Entanglement, fluctuations and discrete symmetries in particle decays

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Quantum entanglement

Consider quantum-mechanical system where states of two or more objects can only be described with reference to each other, even when separated by large distances

Specific, quantum correlation called **entanglement** (*verschränkung,* in Schrödinger's letter to Einstein, 1935)

Lots of fundamental questions in order to understand and quantify entanglement, e.g.

- Result of quantum evolution or initial-state phenomenon?
- Yes-no characteristics?
- Relation to discrete symmetries, e.g. CP, and their violation?
- Cross-correlations in multi-object systems
- ...

Furry's hypothesis - early notion on de-entanglement leading to decoherence

W.H. Furry, PR 49(1936)393

Initially fully entangled state (e.g. two electrons' spins)

$$|\psi(t_1 = t_2 = 0)\rangle = \frac{1}{\sqrt{2}}(|\uparrow\rangle|\downarrow\rangle - |\downarrow\rangle|\uparrow\rangle)$$

becomes statistical mixture of states if spatially separated or if interaction occurs

$$|\psi(t_1 > 0, t_2 > 0)\rangle = |\uparrow\rangle_{t_1}|\downarrow\rangle_{t_2} \quad \text{or} \quad |\downarrow\rangle_{t_1}|\uparrow\rangle_{t_2}$$

with no interference between them, i.e. **no superposition of amplitudes**

Idea incorporated by Bertlman, Grimus & Hiesmayr to systems of neutral mesons Phys.Rev. D60 (1999) 114032

and applied by KLOE to pairs of kaons from decay $\Phi^0(1020) \to K_L K_S$

Phys.Lett. B642 (2006) 315
Found. Phys. 40 (2010) 852

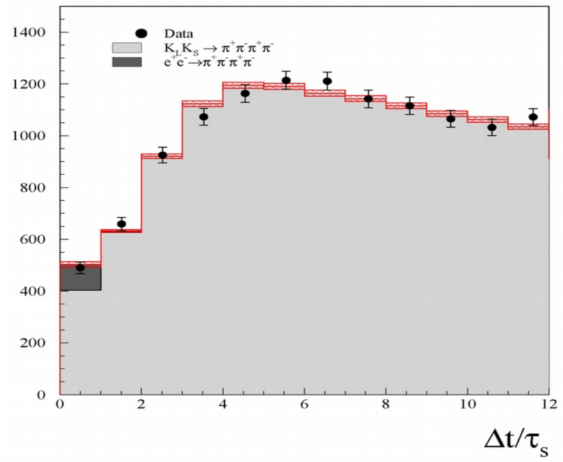
Initial state fully entangled

$$|\psi(t_1 = t_2 = 0)\rangle = \frac{1}{\sqrt{2}}\left(|K_L\rangle_1 |K_S\rangle_2 - |K_S\rangle_1 |K_L\rangle_2\right)$$

Time evolution $|K_{L,S}(t)\rangle = e^{(-im_{L,S} - \Gamma_{L,S}/2)t} |K_{L,S}\rangle$ leads to 2K intensity

$$I(\Delta t) = e^{-\Gamma_L \Delta t} + e^{-\Gamma_S \Delta t} - 2(1-\zeta)e^{-\bar{\Gamma}\Delta t}\cos(\Delta m \Delta t)$$

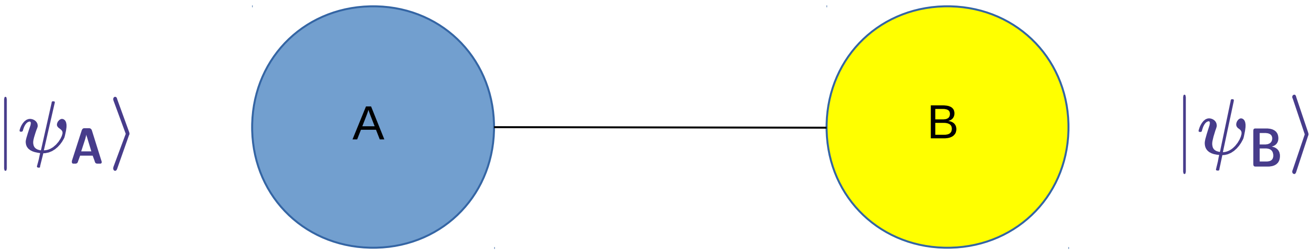
ζ=0 - standard QM
ζ=1- Furry: spontaneous
factorization



ζ =0.003±0.019

Bipartite systems with initial non-perfect entanglement

Consider a coupled bipartite system A x B



where degree of entanglement parametrized by $0 \leq \alpha \leq 1$

$$|\psi_A\rangle|\psi_B\rangle \in \mathbf{H_A} \otimes \mathbf{H_B}$$

Initially

$$|\psi(t_1 = t_2 = 0)\rangle = \sqrt{\alpha}|\psi_A\rangle|\psi_B\rangle - \sqrt{1-\alpha}|\psi_B\rangle|\psi_A\rangle$$

later evolves according to H_A and H_B

$$|\psi_{A,B}(t_{A,B})\rangle = e^{-iH_{A,B}t_{A,B}}|\psi_{A,B}\rangle$$

Density matrix formalism: entanglement entropy

State of the system (any orthonormal basis) $|\psi(t)\rangle$ and define density operator (positive-definite, idempotent, Hermitean, Tr=1)

$$\rho(t) = |\psi(t)\rangle\langle\psi(t)|$$

Define von Neumann entropy

$$S(t) = -\mathrm{Tr}\,\rho(t)\,\ln\rho(t)$$

Bipartite division: the subsystem A and its remainder B

Define **reduced density matrix** and the **entanglement entropy**

$$\rho(t)_A = \mathrm{Tr}_B\,\rho(t) \qquad S_A(t) = -\mathrm{Tr}\,\rho(t)_A \ln\rho(t)_A$$

Tracing over B's degrees of freedom

Appeal to intuition, interpretations and limit cases

Von Neumann entropy: minimum nr of bits to store information about system

Entanglement entropy: **nr of entangled bits between subsystems** A and B

For a simple binary system

$$|\psi\rangle = \sqrt{\alpha}|\uparrow\rangle_A|\downarrow\rangle_B - \sqrt{1-\alpha}|\downarrow\rangle_A|\uparrow\rangle_B \qquad 0 \leq \alpha \leq 1$$

