Study of the $np \to \eta d$ reaction within a three-body model

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The cross section for the process $np \to \eta d$ in the region near threshold is calculated within a three-body model. Non-relativistic and relativistic results are compared. The shape of the cross-section is seen to be determined by the ηd final-state interaction alone, in both relativistic and non-relativistic calculations.

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1 Introduction

Although the variety of meson production reactions from nucleon-nucleon scattering have cross-sections with qualitatively the same dependence on the energy, the reactions which are dominated by the intermediate excitation of a baryonic resonance clearly show enhanced strengths relatively to the others. This is well illustrated by the comparison of $pp \to pn\pi^+$ to $pp \to pp\pi^0$, $pp \to pp\eta$ to $pp \to pp\eta'$ and finally $pp \to p\Lambda K^+$ to $pp \to p\Sigma^0 K^+$. The study of meson production reactions provides thus key information on the behavior of baryonic resonances in the nuclei media. Near the threshold region they additionally probe final state interactions involving the non-stable mesons, as well as short-range nucleonic correlations induced by the high momentum transfers needed for particle production at not sufficiently high energies. For the eta production reactions near threshold the N*(1535) baryon, which couples to the S11 channel of πN scattering, has a very distinctive role. This role comes from the almost equal branching ratios for the decay modes of the N*(1535) into the Nπ and Nη channels, as well as from the vicinity of the resonance excitation energy to the η meson production threshold.

The reaction $np \to \eta d$ has been accurately measured recently in the region near threshold [1, 2]. Since the cross section shows a strong enhancement in that region, as compared with the predictions based on a two-body phase space, it was speculated that this could be a signal for an ηNN quasibound state [3]. However, the solutions of the Faddeev equations for ηd elastic scattering have not supported this hypothesis: the 3-body ηNN amplitude presents a pole a

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few MeV away from threshold, but in the unphysical sheet of the complex plane [4,5], defining a quasivirtual state instead of a quasibound state. This pole is nevertheless present in the reaction $np \rightarrow \eta d$ via the final-state ηd interaction, and visible as an enhancement at threshold. Three-body calculations for the ηNN system had therefore to be established and solved, which is reported in this talk. Also, different ηN dynamical models based upon recent data analysis of the coupled reactions $\pi N \rightarrow \eta N$, $\eta N \rightarrow \eta N$ and $\gamma N \rightarrow \eta N$ were probed. In this presentation firstly we describe the results of a non-relativistic calculation, and secondly how they get modified by a relativistic treatment for kinematics and also dynamics. In common, both the relativistic and non-relativistic calculations show that the shape of the cross-section is essentially determined by the ηd three-body final state interaction alone. One of the main motivations for performing such calculations is that the different data analysis provide a dispersion of the ηN scattering length in the interval $0.27\mathrm{fm} \leq \mathrm{Re}(\mathrm{a}_{\eta\mathrm{N}}) \leq 1.05\mathrm{fm}$. Importantly, the knowledge of this strength is crucial to establish or confirm the possibility for exotic eta-mesic nuclei or nuclear matter, of great astrophysics interest.

2 Three-body calculations and the nucleon-meson models

We consider all the three particles of the ηNN system interacting through pairwise interactions, which we represent by separable potentials. The Faddeev integral equations which we solved sum the S11 isobar multiple scattering with one nucleon. This multiple scattering consists of successive decays and excitations of the S11, mediated by π, η and σ exchange (the latter effectively accounts for the $\pi\pi N$ decay channel of the S11). Labeling the η as particle 1, and the identical nucleons as particles 2 and 3, the coupled-set of integral equations for the half-off-shell scattering transition matrices are written as

$$T_1(q_1;E) = 2\int_0^\infty {q_2}^2 dq_2 K_{12}(q_1,q_2;E)\tau_2(E-{q_2}^2/2\nu_2)T_2(q_2;E). \tag{1}$$

$$\begin{aligned} T_2(q_2;E) &= 2K_{21}(q_2,q_{10};E) + \int_0^\infty {q_2'}^2 dq_2' K_{23}(q_2,q_2';E)\tau_2(E-{q_2'}^2/2\nu_2)T_2(q_2';E) \\ &\quad + \int_0^\infty q_1^2 dq_1 K_{21}(q_2,q_1;E)\tau_1(E-q_1^2/2\nu_1)T_1(q_1;E), \end{aligned} \tag{2}$$

