
Testing discrete symmetries in transitions with entangled neutral kaons



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Testing CPT: introduction

The three discrete symmetries of QM, C (charge conjugation: $q \rightarrow -q$), P (parity: $x \rightarrow -x$), and T (time reversal: $t \rightarrow -t$) are known to be violated in nature both singly and in pairs. Only CPT appears to be an exact symmetry of nature.

CPT theorem holds for any QFT formulated on flat space-time which assumes:
(1) Lorentz invariance (2) Locality (3) Unitarity (i.e. conservation of probability).

Extension of CPT theorem to a theory of quantum gravity far from obvious.
(e.g. CPT violation appears in several QG models)

huge effort in the last decades to study and shed light on QG phenomenology
 \Rightarrow Phenomenological CPTV parameters to be constrained by experiments

Consequences of CPT symmetry: equality of masses, lifetimes, $|q|$ and $|\mu|$ of a particle and its anti-particle.

Neutral meson systems offer unique possibilities to test CPT invariance; e.g. taking as figure of merit the fractional difference between the masses of a particle and its anti-particle:

neutral K system	neutral B system	proton- anti-proton
$\left m_{K^0} - m_{\bar{K}^0} \right / m_K < 10^{-18}$	$\left m_{B^0} - m_{\bar{B}^0} \right / m_B < 10^{-14}$	$\left m_p - m_{\bar{p}} \right / m_p < 10^{-8}$

Other interesting CPT tests: e.g. the study of anti-hydrogen atoms, etc..

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neutral B system

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proton- anti-proton

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Neutral kaons

$$|K_{S,L}\rangle \propto \left[(1 + \varepsilon_{S,L}) |K^0\rangle \pm (1 - \varepsilon_{S,L}) |\bar{K}^0\rangle \right]$$

CP violation:

$$\varepsilon_{S,L} = \varepsilon \pm \delta$$

T violation:

$$\varepsilon = \frac{H_{12} - H_{21}}{2(\lambda_S - \lambda_L)} = \frac{-i\Im M_{12} - \Im \Gamma_{12}/2}{\Delta m + i\Delta\Gamma/2}$$

CPT violation:

$$\delta = \frac{H_{11} - H_{22}}{2(\lambda_S - \lambda_L)} = \frac{1}{2} \frac{(m_{\bar{K}^0} - m_{K^0}) - (i/2)(\Gamma_{\bar{K}^0} - \Gamma_{K^0})}{\Delta m + i\Delta\Gamma/2}$$

- $\delta \neq 0$ implies CPT violation
- $\varepsilon \neq 0$ implies T violation
- $\varepsilon \neq 0$ or $\delta \neq 0$ implies CP violation

$$\Delta m = m_L - m_S \quad , \quad \Delta\Gamma = \Gamma_S - \Gamma_L$$

$$\Delta m = 3.5 \times 10^{-15} \text{ GeV}$$

$$(\text{with a phase convention } \Im \Gamma_{12} = 0) \quad \Delta\Gamma \approx \Gamma_S \approx 2\Delta m = 7 \times 10^{-15} \text{ GeV}$$

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huge amplification factor!!

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- $\varepsilon \neq 0$ implies T violation
- $\varepsilon \neq 0$ or $\delta \neq 0$ implies CP violation

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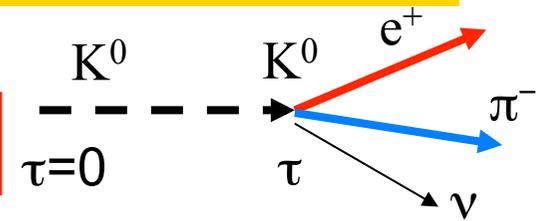
neutral kaons vs other oscillating meson systems

	$\langle m \rangle$ (GeV)	Δm (GeV)	$\langle \Gamma \rangle$ (GeV)	$\Delta \Gamma / 2$ (GeV)
K^0	0.5	3×10^{-15}	3×10^{-15}	3×10^{-15}
D^0	1.9	6×10^{-15}	2×10^{-12}	1×10^{-14}
B^0_d	5.3	3×10^{-13}	4×10^{-13}	$O(10^{-15})$ (SM prediction)
B^0_s	5.4	1×10^{-11}	4×10^{-13}	3×10^{-14}

“Standard” CPT test

Comparing “survival” probabilities of K^0 and \bar{K}^0 measuring semileptonic decays vs time:

$$\Re\delta = (3.0 \pm 3.3 \pm 0.6) \times 10^{-4}$$



CPLEAR

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using the unitarity constraint
(Bell-Steinberger relation)

$$\text{Im } \delta = (-0.7 \pm 1.4) \times 10^{-5}$$

$$2\Im\delta = \Im[\langle K_L | K_S \rangle] = \Im \left[\frac{\sum_f \langle f | T | K_S \rangle \langle f | T | K_L \rangle^*}{i(\lambda_S - \lambda_L^*)} \right]$$

PDG fit (2014)

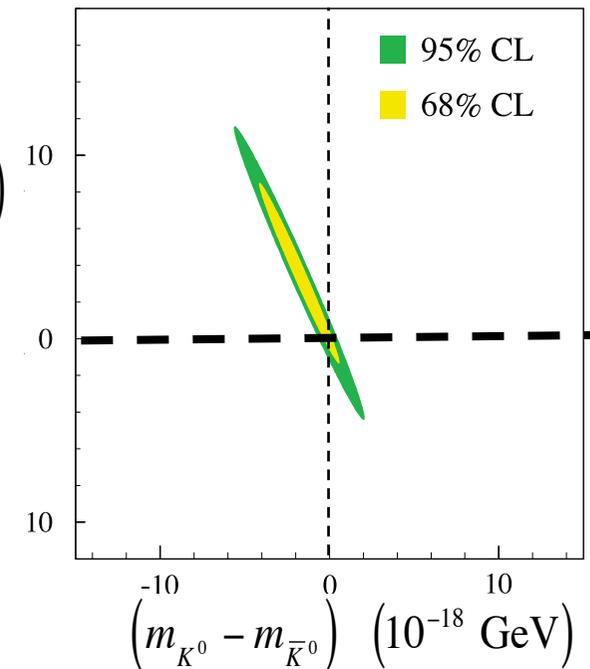
$$\delta = \frac{1}{2} \frac{(m_{\bar{K}^0} - m_{K^0}) - (i/2)(\Gamma_{\bar{K}^0} - \Gamma_{K^0})}{\Delta m + i\Delta\Gamma/2}$$

$$\begin{aligned} &(\Gamma_{K^0} - \Gamma_{\bar{K}^0}) \\ &(\text{in } 10^{-18} \text{ GeV}) \end{aligned}$$

Combining $\text{Re}\delta$ and $\text{Im}\delta$ results

Assuming $(\Gamma_{\bar{K}^0} - \Gamma_{K^0}) = 0$, i.e. no CPT viol. in decay:

$$|m_{\bar{K}^0} - m_{K^0}| < 4.0 \times 10^{-19} \text{ GeV} \quad \text{at 95\% c.l.}$$



Direct CPT test in transitions

Motivations:

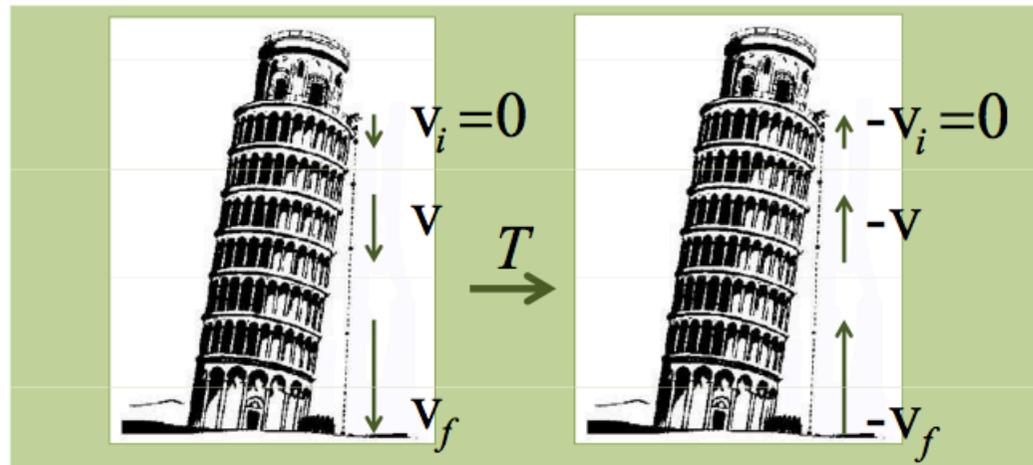
- test the CPT symmetry directly in transition processes between kaon states, rather than comparing masses, lifetimes, or other intrinsic properties of particle and anti-particle states.
- CPT violating effects may not appear at first order in diagonal mass terms (survival probabilities) while they can manifest at first order in transitions (non-diagonal terms).
- In standard WWA the test is related to $\text{Re}\delta$, a genuine CPT violating effect independent of $\Delta\Gamma$, i.e. not requiring the decay as an essential ingredient.
- Clean formulation required. Possible spurious effects induced by CP violation in the decay and/or a violation of the $\Delta S = \Delta Q$ rule have to be well under control.

Probing CPT: J. Bernabeu, A.D.D., P. Villanueva, JHEP 10 (2015) 139

Time-reversal violation: J. Bernabeu, A.D.D., P. Villanueva, NPB 868 (2013) 102

Time Reversal

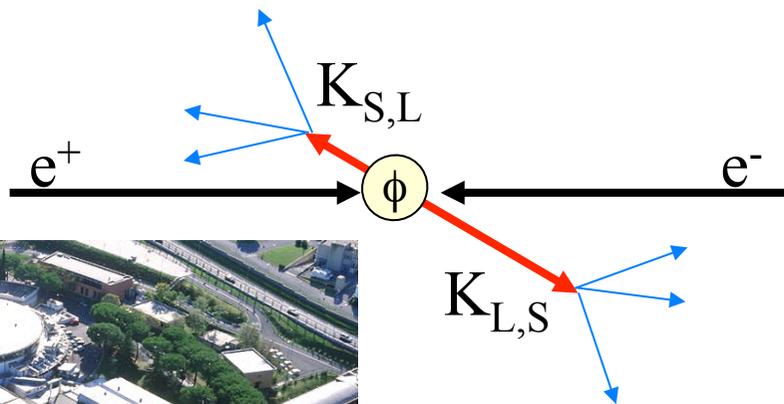
- The transformation of a system corresponding to the inversion of the time coordinate, the formal substitution $t \rightarrow -t$, is usually called ‘**time reversal**’, but a more appropriate name would actually be **motion reversal**.



