Eta bound states in nuclei: a probe of flavour-singlet dynamics

Steven Bass, Innsbruck

Looking for evidence of gluonic degrees of freedom in low energy QCD: Confinement and dynamical (chiral) symmetry breaking SU_L(3) × SU_R(3) × U_A(1) Expect nonet of pseudoscalar Goldstone bosons Pions and Kaons fit in this picture The masses of the eta and eta' are 300-400 MeV too big ! → Famous axial U(1) problem of QCD Additional mass is associated with non-perturbative gluon dynamics Look for possible evidence of singlet dof in eta-nucleon bound states → Mixing doubles the eta-N scattering length and eta binding energies

Cracow, June 16 2010

Chiral symmetry

• QCD Lagrangian with massless quarks exhibits chiral symmetry

$$\mathcal{L}_{QCD} = \sum_{q} ar{q}_L \Big(i \hat{D} - g \hat{A} \Big) q_L + ar{q}_R \Big(i \hat{D} - g \hat{A} \Big) q_R - \sum_{q} m_q \Big(ar{q}_L q_R + ar{q}_R q_L \Big) - rac{1}{2} G_{\mu
u} G^{\mu
u}$$

$$\left(\begin{array}{c} u_L \\ d_L \end{array}\right) \ \mapsto \ e^{i\frac{1}{2}\vec{\alpha}.\vec{\tau}\gamma_5} \left(\begin{array}{c} u_L \\ d_L \end{array}\right) \quad , \quad \left(\begin{array}{c} u_R \\ d_R \end{array}\right) \ \mapsto \ e^{i\frac{1}{2}\vec{\beta}.\vec{\tau}\gamma_5} \left(\begin{array}{c} u_R \\ d_R \end{array}\right)$$

• Noether currents

$$J^{(3)}_{\mu 5} = \left[ar{u} \gamma_{\mu} \gamma_{5} u - ar{d} \gamma_{\mu} \gamma_{5} d
ight] \qquad \qquad \partial^{\mu} J^{(3)}_{\mu 5} = 2m_{u} ar{u} i \gamma_{5} u - 2m_{d} ar{d} i \gamma_{5} d a_{d} a_$$

• No parity doublets in hadron spectrum \rightarrow Spontaneous Chiral symmetry breaking: non zero condensate $\langle vac | \bar{q}q | vac \rangle < 0$ spontaneously breaks the symmetry

 \rightarrow Nonet of near massless Goldstone bosons with J^P = O⁻

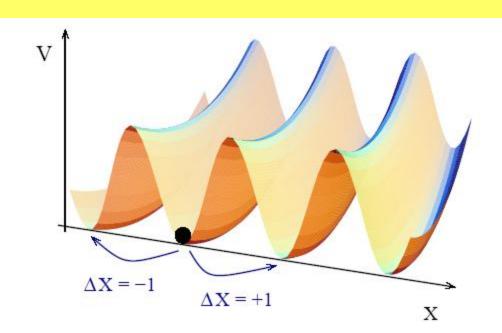
• Identify with pion, kaon, eta with meson mass squared proportional to m_q

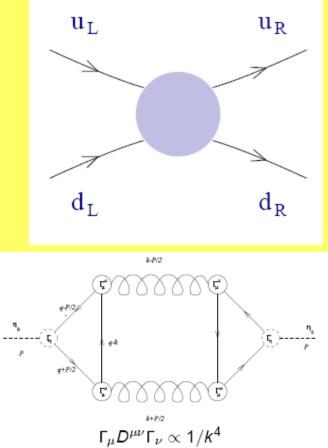
$$m_{\eta_8}^2 = rac{4}{3}m_{
m K}^2 - rac{1}{3}m_{\pi}^2$$

... where is the singlet boson ?

