ETA-MESIC NUCLEUS AND COSY-GEM DATA

Q. Haider[†]

Physics Department
Fordham University
Bronx, New York 10458

Lon-chang Liu

Theoretical Division
Los Alamos National Laboratory
Los Alamos, New Mexico 87545

[†] Talk presented at the International Symposium on Mesic Nuclei, Krakow, Poland, 16 June, 2010

OUTLINE OF THE TALK

- COSY-GEM Experimental Result for $^{25}\mathrm{Mg}_{\eta}$
- Calculated Values of Binding Energy and Half-Width of $^{25}\mathrm{Mg}_{\eta}$
- Reanalysis of COSY-GEM Data
- Summary and Conclusion

COSY-GEM EXPERIMENT

Physical Review **C 79**, 012201(R) (2009)

- The COSY-GEM collaboration searched for η-mesic nucleus by means of a recoil-free transfer reaction $p(^{27}\text{Al},^{3}\text{He})\pi^- p' X$.

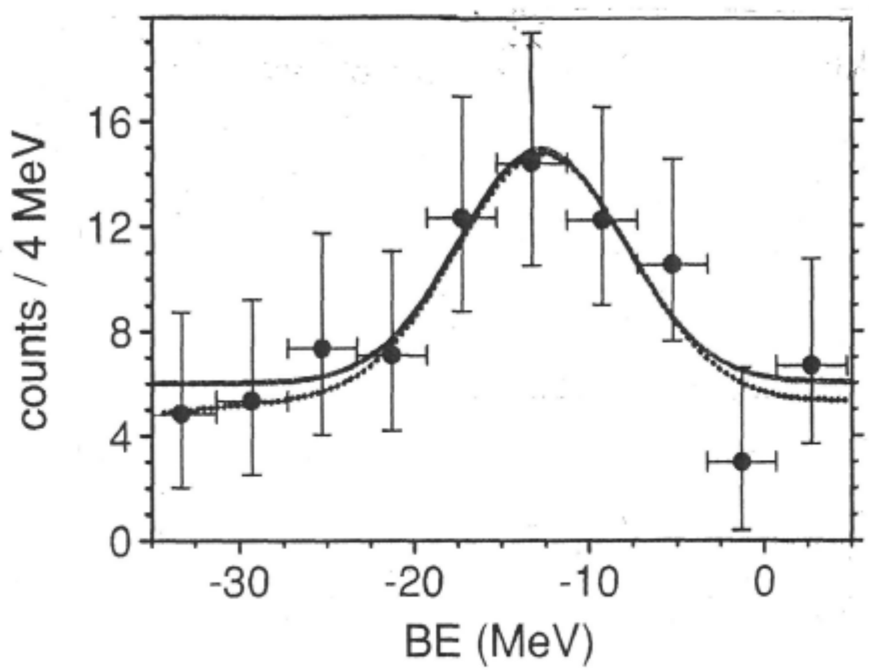
- The multi-step reaction is

$$
\begin{array}{l}
p + {}^{27}\text{Al} \rightarrow \underbrace{\eta + {}^{25}\text{Mg}} + {}^{3}\text{He} \\
\qquad\qquad\quad \downarrow \\
\qquad\qquad\quad {}^{25}\text{Mg}_\eta \\
\qquad\qquad\quad \downarrow \\
\qquad\qquad \overbrace{\eta + {}^{25}\text{Mg}} \rightarrow (\eta + n) + {}^{24}\text{Mg} \rightarrow (\pi^- + p) + \text{X}
\end{array}
$$

- The kinematics was so chosen that the η produced in the intermediate state is nearly at rest, favoring its capture by the residual nucleus ^{25}Mg.

- Because of energy conservation, the bound η cannot reappear as an observable particle in the decay products of the mesic nucleus $^{25}\text{Mg}_\eta$. Instead, it interacts, for example, with a target neutron resulting in the emission of a nearly "back-to-back" $\pi^- p$ pair in the laboratory.

MISSING MASS SPECTRUM



The solid line is a fit to the spectrum with the sum of a background term and a Gaussian

$$|f_b|^2 + |f_g|^2$$

The fit gives

- B.E. $= (-13.13 \pm 1.64)$ MeV
- FWHM $= (10.22 \pm 2.98)$ MeV

Questions:

1. Are the published values of ηN scattering length consistent with the data?

   Different models (fitting πN elastic scattering data and πN$\rightarrow$ ηN cross sections) predict different values of $a_{\eta N}$:

   $$0.27 \leq \mathcal{R}e(a_{\eta N}) \leq 1.05, \quad 0.19 \leq \mathcal{I}m(a_{\eta N}) \leq 0.37$$

   (A reason for this large range: unavailability of data on ηN elastic scattering.)

2. Can the data be explained within the context of our model?

3. Is there any other process that could have contributed to the observed data?

4. In our previous calculations, we neglected the N*(1535)-nucleus interaction. Can the observed spectrum provide information about the nature of N*(1535)-nucleus?

THEORETICAL PREDICTIONS

To obtain the binding energy and half-width of η-mesic nucleus, we solve the relativistic three-dimensional (covariant) integral equation:

$$\frac{k'^2}{2\mu}\psi(\mathbf{k}') + \int d\mathbf{k} < \mathbf{k}' \mid V \mid \mathbf{k} > \psi(\mathbf{k}) = E\psi(\mathbf{k}')$$

First-order low energy η-nucleus optical potential using "on-shell" scattering length as the input is

$$< \mathbf{k}' \mid V \mid \mathbf{k} >= -\frac{1}{4\pi^2\mu}\left(1 + \frac{M_\eta}{M_N}\right) a_{\eta N} f(\mathbf{k} - \mathbf{k}')$$

- $\mathbf{k}$ and $\mathbf{k}'$: initial and final momenta of η in the η-nucleus c.m. frame
- $f(\mathbf{k} - \mathbf{k}')$: nuclear form factor
- $E = \epsilon_\eta - i\Gamma_\eta/2, \quad (\epsilon_\eta < 0)$
- μ: reduced mass of the η-nucleus system
- Only input is the ηN scattering length $a_{\eta N}$
- Gives a general idea of the magnitude of ϵ_η relative to the magnitude of $\Gamma_\eta/2$

Table 1: Binding energy and half-width (both in MeV) of $^{25}\mathrm{Mg}_\eta$ given by the scattering-length approach for two published values of the scattering length $a_{\eta N}$. Form factor used is the 2-parameter Fermi function. The last entry is the fitted value of $a_{\eta N}$.