- Exchange of in \leftrightarrow out states and reversal of all momenta and spins tests time reversal, i.e. the symmetry of the responsible dynamics for the observed process under time reversal $t \rightarrow -t$ (transformation implemented in QM by an antiunitary operator)
- Similarly for CPT tests: the exchange of in \leftrightarrow out states etc.. is required.

Quantum entanglement as a tool

- The in \leftrightarrow out states inversion required in a DIRECT test of these symmetries can be performed exploiting the properties of the quantum entanglement.
- In maximally entangled systems the complete knowledge of the system as a whole is encoded in the state, no information on single subsystems is available.
- Once a measurement is performed on one subsystem, then the information is immediately transferred to its partner; the result of an analogous measurement on it is determined.



$$\begin{aligned}
 |i\rangle &= \frac{1}{\sqrt{2}} \left[|K^0(\vec{p})\rangle |\bar{K}^0(-\vec{p})\rangle - |\bar{K}^0(\vec{p})\rangle |K^0(-\vec{p})\rangle \right] \\
 &= \frac{N}{\sqrt{2}} \left[|K_S(\vec{p})\rangle |K_L(-\vec{p})\rangle - |K_L(\vec{p})\rangle |K_S(-\vec{p})\rangle \right]
 \end{aligned}$$

$$N = \sqrt{\frac{(1 + |\varepsilon_S|^2)(1 + |\varepsilon_L|^2)}{(1 - \varepsilon_S \varepsilon_L)}} \cong 1$$



DAFNE collider

Definition of states

Let us also consider the states $|K_+\rangle$, $|K_-\rangle$ defined as follows: $|K_+\rangle$ is the state filtered by the decay into $\pi\pi$ ($\pi^+\pi^+$ or $\pi^0\pi^0$), a pure CP = +1 state; analogously $|K_-\rangle$ is the state filtered by the decay into $3\pi^0$, a pure CP = -1 state. Their orthogonal states correspond to the states which cannot decay into $\pi\pi$ or $3\pi^0$, defined, respectively, as

$$\begin{aligned} |\tilde{K}_-\rangle &\equiv \tilde{N}_- [|K_L\rangle - \eta_{\pi\pi} |K_S\rangle] \\ |\tilde{K}_+\rangle &\equiv \tilde{N}_+ [|K_S\rangle - \eta_{3\pi^0} |K_L\rangle] \end{aligned}$$

$$\begin{aligned} \eta_{\pi\pi} &= \frac{\langle \pi\pi | T | K_L \rangle}{\langle \pi\pi | T | K_S \rangle} \\ \eta_{3\pi^0} &= \frac{\langle 3\pi^0 | T | K_S \rangle}{\langle 3\pi^0 | T | K_L \rangle} \end{aligned}$$

Orthogonal bases: $\{K_+, \tilde{K}_-\}$ $\{\tilde{K}_+, K_-\}$

Even though the decay products are orthogonal, the filtered $|K_+\rangle$ and $|K_-\rangle$ states can still be nonorthogonal.

Condition of orthogonality:

$$\eta_{\pi\pi} + \eta_{3\pi^0}^* = \epsilon_L + \epsilon_S^* \longrightarrow \begin{aligned} |K_+\rangle &\equiv |\tilde{K}_+\rangle \\ |K_-\rangle &\equiv |\tilde{K}_-\rangle \end{aligned}$$

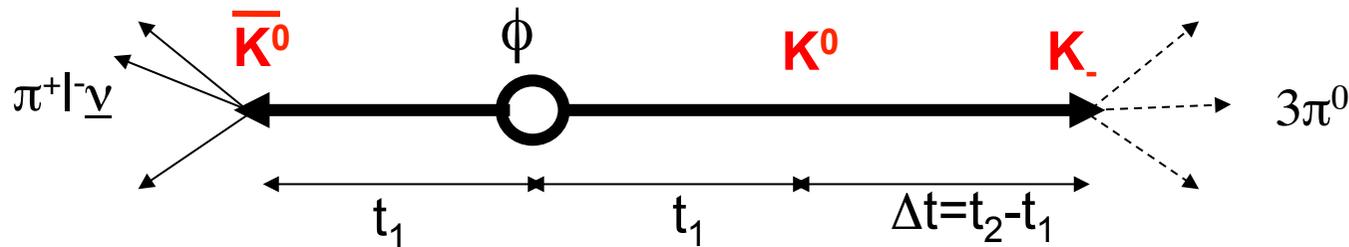
Neglect direct CP violation. Similarly any $\Delta S = \Delta Q$ rule violation for $|K^0\rangle$ and $|\bar{K}^0\rangle$

Entanglement in neutral kaon pairs

- EPR correlations at a ϕ -factory (or B-factory) can be exploited to study transitions involving orthogonal “CP states” K_+ and K_-

$$\begin{aligned}
 |i\rangle &= \frac{1}{\sqrt{2}} \left[|K^0(\vec{p})\rangle |\bar{K}^0(-\vec{p})\rangle - |\bar{K}^0(\vec{p})\rangle |K^0(-\vec{p})\rangle \right] \\
 &= \frac{1}{\sqrt{2}} \left[|K_+(\vec{p})\rangle |K_-(-\vec{p})\rangle - |K_-(-\vec{p})\rangle |K_+(\vec{p})\rangle \right]
 \end{aligned}$$

- decay as filtering measurement
- entanglement \rightarrow preparation of state

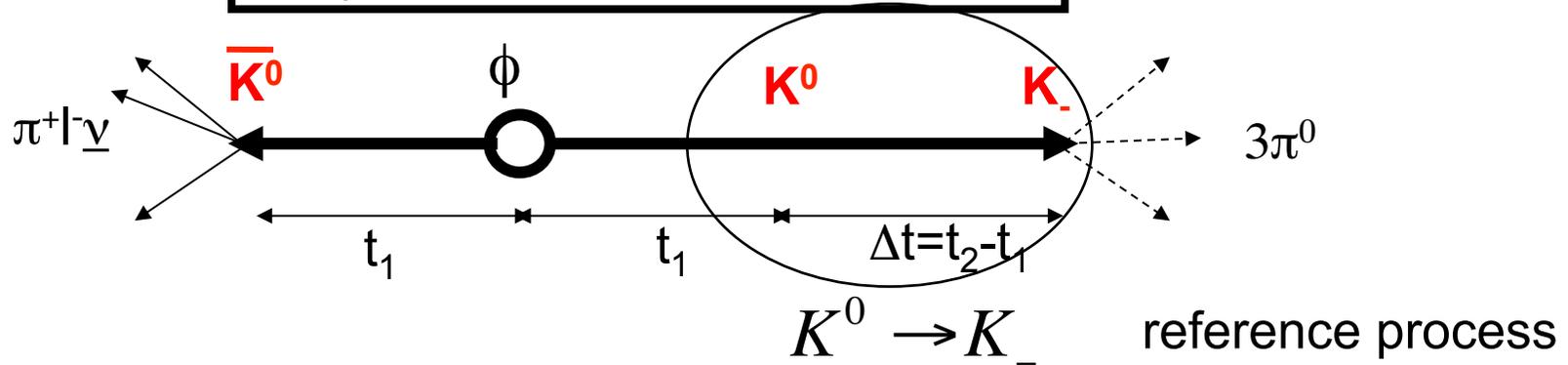


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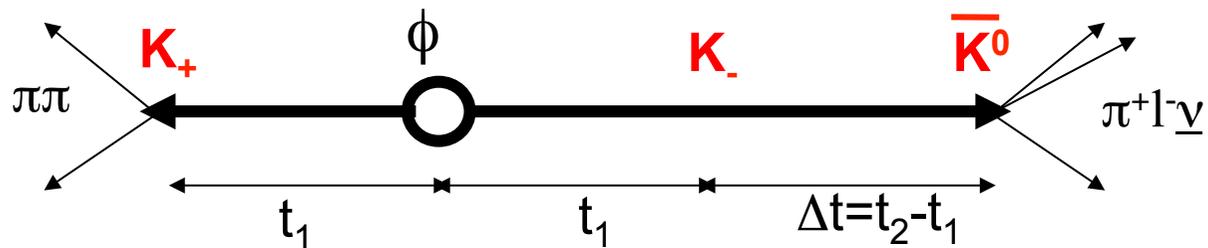
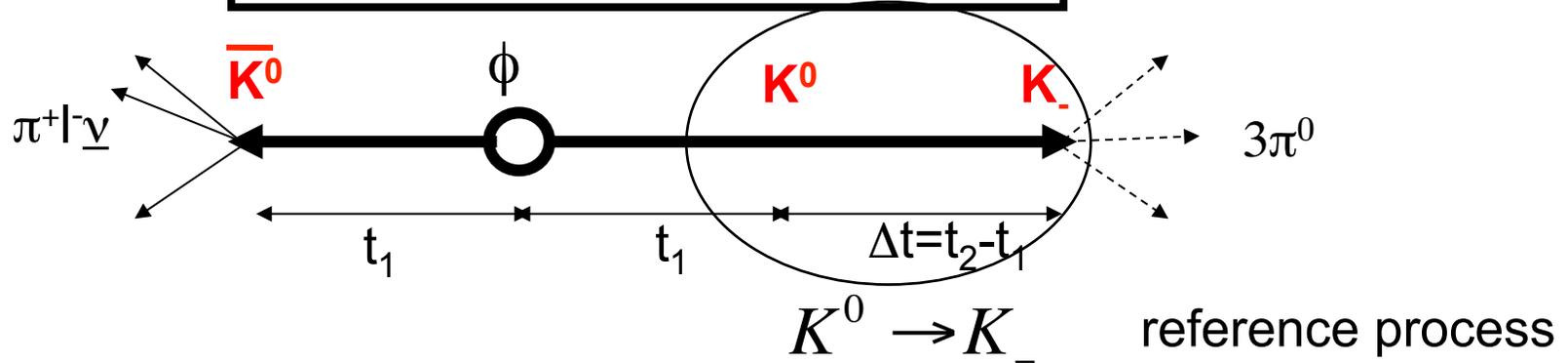