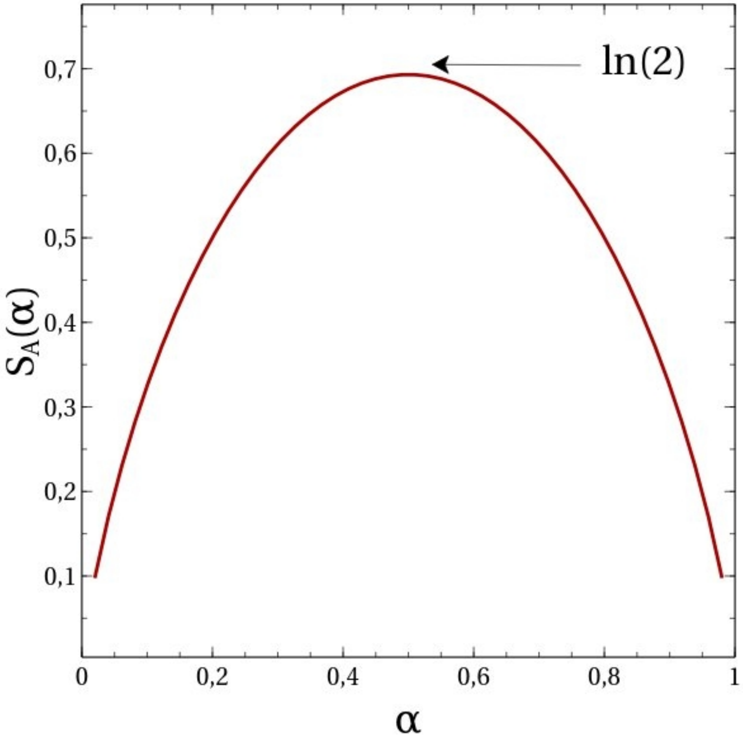
entanglement entropy reads

$$S(\alpha) = -\alpha \ln \alpha - (1-\alpha)\ln(1-\alpha)$$

S(α) is maximal for **maximally entangled state** with α=1/2.

S(α) is minimal (zero) for α=0, 1 corresponding to **factorized states**;

α→0, 1 is spontaneous factorization corresponding to Furry's hypothesis



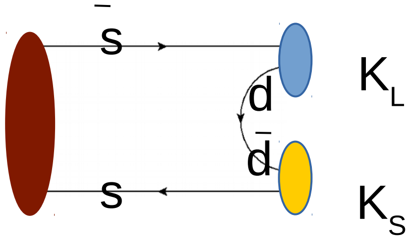
The K_L - K_S system - entropy

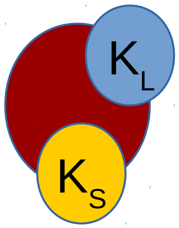
$$|\psi(t_1 = t_2 = 0)\rangle = \sqrt{\alpha}\; K_L\rangle_1\, K_S\rangle_2 - \sqrt{1-\alpha}\; K_S\rangle_1\, K_L\rangle_2$$

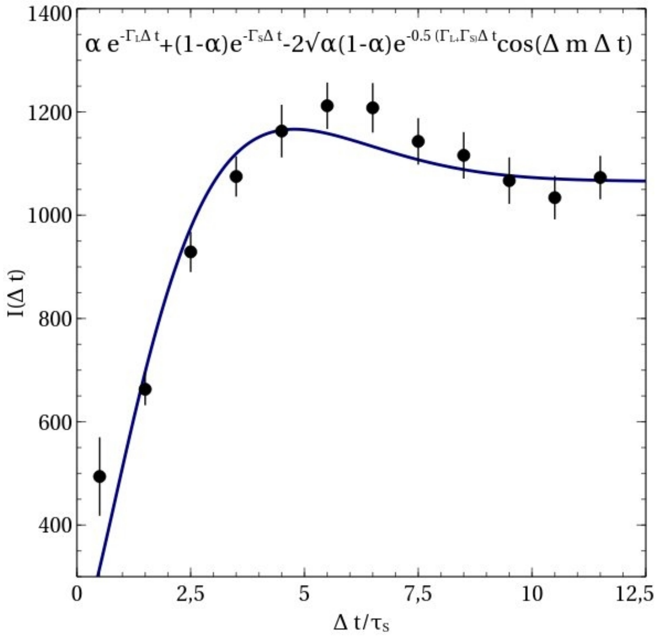
Decay intensity (measured identical K → $\pi^+ \pi^-$ final states)

$$I(\Delta t) = \alpha e^{-\Gamma_L \Delta t} + (1-\alpha) e^{-\Gamma_S \Delta t} - 2\sqrt{\alpha(1-\alpha)}\, e^{-\bar{\Gamma}\Delta t} \cos(\Delta m \Delta t)$$

α characterizes initial-state decay of Φ (lifetime 1.5x10^{-22} s)







KLOE data allow to determine   α=0.72 ± 0.31

Statistical and systematic errors combined

Note, in case of decays of $1^{--} \to 0^- 0^-$ α≠1/2 may violate antisymmetry

Density matrix K_L - K_S system

$$\begin{aligned}
\rho(t_1,t_2) &= |\psi(t_1,t_2)\rangle\langle\psi(t_1,t_2)| \\
& \alpha |K_S(t_1)\rangle_A |K_L(t_2)\rangle_B \; {}_B\langle K_L(t_2)|_A\langle K_S(t_1)| \\
&- \sqrt{\alpha(1-\alpha)} |K_S(t_1)\rangle_A |K_L(t_2)\rangle_B \; {}_B\langle K_S(t_2)|_A\langle K_L(t_1)| \\
&- \sqrt{\alpha(1-\alpha)} |K_L(t_1)\rangle_A |K_S(t_2)\rangle_B \; {}_B\langle K_L(t_2)|_A\langle K_S(t_1)| \\
&+ \alpha |K_L(t_1)\rangle_A |K_S(t_2)\rangle_B \; {}_B\langle K_S(t_2)|_A\langle K_L(t_1)|
\end{aligned}$$

$|K_L\rangle$, $|K_S\rangle$ are physical states but **are not orthogonal**; can mix due to CP violation

One needs to properly define density matrix in orthonormal basis

$$|K_1\rangle, |K_2\rangle$$

Reduced density matrix $\rho_A \sim \mathcal{O}(\varepsilon)$

- tracing over states detected in B detector ${}_B\langle K_{1,2}|\rho|K_{1,2}\rangle_B$
- integrating over evolution time of second particle $\int_0^\infty dt_2\,\rho$

$$\rho_{A_{11}}(t) = \frac{\alpha}{\Gamma_L} e^{-\Gamma_S t}$$

$$\rho_{A_{22}}(t) = \frac{1-\alpha}{\Gamma_S} e^{-\Gamma_L t}$$

$$\rho_{A_{12}}(t) = \frac{\varepsilon^* \alpha}{\Gamma_L} e^{-\Gamma_S t} + \frac{\varepsilon(1-\alpha)}{\Gamma_S} e^{-\Gamma_L t} - \frac{2(\Re\varepsilon)\sqrt{\alpha(1-\alpha}}{\bar{\Gamma}^2 + (\Delta m)^2} e^{-\bar{\Gamma} t} \left(\bar{\Gamma} e^{i\Delta m t} + \Delta m e^{i(\Delta m - \pi/2)}\right)$$