Orbital ($n\ell$)	$a_{\eta N}$ (fm)	$(\epsilon_\eta - i\Gamma_\eta/2)$ (MeV)
$1s$	$0.27 + 0.22i$	$-(10.0 + 18.3i)$
$1s$	$0.43 + 0.39i$ (a)	$-(24.9 + 38.2i)$
$1s$	$0.292 + 0.077i$ (b)	$-(13.2 + 5.11i)$

(a) B. Krusche, Procs. 2nd TAPS Workshop (World Scientific, 1994)
(b) Scattering length that reproduces the COSY-GEM Data

The imaginary part of the fitted value of the scattering length is substantially smaller than the theoretical value.

The need to understand the difference between scattering length (particularly the imaginary part) given by different theories and the one obtained from fitting experimental data is one of the motivations of the present work.

Microscopic (off-shell) Optical Potential

Physical Review **C 66**, 045208 (2002)

The momentum-space matrix elements of the η-nucleus optical potential is

- $< \mathbf{k}' \mid V \mid \mathbf{k} >=< \vec{\kappa'} | t_{\eta N}(W) | \vec{\kappa} > f(\mathbf{k}-\mathbf{k}')$

- $\vec{\kappa}$ and $\vec{\kappa}'$: initial and final ηN relative momenta

- $W = M_\eta + M_N + \langle B_N \rangle$: total energy of the ηN system in its center-of mass frame and $\langle B_N < 0 \rangle$ is the average binding energy of the nucleon

1. $t_{\eta N}$ is the operator for the scattering of η from a nucleon and is calculated using the coupled-channel isobar model of Bhalerao and Liu (Physical Review Letter **54**, 865 (1985)).

2. WHY? The detailed energy dependence of the model is at our disposal and it reproduces the πN S_{11} phase shifts in the energy region where η can be bound in a nucleus.

3. It contains strong-interaction form factors and satisfies off-shell unitarity:

$$< \vec{\kappa}'|t_{\eta N}(W)|\vec{\kappa} >= K\sum_l v_l(\vec{\kappa}', \Lambda_l)\mathcal{A}v_l(\vec{\kappa}, \Lambda_l)$$

K is a kinematic factor, v and Λ are off-shell form factors and range, $\mathcal{A}$ is the energy-dependent amplitude given by

$$\mathcal{A} = \frac{g^2}{2WD(W)}$$

$$D(W) = W - \{\ M^0 + V_{N^*}(W) + \mathcal{R}e[\Sigma^{free}(W)] + i\ \mathcal{I}m[\Sigma^{free}(W) + \Sigma^{abs}(W)]\ \}$$

$$V_{N^*}(W) = W - M^0 + \mathcal{R}e[\Sigma^{free}(W)] - \mathcal{R}e[D(W)]$$

is the real part of the N*-nucleus interaction, M^0 is the bare mass of the resonance and g is the ηNN* coupling constant.

Self Energies:

$\Sigma^{free}(W)$: N* self-energy arising from its decay to the ηN, πN, and $\pi\pi$N channels in free space.

$\Sigma^{abs}(W)$: N* self-energy arising from absorption or annihilation of the pions coming from N* $\rightarrow \pi$N and $\pi\pi$N decays, with $\Sigma^{med} = \Sigma^{free}(W) + \Sigma^{abs}(W)$.

Unitarity requirement of an optical potential is that $\mathcal{I}m[\Sigma^{abs}]$ and $\mathcal{I}m[\Sigma^{free}(W)]$ should have the same sign.

Results

- For $\langle B_N \rangle = 0$, we get $\epsilon_\eta = -10.1$ MeV and $\Gamma_\eta/2 = 16.2$ MeV. These values are close to the result given by the calculation with $a_{\eta N}$ as the input.

- For $\langle B_N \rangle = -30$ MeV, we get $\epsilon_\eta = -7.1$ MeV and $\Gamma_\eta/2 = 7.8$ MeV.

- To reproduce the COSY-GEM values of $\epsilon_\eta = -13$ MeV and $\Gamma_\eta/2 = 5$ MeV requires that the amplitude $\mathcal{A} = -(0.0521+0.0099\,i)$ fm^2 and $W = 1125$ MeV. This implies $\langle B_N \rangle = -360$ MeV, which is unrealistic.

(The above values were obtained with $V_{N^*} = 0$.)

Neither the scattering length calculation, nor the microscopic optical potential can reproduce the binding energy and half-width of ^{25}Mg obtained from the COSY-GEM spectrum.

Is it possible that η got captured in the excited state of ^{25}Mg?

The experimental spectrum as a function of the binding energy E is deduced from measurements of the missing mass (ΔM). Energy conservation and recoil free kinematics give

$$\Delta M \equiv E_p + M_{27} - E_3 = M_{25} + E_x + M_\eta + E$$

E_p: Proton beam energy.

E_3: Total energy of ^{3}He

$E < 0$: Binding energy of η.

$E_x > 0$: Excitation energy of ^{25}Mg.

With known values of the masses, we find that the experimental centroid of the spectrum at $\Delta M = 23.803$ GeV gives

$$E = -(13 + E_x) \quad \text{(MeV)}$$

From this equation, it is obvious that the data suggest $E_x = 0$.

This leads us to infer that there is another process that contributes to the observed spectrum. It is

$$p + {}^{27}\text{Al} \to \underbrace{\eta + {}^{25}\text{Mg}}_{\downarrow} + {}^{3}\text{He}$$

$$\overbrace{\eta + {}^{25}\text{Mg}} \to (\eta + n) + {}^{24}\text{Mg} \to (\pi^- + p) + \text{X} \quad ,$$

i.e., η produced in the intermediate state is scattered by the residual nucleus and emerge as a pion, without forming an η-mesic nucleus.