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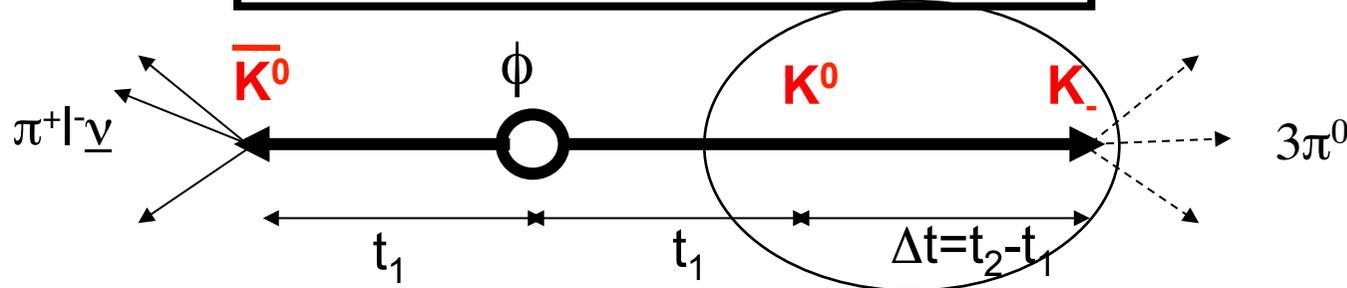


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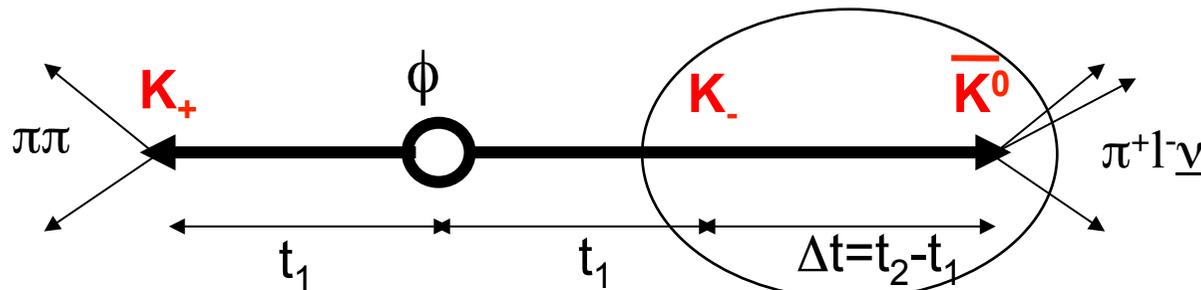


$K^0 \rightarrow K_-$

reference process

$K_- \rightarrow \bar{K}^0$

CPT-conjugated process

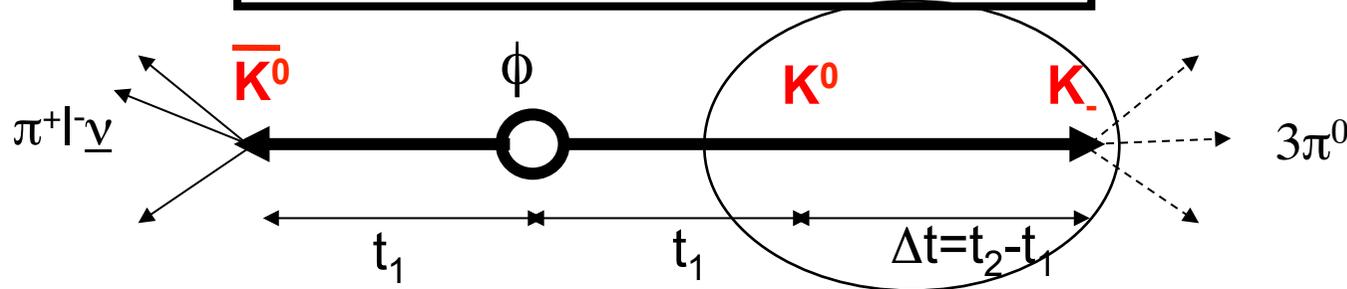


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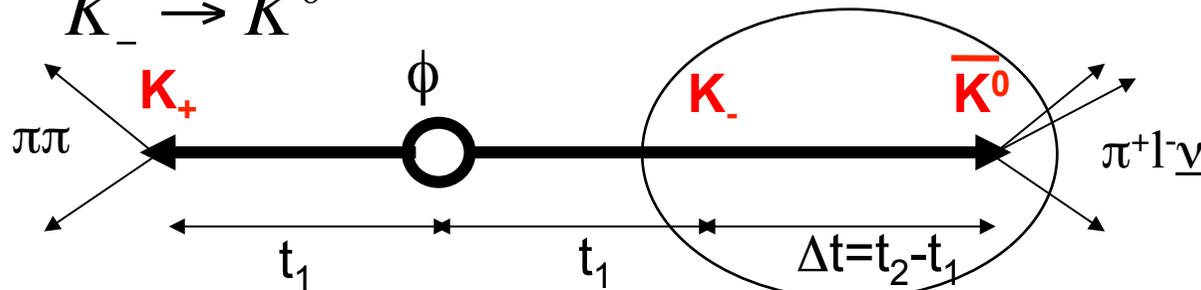


$K^0 \rightarrow K_-$ reference process

Note: CP and T conjugated process

$$\bar{K}^0 \rightarrow K_- \quad K_- \rightarrow K^0$$

$K_- \rightarrow \bar{K}^0$ CPT-conjugated process



Direct test of CPT symmetry in neutral kaon transitions

CPT symmetry test

Reference		\mathcal{CPT} -conjugate	
Transition	Decay products	Transition	Decay products
$K^0 \rightarrow K_+$	$(\ell^-, \pi\pi)$	$K_+ \rightarrow \bar{K}^0$	$(3\pi^0, \ell^-)$
$K^0 \rightarrow K_-$	$(\ell^-, 3\pi^0)$	$K_- \rightarrow \bar{K}^0$	$(\pi\pi, \ell^-)$
$\bar{K}^0 \rightarrow K_+$	$(\ell^+, \pi\pi)$	$K_+ \rightarrow K^0$	$(3\pi^0, \ell^+)$
$\bar{K}^0 \rightarrow K_-$	$(\ell^+, 3\pi^0)$	$K_- \rightarrow K^0$	$(\pi\pi, \ell^+)$

One can define the following ratios of probabilities:

$$R_{1,\mathcal{CPT}}(\Delta t) = P [K_+(0) \rightarrow \bar{K}^0(\Delta t)] / P [K^0(0) \rightarrow K_+(\Delta t)]$$

$$R_{2,\mathcal{CPT}}(\Delta t) = P [K^0(0) \rightarrow K_-(\Delta t)] / P [K_-(0) \rightarrow \bar{K}^0(\Delta t)]$$

$$R_{3,\mathcal{CPT}}(\Delta t) = P [K_+(0) \rightarrow K^0(\Delta t)] / P [\bar{K}^0(0) \rightarrow K_+(\Delta t)]$$

$$R_{4,\mathcal{CPT}}(\Delta t) = P [\bar{K}^0(0) \rightarrow K_-(\Delta t)] / P [K_-(0) \rightarrow K^0(\Delta t)]$$

Any deviation from $R_{i,\mathcal{CPT}}=1$ constitutes a violation of CPT-symmetry

J. Bernabeu, A.D.D., P. Villanueva, JHEP 10 (2015) 139

Direct test of CPT symmetry in neutral kaon transitions

Two observable ratios of double decay intensities

$$R_{2,\text{CPT}}^{\text{exp}}(\Delta t) \equiv \frac{I(\ell^-, 3\pi^0; \Delta t)}{I(\pi\pi, \ell^-; \Delta t)}$$

$$R_{4,\text{CPT}}^{\text{exp}}(\Delta t) \equiv \frac{I(\ell^+, 3\pi^0; \Delta t)}{I(\pi\pi, \ell^+; \Delta t)}$$

for $\Delta t > 0$

$$R_{2,\text{CPT}}^{\text{exp}}(\Delta t) = R_{2,\text{CPT}}(\Delta t) \times D_{\text{CPT}}$$

$$R_{4,\text{CPT}}^{\text{exp}}(\Delta t) = R_{4,\text{CPT}}(\Delta t) \times D_{\text{CPT}}$$

for $\Delta t < 0$

$$R_{2,\text{CPT}}^{\text{exp}}(\Delta t) = R_{1,\text{CPT}}(|\Delta t|) \times D_{\text{CPT}}$$

$$R_{4,\text{CPT}}^{\text{exp}}(\Delta t) = R_{3,\text{CPT}}(|\Delta t|) \times D_{\text{CPT}}$$

with D_{CPT} constant

$$D_{\text{CPT}} = \frac{\text{BR}(K_L \rightarrow 3\pi^0) \Gamma_L}{\text{BR}(K_S \rightarrow \pi\pi) \Gamma_S}$$