The function $T_1(q_1;E)$ represents the ηd transition amplitude, while the function $T_2(q_2;E)$ represents the transition amplitude from the ηd state to a state with a nucleon and a S_{11} isobar. These two half-off-shell scattering matrices depend on the center of mass energy E, which defines the on-shell-momentum q_{10}, and respectively on the η-d relative momentum q_1 and the $S11$-N relative momentum q_2. For the intermediate states this last momentum is represented by the integration variable q_2'. The two-nucleon propagator is τ_1 and the $S11$ propagator is τ_2. The propagators are taken as non-relativistic, and therefore the variables ν_i ($i = 1,2$) are the reduced masses of the spectator particle i and the (jk) pair i.e., $\nu_i = \frac{m_i(m_j+m_k)}{m_i+m_j+m_k}$. The kernels K_{21} and K_{23} describe respectively the exchange of a nucleon and of a meson, the subscripts referring to the spectator particles before and after the exchange. The two kernels are determined by the $S11$-nucleon-meson vertex, the two nucleon interaction vertex, as well as by the intermediate

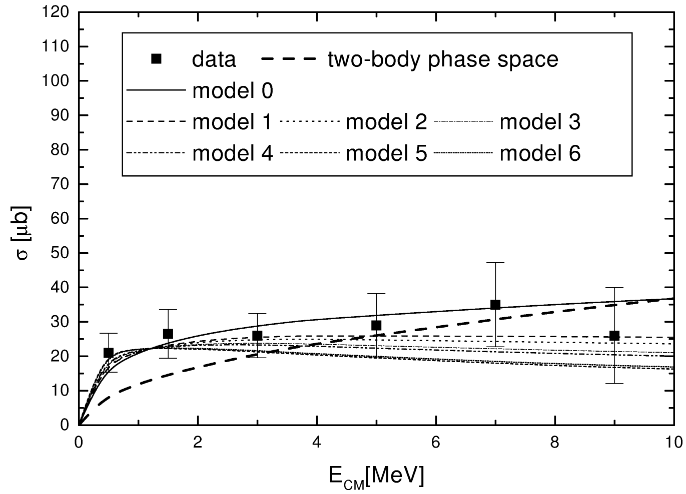

Fig. 1. Results for the non-relativistic calculation of the cross-section for $np \to \eta d$ very near threshold; the phenomenological meson-nucleon interaction models used as input were fitted to different data analysis; the initial NN and final ηd interactions were included

state propagators: the three body propagator of a meson and two nucleons, and the propagator of the $S11$ and one nucleon. Further details on the integral equations and the kernel functions can be found in [6].

To build the input kernels of the equations above, different phenomenological models of the coupled ηN-πN system were fitted to the $\eta N \to \eta N$ amplitude and to $\pi^- p \to \eta n$ cross section, which is a direct source of information on the non-diagonal transition amplitude $\pi N \to \eta N$. The data analysis fitted are from references [7–10]. The models based on the analysis [7–9] are labeled 1-6. The model [10] based on the Jülich data analysis is labeled by 0. As the label of the model increases from 0 to 6, the values for $Re(a_{\eta N})$ (in fm) are respectively 0.42 ,0.72 ,0.75 ,0.83 ,0.87 ,1.05 ,1.07.

In order to use the data analysis [7–10] we built separable potentials describing the meson-nucleon interactions as explained in [6]. The ηN and πN potentials correspond to S waves, while the σN form factor corresponds to a P wave, as required by parity conservation. According to [6], the Lippmann-Schwinger coupled-channel equations fix the off-energy-dependence for the meson-nucleon scattering t-matrices calculated from those separable potentials and give, for a given two body relative energy E and relative initial and final momentum p and p',

$$< p|t_{\eta\eta}(E)|p' >= g_\eta(p)\tau_2(E)g_\eta(p'), \tag{3}$$

$$< p|t_{\pi\pi}(E)|p' >= \frac{\lambda_\pi}{\lambda_\eta} g_\pi(p)\tau_2(E)g_\pi(p'), \tag{4}$$

$$< p|t_{\eta\pi}(E)|p' >= \pm\sqrt{\frac{\lambda_\pi}{\lambda_\eta}} g_\eta(p)\tau_2(E)g_\pi(p'), \tag{5}$$