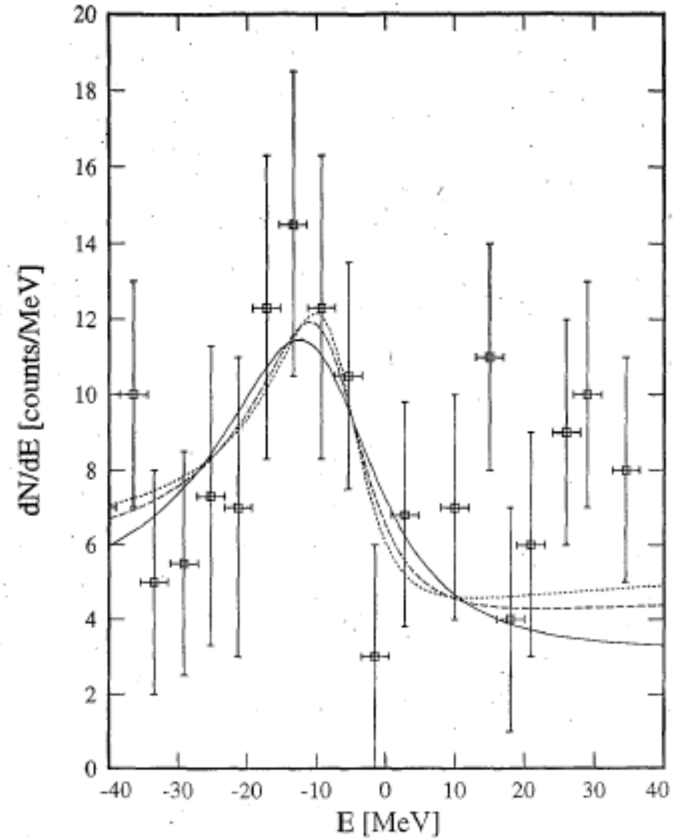
- We fit the experimental spectrum using two amplitudes: f_S for the scattering and f_M for mesic nucleus formation:

$$\alpha \left| f_S + f_M \right|^2 = \alpha \left| \lambda e^{i\theta} + \frac{\Gamma_\eta/2}{E - (\epsilon_\eta - i\Gamma_\eta/2)} \right|^2$$

- α adjusts the overall magnitude

- λ and θ: relative strength and phase between the two processess
  Note: There will be interference effect

- The energy dependence of f_S is neglected because the dependence is much smoother than f_M



Fits to the experimental spectrum by allowing α, λ, and θ to vary while fixing the values of ϵ_η and $\Gamma_\eta/2$. (a) $\epsilon_\eta - i\Gamma_\eta/2 = -(8.0 + 9.6i)$ MeV (dashed curve), (b) $-(10.0 + 14.1i)$ MeV (solid curve), (c) $-(7.1 + 7.8i)$ MeV (dotted curve).

Few remarks:

- Fit (c) is obtained with $\epsilon_\eta = -7.1$ MeV and $\Gamma_\eta/2 = 7.8$ MeV (from our earlier published work), but with interference effect between f_S and f_M included. As can be seen, interference shifted the observed peak from -7.1 MeV to -9.0 MeV.

- Fits (a) and (b) are obtained with $V_{N^*} < 0$. We can see that the final positions of the peak of $|f_S + f_M|^2$ are approximately at -11 MeV [Fit (a)] and -13 MeV [Fit (b)]. They are much closer, and in the case of fit (b) is equal to the experimental centroid.

Table 2: Fitted values of the parameters.

Fit	ϵ_η	$\Gamma_\eta/2$	α	λ	θ	W	$\langle B_N \rangle$	V_{N^*}
(a)	-8.0	9.6	1.91	1.14	0.79	1458	-30	-15
						1438	-50	-40
(b)	-10.0	14.1	2.54	1.17	1.06	1458	-30	-42
						1438	-50	-67
(c)	-7.1	7.8	1.83	1.75	0.62	1458	-30	0

(α is in counts/MeV. λ is dimensionless, θ is in radians, all others are in MeV.)

CONCLUSIONS

1. Two reaction processes are contributing to the observed spectrum.

2. The actual binding of η in ^{25}Mg is weaker than suggested by the centroid of the observed spectrum.

3. Our analysis gives $8 < |\epsilon_\eta| < 10$ MeV and $10 < \Gamma_\eta/2 < 14$ MeV.

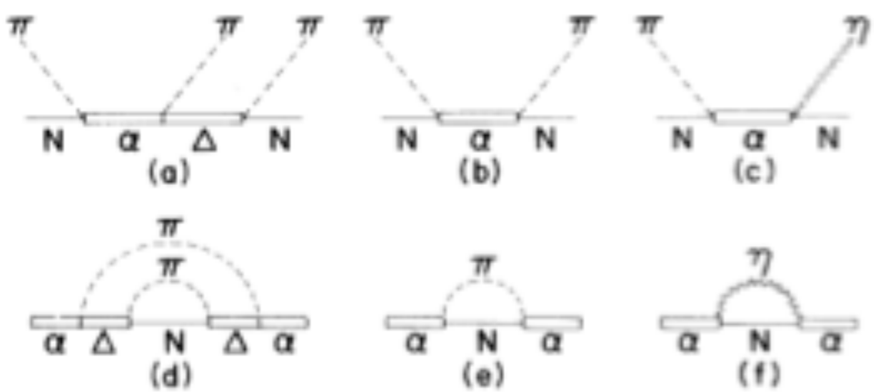
4. The effect of interference is important.
.

