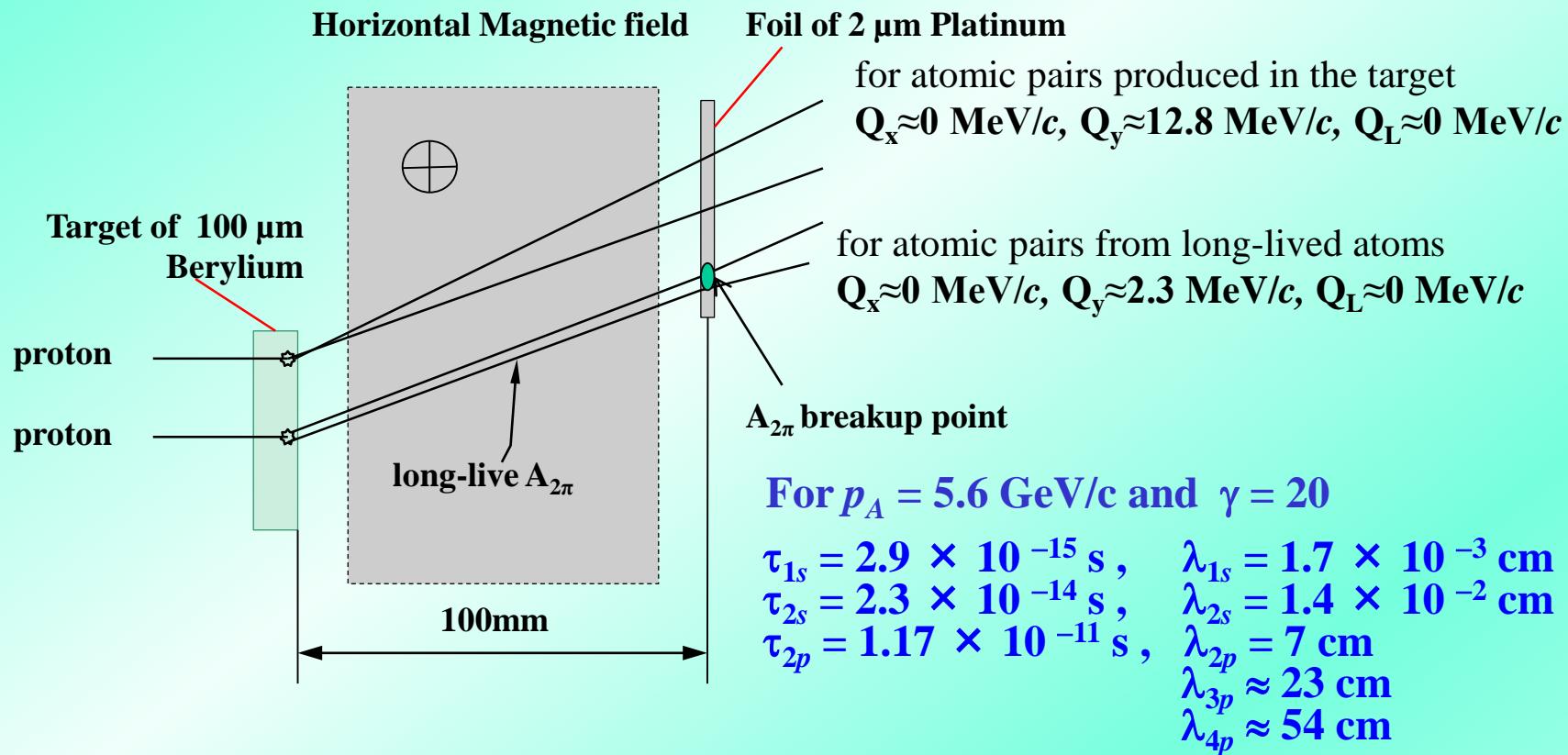
[DIRAC, Physical Review Letters 117, 112001 (2016)  
CERN-EP-2016-128 ; arXiv:1605.06103]