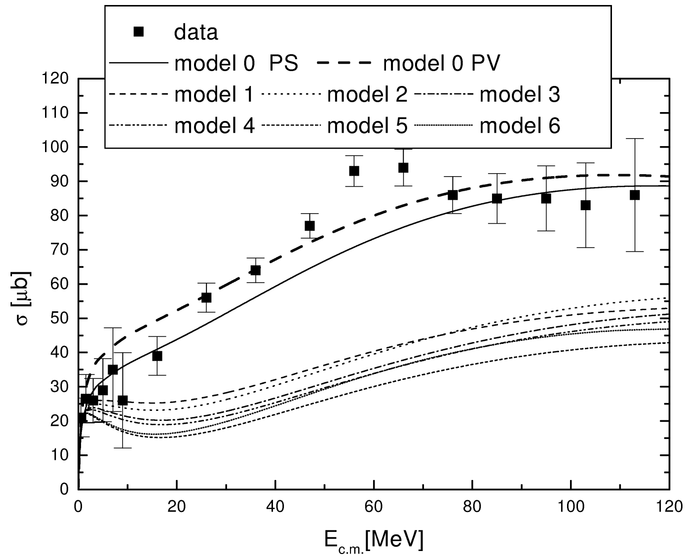

Fig. 2. Results for the non-relativistic calculation of the cross-section, in the region away from threshold. In this figure the heavy full and dashed curves corresponds to model 0 taken respectively with pseudoscalar and pseudovector ηNN coupling in the η exchange kernel K_{23}.

$$< p|t_{\eta\sigma}(E)|p' >= \pm\sqrt{\frac{\lambda_\sigma}{\lambda_\eta}} g_\eta(p)\tau_2(E)g_\sigma(p'), \tag{6}$$

where $\tau_2(E)$ is the S_{11} resonance propagator, its inverse corresponding to a sum of a Dyson series, for the ηN, πN, and σN channels. The parameters λ_m and functions g_m $(m = \eta, \pi, \sigma)$ were fitted to the several $\eta N \to \eta N$ amplitudes of the different models employed. More details are given in reference [6].

We show the results obtained in Figs. 1 and 2. For each model the short-range production mechanism strength was fitted to the very-near-threshold region (see Fig. 1), and from there the cross section at higher energies was predicted as shown in Fig. 2. Only the Jülich model describes the data reasonably well throughout the full energy range. The preference of the data for a model with small Re $a_{\eta N}$ as the Jülich model is to a large extent independent of the production mechanism, indicating that the characteristic shape of the experimental cross section is a signature of the ηd final-state interaction. On the contrary, the absolute value of the cross section depends on the production mechanism.

Given the high threshold energy for η production, the next step was to investigate relativistic effects [11] in the nucleon-nucleon initial state interaction and in the production mechanism of the reaction, for which the meson-nucleon amplitudes are input. Thus, we took covariant meson-nucleon amplitudes with the form ($m=\eta,\pi,\sigma$) [11]:

$$t_{mN\to\eta N}(\vec{p}^{\,2},\vec{p}^{\,'2},M_S) = \frac{(2\pi)^2}{M}\sqrt{\omega_m(\vec{p}^{\,2})\omega_\eta(\vec{p}^{\,'2})E_N(\vec{p}^{\,2})E_N(\vec{p}^{\,'2})} \times h_m(\vec{p}^{\,2})\frac{\not{k}_m+\not{k}_2+M_S}{2M_S}h_\eta(\vec{p}^{\,'2})\tau_2(M_S), \quad (7)$$

where $\vec{p}$ and $\vec{p}^{\,'}$ are the relative initial and final relative πN three-momenta, ω_m and E_N are the on-shell energies respectively of the m meson and of the nucleon in the c.m. frame. The functions h_m include the vertex functions g_m, and the isobar mass M_S is calculated from the nucleon momenta before and after η production, respectively k_2 and k_3, and the m meson and η momenta, k_m and k_η, as

$$M_S=\sqrt{(k_m+k_2)^2}=\sqrt{(k_\eta+k_3)^2}. \quad (8)$$

To further implement relativity in the calculation, a relativistic version of the 3-body equations was used, incorporating relativistic kinematics and the boost of the 2-body meson-nucleon and nucleon-nucleon interactions. Reference [12] supplies the technical details of the relativistic formalism used by us. It starts from a form of field theory where, alike Time Ordered Field Theory, all particles are on their mass-shells at every stage. This choice implies a 3-dimensional theory from the beginning, and a straightforward generalization of the non-relativistic Faddeev equations.

In that formalism, if the variable $\vec{p}_i$ stands for the relative momentum of the pair jk (made of the particles with mass m_j and m_k) measured in the c.m. frame of that pair (where particle j and particle k have total momentum 0), and $\vec{q}_i$ for the relative momentum between the pair jk and the spectator particle i, measured in the c.m. frame of the three particles, then the energy of the jk pair in its c.m. frame is $\omega(p_i)=\sqrt{m_j^2+p_i^2}+\sqrt{m_k^2+p_i^2}$ and the total energy of the pair $W_i(p_iq_i)=\sqrt{\omega^2(p_i)+q_i^2}$. As for the invariant energy of the three particles, it is written as $W(p_iq_i)=\omega_i(q_i)+W_i(p_iq_i)$, with $\omega_i(q_i)=\sqrt{m_i^2+q_i^2}$.