5. Real part of the interaction between N^* and medium-mass nuclei is attractive.

Future Work:

$$f_s \equiv < \vec{k}' | V(E) | \vec{k} >$$

$$f_M \equiv \frac{< \vec{k}' | V(E) | \psi >< \Psi | V(E) | \vec{k} >}{E - (\epsilon_\eta - i\Gamma/2)}$$


π
π
π
N
α
Δ
N
(a)
π
π
N
α
N
(b)
π
η
N
α
N
(c)
π
π
α
Δ
N
Δ
α
(d)
π
α
N
α
(e)
η
α
N
α
(f)

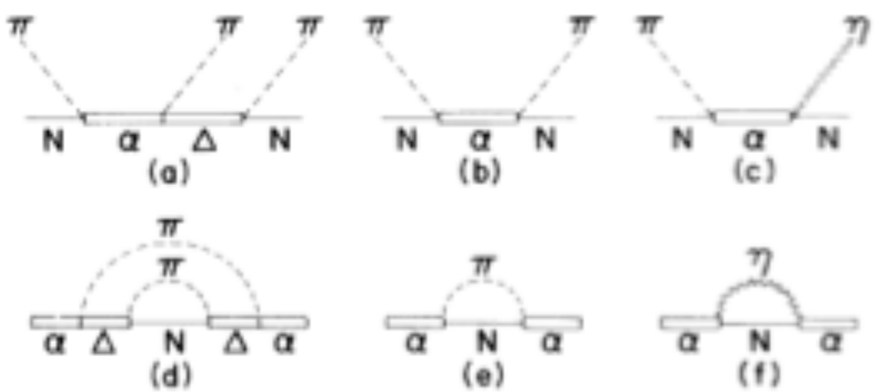
COUPLED-CHANNEL MODEL

OF BHALERAO AND LIU

(Phys. Rev. Lett **54**, 865 (1985))

Features of the Model:

- The model was developed in 1985 to study simultaneously πN elastic scattering, $\pi N \to \pi\pi N$ and $\pi N \to \eta N$ reactions.
- It contains strong-interaction form factors and satisfies off-shell unitarity.
- The basic interactions of the model are


π π π
N α Δ N
(a)
π π
N α N
(b)
π η
N α N
(c)
π
π
α Δ N Δ α
(d)
π
α N α
(e)
η
α N α
(f)

The first three figures represent the matrix elements of the coupled-channel model:

1. α – denotes isospin-$\frac{1}{2}$ πN resonance.

2. For c.m. energy $1470 < \sqrt{s} < 1600$ MeV, only one such resonance has to be considered for each meson-nucleon partial-wave amplitude:

   - N*(1535) for the s-wave (S_{11})
   - N*(1440) for the p-wave (P_{11})
   - N*(1520) for the d-wave (D_{13})

3. ηNN channel is not included in the model because $\sqrt{s}$ is far from the nucleon mass

   Also $g_{\eta NN}$ is negligibly small.

   (Grein and Kroll, Nucl. Phys. **A338**, 332 (1980))

The remaining three figures represent the self-energies, Σ^{α} (complex), of the resonance α due to coupling to the $\pi\pi$N, πN and ηN channels.

- Sum of the real parts of $\Sigma \rightarrow$ resonance energy M_{α} (energy-dependent)
- Sum of the imaginary parts of $\Sigma \rightarrow$ resonance half-width Γ_{α} (energy-dependent)

The radial part of the ηN scattering amplitude:

$$< p' \mid T^{\alpha}_{\eta N,\eta N}(\sqrt{s}) \mid p >= \frac{g^2_{\eta N\alpha} v_{\eta N\alpha}(p', \Lambda_{\eta N\alpha}) v_{\eta N\alpha}(p, \Lambda_{\eta N\alpha})}{D}$$

$$D = \sqrt{s} - M^{\alpha}_{B} - M_{\alpha}(\sqrt{s}) - \frac{i\Gamma_{\alpha}(\sqrt{s})}{2}$$

- $g_{\eta N\alpha}$: coupling constant
- $\Lambda_{\eta N\alpha}$: range parameter
- M^{α}_{B}: bare mass of the resonance
- $v_{\eta N\alpha} \propto \Lambda^2_{\eta N\alpha}/(\Lambda^2_{\eta N\alpha} + q^2)$ is the vertex function

Parameters of the model:

- Coupling constant $g_{\eta N\alpha}$
- Range $\Lambda_{\eta N\alpha}$
- Bare mass M_{α}

They were determined by fitting the πN P_{33} phase shifts data of Arndt (CERN)

Deduced $s-$wave scattering lengths:

$a_{\eta N} = (0.27 + 0.22i)$ fm (Arndt et al.) and $a_{\eta N} = (0.28 + 0.19i)$ (CERN data)

ETA-MESIC NUCLEUS

Relativistic three-dimensional (covariant) integral equation:

$$\frac{k'^2}{2\mu}\psi(\mathbf{k}') + \int d\mathbf{k} < \mathbf{k}' \mid V \mid \mathbf{k} > \psi(\mathbf{k}) = E\psi(\mathbf{k}')$$

- $< \mathbf{k}' \mid V \mid \mathbf{k} > \equiv$ momentum-space matrix elements of the η-nucleus optical potential
- $\mathbf{k}$ and $\mathbf{k}'$: initial and final momenta of η in the η-nucleus c.m. frame
- μ: reduced mass of the η-nucleus system
- $E = \epsilon_\eta + i\Gamma_\eta/2, \quad \epsilon_\eta < 0$: binding energy, $\Gamma_\eta < 0$: width

Solved the integral equation by using "inverse-iteration method" of Tabakin

Microscopic Potential:

First-order η-nucleus optical potential after using the covariant reduction scheme of Liu et al. (Phys. Rev. C10, 398 (1974))

$$< k' \mid V \mid k >= \sum_j \int d\mathbf{Q} < \mathbf{k}', -(\mathbf{k}' + \mathbf{Q}) \mid t(\sqrt(s_j)_{\eta N \to \eta N} \mid \mathbf{k}, -(\mathbf{k} + \mathbf{Q}) >$$
$$\times \, \phi_j^*(-\mathbf{k}' - \mathbf{Q}) \phi_j(-\mathbf{k} - \mathbf{Q})$$

- ϕ_j: nuclear wave function corresponding to having nucleon j at momentum $-(\mathbf{k} + \mathbf{Q})$ and $-(\mathbf{k}' + \mathbf{Q})$ - derived from the experimental charge form factors with the proton finite size corrected for

- $\sqrt{s_j}$: the ηN invariant mass in the c.m. frame of η and nucleon j

$$s_j = \left[M_\eta + M_N - \mid \epsilon_j \mid - \frac{\mathbf{Q}^2}{2M_{C,j}} \left(\frac{M_\eta + M_A}{M_\eta + M_N} \right) \right]^2$$