Direct test of CPT symmetry in neutral kaon transitions

Explicitly in standard Wigner Weisskopf approach for $\Delta t > 0$:

$$R_{2,\text{CPT}}^{\text{exp}}(\Delta t) = \frac{P[\text{K}^0(0) \rightarrow \text{K}_-(\Delta t)]}{P[\text{K}_-(0) \rightarrow \bar{\text{K}}^0(\Delta t)]} \times D_{\text{CPT}}$$

$$\simeq |1 - 2\delta|^2 \left| 1 + 2\delta e^{-i(\lambda_S - \lambda_L)\Delta t} \right|^2 \times D_{\text{CPT}}$$

$$R_{4,\text{CPT}}^{\text{exp}}(\Delta t) = \frac{P[\bar{\text{K}}^0(0) \rightarrow \text{K}_-(\Delta t)]}{P[\text{K}_-(0) \rightarrow \text{K}^0(\Delta t)]} \times D_{\text{CPT}}$$

$$\simeq |1 + 2\delta|^2 \left| 1 - 2\delta e^{-i(\lambda_S - \lambda_L)\Delta t} \right|^2 \times D_{\text{CPT}}$$

For comparison the ratio of survival probabilities: Vanishes for $\Delta\Gamma \rightarrow 0$

$$\frac{I(\ell^-, \ell^+; \Delta t)}{I(\ell^+, \ell^-; \Delta t)} = \frac{P[\text{K}^0(0) \rightarrow \text{K}^0(\Delta t)]}{P[\bar{\text{K}}^0(0) \rightarrow \bar{\text{K}}^0(\Delta t)]}$$

$$\simeq |1 - 4\delta|^2 \left| 1 + \frac{8\delta}{1 + e^{+i(\lambda_S - \lambda_L)\Delta t}} \right|^2$$

As an illustration of the different sensitivity: it vanishes up to second order in CPTV and decoherence parameters α, β, γ (Ellis, Mavromatos et al. PRD1996)

Impact of the approximations

In general K_+ and K_-
(and K_0 and \bar{K}_0)
can be non-orthogonal

Direct CP (CPT) violation

$$\eta_{\pi\pi} = \epsilon_L + \epsilon'_{\pi\pi}$$

$$\eta_{3\pi^0} = \epsilon_S + \epsilon'_{3\pi^0}$$

CPT cons. and CPT viol.

$\Delta S = \Delta Q$ violation

$$x_+, x_-$$

Orthogonal
bases

$$\{K_+, \tilde{K}_-\}$$

$$\{\tilde{K}_+, K_-\}$$

$$\{\tilde{K}_0, K_{\bar{0}}\} \text{ and } \{\tilde{K}_{\bar{0}}, K_0\}$$

Explicitly for $\Delta t > 0$:

$$R_{2,\text{CPT}}^{\text{exp}}(\Delta t) = \frac{P[\tilde{K}_0(0) \rightarrow K_-(\Delta t)]}{P[\tilde{K}_-(0) \rightarrow K_{\bar{0}}(\Delta t)]} \times D_{\text{CPT}}$$

$$= |1 - 2\delta + 2x_+^* - 2x_-^*|^2 \left| 1 + (2\delta + \epsilon'_{3\pi^0} - \epsilon'_{\pi\pi}) e^{-i(\lambda_S - \lambda_L)\Delta t} \right|^2 \times D_{\text{CPT}}$$

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$$= |1 + 2\delta + 2x_+ + 2x_-|^2 \left| 1 - (2\delta + \epsilon'_{3\pi^0} - \epsilon'_{\pi\pi}) e^{-i(\lambda_S - \lambda_L)\Delta t} \right|^2 \times D_{\text{CPT}}$$

Impact of the approximations

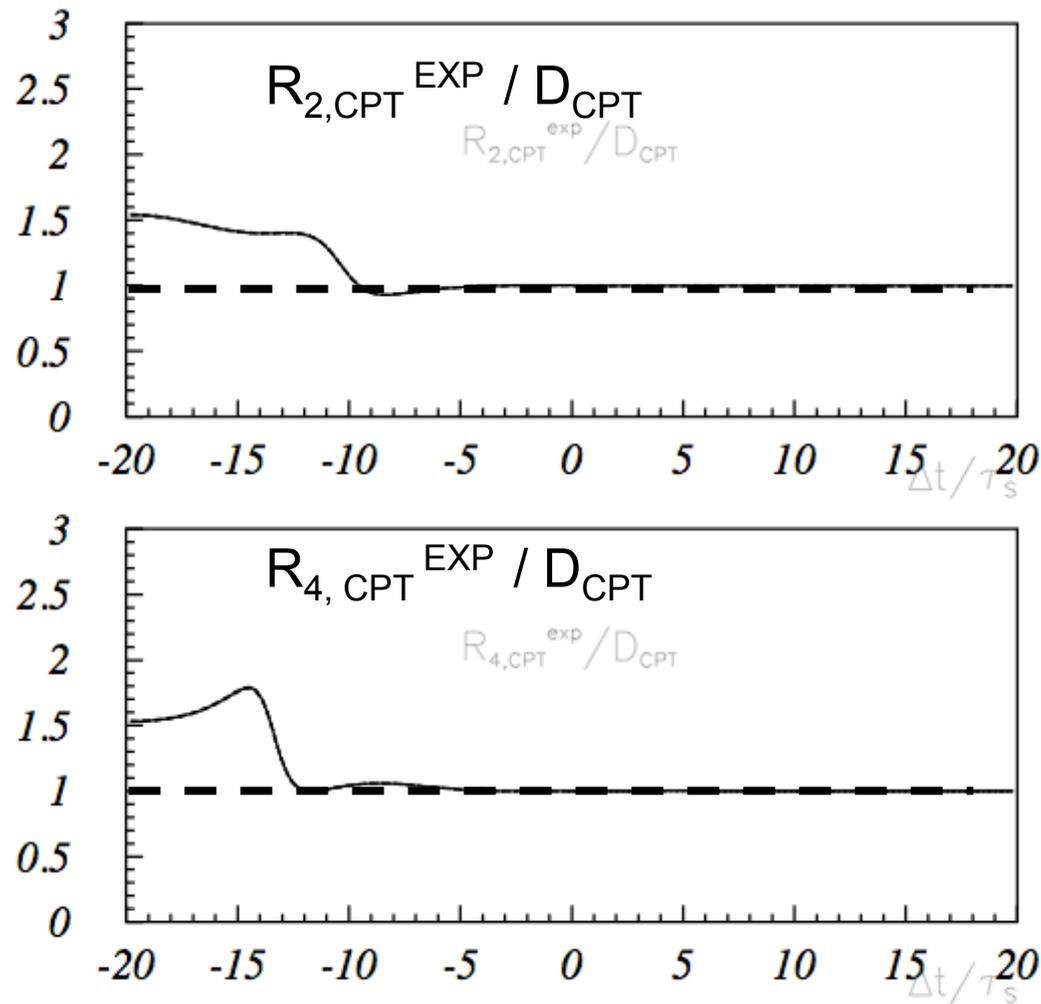
$$\begin{aligned}\frac{R_{2,\text{CPT}}^{\text{exp}}(\Delta t)}{R_{4,\text{CPT}}^{\text{exp}}(\Delta t)} &\simeq (1 - 8\Re\delta - 8\Re x_-) \left| 1 + 2(\eta_{3\pi^0} - \eta_{\pi\pi}) e^{-i(\lambda_S - \lambda_L)\Delta t} \right|^2 \\ &= (1 - 8\Re\delta - 8\Re x_-) \left| 1 + 2(2\delta + \epsilon'_{3\pi^0} - \epsilon'_{\pi\pi}) e^{-i(\lambda_S - \lambda_L)\Delta t} \right|^2\end{aligned}$$

The double ratio constitutes one of the most robust observables for the proposed CPT test. In the limit $\Delta t \gg \tau_S$ it exhibits a pure and genuine CPT violating effect, even without assuming negligible contaminations from direct CP violation and/or $\Delta S = \Delta Q$ rule violation.

$$\frac{R_{2,\text{CPT}}^{\text{exp}}(\Delta t \gg \tau_S)}{R_{4,\text{CPT}}^{\text{exp}}(\Delta t \gg \tau_S)} = 1 - 8\Re\delta - 8\Re x_-$$

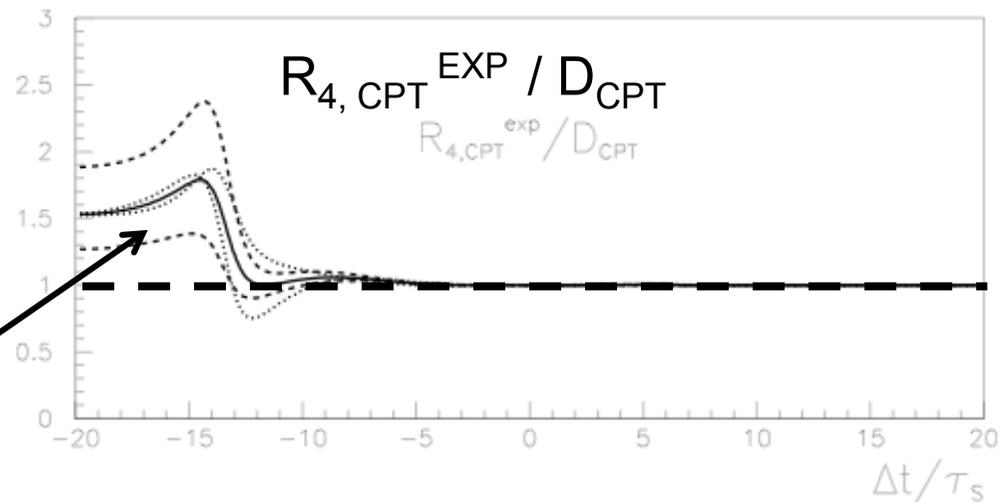
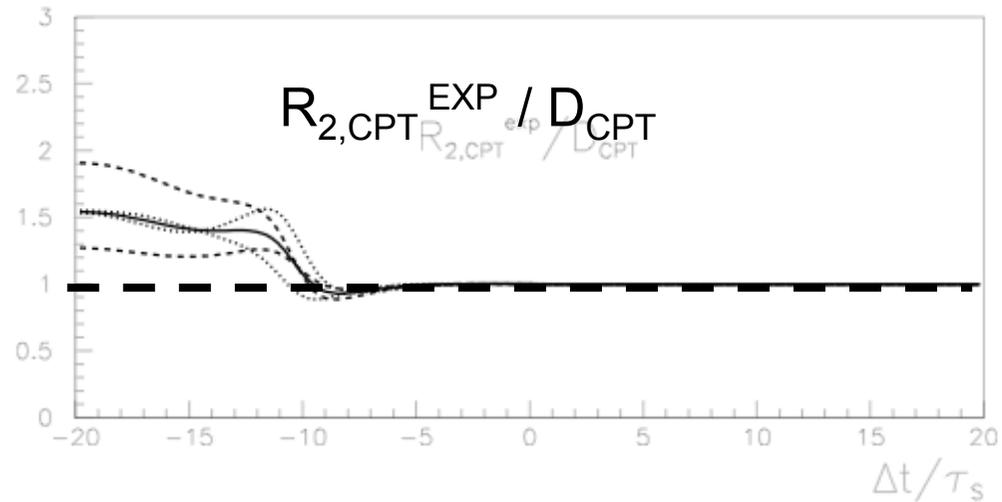
Direct test of CPT in transitions with neutral kaons