Chirality and anomalous glue

- Perturbative QCD conserves chirality for massless quarks
- Confinement and vacuum tunneling processes (instantons, ...) connect left and right handed quarks





Eta and Etaprime masses with mixing

Mass matrix

$$M_{\eta-\eta'}^2 = \begin{pmatrix} \frac{4}{3}m_{\rm K}^2 - \frac{1}{3}m_{\pi}^2 & -\frac{2}{3}\sqrt{2}(m_{\rm K}^2 - m_{\pi}^2) \\ \\ -\frac{2}{3}\sqrt{2}(m_{\rm K}^2 - m_{\pi}^2) & [\frac{2}{3}m_{\rm K}^2 + \frac{1}{3}m_{\pi}^2 + \tilde{m}_{\eta_0}^2] \end{pmatrix}$$

- $egin{array}{rcl} |\eta
 angle &=& \cos heta \; |\eta_8
 angle \sin heta \; |\eta_0
 angle \ |\eta'
 angle &=& \sin heta \; |\eta_8
 angle + \cos heta \; |\eta_0
 angle \end{array}$
- Diagonalize

$$m_{\eta',\eta}^2 = (m_{\rm K}^2 + \tilde{m}_{\eta_0}^2/2) \pm \frac{1}{2} \sqrt{(2m_{\rm K}^2 - 2m_{\pi}^2 - \frac{1}{3}\tilde{m}_{\eta_0}^2)^2 + \frac{8}{9}\tilde{m}_{\eta_0}^4}$$

• Eigenvalues

$$m_\eta^2 + m_{\eta'}^2 = 2m_K^2 + ilde{m}_{\eta_0}^2.$$

• With no glue: chiral symmetry "predicts" eigenstates with masses 300 MeV "too small" » "eta" $(\frac{1}{\sqrt{2}}|\bar{u}u+\bar{d}d\rangle)$ degenerate with the pion

» "etaprime"
$$|ar{s}s
angle$$
 with mass $\sqrt{2m_K^2-m_\pi^2}$

Axial U(1) symmetry

• Extra gluonic mass term is associated with the QCD axial anomaly

$$J_{\mu5}=\left[ar{u}\gamma_{\mu}\gamma_{5}u+ar{d}\gamma_{\mu}\gamma_{5}d+ar{s}\gamma_{\mu}\gamma_{5}s
ight]$$

$$\partial^{\mu} J_{\mu 5} = \sum_{k=1}^{f} 2i \left[m_k \bar{q}_k \gamma_5 q_k \right] + N_f \left[\frac{\alpha_s}{4\pi} G_{\mu\nu} \tilde{G}^{\mu\nu} \right]$$

- plus gluon topology (note the difference with "perturbative glue")
- *´*t Hooft, Veneziano, Witten, Crewther, ...
 - possible connection to confinement (Kogut and Susskind)

Can we observe physical manifestation of this anomalous glue in low-energy physical processes involving eta and eta´ mesons ? → For review see SDB, Acta Phys Pol B Suppl 2 (2009) 11.

Eta bound states in nuclei

[SDB + AW Thomas, Phys Lett B634 (2006) 368]

- New experiments + big effort ...
- Binding energies and effective masses in nuclei are sensitive to
 - Coupling to scalar sigma field in the nuclei in mean field approximation
 - Nucleon-nucleon and nucleon-hole excitations in the medium
- TH: Solve for the meson self-energy in the medium

$$k^2-m^2={\rm Re}~\Pi(E,\vec{k},\rho)$$

$$\Pi(E,\vec{k},\rho)\bigg|_{\{\vec{k}=0\}} = -4\pi\rho\bigg(\frac{b}{1+b\langle\frac{1}{r}\rangle}\bigg), \qquad b=a(1+\frac{m}{M})$$

- Where a is the "eta-nucleon scattering length"

Eta bound-states in nuclei

- Sigma mean field couples to light quarks and not to strange quarks
 → Flavour-singlet component is important !