  - $M_{C,j}$ is the mass of the core nucleus obtained from removing a nucleon j of momentum $-(\mathbf{k} + \mathbf{Q})$ and binding energy $\mid \epsilon_j \mid$ from the target nucleus

- The calculation of V involves full off-shell kinematics and integration over the Fermi motion variable Q

- Used the off-shell ηN model of Balerao and Liu to obtain $t_{\eta N}$

- All kinematic quantities are calculated from $\mathbf{k}'$, $\mathbf{k}$, and Q using well established Lorentz transformations

- For near threshold ηN interaction, only the $l = 0$ (S_{11}), $l = 1$ (P_{11}), and $l = 2$ (D_{13}) isobars need to be considered

- After including $s-$, $p-$, and $d-$wave ηN interactions, η-nucleus interaction remains attractive at low energies

TABLE II. Binding energies and half-widths (both in MeV) of η-mesic nuclei given by the full off-shell calculation. The solutions were obtained with the ηN interaction parameters determined from the πN phase shifts of Arndt *et al.* (Ref. [38]). No bound state solutions of Eq. (1) were found for $A < 12$.

Nucleus	Orbital ($n\ell$)	$\epsilon_\eta + i\Gamma_\eta/2$
^{12}C	$1s$	$-(1.19+3.67i)$
^{16}O	$1s$	$-(3.45+5.38i)$
^{26}Mg	$1s$	$-(6.39+6.60i)$
^{40}Ca	$1s$	$-(8.91+6.80i)$
^{90}Zr	$1s$	$-(14.80+8.87i)$
	$1p$	$-(4.75+6.70i)$
^{208}Pb	$1s$	$-(18.46+10.11i)$
	$2s$	$-(2.37+5.82i)$
	$1p$	$-(12.28+9.28i)$
	$1d$	$-(3.99+6.90i)$

Observations:

- η can be bound into a nuclear orbital with mass number $A > 10$.

- The interaction is not strong enough to have a bound state in lighter nuclei

- There is a reduction in the strength of the η-nucleus interaction at subthreshold energies

- The number of nuclear orbital in which the η is bound increases with increasing mass number A

- Calculations do not include the effects of Pauli blocking of the nucleon on the self-energies Σ^{α}

- Estimate of the blocking using local density approximation shows a 5% reduction in the widths (Phys. Lett. **172B**, 257 (1986))

Condition for at least one s-wave bound state

For an equivalent square-well (complex) potential of depth V_0 and range $R = r_0 A^{1/3}$, the condition for the nucleus to have one $s-$wave bound state is (Quantum Mechanics by Schiff or Gasiorowicz)

$$\frac{\pi^2}{8\mu} < V_0 R^2 < \frac{9\pi^2}{8\mu}$$

In terms of the scattering length the condition is

$$X < \Re(a_{\eta N}) < 9X, \quad X = \frac{\pi^2 R}{12A}\left(1 + \frac{M_\eta}{M_N}\right)^{-1}$$

The potential is given by

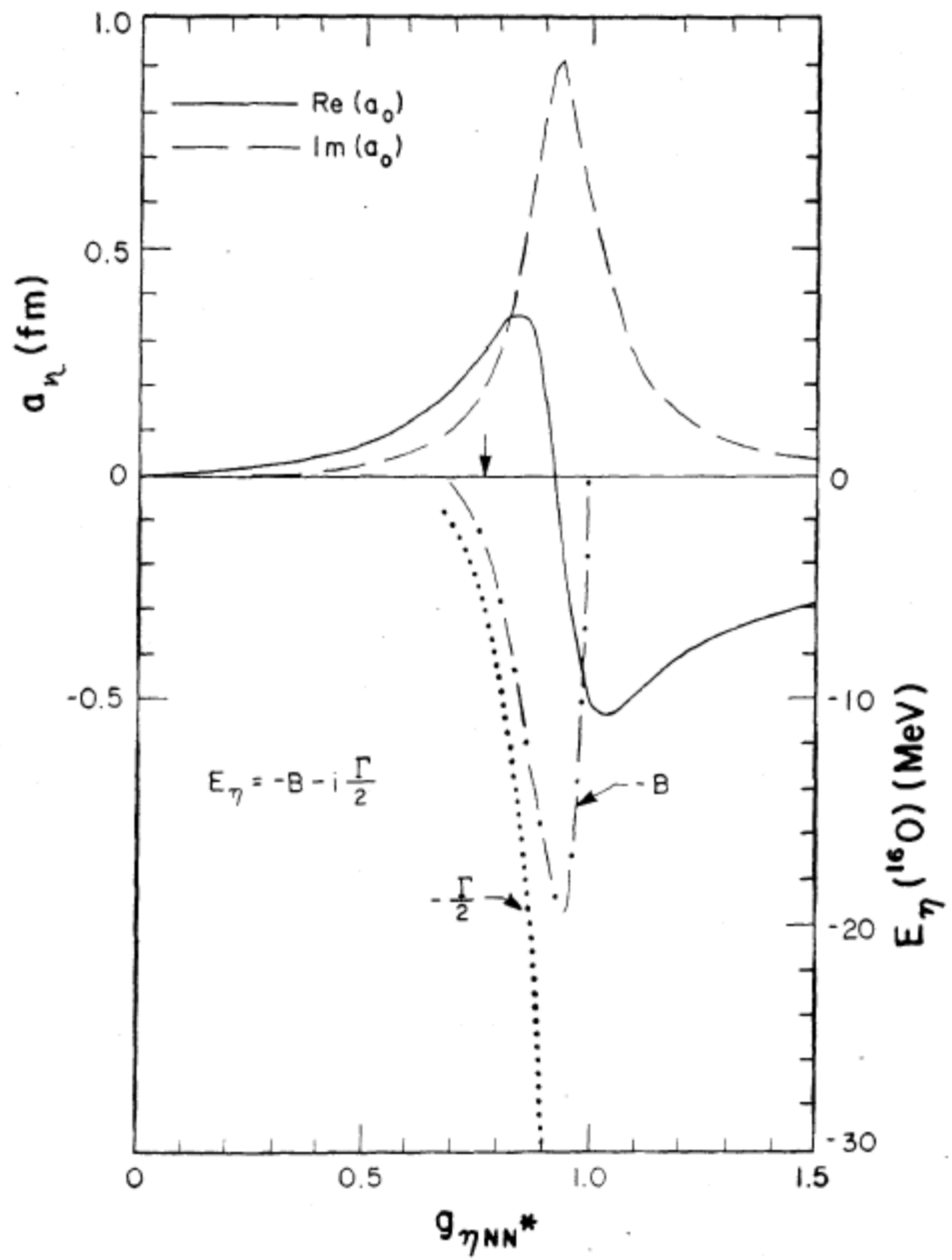
$$V_0 = -197.3\left(\frac{3Aa_{\eta N}}{2R^3}\right)\left(1 + \frac{M_\eta}{M_N}\right)\left(\frac{M_\eta + M_A}{M_\eta M_A}\right)$$