$$\rho_{A_{21}}(t) = \rho_{A_{12}}(t)^*$$

Explicit form of the off-diagonal complex elements

$$
\begin{aligned}
\Re\rho_{A_{12}} &= (\Re\varepsilon)\Big(\frac{\alpha}{\Gamma_L}e^{-\Gamma_S t} + \frac{1-\alpha}{\Gamma_S}e^{-\Gamma_L t}\Big) \\
&- 2(\Re\varepsilon)\frac{\sqrt{\alpha(1-\alpha)}}{\bar{\Gamma}^2+(\Delta m)^2}e^{-\bar{\Gamma}t}\Big(\bar{\Gamma}\cos(\Delta m t) + \Delta m\sin(\Delta m t)\Big)
\end{aligned}
$$

$$
\begin{aligned}
\Im\rho_{A_{12}} &= (\Im\varepsilon)\Big(-\frac{\alpha}{\Gamma_L}e^{-\Gamma_S t} + \frac{1-\alpha}{\Gamma_S}e^{-\Gamma_L t}\Big) \\
&- 2(\Re\varepsilon)\frac{\sqrt{\alpha(1-\alpha)}}{\bar{\Gamma}^2+(\Delta m)^2}e^{-\bar{\Gamma}t}\Big(\bar{\Gamma}\sin(\Delta m t) - \Delta m\cos(\Delta m t)\Big)
\end{aligned}
$$

Note general requirement $\mathbf{Tr}\ \boldsymbol{\rho} = \mathbf{1}$
meaning that for decaying system one needs to renormalize density matrix

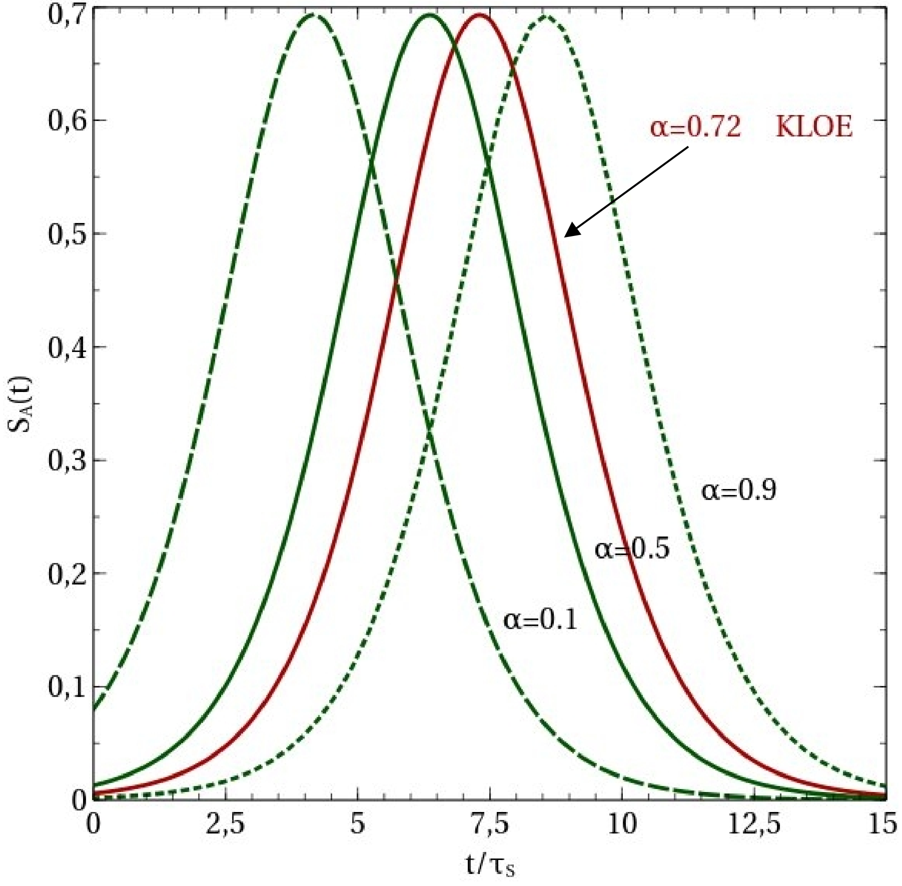
$$\rho_{A}(t) \rightarrow \frac{\rho_{A}(t)}{\mathrm{Tr}\ \rho_{A}(t)}$$

Entanglement entropy

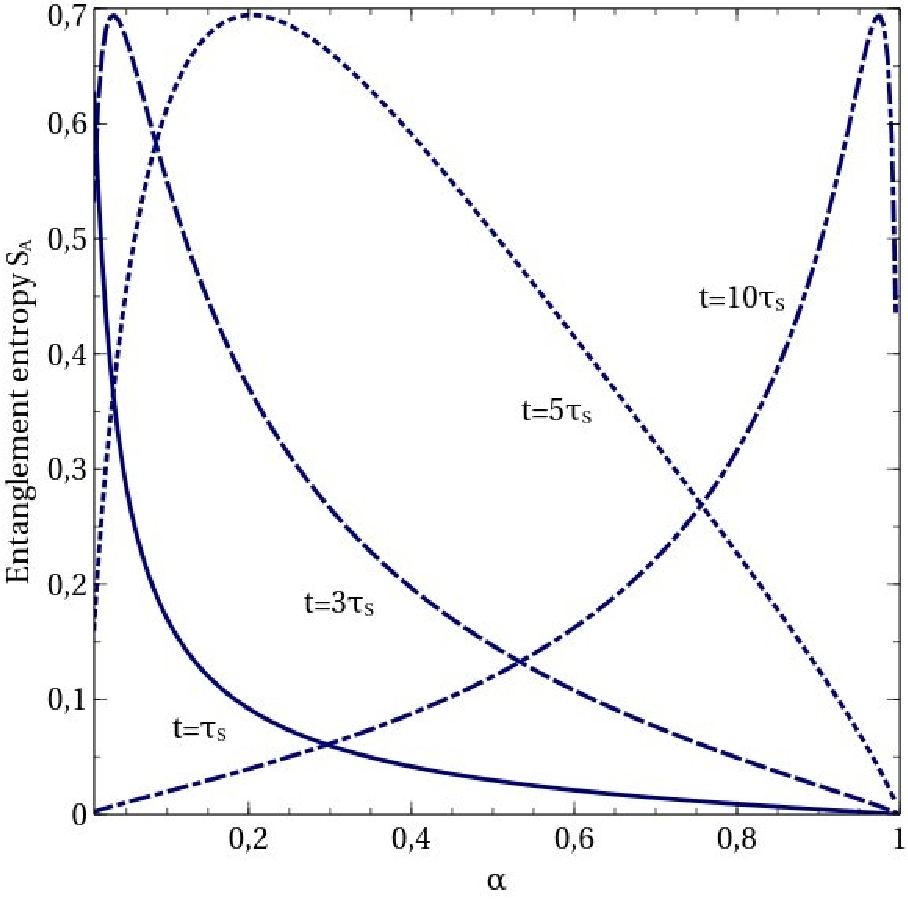
$$\begin{aligned} S_{A}(t) &= -\mathrm{Tr}\ \hat{\rho}_{A}(t)\ \ln\ \hat{\rho}_{A}(t) \\ &= -\rho_{A_{11}} \ln \rho_{A_{11}} - \rho_{A_{22}} \ln \rho_{A_{22}} - \rho_{A_{12}} \ln \rho^{*}_{A_{12}} - \rho^{*}_{A_{12}} \ln \rho_{A_{12}} \\ &= -\rho_{A_{11}} \ln \rho_{A_{11}} - \rho_{A_{22}} \ln \rho_{A_{22}} - 2\left[\left(\Re\rho_{A_{12}}\right) \ln \left|\rho_{A_{12}}\right| + \left(\Im\rho_{A_{12}}\right) \arg\ \rho_{A_{12}}\right] \end{aligned}$$

$$\arg\ \rho_{A_{12}} = \arctan \frac{\Im\rho_{A_{12}}}{\Re\rho_{A_{12}}}$$

Maximal entanglement ln(2)=0.69



Time of maximal entanglement is ɑ-dependent



Value of $ɑ_{max}$ depends on time

Entanglement of beauty mesons

Much heavier than kaons, shorter living, not much different lifetimes between CP-even and CP-odd states.

