Discrete symmetries in gravitational fields

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Motivation

Last 20 years: tremendous interest and progress in testing Lorentz invariance.

Why? Two motivations.

1. Increase of precision of experiments due to advances in technology.
2. Theoretical suggestions that Lorentz symmetry may not be exact at all energies — search for fundamental theory valid at Planck energies

$$E_P = \sqrt{\frac{\hbar c^5}{G}} = 1,2 \cdot 10^{19} \mathrm{GeV}.$$

These are quantum gravity theory (QG) and string theory (,,theory of everything").

Lorentz symmetry violation (LSV) in QG is a conjecture — no model of QG does predict LSV uniquely.

Two possibilities:
1) Full QG does predict LSV at $E \approx E_P \Rightarrow$ by continuity small LSV occurs at low energies.
2) Full QG is Lorentz invariant (LI) and admits tensor fields having vacuum expectation values at low energies VEV$\neq 0$. These fields *spontaneously* break Lorentz symmetry by VEV$\neq 0$.
This means: QG is LI, but the low energy quantum state we live in, is not LI.
Conclusion:
low energy experiments cannot establish whether or not exact QG breaks LI.

Here I am dealing with the low–energy approximation to QG $\equiv$ Effective Field Theory (EFT).
In practice: this is a *non–quantum* field theory with the Lagrangian of the Standard Model plus low energy additional terms involving Lorentz symmetry braking operators and a gravitational sector $\equiv$ Standard Model Extension (SME).

Continuous LSV

Def.
LSV means violation of the explicit form–invariance of the dynamical action for a physical field by Lorentz transformations (continuous or CPT).
Example.
Minkowski space $\mathcal{M}_4$ with $\eta_{\mu\nu}$ and a *fixed constant* symmetric tensor field $\zeta^{\mu\nu}$ =const. $\phi(x)$ — a massless scalar field coupled to $\zeta^{\mu\nu}$,

$$S = \int_D d^4x \left[\eta^{\mu\nu} \frac{\partial\phi}{\partial x^\mu}\frac{\partial\phi}{\partial x^\nu} + \zeta^{\mu\nu}\frac{\partial\phi}{\partial x^\mu}\frac{\partial\phi}{\partial x^\nu}\right],$$

S and integrand terms — scalars. L — active Lorentz transf.,
$L: \; x \in \mathcal{M}_4 \longmapsto y = L^{-1}x \in \mathcal{M}_4$ or $x = Ly$,
L maps D onto $D' = L(D)$. ϕ is a scalar:

$$\phi(x) = \phi(Ly) \equiv \phi'(y).$$

S remains unchanged:

$$S = \int_{D'} d^4y \left[\eta^{\mu\nu} \frac{\partial \phi'}{\partial y^\mu} \frac{\partial \phi'}{\partial y^\nu} + \zeta'^{\mu\nu} \frac{\partial \phi'}{\partial y^\mu} \frac{\partial \phi'}{\partial y^\nu} \right],$$

$\eta^{\mu\nu} = \eta'^{\mu\nu} = \mathrm{diag}[1, -1, -1, -1]$ — form–invariant,

$$\zeta'^{\mu\nu} = (L^{-1})^\mu{}_\alpha (L^{-1})^\nu{}_\beta \zeta^{\alpha\beta} \neq \zeta^{\mu\nu}$$

is *not* form–invariant (two different matrices).
The scalar term $\zeta^{\mu\nu}\partial_\mu\phi\partial_\nu\phi$ in S causes LSV if it is not form–invariant under L — it contains a non–dynamical $\zeta^{\mu\nu}$ which is *frame–dependent*.

$\zeta^{\mu\nu}$ generates LSV in *all* inertial reference frames.
There are *preferred* frames: $\zeta^{\mu\nu}$ has simplest form and/or is small — *concordant frames*.
The existence of preferred frames (directions in spacetime) is *not* necessary — only in simple models of SME.
If two LI violating tensors, $\zeta^{\mu\nu}$ and $\chi^{\mu\nu}$ $\Rightarrow$ *no* preferred frames.

CPT symmetry

Special relativity: continuous Lorentz symmetry is necessary to prove the CPT theorem in QFT. Continuous LSV allows for CPT violation, but does not require it. CPT violation implies continuous LSV in local QFT.
In $\mathcal{M}_4$ discrete symmetries C, P, T and CPT are *well defined*.
General relativity (GR): curved spacetime $\Rightarrow$ arbitrary reference frames (no preferred frames) $\Rightarrow$ arbitrary coordinates — have *no direct geometrical* (physical) meaning $\Rightarrow$ just 4 numbers labelling points (events).
Slicing of spacetime by spaces (3–dim. sets of simultaneous events) labelled by a time coordinate — is arbitrary to a large extent.
Time coordinate — does *not* measure proper time,
spatial coordinates — do *not* determine distances.

Spacetime geometry (metric) evolves in time and space — inhomogeneous and anisotropic, no symmetries.
No translational invariance in time $\Rightarrow$ no time reversal T,
no translational invariance in space $\Rightarrow$ no space inversion P.
Only future and past directions are defined:
future of an event p is the collection of all points inside the future light cone of p.
In GR the *local* Lorentz invariance holds: at each point there is the whole set of local Lorentz frames and none of them is preferred and there are no distinguished directions at this point.
In GR continuous LSV occurs due to the covariant form of tensors (operators) in the action generating LSV in Minkowski spacetime.