Used $a_{\eta N} = (0.28 + 0.19i)$

The widths of the η-mesic nuclei are compatible with the imaginary part of V_0

Nucleus	V_0 (MeV)	9X (fm)	X (fm)
^{6}Li	-(5,5 + 3.7i)	2.5	0.26
^{12}C	-(8.9 + 6.0i)	1.3	0.14
^{16}O	-(17 + 12i)	1.0	0.11
^{40}Ca	-(20 + 14i)	0.53	0.06
^{90}Zr	-(24 + 16i)	0.29	0.03
^{208}Pb	-(29 + 20i)	0.15	0.02

The dependence of ηN scattering length a_{η} on the coupling constant g, and the binding energy B and half-width $\Gamma/2$ of $^{16}_{\eta}O$ as functions of g.



Binding Energy and the Coupling Constant:

- Value of $g_{\eta NN^*}$ determined by Bhalerao and Liu is 0.77, corresponding to $a_{\eta N} = (0.28 + 0.19i$ fm)

- Binding energy is very sensitive to the coupling constant $g_{\eta NN^*}$ of the ηN interaction

- The width of $^{15}O_\eta$ increases rapidly with $g_{\eta NN^*}$, but binding energy does not

- Binding energy first increases, then decreases for $g_{\eta NN^*} > 0.94$

- For $g_{\eta NN^*} > 1.0$, bound-state ceases to exist

- Real part of $a_{\eta N}$, and hence, the η-nucleus interaction decreases for $g_{\eta NN^*} > 0.85$ and becomes repulsive at large $g_{\eta NN^*}$

- This is because ηN scattering amplitude has a nonlinear dependence on the coupling constant

  - The nonlinearity arises because $g^2_{\eta NN*}$ appears both in the numerator and denominator (through the self-energies Σ) in the expression of the amplitude

  - The physics that governs this nonlinear behavior is as follows: although the ηN scattering via the formation of of N* is proportional to $g^2_{\eta NN*}$, the instability of N* arising from its decay to the ηN channel also increases with $g^2_{\eta NN*}$. This reflects the competition between the formation and decay of a resonance

Factorization Approximation (FA)

- The ηN scattering amplitude in the expression for V is taken out of the integration and evaluated at fixed momentum $< Q >$ and the interaction is given by an energy $\sqrt{\bar{s}}$.

$$< Q >= -\left(\frac{A-1}{2A}\right)(\mathbf{k}' - \mathbf{k})$$

  - This corresponds to a motionless target nucleon fixed before and after the ηN interaction
  - It preserves the symmetry of the t matrix with respect to interchange of $\mathbf{k}$ and $\mathbf{k}'$

- Guided by the expression for $\sqrt{s_j}$, assumed $\sqrt{\bar{s}} = M_\eta + M_N - \Delta$

  Δ: energy shift parameter (a downward shift of 30 MeV is used to fit πN scattering data)

With the above approximations

$$< \mathbf{k}' \mid V_{FA} \mid \mathbf{k} >=< \mathbf{k}', -(\mathbf{k}' + < \mathbf{Q} >) \mid t(\sqrt{\bar{s}})_{\eta N \to \eta N} \mid \mathbf{k}, -(\mathbf{k} + < \mathbf{Q} >) > f(\mathbf{k}' - \mathbf{k})$$

f is the nuclear form factor normalized to the mass number A

A	Orbital	$\Delta = 0$	$\Delta = 10$	$\Delta = 20$	$\Delta = 30$	$\Delta = 35$	$\Delta = 40$
^{12}C	$1s$	$-(2.74 + 10.94i)$	$-(2.25 + 7.51i)$	$-(1.80 + 5.76i)$	$-(1.43 + 4.58i)$	$-(1.24 + 4.11i)$	$-(1.10 + 3.70i)$
^{16}O	$1s$	$-(5.22 + 12.50i)$	$-(4.44 + 8.81i)$	$-(3.78 + 6.91i)$	$-(3.23 + 5.62i)$	$-(2.96 + 5.11i)$	$-(2.75 + 4.66i)$
^{26}Mg	$1s$	$-(10.15 + 15.33i)$	$-(8.89 + 11.11i)$	$-(7.89 + 8.93i)$	$-(7.06 + 7.42i)$	$-(6.65 + 6.83i)$	$-(6.33 + 6.31i)$
^{40}Ca	$1s$	$-(13.45 + 16.10i)$	$-(11.95 + 11.85i)$	$-(10.80 + 9.64i)$	$-(9.83 + 8.11i)$	$-(9.36 + 7.50i)$	$-(8.99 + 6.97i)$
^{90}Zr	$1s$	$-(20.84 + 18.18i)$	$-(18.71 + 13.66i)$	$-(17.17 + 11.28i)$	$-(15.90 + 9.62i)$	$-(15.30 + 8.97i)$	$-(14.81 + 8.38i)$
	$1p$	$-(8.15 + 15.49i)$	$-(7.05 + 11.36i)$	$-(6.09 + 9.20i)$	$-(5.27 + 7.71i)$	$-(4.87 + 7.12i)$	$-(4.55 + 6.60i)$
^{208}Pb	$1s$	$-(25.74 + 20.45i)$	$-(23.40 + 15.40i)$	$-(21.69 + 12.75i)$	$-(20.29 + 10.91i)$	$-(19.61 + 10.18i)$	$-(19.06 + 9.53i)$
	$2s$	$-(4.55 + 11.06i)$	$-(3.44 + 8.36i)$	$-(2.65 + 6.87i)$	$-(2.00 + 5.80i)$	$-(1.69 + 5.37i)$	$-(1.45 + 4.98i)$
	$1p$	$-(19.59 + 19.93i)$	$-(17.46 + 15.01i)$	$-(15.87 + 12.41i)$	$-(14.56 + 10.60i)$	$-(13.92 + 9.88i)$	$-(13.41 + 9.24i)$
	$1d$	$-(8.46 + 14.56i)$	$-(6.92 + 10.96i)$	$-(5.80 + 9.04i)$	$-(4.87 + 7.70i)$	$-(4.42 + 7.16i)$	$-(4.07 + 6.68i)$

Values of binding energies and half-widths (both in MeV) in factorization approximation calculations.