Also, CP-violation effects in b-mesons are smaller than in strange mesons

Beauty non-strange pairs

$$\Upsilon(10580) = \Upsilon(4S) \rightarrow B^0\bar{B}^0$$

with branching fraction 49%

m(B^0)=5280 MeV

Beauty strange pairs are harder to observe

$$\Upsilon(10860) \rightarrow B_s^0\bar{B}_s^0$$

with branching fraction only 0.1%

m(B$^0{}_s$)=5367 MeV,
exceeds ½ m(Y(4S))

Properties

$$\mathbf{B^0} = (\mathbf{d\bar{b}})$$

$$\Delta\Gamma = \Gamma_H - \Gamma_L = 0.002 \pm 0.011 \text{ ps}^{-1}$$

$$\tau_B = 1/\Gamma = 1.52 \text{ ps}$$

$$\Delta m = m_H - m_L = 0.506 \text{ ps}^{-1}$$

Important difference between K- and B- systems:

Lifetimes of K_L and K_S much different (much different phase space between decay modes)

Lifetimes of B_H and B_L similar

Observable to determine entanglement parameter α

$$|\Upsilon(4S)\rangle \quad \rightarrow \quad \frac{1}{\sqrt{2}}\left(|B^0\rangle_1|\bar{B}^0\rangle_2 - |\bar{B}^0\rangle_1|B^0\rangle_2\right)$$

$$= \quad \frac{1}{\sqrt{2}}\left(|B_H\rangle_1|B_L\rangle_2 - |B_L\rangle_1|B_H\rangle_2\right)$$

Small CP violation effects < 10^{-3}
neglected here; B_H and B_L orthogonal
with good accuracy; therefore =

$\alpha^{½}$

$(1-\alpha)^{½}$

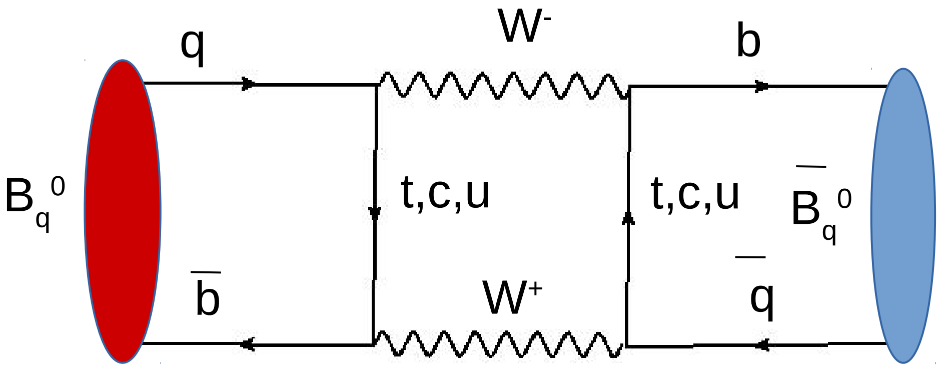
Entanglement parameters

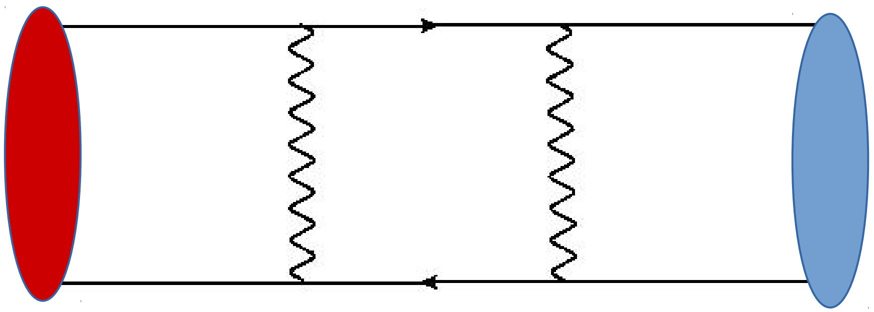
Dependence on individual proper times t_1 and t_2

$$|B_{H,L}(t_{1,2})\rangle = e^{-im_{H,L}\,t_{1,2} - \frac{\Gamma_{H,L}}{2}\,t_{1,2}}|B_{H,L}\rangle$$

Integration over t_1+t_2
dependence on $\Delta t = t_2 - t_1$ considered

Time evolution of final state due to mixing: $B_q^0 \rightarrow \bar{B}_q^0$ transitions (q=d,s)





B flavour identified using semi-leptonic decays

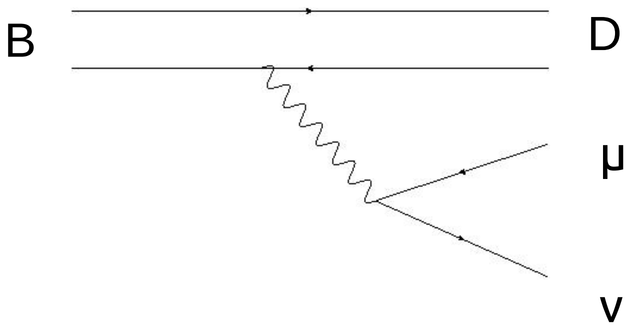
$$B^0 \rightarrow D^{(*)-}\mu^+\nu_\mu$$

- sign for anti-B

$$D^- \rightarrow K^+\pi^-\pi^-$$

$$D^{*-} \rightarrow \bar{D}^0\pi^-$$

$$\bar{D}^0 \rightarrow K^+\pi^-$$



Rate asymmetry between unmixed and mixed final states used to investigate entanglement

$$A(\Delta t) = \frac{N_u(\Delta t) - N_m(\Delta t)}{N_u(\Delta t) + N_m(\Delta t)}$$

where

$$\begin{aligned} N_u(\Delta t) &= \int_{\Delta t}^{\infty} dT \,|\langle B(\Delta t, T)\bar{B}(\Delta t, T)|\Upsilon(\Delta t, T)\rangle|^2 \\ &+ \int_{\Delta t}^{\infty} dT \,|\langle \bar{B}(\Delta t, T)B(\Delta t, T)|\Upsilon(\Delta t, T)\rangle|^2 \end{aligned}$$