 The bigger the eta-eta' mixing angle, the bigger the singlet component in the eta
 - \rightarrow greater the attraction
 - \rightarrow more binding
 - \rightarrow bigger eta-N scattering length

QCD arguments

 \rightarrow gluonic mass term is suppressed in the medium

but TH technology to calculate the size of the effect direct from QCD still some time away

 \rightarrow look at QCD inspired models

U(1) extended chiral Lagrangian

Low energy effective Lagrangian

$$\mathcal{L}_{\mathrm{m}} = \frac{F_{\pi}^2}{4} \mathrm{Tr}(\partial^{\mu} U \partial_{\mu} U^{\dagger}) + \frac{F_{\pi}^2}{4} \mathrm{Tr}\Big[\chi_0 \left(U + U^{\dagger}\right)\Big] + \frac{1}{2} i Q \mathrm{Tr}\Big[\log U - \log U^{\dagger}\Big] + \frac{3}{\tilde{m}_{\eta_0}^2 F_0^2} Q^2 + \frac{1}{2} i Q \mathrm{Tr}\Big[\log U - \log U^{\dagger}\Big] + \frac{1}{2} i Q \mathrm{Tr}\Big[\log U - \log U^{\dagger}\Big] + \frac{1}{2} i Q \mathrm{Tr}\Big[\log U - \log U^{\dagger}\Big] + \frac{1}{2} i Q \mathrm{Tr}\Big[\log U - \log U^{\dagger}\Big] + \frac{1}{2} i Q \mathrm{Tr}\Big[\log U - \log U^{\dagger}\Big] + \frac{1}{2} i Q \mathrm{Tr}\Big[\log U - \log U^{\dagger}\Big] + \frac{1}{2} i Q \mathrm{Tr}\Big[\log U - \log U^{\dagger}\Big] + \frac{1}{2} i Q \mathrm{Tr}\Big[\log U - \log U^{\dagger}\Big] + \frac{1}{2} i Q \mathrm{Tr}\Big[\log U - \log U^{\dagger}\Big] + \frac{1}{2} i Q \mathrm{Tr}\Big[\log U - \log U^{\dagger}\Big] + \frac{1}{2} i Q \mathrm{Tr}\Big[\log U - \log U^{\dagger}\Big] + \frac{1}{2} i Q \mathrm{Tr}\Big[\log U - \log U^{\dagger}\Big] + \frac{1}{2} i Q \mathrm{Tr}\Big[\log U - \log U^{\dagger}\Big] + \frac{1}{2} i Q \mathrm{Tr}\Big[\log U - \log U^{\dagger}\Big] + \frac{1}{2} i Q \mathrm{Tr}\Big[\log U - \log U^{\dagger}\Big] + \frac{1}{2} i Q \mathrm{Tr}\Big[\log U - \log U^{\dagger}\Big] + \frac{1}{2} i Q \mathrm{Tr}\Big[\log U - \log U^{\dagger}\Big] + \frac{1}{2} i Q \mathrm{Tr}\Big[\log U - \log U^{\dagger}\Big] + \frac{1}{2} i Q \mathrm{Tr}\Big[\log U - \log U^{\dagger}\Big] + \frac{1}{2} i Q \mathrm{Tr}\Big[\log