In GR there are no separate symmetries P and T.
There is *no* clear definition of CPT symmetry $\Rightarrow$ no CPT theorem in QFT (actually there is no QFT in curved spacetimes).
Only an indirect indication that an analogue of CPT symmetry might be broken.
Conjecture:
in gravitational field CPT symmetry is broken by the covariant form of those terms in S which break CPT symmetry in Minkowski $\mathcal{M}_4$.

Consequences of CPT violation in gravit. field

(Then continuous Lorentz symmetry is also broken.)
1) In some models the fundamental axioms of GR, the Weak and Strong Equivalence Principles are violated: particle accelerations are *mass dependent*;
2) troubles with causality (model dependent): no universal light cone that all phys. fields must propagate within;
3) existence of a stable ground state in EFT is doubtful: perturbations with $E > 0$ in one reference frame may turn out $E < 0$ modes in another frame $\Rightarrow$ there are *inequivalent* ground state solutions.

Properties of Lorentz symmetry violating tensors in GR

$\zeta^{\mu_1 \cdots \mu_n}$ =const, $n = 1, 2, \ldots, k$ — Lorentz symmetry violating tensor in $\mathcal{M}_4$.
GR: constant ζ replaced by a point–dependent field $\zeta^{\mu_1 \cdots \mu_n}(x)$.
Two possibilities:
1) $\zeta(x)$ — a *fixed non–dynamical* field (additional geometrical structure of the spacetime);
2) $\zeta(x)$ — a *dynamical* (physical) field with its own equations of motion.
Both cases generate severe problems.

Case 1) — fixed non-dynamical operator ζ. Dynamics is (or may be) *inconsistent*.

Choice of the point dependence of $\zeta(x)$ is unclear:

does $\partial_\mu \zeta = 0$ in $\mathcal{M}_4$ imply $\nabla_\mu \zeta(x) = 0$ in GR?

In most curved spacetimes this is *inconsistent*: $\nabla_\mu \zeta \neq 0$.

The tensor field $\zeta(x)$ must be carefully adjusted.

Assume: $\zeta(x)$ is adjusted in the given spacetime so that $\zeta =$const in the flat spacetime limit.

Then in general Einstein field equations are *inconsistent*.

Simplest example:

scalar massless $\phi(x)$, adjusted $\zeta^{\mu\nu}(x)$ — no CPT violation, only continuous LSV.

Action

$$S = \int_D d^4x \sqrt{-g} \left[\frac{1}{16\pi G} R + (g^{\mu\nu} + \zeta^{\mu\nu}(x)) \frac{\partial\phi}{\partial x^\mu} \frac{\partial\phi}{\partial x^\nu} \right].$$

Eqs. of motion for ϕ:

$$\Box\phi + \nabla_\mu \left(\zeta^{\mu\nu} \frac{\partial\phi}{\partial x^\nu} \right) = 0,$$

Einstein field eqs.:

$$\begin{aligned} G_{\alpha\beta} &= 8\pi G T_{\alpha\beta} \equiv \\ &\equiv 8\pi G \left[2 \frac{\partial\phi}{\partial x^\alpha} \frac{\partial\phi}{\partial x^\beta} - (g^{\mu\nu} + \zeta^{\mu\nu}(x)) \frac{\partial\phi}{\partial x^\mu} \frac{\partial\phi}{\partial x^\nu} g_{\alpha\beta} \right]. \end{aligned}$$

Bianchi identity

$$\nabla_\beta G^\beta_\alpha \equiv 0 \quad \text{implies} \quad \nabla_\beta T^\beta_\alpha = 0,$$

but for this T one gets

$$\nabla_\beta\, T^\beta_\alpha = -2\frac{\partial\phi}{\partial x^\alpha}\nabla_\mu\left(\zeta^{\mu\nu}\frac{\partial\phi}{\partial x^\nu}\right) - \nabla_\alpha\left(\zeta^{\mu\nu}\frac{\partial\phi}{\partial x^\mu}\frac{\partial\phi}{\partial x^\nu}\right) \neq 0$$

for adjusted $\zeta^{\mu\nu}$ and solutions for $\phi \Rightarrow$ *inconsistency*.
The divergence might be 0 only for very special solutions ϕ.
Conclusion:
in the presence of gravitation the Lorentz symmetry breaking field $\zeta^{\mu\nu}$ *must be dynamical* $\Rightarrow$ consistency is restored.
$\zeta^{\mu\nu}(x)$ — what is it?

Standard Model Extension (SME)

Action

$$S_{\rm SME} = S_{\rm SM} + S_{\rm LV} + S_g + \text{higher order}.$$

$S_{\rm SM}$ — SM action in covariant form (minimally coupled to gravity), $SU(3) \times SU(2) \times U(1)$ gauge invariant, Lagrangian

$$\mathcal{L}_{\rm SM} = \text{leptons} + \text{quarks} + \text{Yukawa coupl.} + \text{Higgs} + \text{gauge}.$$

$S_{\rm LV}$ — SM fields coupled to continuous Lorentz symmetry violating operators and to CPT violating operators,
CPT is violated in sectors: lepton, quark, Higgs and gauge.
S_g — Einstein gravity plus symmetry violating operators coupled to the curvature,

$$\mathcal{L}_g = R - 2\Lambda + \mathcal{L}(R_{\alpha\beta\mu\nu}, \zeta),$$

$\mathcal{L}(R_{\alpha\beta\mu\nu}, \zeta)$ — continuous Lorentz symmetry violating terms, *no* CPT violation.

Conclusions

I.

Continuous LSV in the gravit. sector of SME — observable (?) effects near black holes, very early Universe, strong gravit. waves.

II.

CPT violation possible in SME in *all* particle sectors. Unclear what might be observed.