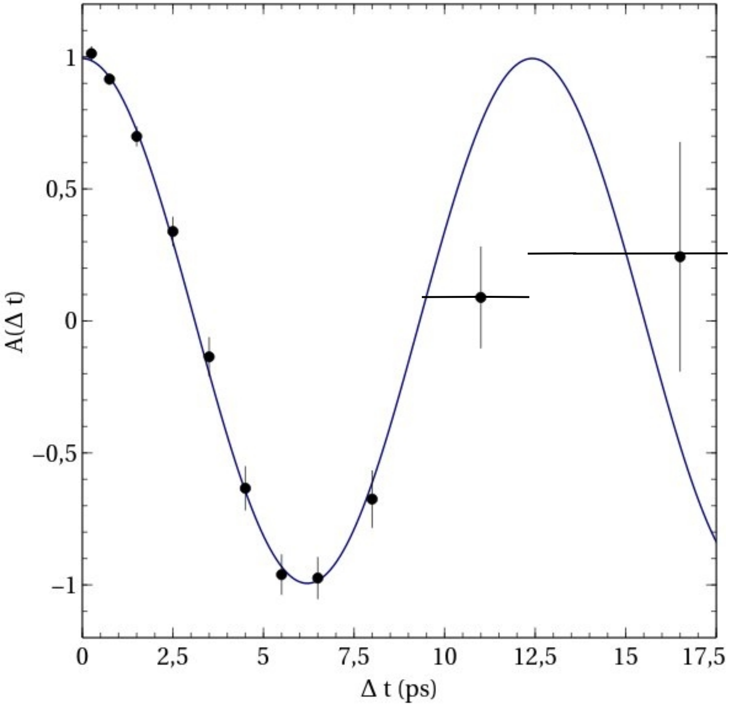
$$\begin{aligned} N_m(\Delta t) &= \int_{\Delta t}^{\infty} dT \,|\langle B(\Delta t, T)B(\Delta t, T)|\Upsilon(\Delta t, T)\rangle|^2 \\ &+ \int_{\Delta t}^{\infty} dT \,|\langle \bar{B}(\Delta t, T)\bar{B}(\Delta t, T)|\Upsilon(\Delta t, T)\rangle|^2 \end{aligned}$$

$$A(\Delta t) = \frac{2\sqrt{\alpha(1-\alpha)}\cos(\Delta m \Delta t)}{\alpha e^{-\Delta\Gamma \Delta t/2} + (1-\alpha)e^{\Delta\Gamma \Delta t/2}}$$