Preliminary

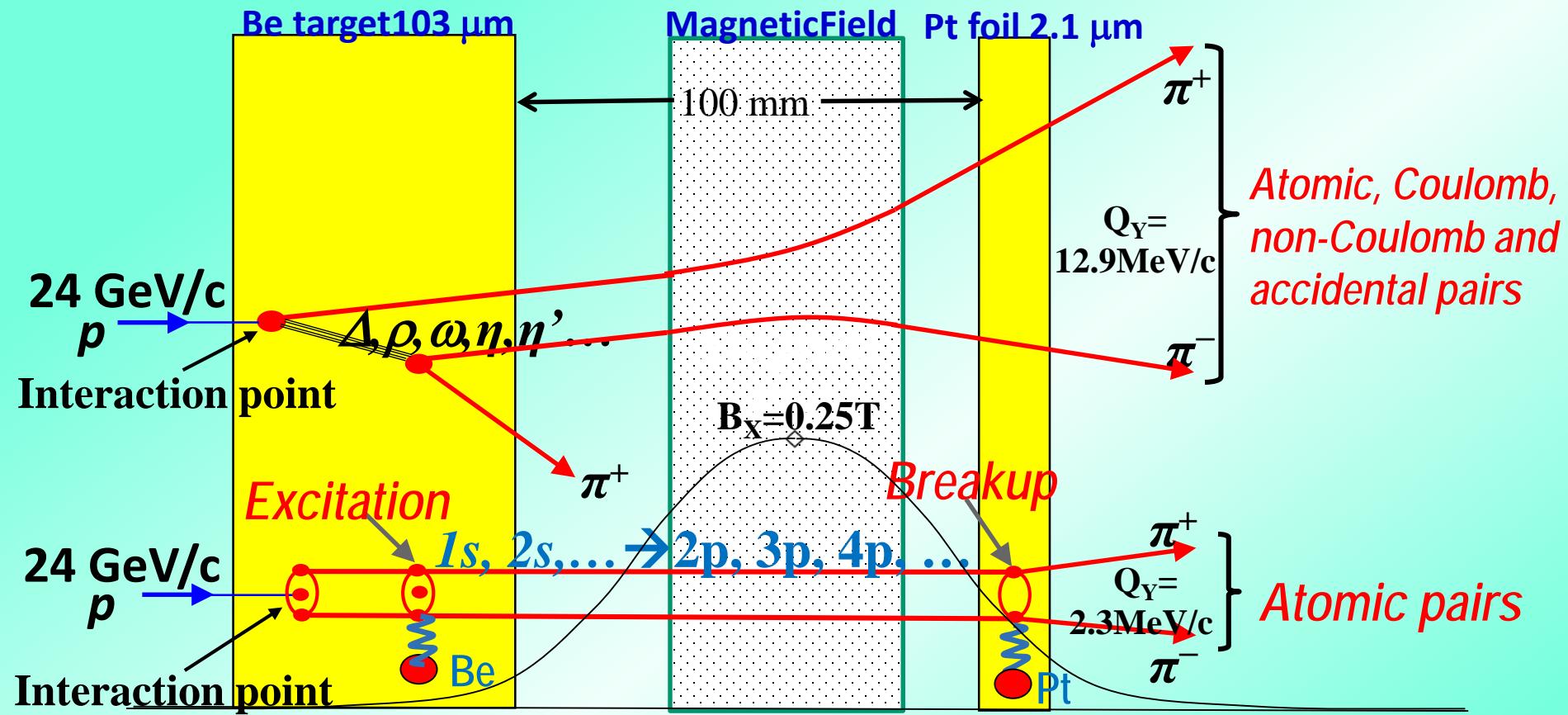
$$\tau = (5.5^{+5.0}_{-2.8}|\text{tot}) \text{ fs} \quad 1/3 |a_{1/2} - a_{3/2}| = (0.072^{+0.031}_{-0.020}) M_{\pi^+}^{-1}$$

# *Search for long-lived states of $\pi^+\pi^-$ atoms*

*During 2011-2012 the data were collected for observation of the long-lived states of  $\pi^+\pi^-$  atom. This observation opens the future possibility to measure the energy difference between ns and np states  $\Delta E(ns-np)$  and the value of  $\pi\pi$  scattering length combination  $|2a_0+a_2|$ .*