U - \log U^{\dagger}\Big] + \frac{1}{2} i Q \mathrm{Tr}\Big[\log U - \log U^{\dagger}\Big] + \frac{1}{2} i Q \mathrm{Tr}\Big[\log U - \log U^{\dagger}\Big] + \frac{1}{2} i Q \mathrm{Tr}\Big[\log U - \log U^{\dagger}\Big] + \frac{1}{2} i Q \mathrm{Tr}\Big[\log U - \log U^{\dagger}\Big] + \frac{1}{2} i Q \mathrm{Tr}\Big[\log U - \log U^{\dagger}\Big] + \frac{1}{2} i Q \mathrm{Tr}\Big[\log U - \log U^{\dagger}\Big] + \frac{1}{2} i Q \mathrm{Tr}\Big[\log U - \log U^{\dagger}\Big] + \frac{1}{2} i Q \mathrm{Tr}\Big[\log U - \log U^{\dagger}\Big] + \frac{1}{2} i Q \mathrm{Tr}\Big[\log U - \log U^{\dagger}\Big] + \frac{1}{2} i Q \mathrm{Tr}\Big[\log U - \log U^{\dagger}\Big] + \frac{1}{2} i Q \mathrm{Tr}\Big[\log U - \log U^{\dagger}\Big] + \frac{1}{2} i Q \mathrm{Tr}\Big[\log U - \log U^{\dagger}\Big] + \frac{1}{2} i Q \mathrm{Tr}\Big[\log U - \log U^{\dagger}\Big] + \frac{1}{2} i Q \mathrm{Tr}\Big[\log U - \log U^{\dagger}\Big] + \frac{1}{2} i Q \mathrm{Tr}\Big[\log U - \log U^{\dagger}\Big] + \frac{1}{2} i Q \mathrm{Tr}\Big[\log U - \log U^{\dagger}\Big] + \frac{1}{2} i Q \mathrm{Tr}\Big[\log U - \log U^{\dagger}\Big] + \frac{1}{2} i Q \mathrm{Tr}\Big[\log U - \log U^{\dagger}\Big] + \frac{1}{2} i Q \mathrm{Tr}\Big[\log U - \log U^{\dagger}\Big] + \frac{1}{2} i Q \mathrm{Tr}\Big[\log U - \log U^{\dagger}\Big] + \frac{1}{2} i Q \mathrm{Tr}\Big[\log U - \log U^{\dagger}\Big] + \frac{1}{2} i Q \mathrm{Tr}\Big[\log U - \log U^{\dagger}\Big] + \frac{1}{2} i Q \mathrm{Tr}\Big[\log U - \log U^{\dagger}\Big] + \frac{1}{2} i Q \mathrm{Tr}\Big[\log U - \log U^{\dagger}\Big] + \frac{1}{2} i Q \mathrm{Tr}\Big[\log U - \log U^{\dagger}\Big] + \frac{1}{2} i Q \mathrm{Tr}\Big[\log U - \log U^{\dagger}\Big] + \frac{1}{2} i Q \mathrm{Tr}\Big[\log U - \log U^{\dagger}\Big] + \frac{1}{2} i Q \mathrm{Tr}\Big[\log U - \log U^{\dagger}\Big] + \frac{1}{2} i Q \mathrm{Tr}\Big[\log U - \log U^{\dagger}\Big] + \frac{1}{2} i Q \mathrm{Tr}\Big[\log U - \log U^{\dagger}\Big] + \frac{1}{2} i Q \mathrm{Tr}\Big[\log U - \log$$