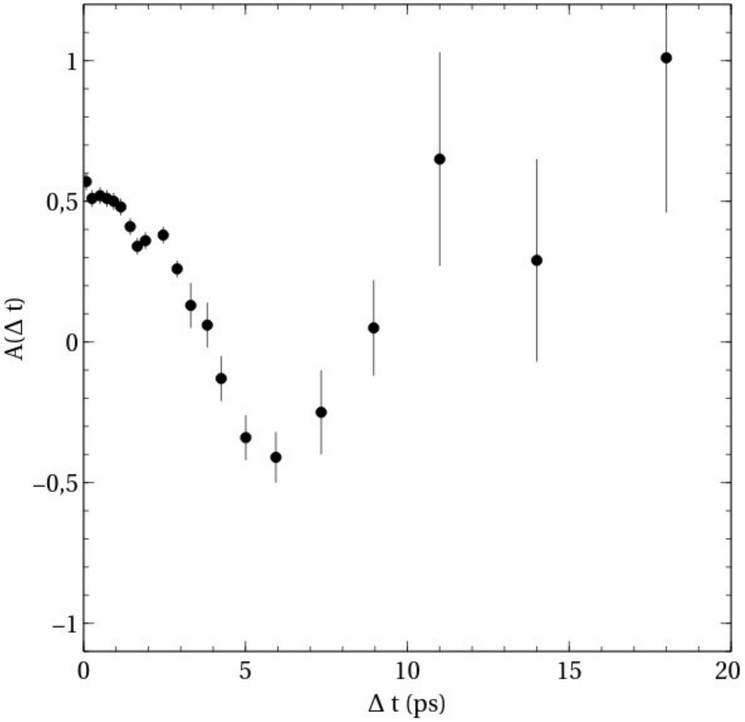
Small number

Two sets of data available experimentally: Belle and BaBar experiments

Phys.Rev.Lett. 99 (2007) 131802



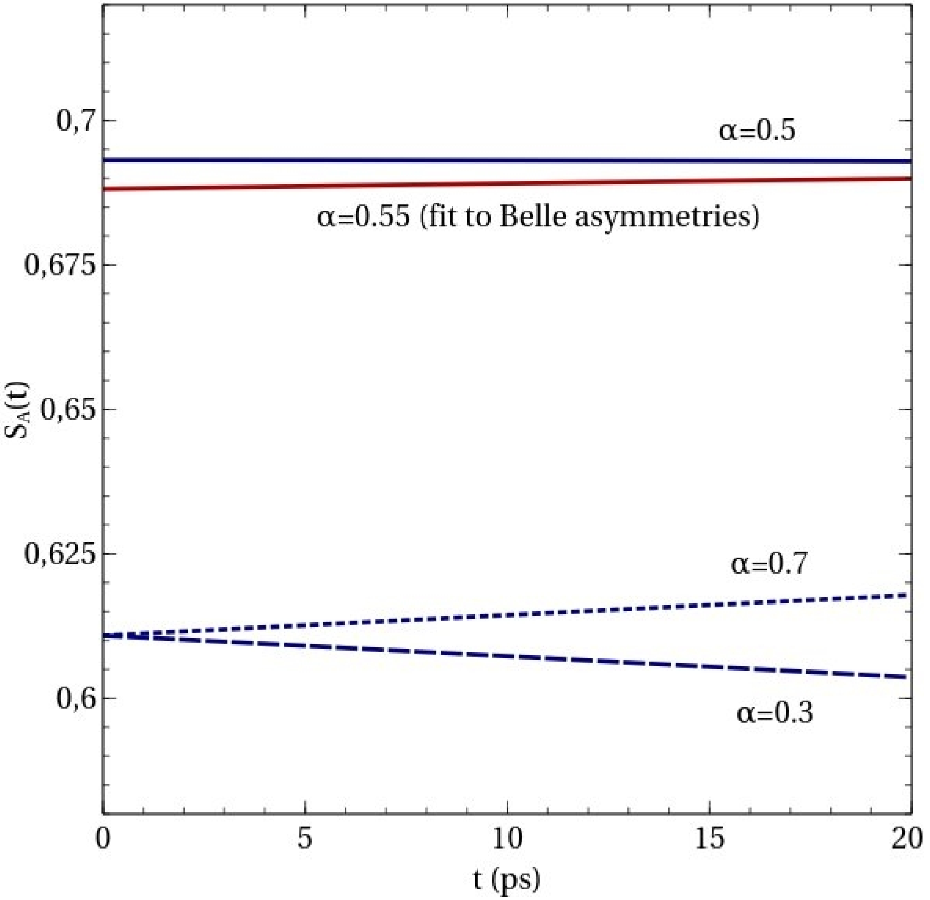
Phys.Rev. D66 (2002) 032003

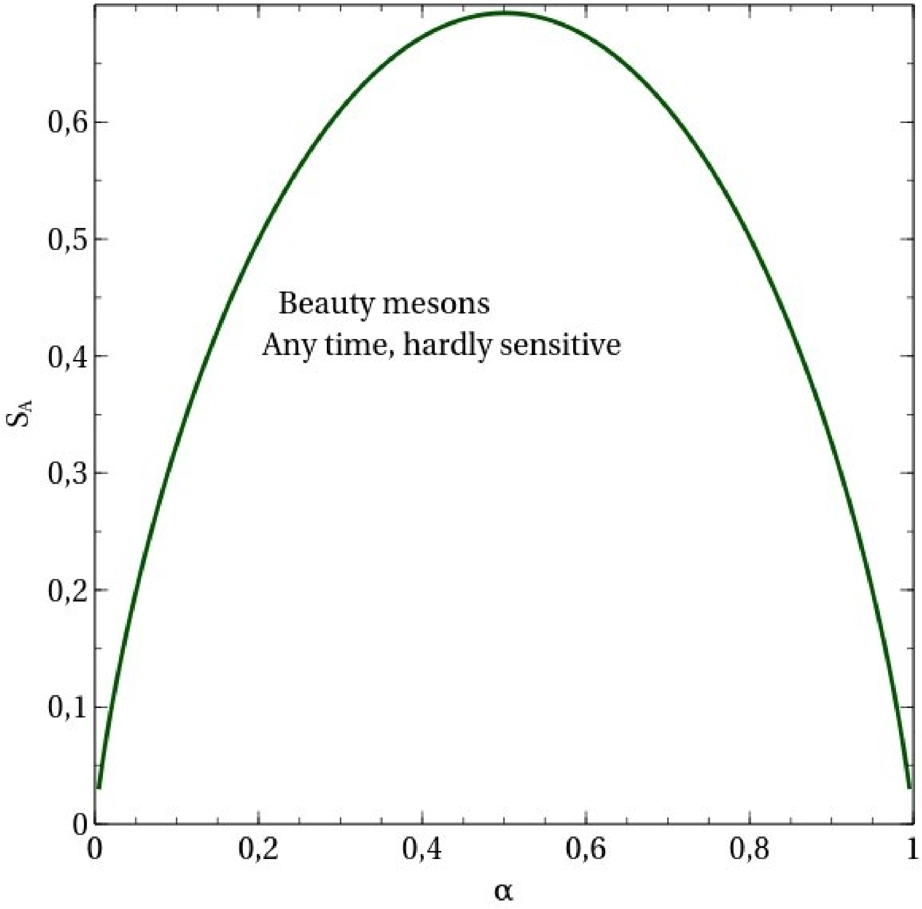


α = 0.55 ± 0.07

Cannot fit, no access to tagging efficiencies

The B – B system - entropy





Weak time dependence compared to kaons; small ΔΓ

Cumulants and fluctuations of CP

Dynamics of meson decays affects both entanglement of a pair and fluctuations of CP detected;
CP fluctuations are fluctuations of nr kaons of given CP, K_1 or K_2

Approach to measure it
define time-dependent moment generating function

Expected value in state ψ(t)

$$\begin{aligned} \chi(\lambda, t) &= \langle \exp(i\lambda \cdot CP_A(t)) \rangle \\ &= \sum_m P(CP_A = m) e^{i\lambda m} \end{aligned}$$

and cumulants given by differentiation

$$C_n = \left(-i \frac{\partial}{\partial \lambda} \right)^n \ln \chi(\lambda, t) \Big|_{\lambda=0}$$

C_1 – mean value

C_2 – variance

Back to two-kaon system

$$\begin{aligned}
|\psi(t_1,t_2)\rangle &= \sqrt{\alpha}|K_S(t_1)\rangle|K_L(t_2)\rangle - \sqrt{1-\alpha}|K_L(t_1)K_S(t_2)\rangle \\
&= \sqrt{\alpha}e^{(-im_S+\Gamma_S)t_1}e^{(-im_L+\Gamma_L)t_2}(|K_1\rangle_A + \varepsilon|K_2\rangle_A)(\varepsilon|K_1\rangle_B + |K_2\rangle_B) \\
&- \sqrt{1-\alpha}e^{(-im_L+\Gamma_L)t_1}e^{(-im_S+\Gamma_S)t_2}(\varepsilon|K_1\rangle_A + |K_2\rangle_A)(|K_1\rangle_B + \varepsilon|K_2\rangle_B)
\end{aligned}$$

1. Integration over t_2

2. O(ε)

3. Amplitudes for kaon-detector configurations

f_{11} K_1 detector A, K_1 detector B
f_{12} K_1 detector A, K_2 detector B
… etc.

Cumulant generating function and cumulants

$$
\begin{aligned}
\chi(\lambda, t) &= P(CP_A = +1)e^{i\lambda} + P(CP = -1)e^{-i\lambda} \\
&= (|f_{11}(t)|^2 + |f_{12}|^2)e^{i\lambda} + (|f_{22}(t)|^2 + |f_{21}|^2)e^{-i\lambda} \\
&= \frac{\alpha e^{-\Gamma_S t} e^{i\lambda}}{\Gamma_L^2/4 + m_L^2} + \frac{(1-\alpha)e^{-\Gamma_L t} e^{-i\lambda}}{\Gamma_S^2/4 + m_S^2}
\end{aligned}
$$

$$\chi(\lambda, t) = A(t)e^{i\lambda} + B(t)e^{-i\lambda}$$

In our case, all these depend on degree of entanglement α

CP mean value

$$\begin{aligned} C_1(t,\alpha) &= \langle CP(t) \rangle \\ &= \frac{A(t) - B(t)}{A(t) + B(t)} \\ &= \frac{\alpha - (1-\alpha)\frac{\Gamma_L^2+4m_L^2}{\Gamma_S^2+4m_S^2} e^{\Delta\Gamma t}}{\alpha + (1-\alpha)\frac{\Gamma_L^2+4m_L^2}{\Gamma_S^2+4m_S^2} e^{\Delta\Gamma t}} \end{aligned}$$