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 $\text{Re}(\delta)=3.3 \cdot 10^{-4}$ $\text{Im}(\delta)=1.6 \cdot 10^{-5}$



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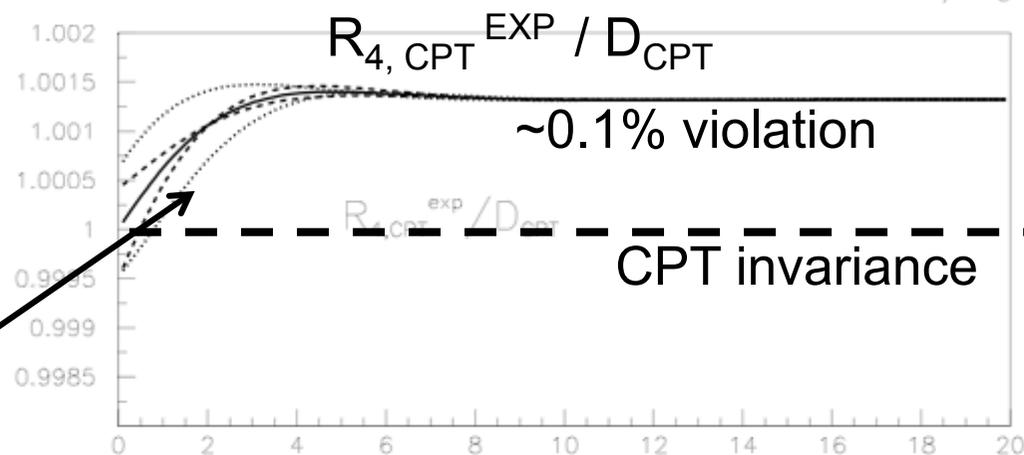
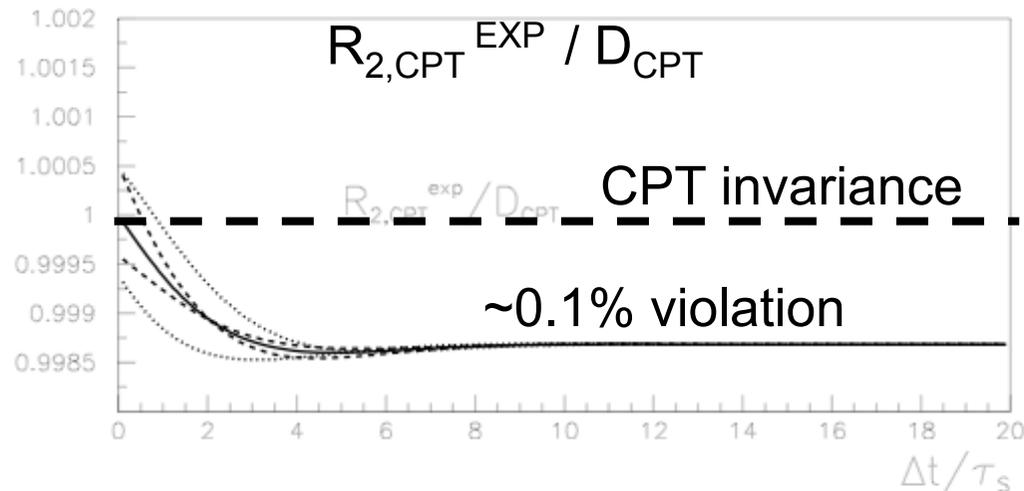
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Modifications due to direct CP violation effects (unrealistically amplified $\sim x100$)

Direct test of CPT in transitions with neutral kaons

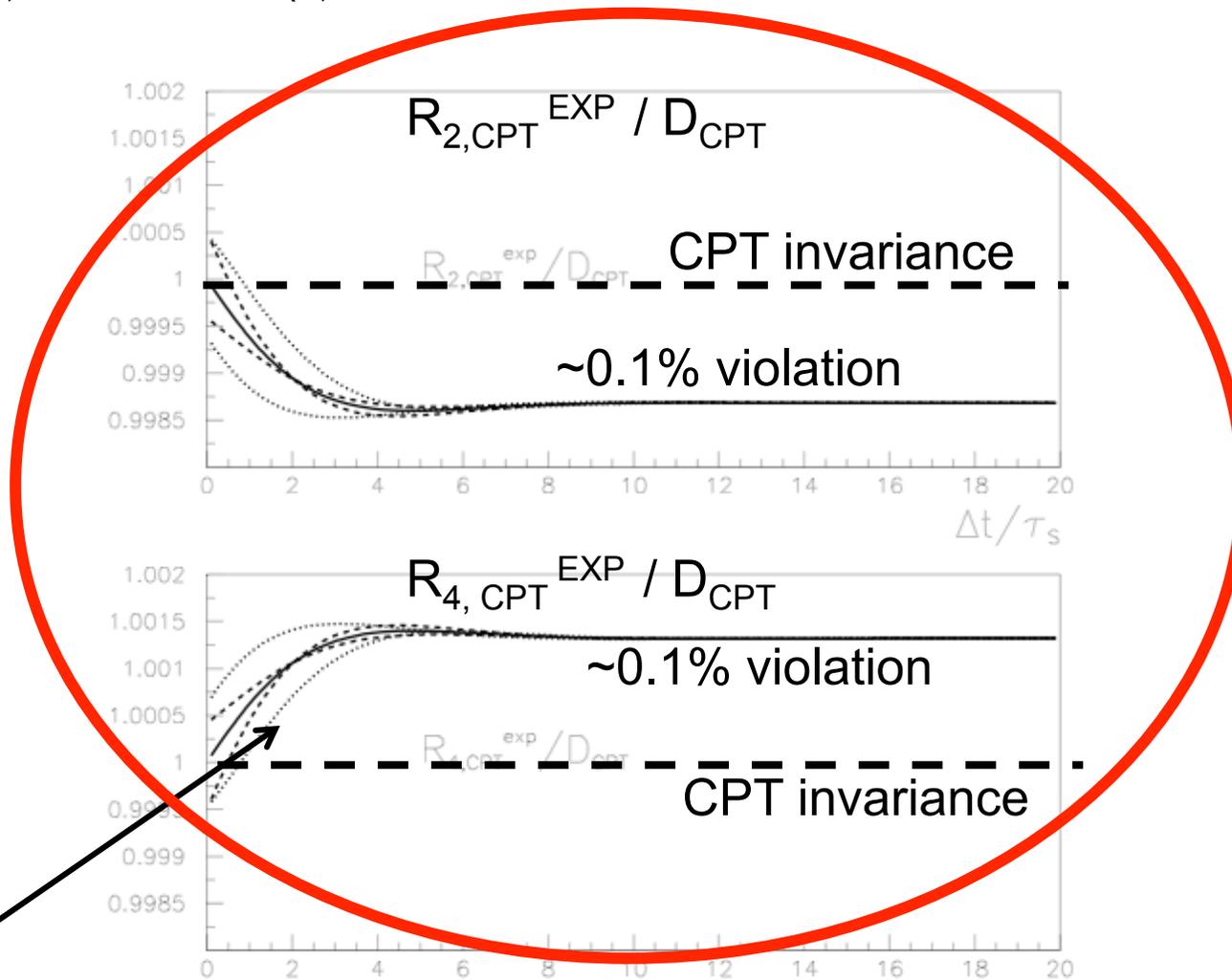
for visualization purposes, plots with
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Modifications due to direct CP violation effects (unrealistically amplified $\sim x100$)

Direct test of CPT symmetry with neutral kaons

for visualization purposes, plots with
 $\text{Re}(\delta)=3.3 \cdot 10^{-4}$ $\text{Im}(\delta)=1.6 \cdot 10^{-5}$



measurable
at KLOE-2

see A. Gajos's talk

Modifications due to direct CP violation effects (unrealistically amplified $\sim x100$)