$$U = \exp\left(i\frac{\phi}{F_{\pi}} + i\sqrt{\frac{2}{3}}\frac{\eta_0}{F_0}\right)$$

• Q represents the topological charge density. The gluonic potential

$$\frac{1}{2}iQ \mathrm{Tr} \left[\log U - \log U^{\dagger} \right] + \frac{3}{\tilde{m}_{\eta_0}^2 F_0^2} Q^2 \quad \mapsto \quad -\frac{1}{2} \tilde{m}_{\eta_0}^2 \eta_0^2$$

yields the gluonic contribution to the etaprime mass term

• Couple to sigma mean field and repeat ...

$$\mathcal{L}_{\sigma} = \frac{F_{\pi}^2}{4} \text{Tr} M ~(U + U^{\dagger}) ~g_{\sigma}^M \sigma + Q^2 ~g_{\sigma}^Q \sigma$$

$$\tilde{m}_{\eta_0}^2 \mapsto \tilde{m}_{\eta_0}^{*2} = \tilde{m}_{\eta_0}^2 \frac{1+2x}{(1+x)^2} < \tilde{m}_{\eta_0}^2$$

where

$$x = \frac{1}{3} g^Q_\sigma \sigma \ \tilde{m}^2_{\eta_0} F_0^2.$$

QCD Inspired Models

- Phenomenological fits to EP data
 - » On-shell Re[a_eta] ~ 0.9 fm [Green + Wycech, Arndt et al]
 - » COSY-11 ~ 0.7 fm from FSI in pp → pp eta
- Chiral coupled channels treating the eta as a pure octet state
 - » Small mass shift and small Re[a_eta] ~ 0.2 fm
- Quark Meson Coupling Model:
 - Can vary the mixing angle !
 - Use large eta and eta' masses to treat the eta and eta' as MIT Bags embedded in the medium with coupling between the light-quarks and the sigma mean field
 - Solve for in-medium mass and binding energy
 - \rightarrow Extract an "effective" scattering length for the model
 - \rightarrow Increases with increasing singlet component in the eta !

	$m(\mathrm{MeV})$	m^* (Me V)	Re a (fm)
η ₈	547.75	500.0	0.43
η (-10°)	547.75	474.7	0.64
η (-20°)	547.75	449.3	0.85
η ₀	958	878.6	0.99
$\eta^{i}(-10^{\circ})$	958	899.2	0.74
$\eta'(-20^{\circ})$	958	921.3	0.47

Bound states in finite nuclei



10 December 1998

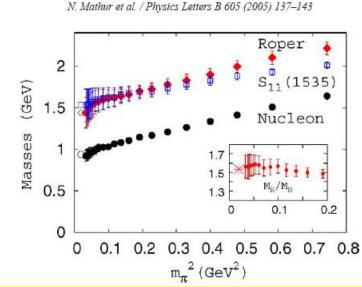
NH	10 December 1998		11.1								
		YSICS LETTERS B	ble 1	• • •							
	rn.	ISICS LETTERS B	ω and η' bound	1 state	e energies	(in Me	eV), $E_j =$	= Re(E	$E_j^* - m_j) \left(j = \eta, \omega, \eta \right)$		
SEVIER	Physics Letters B 443 (1998) 26-32		iths for the η' a	are set	to zero.	The eig	enenergie	s are g	given by, $E_j^* = E_j +$		
					$\gamma_{\eta} = 0.5$		$\gamma_{\omega}=0.2$		$\gamma_{\eta'} = 0$		
					E_{η}	Γ_{η}	E_{ω}	Γ_{ω}	$E_{\eta'}$		
Are η - and ω -nuclear states bound ?			⁶ He	1s	-10.7	14.5	-55.6	24.7	* (not calculated)		
			in B	1s	-24.5	22.8	-80.8	28.8	*		
K. Tsushima ^{a,1} , D.H. Lu ^{a,2} , A.W. Thomas ^{a,3} , K. Saito ^{b,4}			$_{j}^{26}Mg$	1s	-38.8	28.5	-99.7	31.1	*		
			, -	1p	-17.8	23.1	-78.5	29.4	*		
				2s			-42.8	24.8	*		
NH			$_{i}^{16}O$	1s	-32.6	26.7	-93.4	30.6	-41.3		
国際			,	1p	-7.72	18.3	-64.7	27.8	-22.8		
			$_{i}^{40}$ Ca	1s	-46.0	31.7	-111	33.1	-51.8		
ELSEVIER	Nuclear Physics A670 (2000) 198c-	201c	,	1p	-26.8	26.8	-90.8	31.0	-38.5		
				2s	-4.61	17.7	-65.5	28.9	-21.9		
Study of ω -, η -, η '- and D mesic nuclei			$_{i}^{90}$ Zr	1s	-52.9	33.2	-117		-56.0		
			,	1p	-40.0	30.5	-105		-47.7		
K. Tsushima ^a *				2s	-21.7	26.1	-86.4	30.7			
			²⁰⁸ Pb	1s	-56.3	33.2	-118	33.1	-57.5		
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	Physic	cs Division, To	ohoku College of Pha	rmacy,	Sendai 981,	, Japan					
		K. Tsushima [†] and A. W. Thomas [‡]									
	Department of Physics and Mat	-	-		-		omic Structi	ire of M	latter,		
		University of I	<i>Adelaide, South Austr</i> (Received 6 March		005, AUSTRALI	ia					
			(received o iviarch	1997)							
	The quark-meson coupli										
	carbon nuclei. The average	mass of a p-m	neson formed in ^{3,4} He	and 12	C is expecte	d to be a	round 730,	690, and	1		