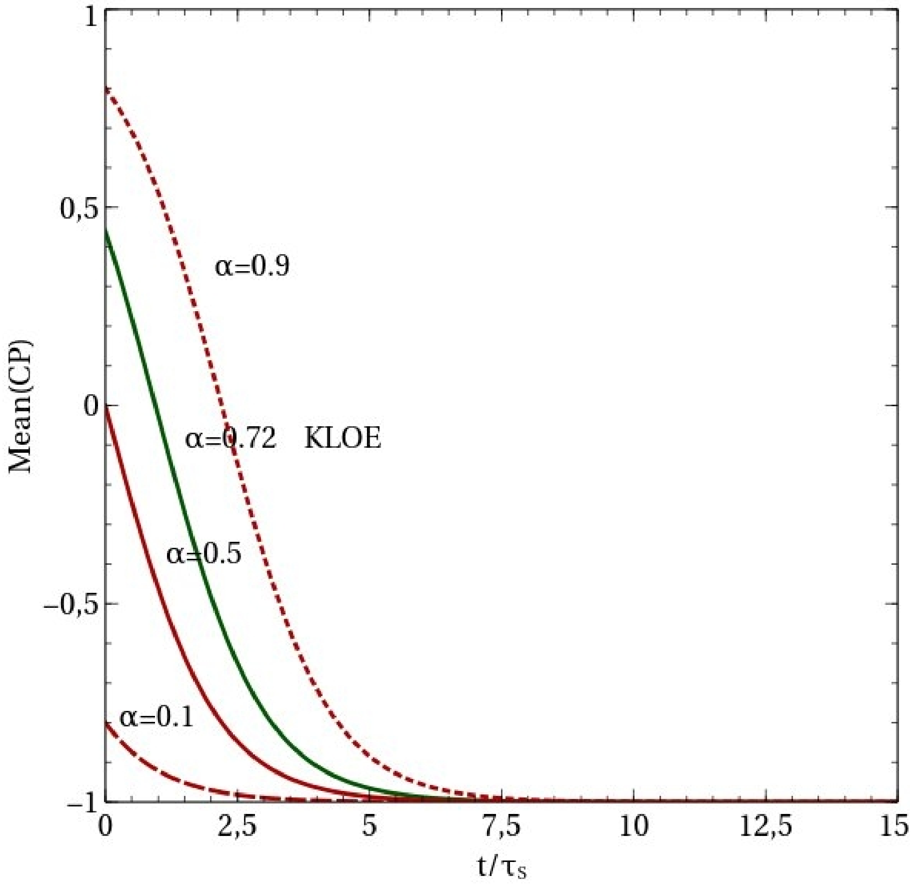
$$\Delta\Gamma = \Gamma_S - \Gamma_L$$

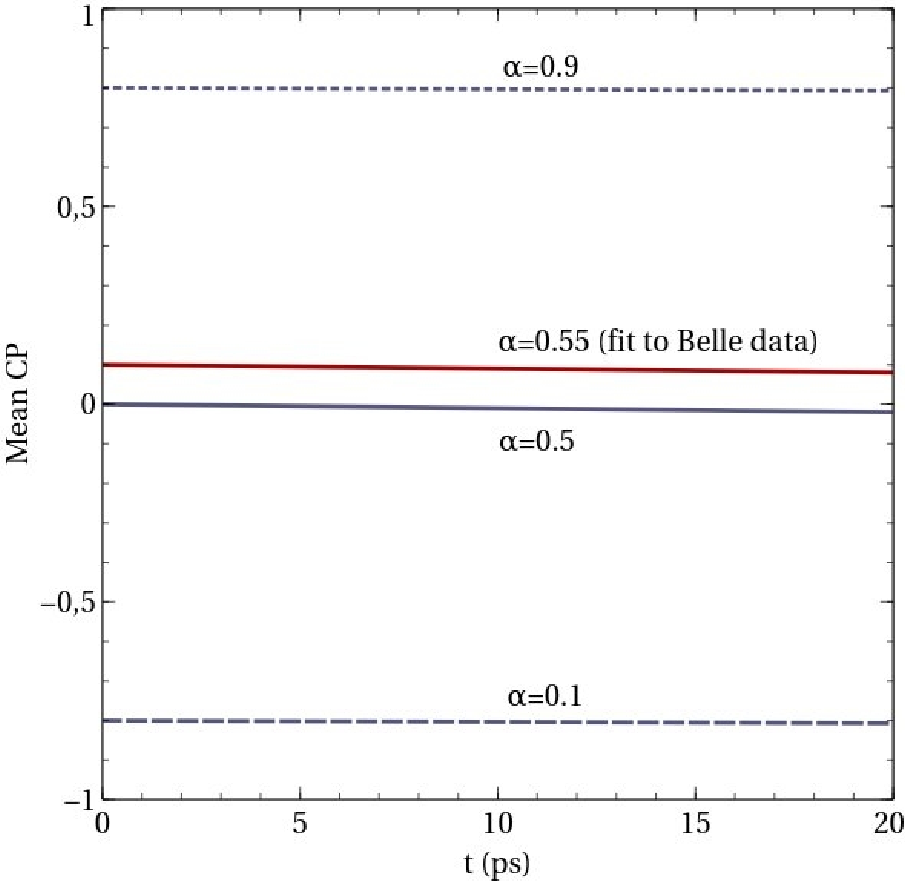
$$C_1(t,\alpha) \xrightarrow[t\to 0]{} 2\alpha - 1$$

$$C_1(t,\alpha) \xrightarrow[t\to\infty]{} -1$$

For large time only long-living K_2 component with CP=-1 survives, all short-living K_1 with CP=+1 die out

Mean CP





K-mesons
Asymptotics well seen, only K_L survive

B-mesons
Asymptotics not seen, same lifetimes of B_H and B_L, CP determined mainly by ɑ and not evolution time