According to reference [12] these kinematic relations determine the boost of the 2-body interactions, i.e., the transformation of the matrix elements of the two-body potential V from the two-body c.m. frame to the three-body c.m. reference frame. In turn, from the boosted potentials V one obtains the boosted matrix elements of the two-body t-matrices $t(\vec{p}_i,\vec{p}_i^{\,\prime};q_i)$ which satisfy the Lippmann-Schwinger equation with a propagator corresponding to relativistic kinematics,

$$t(\vec{p}_i,\vec{p}_i^{\,\prime};q_i) = V(\vec{p}_i,\vec{p}_i^{\,\prime})+\int d\vec{p}_i^{\,\prime\prime}V(\vec{p}_i,\vec{p}_i^{\,\prime\prime}) \times\frac{1}{W_0-W(p_i^{\prime\prime}q_i)+i\epsilon}t(\vec{p}_i^{\,\prime\prime},\vec{p}_i^{\,\prime};q_i), \quad (9)$$

where the variable W_0 is the invariant energy of the system, and p_i, p_i' for the initial and final relative momentum in pair jk. As for the driving terms of the Faddeev equations for ηd elastic scattering, they were also modified by the inclusion of relativistic kinematics, as explained in [11, 12].

In our calculations the initial state interaction was treated exactly. This means that the Lippmann-Schwinger equation with relativistic kinematics given by the formulas above was

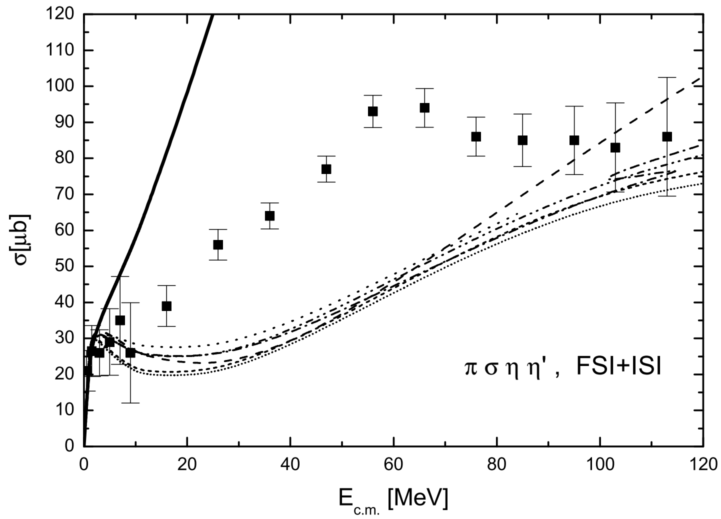

Fig. 3. Results for the relativistic calculation of the cross-section for $np \to \eta d$ very near threshold.

solved for the nucleon-nucleon interaction, and its solution used in convolution integrals with the production operator matrix element, thereby distorted. We verified that model 0 is the one affected to the least extent by the initial state interaction, consistently with the specific content of that model: the pion momentum range parameter α_π and the scattering length $a_{\eta N}$ for the short-range ηN interaction are smaller than in other models. Due to these features, and since the initial state interaction is induced by large three-momentum transfer (short-distance interactions) between the 2 nucleons, in model 0, both the pion and the heavier mesons are less affected by the initial state reduction. We additionally checked that the relativistic boosts in the meson-nucleon amplitudes are negligible, as naturally expected in the vicinity of the production threshold.

We show in Fig. 3 the results of the relativistic calculation. When the strength of the production mechanisms is adjusted so as to reproduce the cross section near threshold, the Jülich model (model 0) fails in the high energy region, as Fig. 3 demonstrates. To understand this result versus the successful non-relativistic description obtained with the Jülich model, one should mention that the relativistic version of model 0 does have the pion range parameter reduced relatively to its non-relativistic version. The results in Fig. 3 are however indicative that a reasonable description of the data could be obtained with a model in between model 0 and model 1, i.e., with a ηN scattering length larger than $0.42\ \mathrm{fm}$ and smaller than $0.72\ \mathrm{fm}$.

Our findings call for further studies on the ηN dynamics, namely within a comprehensive description of η production, both on the nucleon and on the deuteron, very near as well as away from the threshold energy. A finer knowledge of the balance between the pion exchange momentum range and contribution at low momenta transfers is still needed. Nevertheless, the relativistic calculation confirms the non-relativistic results showing that the shape of the cross-section near threshold is essentially determined by the ηd final state interaction alone.

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