720 MeV, respectively. [S0556-2813(97)04007-7]

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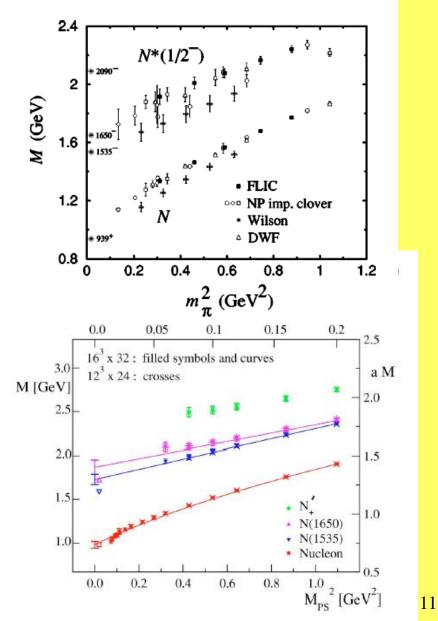
The $S_{11}(1535)$ resonance

- 3 quark state (1s)²(1p) ?
 in Quark model and lattice calculations or
- K-Sigma quasi-bound state ?
 Chiral coupled channels in octet approx.



 In data and in both QMC and chiral coupled channels models, negligible shift in excitation energy in nuclei

EXCITED BARYONS IN LATTICE QCD

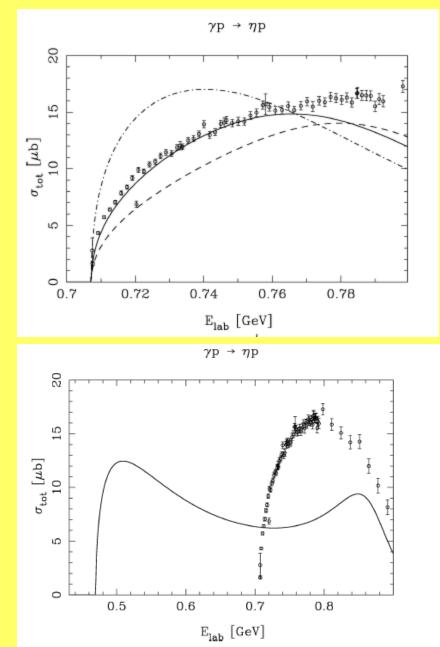


The N*(1535): fun with coupled channels

SDB, Wetzel and Weise (2000)

Octet eta → dynamical generated in K-Sigma rescattering

- Turn on eta-etaprime mixing → lots of new axial U(1) parameters, not well constrained
- Modest changes within "respectable" range can change the shape of the eta-production cross-section (e.g. resonance gets washed out, splits in 2, cusps can appear ...!)
- → Suggests dynamical resonance interpretation an artifact of the octet approximation



Outlook and Conclusions

- Eta and etaprime physics probes the role of long range gluonic dynamics
- Etas and etaprimes in nuclei:
 - Binding energies and scattering lengths sensitive to the flavoursinglet component in the eta
 - QMC model:
 - » Factor of 2 increase in the eta-nucleon scattering length and binding energy in nuclei with eta-etaprime mixing cf. Theory prediction with a pure octet eta
 - » N*(1535) as 3 quark state (1s)²(1p)
 - » For densities between 50% and 100% nuclear matter

$$rac{m_\eta^*}{m_\eta}\simeq$$
1 $-$ 0.17 $rac{
ho}{
ho_0}$

... Awaits experimental input!