# Method for observing long-lived $\pi^+\pi^-$ atom with breakup Pt foil



for  
 $\gamma = 16$

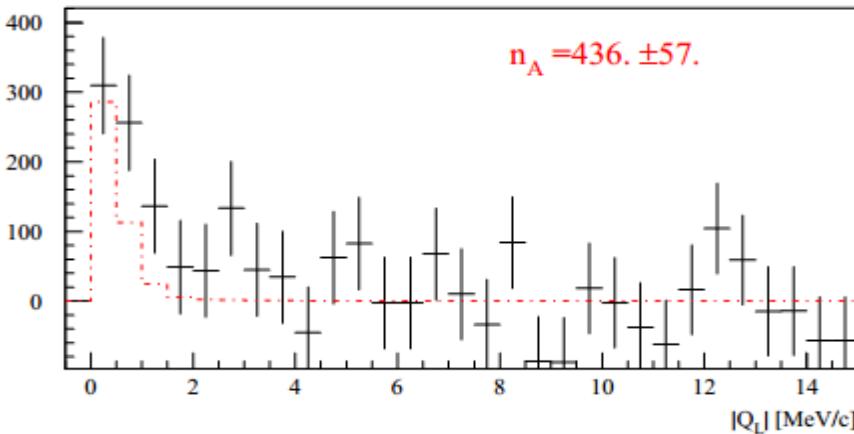
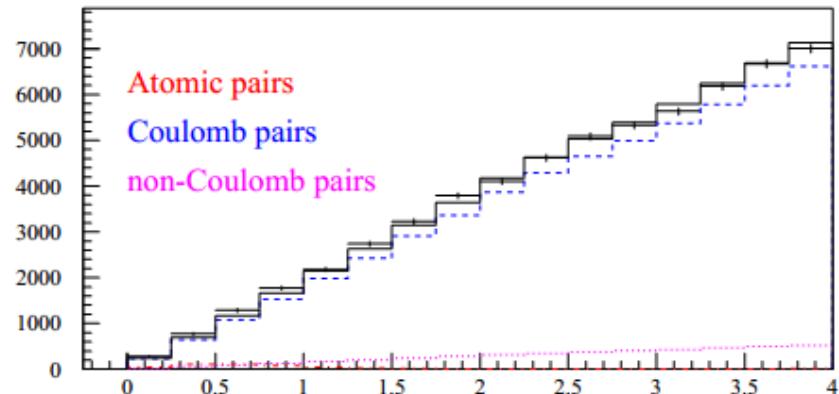
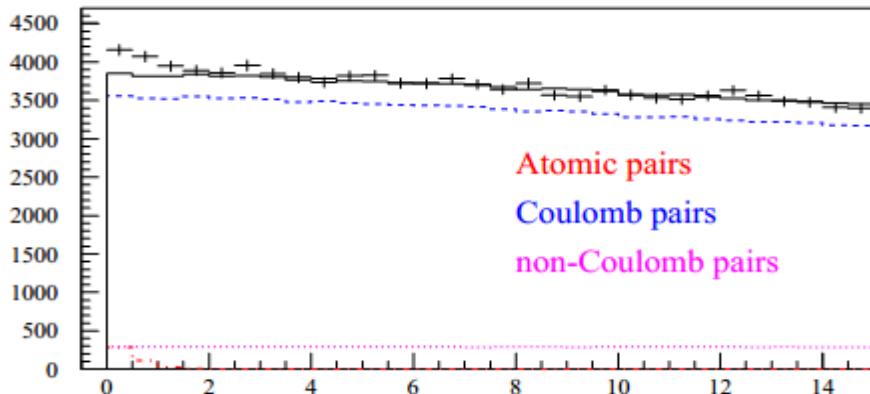
$$l(2p) = 5.6 \text{ cm}, l(3p) = 19 \text{ cm}, l(4p) = 43 \text{ cm}, l(5p) = 84 \text{ cm}$$

$$l(2s) = 0.11 \text{ mm}, l(3s) = 0.38 \text{ mm}, l(4s) = 0.89 \text{ mm}, l(5s) = 1.74 \text{ mm}$$

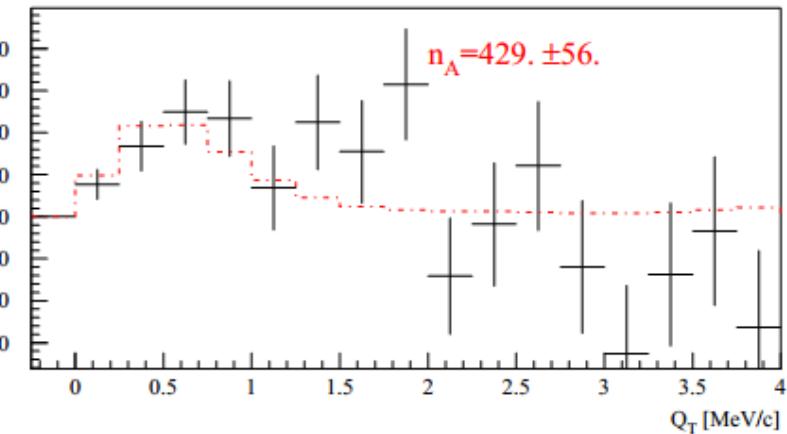
n	2	3	4	5	>2
$\varepsilon_n(\text{Be}) \times 10^2$	$2.48 \pm 0(10^{-3})$	$1.54 \pm 0.01$	$0.86 \pm 0.03$	$0.56 \pm 0.06$	$7.1 \pm 0.8$
$\varepsilon_n(\text{Pt}) \times 10^2$	$0.52 \pm 0(10^{-4})$	$1.10 \pm 0(10^{-3})$	$0.78 \pm 0.03$	$0.54 \pm 0.06$	$4.6 \pm 0.8$

# *Observation of long-lived $\pi^+\pi^-$ atoms*

Two-dimensional distribution over  $|Q_L| Q_T$ , have been fitted with  $\chi^2/\text{ndf} = 138/140$ . Projections to  $|Q_L|$  and  $Q_T$  are presented.