Direct test of symmetries with neutral kaons

Reference	T -conjugate	CP -conjugate	CPT -conjugate
$K^0 \rightarrow K^0$	$K^0 \rightarrow K^0$	$\bar{K}^0 \rightarrow \bar{K}^0$	$\bar{K}^0 \rightarrow \bar{K}^0$
$K^0 \rightarrow \bar{K}^0$	$\bar{K}^0 \rightarrow K^0$	$\bar{K}^0 \rightarrow K^0$	$K^0 \rightarrow \bar{K}^0$
$K^0 \rightarrow K_+$	$K_+ \rightarrow K^0$	$\bar{K}^0 \rightarrow K_+$	$K_+ \rightarrow \bar{K}^0$
$K^0 \rightarrow K_-$	$K_- \rightarrow K^0$	$\bar{K}^0 \rightarrow K_-$	$K_- \rightarrow \bar{K}^0$
$\bar{K}^0 \rightarrow K^0$	$K^0 \rightarrow \bar{K}^0$	$K^0 \rightarrow \bar{K}^0$	$\bar{K}^0 \rightarrow K^0$
$\bar{K}^0 \rightarrow \bar{K}^0$	$\bar{K}^0 \rightarrow \bar{K}^0$	$K^0 \rightarrow K^0$	$K^0 \rightarrow K^0$
$\bar{K}^0 \rightarrow K_+$	$K_+ \rightarrow \bar{K}^0$	$K^0 \rightarrow K_+$	$K_+ \rightarrow K^0$
$\bar{K}^0 \rightarrow K_-$	$K_- \rightarrow \bar{K}^0$	$K^0 \rightarrow K_-$	$K_- \rightarrow K^0$
$K_+ \rightarrow K^0$	$K^0 \rightarrow K_+$	$K_+ \rightarrow \bar{K}^0$	$\bar{K}^0 \rightarrow K_+$
$K_+ \rightarrow \bar{K}^0$	$\bar{K}^0 \rightarrow K_+$	$K_+ \rightarrow K^0$	$K^0 \rightarrow K_+$
$K_+ \rightarrow K_+$	$K_+ \rightarrow K_+$	$K_+ \rightarrow K_+$	$K_+ \rightarrow K_+$
$K_+ \rightarrow K_-$	$K_- \rightarrow K_+$	$K_+ \rightarrow K_-$	$K_- \rightarrow K_+$
$K_- \rightarrow K^0$	$K^0 \rightarrow K_-$	$K_- \rightarrow \bar{K}^0$	$\bar{K}^0 \rightarrow K_-$
$K_- \rightarrow \bar{K}^0$	$\bar{K}^0 \rightarrow K_-$	$K_- \rightarrow K^0$	$K^0 \rightarrow K_-$
$K_- \rightarrow K_+$	$K_+ \rightarrow K_-$	$K_- \rightarrow K_+$	$K_+ \rightarrow K_-$
$K_- \rightarrow K_-$	$K_- \rightarrow K_-$	$K_- \rightarrow K_-$	$K_- \rightarrow K_-$

Direct test of symmetries with neutral kaons

Conjugate=
reference

Reference	T -conjugate	CP -conjugate	CPT -conjugate
$K^0 \rightarrow K^0$	$K^0 \rightarrow K^0$	$\bar{K}^0 \rightarrow \bar{K}^0$	$\bar{K}^0 \rightarrow \bar{K}^0$
$K^0 \rightarrow \bar{K}^0$	$\bar{K}^0 \rightarrow K^0$	$\bar{K}^0 \rightarrow K^0$	$K^0 \rightarrow \bar{K}^0$
$K^0 \rightarrow K_+$	$K_+ \rightarrow K^0$	$\bar{K}^0 \rightarrow K_+$	$K_+ \rightarrow \bar{K}^0$
$K^0 \rightarrow K_-$	$K_- \rightarrow K^0$	$\bar{K}^0 \rightarrow K_-$	$K_- \rightarrow \bar{K}^0$
$\bar{K}^0 \rightarrow K^0$	$K^0 \rightarrow \bar{K}^0$	$K^0 \rightarrow \bar{K}^0$	$\bar{K}^0 \rightarrow K^0$
$\bar{K}^0 \rightarrow \bar{K}^0$	$\bar{K}^0 \rightarrow \bar{K}^0$	$K^0 \rightarrow K^0$	$K^0 \rightarrow K^0$
$\bar{K}^0 \rightarrow K_+$	$K_+ \rightarrow \bar{K}^0$	$K^0 \rightarrow K_+$	$K_+ \rightarrow K^0$
$\bar{K}^0 \rightarrow K_-$	$K_- \rightarrow \bar{K}^0$	$K^0 \rightarrow K_-$	$K_- \rightarrow K^0$
$K_+ \rightarrow K^0$	$K^0 \rightarrow K_+$	$K_+ \rightarrow \bar{K}^0$	$\bar{K}^0 \rightarrow K_+$
$K_+ \rightarrow \bar{K}^0$	$\bar{K}^0 \rightarrow K_+$	$K_+ \rightarrow K^0$	$K^0 \rightarrow K_+$
$K_+ \rightarrow K_+$	$K_+ \rightarrow K_+$	$K_+ \rightarrow K_+$	$K_+ \rightarrow K_+$
$K_+ \rightarrow K_-$	$K_- \rightarrow K_+$	$K_- \rightarrow K_-$	$K_- \rightarrow K_+$
$K_- \rightarrow K^0$	$K^0 \rightarrow K_-$	$K_- \rightarrow \bar{K}^0$	$\bar{K}^0 \rightarrow K_-$
$K_- \rightarrow \bar{K}^0$	$\bar{K}^0 \rightarrow K_-$	$K_- \rightarrow K^0$	$K^0 \rightarrow K_-$
$K_- \rightarrow K_+$	$K_+ \rightarrow K_-$	$K_+ \rightarrow K_+$	$K_+ \rightarrow K_-$
$K_- \rightarrow K_-$	$K_- \rightarrow K_-$	$K_- \rightarrow K_-$	$K_- \rightarrow K_-$

Direct test of symmetries with neutral kaons

Conjugate=
reference

already in the
table with
conjugate as
reference

Reference	<i>T</i> -conjugate	<i>CP</i> -conjugate	<i>CPT</i> -conjugate
$K^0 \rightarrow K^0$	$K^0 \rightarrow K^0$	$\bar{K}^0 \rightarrow \bar{K}^0$	$\bar{K}^0 \rightarrow \bar{K}^0$
$K^0 \rightarrow \bar{K}^0$	$\bar{K}^0 \rightarrow K^0$	$\bar{K}^0 \rightarrow K^0$	$K^0 \rightarrow \bar{K}^0$
$K^0 \rightarrow K_+$	$K_+ \rightarrow K^0$	$\bar{K}^0 \rightarrow K_+$	$K_+ \rightarrow \bar{K}^0$
$K^0 \rightarrow K_-$	$K_- \rightarrow K^0$	$\bar{K}^0 \rightarrow K_-$	$K_- \rightarrow \bar{K}^0$
$\bar{K}^0 \rightarrow K^0$	$K^0 \rightarrow \bar{K}^0$	$K^0 \rightarrow \bar{K}^0$	$\bar{K}^0 \rightarrow K^0$
$\bar{K}^0 \rightarrow \bar{K}^0$	$\bar{K}^0 \rightarrow \bar{K}^0$	$K^0 \rightarrow K^0$	$K^0 \rightarrow K^0$
$\bar{K}^0 \rightarrow K_+$	$K_+ \rightarrow \bar{K}^0$	$K^0 \rightarrow K_+$	$K_+ \rightarrow K^0$
$\bar{K}^0 \rightarrow K_-$	$K_- \rightarrow \bar{K}^0$	$K^0 \rightarrow K_-$	$K_- \rightarrow K^0$
$K_+ \rightarrow K^0$	$K^0 \rightarrow K_+$	$K_+ \rightarrow \bar{K}^0$	$\bar{K}^0 \rightarrow K_+$
$K_+ \rightarrow \bar{K}^0$	$\bar{K}^0 \rightarrow K_+$	$K_+ \rightarrow K^0$	$K^0 \rightarrow K_+$
$K_+ \rightarrow K_+$	$K_+ \rightarrow K_+$	$K_+ \rightarrow K_+$	$K_+ \rightarrow K_+$
$K_+ \rightarrow K_-$	$K_- \rightarrow K_+$	$K_- \rightarrow K_-$	$K_- \rightarrow K_+$
$K_- \rightarrow K^0$	$K^0 \rightarrow K_-$	$K_- \rightarrow \bar{K}^0$	$\bar{K}^0 \rightarrow K_-$
$K_- \rightarrow \bar{K}^0$	$\bar{K}^0 \rightarrow K_-$	$K_- \rightarrow K^0$	$K^0 \rightarrow K_-$
$K_- \rightarrow K_+$	$K_+ \rightarrow K_-$	$K_+ \rightarrow K_+$	$K_+ \rightarrow K_-$
$K_- \rightarrow K_-$	$K_- \rightarrow K_-$	$K_- \rightarrow K_-$	$K_- \rightarrow K_-$

Direct test of symmetries with neutral kaons

Conjugate=
reference



already in the
table with
conjugate as
reference



Two identical
conjugates
for one reference



Reference	<i>T</i> -conjugate	<i>CP</i> -conjugate	<i>CPT</i> -conjugate
$K^0 \rightarrow K^0$	$K^0 \rightarrow K^0$	$\bar{K}^0 \rightarrow \bar{K}^0$	$\bar{K}^0 \rightarrow \bar{K}^0$
$K^0 \rightarrow \bar{K}^0$	$\bar{K}^0 \rightarrow K^0$	$\bar{K}^0 \rightarrow K^0$	$K^0 \rightarrow \bar{K}^0$
$K^0 \rightarrow K_+$	$K_+ \rightarrow K^0$	$\bar{K}^0 \rightarrow K_+$	$K_+ \rightarrow \bar{K}^0$
$K^0 \rightarrow K_-$	$K_- \rightarrow K^0$	$\bar{K}^0 \rightarrow K_-$	$K_- \rightarrow \bar{K}^0$
$\bar{K}^0 \rightarrow K^0$	$K^0 \rightarrow \bar{K}^0$	$K^0 \rightarrow \bar{K}^0$	$\bar{K}^0 \rightarrow K^0$
$\bar{K}^0 \rightarrow \bar{K}^0$	$\bar{K}^0 \rightarrow \bar{K}^0$	$K^0 \rightarrow K^0$	$K^0 \rightarrow K^0$
$\bar{K}^0 \rightarrow K_+$	$K_+ \rightarrow \bar{K}^0$	$K^0 \rightarrow K_+$	$K_+ \rightarrow K^0$
$\bar{K}^0 \rightarrow K_-$	$K_- \rightarrow \bar{K}^0$	$K^0 \rightarrow K_-$	$K_- \rightarrow K^0$
$K_+ \rightarrow K^0$	$K^0 \rightarrow K_+$	$K_+ \rightarrow \bar{K}^0$	$\bar{K}^0 \rightarrow K_+$
$K_+ \rightarrow \bar{K}^0$	$\bar{K}^0 \rightarrow K_+$	$K_+ \rightarrow K^0$	$K^0 \rightarrow K_+$
$K_+ \rightarrow K_+$	$K_+ \rightarrow K_+$	$K_+ \rightarrow K_+$	$K_+ \rightarrow K_+$
$K_+ \rightarrow K_-$	$K_- \rightarrow K_+$	$K_+ \rightarrow K_-$	$K_- \rightarrow K_+$
$K_- \rightarrow K^0$	$K^0 \rightarrow K_-$	$K_- \rightarrow \bar{K}^0$	$\bar{K}^0 \rightarrow K_-$
$K_- \rightarrow \bar{K}^0$	$\bar{K}^0 \rightarrow K_-$	$K_- \rightarrow K^0$	$K^0 \rightarrow K_-$
$K_- \rightarrow K_+$	$K_+ \rightarrow K_-$	$K_+ \rightarrow K_+$	$K_+ \rightarrow K_-$
$K_- \rightarrow K_-$	$K_- \rightarrow K_-$	$K_- \rightarrow K_-$	$K_- \rightarrow K_-$