U(1) extended chiral Lagrangian

- Low energy effective Lagrangian
 - constructed to reproduce the axial anomaly in the anomalous divergence equation and the gluonic mass term for the singlet boson

$$\mathcal{L}_{\mathrm{m}} = \frac{F_{\pi}^2}{4} \mathrm{Tr}(\partial^{\mu} U \partial_{\mu} U^{\dagger}) + \frac{F_{\pi}^2}{4} \mathrm{Tr}\Big[\chi_0 \left(U + U^{\dagger}\right)\Big] + \frac{1}{2} i Q \mathrm{Tr}\Big[\log U - \log U^{\dagger}\Big] + \frac{3}{\tilde{m}_{\eta_0}^2 F_0^2} Q^2 U^2 + \frac{1}{2} i Q \mathrm{Tr}\Big[\log U - \log U^{\dagger}\Big] + \frac{1}{2} i Q \mathrm{Tr}\Big[\log U - \log U^{\dagger}\Big] + \frac{1}{2} i Q \mathrm{Tr}\Big[\log U - \log U^{\dagger}\Big] + \frac{1}{2} i Q \mathrm{Tr}\Big[\log U - \log U^{\dagger}\Big] + \frac{1}{2} i Q \mathrm{Tr}\Big[\log U - \log U^{\dagger}\Big] + \frac{1}{2} i Q \mathrm{Tr}\Big[\log U - \log U^{\dagger}\Big] + \frac{1}{2} i Q \mathrm{Tr}\Big[\log U - \log U^{\dagger}\Big] + \frac{1}{2} i Q \mathrm{Tr}\Big[\log U - \log U^{\dagger}\Big] + \frac{1}{2} i Q \mathrm{Tr}\Big[\log U - \log U^{\dagger}\Big] + \frac{1}{2} i Q \mathrm{Tr}\Big[\log U - \log U^{\dagger}\Big] + \frac{1}{2} i Q \mathrm{Tr}\Big[\log U - \log U^{\dagger}\Big] + \frac{1}{2} i Q \mathrm{Tr}\Big[\log U - \log U^{\dagger}\Big] + \frac{1}{2} i Q \mathrm{Tr}\Big[\log U - \log U^{\dagger}\Big] + \frac{1}{2} i Q \mathrm{Tr}\Big[\log U - \log U^{\dagger}\Big] + \frac{1}{2} i Q \mathrm{Tr}\Big[\log U - \log U^{\dagger}\Big] + \frac{1}{2} i Q \mathrm{Tr}\Big[\log U - \log U^{\dagger}\Big] + \frac{1}{2} i Q \mathrm{Tr}\Big[\log U - \log U^{\dagger}\Big] + \frac{1}{2} i Q \mathrm{Tr}\Big[\log U - \log U^{\dagger}\Big] + \frac{1}{2} i Q \mathrm{Tr}\Big[\log U - \log U^{\dagger}\Big] + \frac{1}{2} i Q \mathrm{Tr}\Big[\log U - \log U^{\dagger}\Big] + \frac{1}{2} i Q \mathrm{Tr}\Big[\log U - \log U^{\dagger}\Big] + \frac{1}{2} i Q \mathrm{Tr}\Big[\log U - \log U^{\dagger}\Big] + \frac{1}{2} i Q \mathrm{Tr}\Big[\log U - \log U^{\dagger}\Big] + \frac{1}{2} i Q \mathrm{Tr}\Big[\log U - \log U^{\dagger}\Big] + \frac{1}{2} i Q \mathrm{Tr}\Big[\log U - \log U^{\dagger}\Big] + \frac{1}{2} i Q \mathrm{Tr}\Big[\log U - \log U^{\dagger}\Big] + \frac{1}{2} i Q \mathrm{Tr}\Big[\log U - \log U^{\dagger}\Big] + \frac{1}{2} i Q \mathrm{Tr}\Big[\log U - \log U^{\dagger}\Big] + \frac{1}{2} i Q \mathrm{Tr}\Big[\log U - \log U^{\dagger}\Big] + \frac{1}{2} i Q \mathrm{Tr}\Big[\log U - \log U^{\dagger}\Big] + \frac{1}{2} i Q \mathrm{Tr}\Big[\log U - \log U^{\dagger}\Big] + \frac{1}{2} i Q \mathrm{Tr}\Big[\log U - \log U^{\dagger}\Big] + \frac{1}{2} i Q \mathrm{Tr}\Big[\log U - \log U^{\dagger}\Big] + \frac{1}{2} i Q \mathrm{Tr}\Big[\log U - \log U^{\dagger}\Big] + \frac{1}{2} i Q \mathrm{Tr}\Big[\log U - \log U^{\dagger}\Big] + \frac{1}{2} i Q \mathrm{Tr}\Big[\log U - \log U^{\dagger}\Big] + \frac{1}{2} i Q \mathrm{Tr}\Big[\log U - \log U^{\dagger}\Big] + \frac{1}{2} i Q \mathrm{Tr}\Big[\log U - \log U^{\dagger}\Big] + \frac{1}{2} i Q \mathrm{Tr}\Big[\log U - \log U^{\dagger}\Big] + \frac{1}{2} i Q \mathrm{Tr}\Big[\log U - \log U^{\dagger}\Big] + \frac{1}{2} i Q \mathrm{Tr}\Big[\log U - \log U^{\dagger}\Big] + \frac{1}{2} i Q \mathrm{Tr}\Big[\log U - \log U^{\dagger}\Big] + \frac{1}{2} i Q \mathrm{Tr}\Big[\log U - \log U^{\dagger}\Big] + \frac{1}{2} i Q \mathrm{Tr}\Big[\log U - \log U^{\dagger}\Big] + \frac{1}{2} i Q \mathrm{Tr}\Big[\log U - \log U^{\dagger}\Big] + \frac{1}{2} i Q \mathrm{Tr}\Big[\log U - \log U^{\dagger}\Big] + \frac{1}{2} i Q \mathrm{Tr}\Big[\log U - \log U^{\dagger}\Big] + \frac{1}{2} i Q \mathrm{Tr}\Big[\log U - \log U^{\dagger}\Big] + \frac{1}{2} i Q \mathrm{Tr}\Big[\log U - \log U^{\dagger}\Big] + \frac{1}{2} i Q \mathrm{Tr}\Big[\log U - \log U^{\dagger}\Big] + \frac{1}{2} i Q \mathrm{Tr}\Big[\log U$$

$$U = \exp\left(irac{\phi}{F_{\pi}} + i\sqrt{rac{2}{3}}rac{\eta_0}{F_0}
ight)$$

• Q represents the topological charge density. The gluonic potential

$$\frac{1}{2}iQ\mathrm{Tr}\Big[\log U - \log U^{\dagger}\Big] + \frac{3}{\tilde{m}_{\eta_0}^2 F_0^2}Q^2 \ \mapsto \ -\frac{1}{2}\tilde{m}_{\eta_0}^2\eta_0^2$$

yields the gluonic contribution to the etaprime mass term

• Singlet decay constant from etaprime \rightarrow 2 photons

$$\frac{2\alpha}{\pi} = \sqrt{\frac{3}{2}} F_0 \Big(g_{\eta'\gamma\gamma} - g_{Q\gamma\gamma} \Big)$$