$|Q_L|$  for  $Q_T < 2.0$  MeV/c



$Q_T$  for  $|Q_L| < 2.0$  MeV/c

# *Observation of long-lived $\pi^+\pi^-$ atoms*

$Q_T$ cut	$n_A$	$n_A^{tot}$	Background	$\chi^2/ndf$
$Q_T < 2.0 \text{ MeV}/c$	$436 \pm 57$ $(\sim 7.6\sigma)$	$488 \pm 64$	16719	138/140

1-dimensional fit over  $Q_L$

$Q_T < 0.5 \text{ MeV}/c$	$152 \pm 29$ $(\sim 5.2\sigma)$	$467 \pm 88$	971	29/27
$Q_T < 1.0 \text{ MeV}/c$	$349 \pm 53$ $(\sim 6.6\sigma)$	$489 \pm 75$	3692	19/27
$Q_T < 1.5 \text{ MeV}/c$	$386 \pm 78$ $(\sim 4.9\sigma)$	$454 \pm 91$	9302	22/27
$Q_T < 2.0 \text{ MeV}/c$	$442 \pm 105$ $(\sim 4.2\sigma)$	$495 \pm 117$	16774	22/27

# *Observation of long-lived $\pi^+\pi^-$ atoms*

Systematic errors of number of long-lived “atomic pairs”

Sources of systematic errors	$\sigma^{\text{syst}}$
Uncertainty in correction on $\Lambda$ -width	4.4
Uncertainty of Platinum foil thickness	22
Total	23

$$n_A^L = 436 \pm 57(\text{stat}) \pm 23(\text{syst}) = 436 \pm 61(\text{tot})$$

Expected number  $\rightarrow 653 \pm 110$  ( $453 \div 845$ )

B.Adeva et al., Phys. Lett. B 751 (2015) 12

# **Experiment DIRAC at SPS CERN**

*In 2013 DIRAC setup has been dismantled from the experimental hall of PS CERN. All detectors are stored for using in the future experiment.*

*DIRAC collaboration is planning to continue investigation of  $\pi^-K^+$ ,  $\pi^+K^-$  and  $\pi^+\pi^-$  atoms at SPS accelerator at CERN. The correspondent gains in production rates of these atoms at SPS relative to PS (450 GeV vs. 24 GeV) are 18, 24 and 12. This allows to increase significantly the collected data and to check the precise prediction of Low-Energy QCD at a higher accuracy. Now the collaboration is planning to submit the Letter of Intend for study  $\pi K$  and  $\pi^+\pi^-$  atoms at SPS to SPSC CERN.*

**Thank you  
for your attention!**

# $\pi^+\pi^-$ experimental results

**K $\rightarrow$ 3 $\pi$ :**

(scattering length in  $m_\pi^{-1}$ )

2009 NA48/2 (EPJ C64, 589)

$$\Rightarrow a_0 - a_2 = 0.2571 \pm 0.0048 \Big|_{stat} \pm 0.0025 \Big|_{syst} \pm 0.0014 \Big|_{ext} = \dots \pm 2.2\%$$

**Ke4:**

plus additional 3.4% theory uncertainty

2010 NA48/2 (EPJ C70, 635)

$$\Rightarrow a_0 = 0.2220 \pm 0.0128 \Big|_{stat} \pm 0.0050 \Big|_{syst} \pm 0.0037 \Big|_{theo} = \dots \pm 6.4\%$$

$$\Rightarrow a_2 = -0.0432 \pm 0.0086 \Big|_{stat} \pm 0.0034 \Big|_{syst} \pm 0.0028 \Big|_{theo} = \dots \pm 22\%$$

**$\pi^+\pi^-$  atom:**

2011 DIRAC (PLB 704, 24)

$$\Rightarrow |a_0 - a_2| = 0.2533 \begin{array}{l} +0.0078 \\ -0.0080 \end{array} \Big|_{stat} \begin{array}{l} +0.0072 \\ -0.0077 \end{array} \Big|_{syst} = \dots \begin{array}{l} +4.2\% \\ -4.4\% \end{array}$$