Initial values for t=0 everywhere 2ɑ-1

Variance of CP

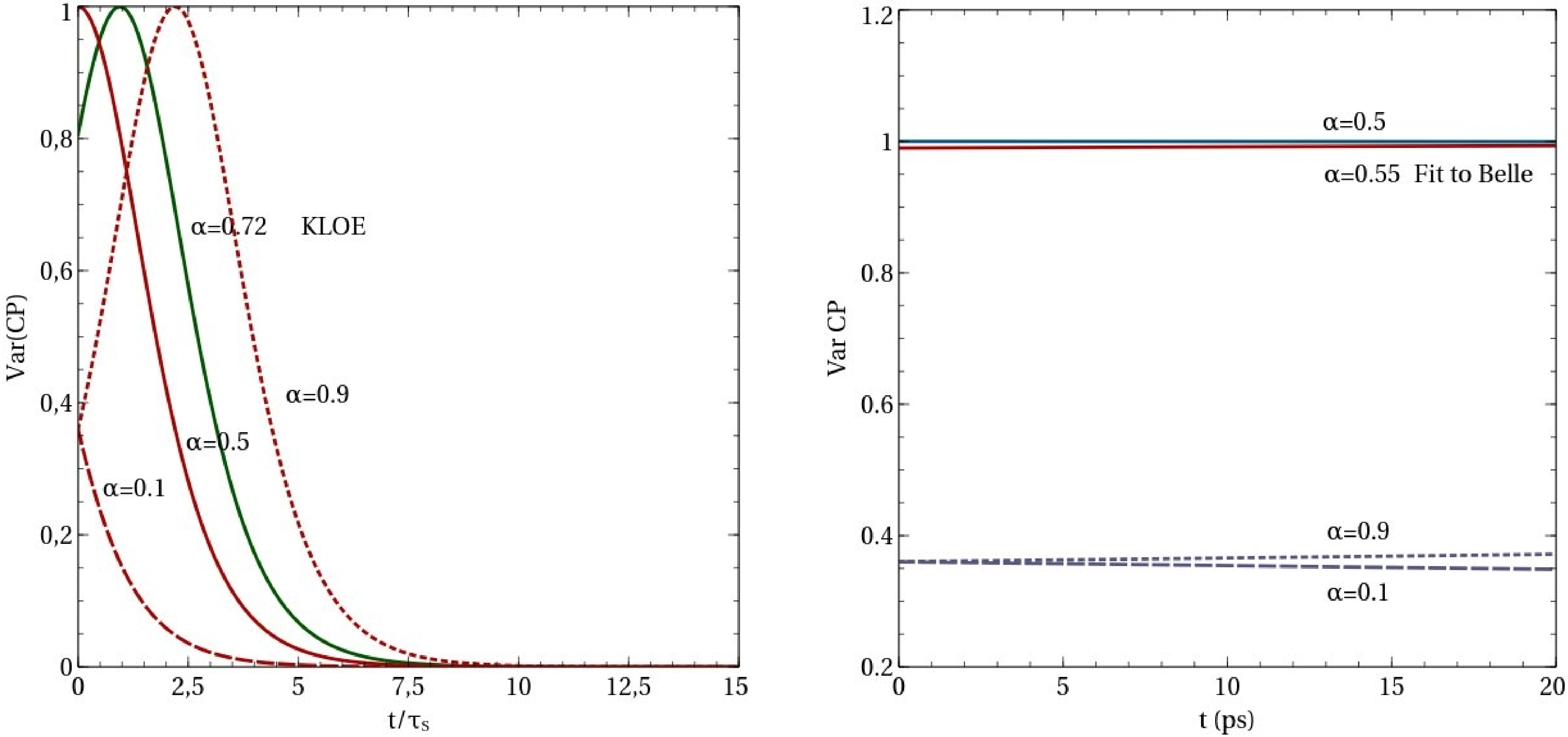
$$\begin{aligned} C_2(t,\alpha) &= \langle CP(t)^2\rangle - \langle CP(t)\rangle^2 \\ &= 1 - \left[\frac{A(t) - B(t)}{A(t) + B(t)}\right]^2 \\ &= 1 - \left[\frac{\alpha - (1-\alpha)\frac{\Gamma_L^2+4m_L^2}{\Gamma_S^2+4m_S^2}e^{\Delta\Gamma t}}{\alpha + (1-\alpha)\frac{\Gamma_L^2+4m_L^2}{\Gamma_S^2+4m_S^2}e^{\Delta\Gamma t}}\right]^2 \end{aligned}$$

$$C_2(t,\alpha) \xrightarrow[t\to 0]{} 4\alpha(1-\alpha)$$

$$C_2(t,\alpha) \xrightarrow[t\to \infty]{} 0$$

For large time only long-living K_2 survives and CP distribution narrows down to sharp value; short-living K_1 die out

Variance of CP



Same differences in asymptotics between KK and BB, as for mean CP;

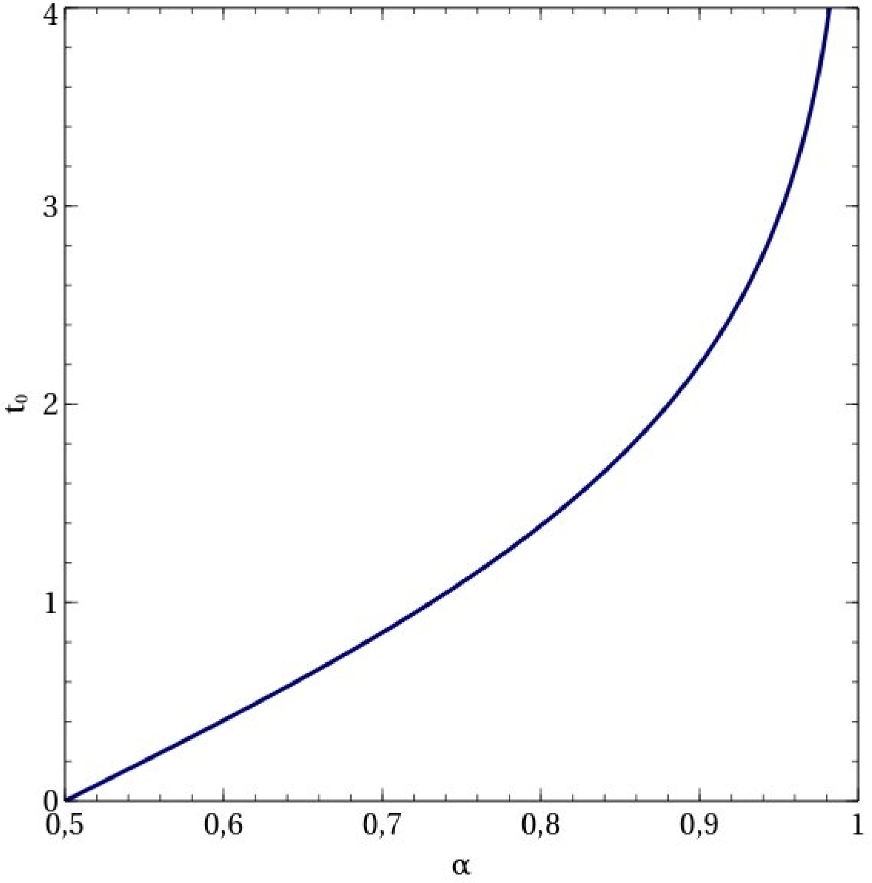
KK – CP variance becomes asymptotically sharp

Interesting feature – KK only

Variance of CP can be non-monotonic and have a maximum at

$$t_0 = \frac{1}{\Delta\Gamma} \ln\left(\frac{\Gamma_S^2+4m_S^2}{\Gamma_L^2+4m_L^2}\frac{\alpha}{1-\alpha}\right)$$

about 1



$$t_0 > 0 \iff \alpha \gtrsim 1/2$$

Location of the maximum variance of CP Moves

$$t_0 \xrightarrow[\alpha\to 1]{} \infty$$

but its value remains ~1

Concluding remarks

- Neutral meson systems (KK, BB, ..) are natural examples of quantum bi-partite systems
- Degree of initial entanglement α is a parameter determining degree of entanglement in future evolution, quantified by entanglement entropy S_A

May be also related to violation of state symmetry

- Presented approach does not describe dissipative decoherence, though extensions in this direction are possible
- Interesting dependence of detected CP distribution on α can be observed

Extensions and further interest

- Sensitivity of S_A evolution to CPT (δ, ω parameters)
- Renyi entropies approach allowing to exploit some thermodynamic analogues
- Tri-partite systems carrying same quantum nrs (e.g. 3γ from orthopositronium) and question on nature of correlations there (e.g. Borromean vs. Hopfian)