A	Orbital	$\Delta = 0$	$\Delta = 10$	$\Delta = 20$	$\Delta = 30$	$\Delta = 35$	$\Delta = 40$
^{20}Ne	$1s$	$-(6.52 + 12.86i)$	$-(5.63 + 9.16i)$	$-(4.90 + 7.26i)$	$-(4.29 + 5.96i)$	$-(3.98 + 5.44i)$	$-(3.75 + 5.00i)$
^{24}Mg	$1s$	$-(9.26 + 14.90i)$	$-(8.09 + 10.75i)$	$-(7.15 + 8.60i)$	$-(6.37 + 7.13i)$	$-(5.99 + 6.55i)$	$-(5.69 + 6.04i)$
^{28}Si	$1s$	$-(10.84 + 15.70i)$	$-(9.53 + 11.40i)$	$-(8.49 + 9.18i)$	$-(7.62 + 7.65i)$	$-(7.20 + 7.05i)$	$-(6.87 + 6.52i)$
^{32}S	$1s$	$-(11.94 + 16.19i)$	$-(10.56 + 11.80i)$	$-(9.47 + 9.53i)$	$-(8.55 + 7.97i)$	$-(8.11 + 7.35i)$	$-(7.75 + 6.81i)$
^{46}Ti	$1s$	$-(15.40 + 17.12i)$	$-(13.73 + 12.66i)$	$-(12.46 + 10.34i)$	$-(11.40 + 8.73i)$	$-(10.89 + 8.09i)$	$-(10.48 + 7.53i)$
^{52}Cr	$1s$	$-(16.42 + 16.99i)$	$-(14.65 + 12.63i)$	$-(13.33 + 10.35i)$	$-(12.24 + 8.77i)$	$-(11.71 + 8.15i)$	$-(11.28 + 7.59i)$
	$1p$	$-(2.10 + 15.09i)$	$-(1.40 + 10.54i)$	$-(0.75 + 8.21i)$	$-(0.19 + 6.62i)$	–	–
^{56}Fe	$1s$	$-(16.73 + 16.81i)$	$-(14.94 + 12.53i)$	$-(13.60 + 10.28i)$	$-(12.51 + 8.72i)$	$-(11.98 + 8.11i)$	$-(11.55 + 7.56i)$
	$1p$	$-(2.68 + 15.00i)$	$-(2.00 + 10.53i)$	$-(1.34 + 8.25i)$	$-(0.76 + 6.69i)$	$-(0.47 + 6.07i)$	$-(0.26 + 5.53i)$
^{58}Ni	$1s$	$-(17.04 + 16.88i)$	$-(15.23 + 12.59i)$	$-(13.88 + 10.34i)$	$-(12.77 + 8.78i)$	$-(12.24 + 8.16i)$	$-(11.80 + 7.61i)$
	$1p$	$-(3.17 + 14.82i)$	$-(2.43 + 10.47i)$	$-(1.73 + 8.24i)$	$-(1.13 + 6.71i)$	$-(0.83 + 6.10i)$	$-(0.61 + 5.58i)$
^{70}Zn	$1s$	$-(18.57 + 17.34i)$	$-(16.62 + 12.98i)$	$-(15.20 + 10.69i)$	$-(14.03 + 9.10i)$	$-(13.46 + 8.47i)$	$-(13.01 + 7.90i)$
	$1p$	$-(5.02 + 15.29i)$	$-(4.20 + 10.94i)$	$-(3.42 + 8.71i)$	$-(2.75 + 7.18i)$	$-(2.41 + 6.58i)$	$-(2.15 + 6.05i)$

Values of the binding energies and half-widths (both in MeV) given by the factorization approximation method. A dash indicates when no bound states were found.

- Interaction parameters are same as those used for the off-shell calculations

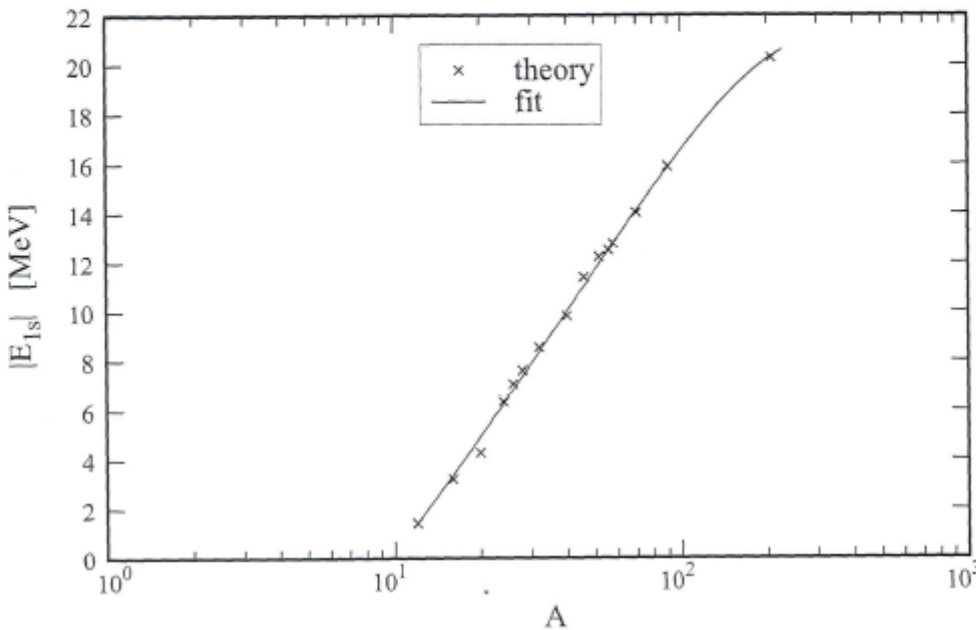
- The FA results with $\Delta = 30$ MeV are very close to the off-shell results