Direct test of symmetries with neutral kaons

Conjugate=
reference

Reference	<i>T</i> -conjugate	<i>CP</i> -conjugate	<i>CPT</i> -conjugate
$K^0 \rightarrow K^0$	$K^0 \rightarrow K^0$	$\bar{K}^0 \rightarrow \bar{K}^0$	$\bar{K}^0 \rightarrow \bar{K}^0$
$K^0 \rightarrow \bar{K}^0$	$\bar{K}^0 \rightarrow K^0$	$\bar{K}^0 \rightarrow K^0$	$K^0 \rightarrow \bar{K}^0$
$K^0 \rightarrow K_+$	$K_+ \rightarrow K^0$	$\bar{K}^0 \rightarrow K_+$	$K_+ \rightarrow \bar{K}^0$
$K^0 \rightarrow K_-$	$K_- \rightarrow K^0$	$\bar{K}^0 \rightarrow K_-$	$K_- \rightarrow \bar{K}^0$
$\bar{K}^0 \rightarrow K^0$	$K^0 \rightarrow \bar{K}^0$	$K^0 \rightarrow \bar{K}^0$	$\bar{K}^0 \rightarrow K^0$
$\bar{K}^0 \rightarrow \bar{K}^0$	$\bar{K}^0 \rightarrow \bar{K}^0$	$K^0 \rightarrow K^0$	$K^0 \rightarrow K^0$
$\bar{K}^0 \rightarrow K_+$	$K_+ \rightarrow \bar{K}^0$	$K^0 \rightarrow K_+$	$K_+ \rightarrow K^0$
$\bar{K}^0 \rightarrow K_-$	$K_- \rightarrow \bar{K}^0$	$K^0 \rightarrow K_-$	$K_- \rightarrow K^0$
$K_+ \rightarrow K^0$	$K^0 \rightarrow K_+$	$K_+ \rightarrow \bar{K}^0$	$\bar{K}^0 \rightarrow K_+$
$K_+ \rightarrow \bar{K}^0$	$\bar{K}^0 \rightarrow K_+$	$K_+ \rightarrow K^0$	$K^0 \rightarrow K_+$
$K_+ \rightarrow K_+$	$K_+ \rightarrow K_+$	$K_+ \rightarrow K_+$	$K_+ \rightarrow K_+$
$K_+ \rightarrow K_-$	$K_- \rightarrow K_+$	$K_+ \rightarrow K_-$	$K_- \rightarrow K_+$
$K_- \rightarrow K^0$	$K^0 \rightarrow K_-$	$K_- \rightarrow \bar{K}^0$	$\bar{K}^0 \rightarrow K_-$
$K_- \rightarrow \bar{K}^0$	$\bar{K}^0 \rightarrow K_-$	$K_- \rightarrow K^0$	$K^0 \rightarrow K_-$
$K_- \rightarrow K_+$	$K_+ \rightarrow K_-$	$K_+ \rightarrow K_+$	$K_+ \rightarrow K_+$
$K_- \rightarrow K_-$	$K_- \rightarrow K_-$	$K_- \rightarrow K_-$	$K_- \rightarrow K_-$

already in the
table with
conjugate as
reference

4 distinct tests
of *T* symmetry

4 distinct tests
of *CP* symmetry

4 distinct tests
of *CPT* symmetry

Two identical
conjugates
for one reference

Direct test of Time Reversal symmetry with neutral kaons

T symmetry test

Reference		T -conjugate	
Transition	Final state	Transition	Final state
$\bar{K}^0 \rightarrow K_-$	$(\ell^+, \pi^0 \pi^0 \pi^0)$	$K_- \rightarrow \bar{K}^0$	$(\pi^0 \pi^0 \pi^0, \ell^-)$
$K_+ \rightarrow K^0$	$(\pi^0 \pi^0 \pi^0, \ell^+)$	$K^0 \rightarrow K_+$	$(\ell^-, \pi \pi)$
$\bar{K}^0 \rightarrow K_+$	$(\ell^+, \pi \pi)$	$K_+ \rightarrow \bar{K}^0$	$(\pi^0 \pi^0 \pi^0, \ell^-)$
$K_- \rightarrow K^0$	$(\pi \pi, \ell^+)$	$K^0 \rightarrow K_-$	$(\ell^-, \pi \pi)$

One can define the following ratios of probabilities:

$$\begin{aligned}
 R_1(\Delta t) &= P [K^0(0) \rightarrow K_+(\Delta t)] / P [K_+(0) \rightarrow K^0(\Delta t)] \\
 R_2(\Delta t) &= P [K^0(0) \rightarrow K_-(\Delta t)] / P [K_-(0) \rightarrow K^0(\Delta t)] \\
 R_3(\Delta t) &= P [\bar{K}^0(0) \rightarrow K_+(\Delta t)] / P [K_+(0) \rightarrow \bar{K}^0(\Delta t)] \\
 R_4(\Delta t) &= P [\bar{K}^0(0) \rightarrow K_-(\Delta t)] / P [K_-(0) \rightarrow \bar{K}^0(\Delta t)] .
 \end{aligned}$$

Any deviation from $R_i=1$ constitutes a violation of T-symmetry

J. Bernabeu, A.D.D., P. Villanueva: NPB 868 (2013) 102

Direct test of Time Reversal symmetry with neutral kaons

Two observable ratios of double decay intensities

$$R_{2,\mathcal{T}}^{\text{exp}}(\Delta t) \equiv \frac{I(\ell^-, 3\pi^0; \Delta t)}{I(\pi\pi, \ell^+; \Delta t)}$$

$$R_{4,\mathcal{T}}^{\text{exp}}(\Delta t) \equiv \frac{I(\ell^+, 3\pi^0; \Delta t)}{I(\pi\pi, \ell^-; \Delta t)}$$

Direct test of Time Reversal symmetry with neutral kaons

Explicitly in standard
Wigner Weisskopf
approach
for $\Delta t > 0$:

$$\begin{aligned} R_{2,\mathcal{T}}^{\text{exp}}(\Delta t) &= \frac{P[\text{K}^0(0) \rightarrow \text{K}_-(\Delta t)]}{P[\text{K}_-(0) \rightarrow \text{K}^0(\Delta t)]} \times D_{\mathcal{T},2} \\ &= (1 - 4\Re\epsilon) \left| 1 + 2\epsilon e^{-i(\lambda_S - \lambda_L)\Delta t} \right|^2 \times D_{\mathcal{CPT}} \end{aligned}$$

$$\begin{aligned} R_{4,\mathcal{T}}^{\text{exp}}(\Delta t) &= \frac{P[\bar{\text{K}}^0(0) \rightarrow \text{K}_-(\Delta t)]}{P[\text{K}_-(0) \rightarrow \bar{\text{K}}^0(\Delta t)]} \times D_{\mathcal{T},4} \\ &= (1 + 4\Re\epsilon) \left| 1 - 2\epsilon e^{-i(\lambda_S - \lambda_L)\Delta t} \right|^2 \times D_{\mathcal{CPT}} \end{aligned}$$

Impact of the approximations

In general K_+ and K_-
(and K_0 and \bar{K}_0)
can be non-orthogonal

Direct CP (CPT) violation

$$\eta_{\pi\pi} = \epsilon_L + \epsilon'_{\pi\pi}$$

$$\eta_{3\pi^0} = \epsilon_S + \epsilon'_{3\pi^0}$$

CPT cons. and CPT viol.

$\Delta S = \Delta Q$ violation

$$x_+, x_-$$

Orthogonal
bases

$$\{K_+, \tilde{K}_-\}$$

$$\{\tilde{K}_+, K_-\}$$

$$\{\tilde{K}_0, K_{\bar{0}}\} \text{ and } \{\tilde{K}_{\bar{0}}, K_0\}$$

Explicitly for $\Delta t > 0$:

$$R_{2,\mathcal{T}}^{\text{exp}}(\Delta t) = \frac{P[\tilde{K}_0(0) \rightarrow K_-(\Delta t)]}{P[\tilde{K}_-(0) \rightarrow K_0(\Delta t)]} \times D_{\mathcal{T},2}$$

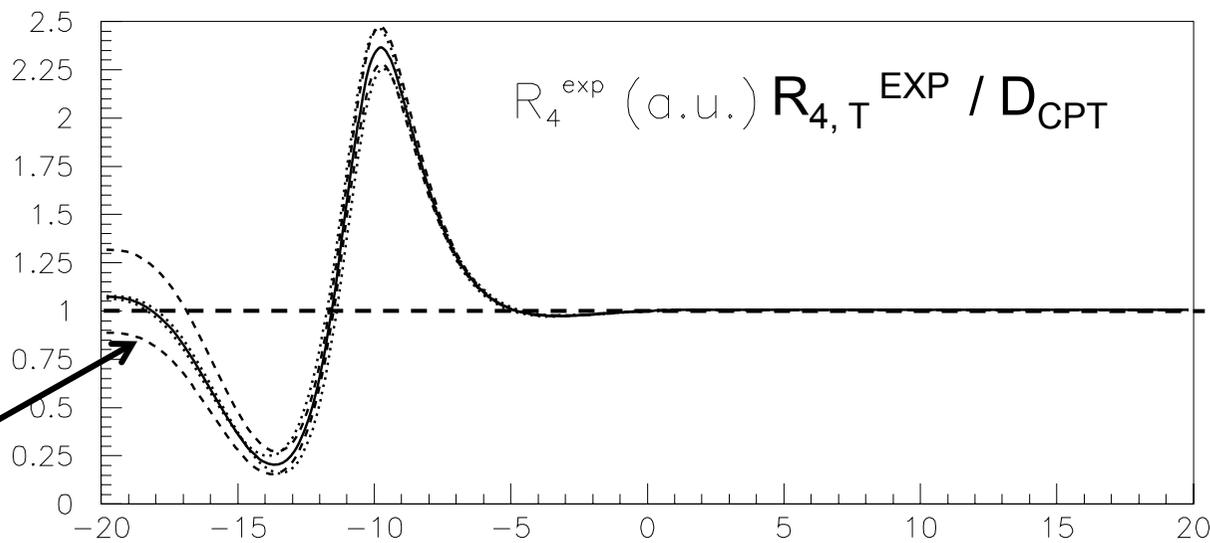
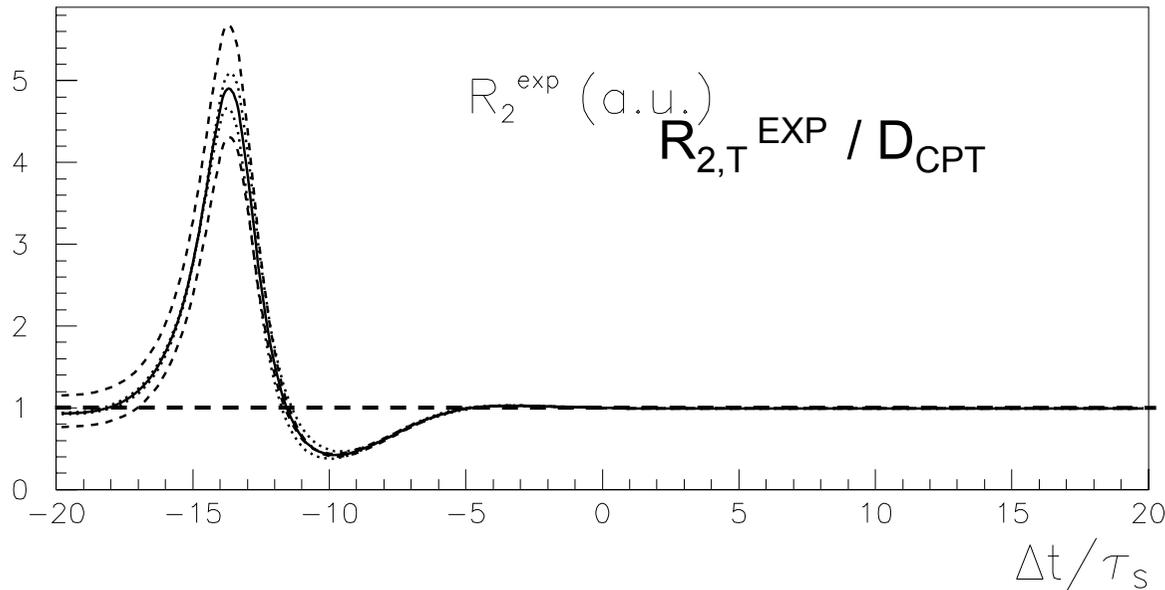
$$= (1 - 4\Re\epsilon + 4\Re x_+ + 4\Re y) \left| 1 + (2\epsilon + \epsilon'_{3\pi^0} + \epsilon'_{\pi\pi}) e^{-i(\lambda_S - \lambda_L)\Delta t} \right|^2 \times D_{\text{CPT}}$$

$$R_{4,\mathcal{T}}^{\text{exp}}(\Delta t) = \frac{P[\tilde{K}_{\bar{0}}(0) \rightarrow K_-(\Delta t)]}{P[\tilde{K}_-(0) \rightarrow K_{\bar{0}}(\Delta t)]} \times D_{\mathcal{T},4}$$

$$= (1 + 4\Re\epsilon + 4\Re x_+ - 4\Re y) \left| 1 - (2\epsilon + \epsilon'_{3\pi^0} + \epsilon'_{\pi\pi}) e^{-i(\lambda_S - \lambda_L)\Delta t} \right|^2 \times D_{\text{CPT}}$$

Direct test of Time Reversal symmetry with neutral kaons

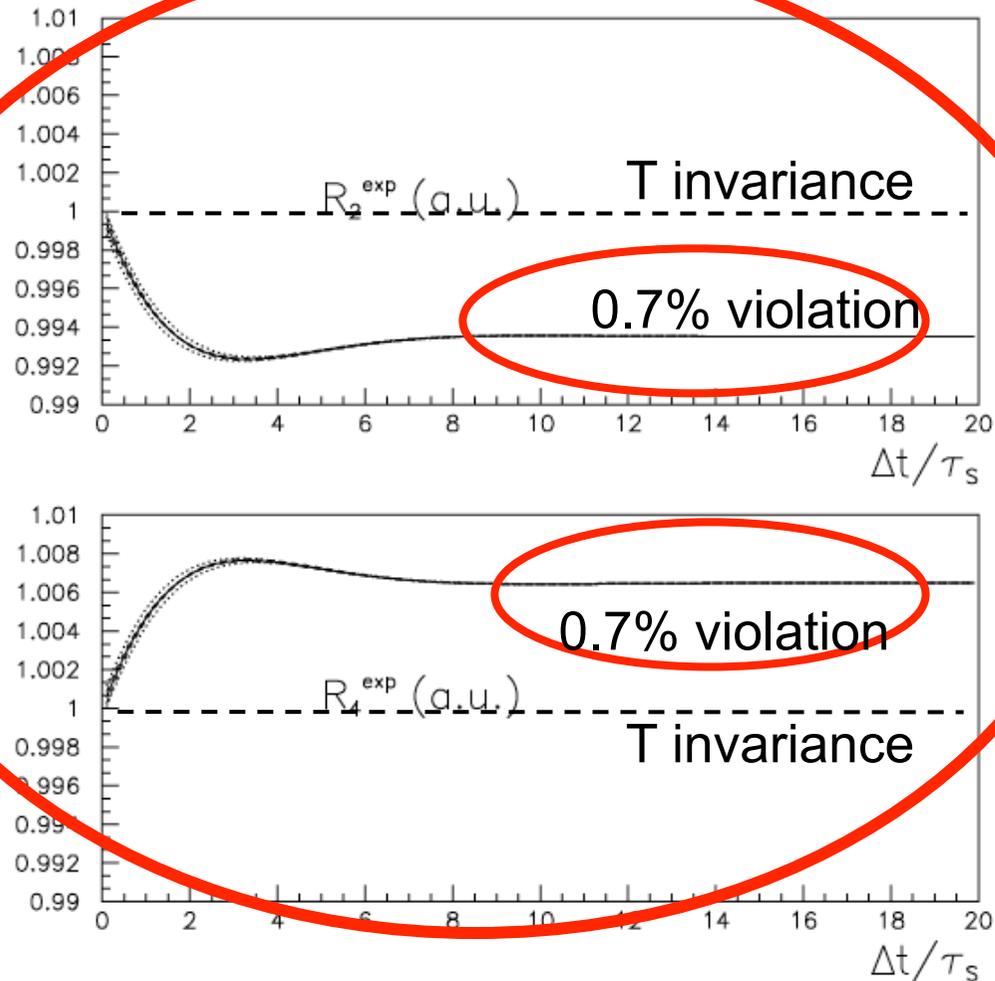
plots with CPV
 $\text{Re}\epsilon$ and $\text{Im}\epsilon$
 values



Modifications due to direct CP violation effects (unrealistically amplified $\sim \times 100$)

Direct test of Time Reversal symmetry with neutral kaons

plots with CPV
 $\text{Re}\epsilon$ and $\text{Im}\epsilon$
 values



measurable
 at KLOE-2

see A. Gajos's talk

$$R_2(\Delta t \gg \tau_S) = 1 - 4\text{Re}(\epsilon) \sim 0.993$$

$$R_4(\Delta t \gg \tau_S) = 1 + 4\text{Re}(\epsilon) \sim 1.007$$

Conclusions

- It is possible to directly test T and CPT in transition processes for the first time between neutral kaon states.
- Maximal entanglement of the initial state is assumed (impact of possible loss of coherence => ω effect; stringent limits measured at KLOE).
- KLOE data analysis ongoing (see A. Gajos's talk); KLOE-2 could reach a statistical sensitivity of $O(10^{-3})$ on these new observables.
- In B meson system similar tests (see Bevan's talk);

CPT test:

- The proposed CPT test is model independent and fully robust. (It can then be translated in terms of δ , α , β , γ , Δa_μ etc..).
- In standard WWA the test is related to $\text{Re}\delta$, a genuine CPT violating effect independent of $\Delta\Gamma$ and not requiring the decay as an essential ingredient.
- **VERY CLEAN TEST.** Possible spurious effects induced by CP violation in the decay and/or a violation of the $\Delta S = \Delta Q$ rule have been shown to be well under control.
- There exists a connection with charge semileptonic asymmetries of K_S and K_L

T test:

- It is possible to perform a direct test of the time reversal symmetry, independently from CP violation and CPT invariance constraints.
- Clean test, no impact of direct CPV; $\Delta S = \Delta Q$ and no CPT viol. in semilep. decay assumed.
- The constant D_{CPT} needs to be measured with $\sim 0.1\%$ precision.
- in the “plateau” region effect proportional to $\text{Re}(\varepsilon)$, not independent of $\Delta\Gamma$.