- A 30 MeV downward shift of the hadron-nucleon interaction is consistent with the one found in πN elastic scattering

- This indicates η bound-state formation takes place at energies 30 MeV below the free-space threshold

$a A + d A^{1/3} + b A^{2/3} + c$; (Delta= -30 MeV)

a= 0.0003, d= 13.06, b= -0.9562, c= - 23.46



The interpretation of a, b, c is easy. a=volume energy, b=surface energy, c=thereshold effect. However, the interpretation of d has not been found. From numerical point of view, setting d=0 will not fit the theory. What is the physics of d? We can try to think about it.

In the meantime, can we find other functions?

On-Shell Optical Potential:

First-order low energy η-nucleus "on-shell" optical potential

$$< \mathbf{k}' \mid V \mid \mathbf{k} >= -\frac{1}{4\pi^2\mu}\left(1+\frac{M_\eta}{M_N}\right) a_{\eta N} f(\mathbf{k}'-\mathbf{k})$$

- It corresponds to V_{FA} with no energy shift ($\Delta = 0$)
- Gives an upper limit to the value of ϵ_η
- Only input is the ηN scattering length
- $a_{\eta N}$ is not directly measurable ; its value model dependent
- Different models (fitting πN elastic scattering data and πN$\rightarrow \eta$N cross sections) predict different values of a_η
- $0.27 \leq \Re(a_{\eta N}) \leq 1.05, \;\; 0.19 \leq \Im(a_{\eta N}) \leq 0.37$
- A reason for this large range: unavailability of data on ηN elastic scattering

TABLE I. η-nucleon s-wave scattering lengths $a_{\eta N}$.

$a_{\eta N}$ (fm)	Formalism or reaction	Reference
$0.270+0.220i$	Isobar model	Bhalerao and Liu [2]
$0.280+0.190i$	Isobar model	Bhalerao and Liu [2]
$0.281+0.360i$	Photoproduction of η	Krusche [23]
$0.430+0.394i$		Krusche [23]
$0.579+0.399i$		Krusche [23]
$0.476+0.279i$	Electroproduction of η	Tiator *et al.* [22]
$0.500+0.330i$	$pd \rightarrow {}^3\mathrm{He}\, e\eta$	Wilkin [24]
$0.510+0.210i$	Isobar model	Sauermann *et al.* [14]
$0.550+0.300i$		Sauermann *et al.* [14]
$0.620+0.300i$	Coupled T matrices	Abaev and Nefkens [16]
$0.680+0.240i$	Effective Lagrangian	Kaiser *et al.* [17]
$0.750+0.270i$	Coupled K matrices	Green and Wycech [12]
$0.870+0.270i$	Coupled K matrices	Green and Wycech [13]
$1.050+0.270i$		Green and Wycech [13]
$0.404+0.343i$	Coupled T matrices	Batinić *et al.* [18]
$0.876+0.274i$		Batinić and Švarc [19]
$0.886+0.274i$		Batinić and Švarc [19]
$0.968+0.281i$		Batinić *et al.* [20]
$0.980+0.370i$	Coupled T matrices	Arima *et al.* [21]

TABLE V. Binding energies and half-widths (both in MeV) of η-mesic nuclei given by the scattering-length approach for two different values of the scattering length $a_{\eta N}$. A blank entry indicates the absence of bound state. No bound state exists in ^{3}He.

Nucleus	Orbital ($n\ell$)	$a_{\eta N}$=(0.28+0.19i) fm	$a_{\eta N}$=(0.51+0.21i) fm
^{4}He[a]	1s		−(6.30+11.47i)
^{6}Li	1s		−(3.47+6.79i)
^{9}Be	1s		−(13.78+12.45i)
^{10}B	1s	−(0.93+8.70)	−(15.85+13.05i)
^{11}B	1s	−(2.71+10.91i)	−(20.78+15.42i)
^{12}C	1s	−(2.91+10.22i)	−(19.61+14.20i)
^{16}O	1s	−(5.42+11.43i)	−(23.26+14.86i)
	1p		−(0.95+7.72i)
^{26}Mg	1s	−(11.24+14.76i)	−(33.11+17.73i)
	1p		−(13.41+12.33i)
^{40}Ca	1s	−(15.46+16.66i)	−(38.85+19.16i)
	2s		−(5.59+6.14i)
	1p	−(1.22+10.58i)	−(22.84+14.32i)
	1d		−(4.28+9.52i)
^{90}Zr	1s	−(22.41+19.97i)	−(48.40+22.60i)
	2s		−(26.07+10.07i)
	1p	−(10.18+14.33i)	−(31.53+15.93i)
	2p		−(18.51+8.57i)
^{208}Pb	1s	−(24.55+19.57i)	−(50.27+21.42i)
	2s	−(10.56+13.32i)	−(22.27+11.50i)
	1p	−(20.19+19.05i)	−(34.03+10.03i)
	2p		−(1.89+3.75i)
	1d	−(12.22+16.07i)	−(27.89+12.17i)

[a]Form factor used is the three-parameter Fermi.

TABLE IV. Nuclear form factors used in the factorization approach and scattering-length approach.

Nucleus	Form factor	Parameters[a]
^{3}He	Hollow exponential	$a=1.82$ fm
	Gaussian	$a=1.77$ fm
^{4}He	Three-parameter Fermi	$c=1.01$ fm, $z=0.327$ fm, $w=0.445$ fm
	Frosch model	$a=0.316$ fm, $b=0.680$ fm
^{6}Li	Modified harmonic well	$a_1=1.71$ fm, $a_2=2.08$ fm
^{9}Be	Harmonic well	$\alpha=2/3$, $a=2.42$ fm
^{10}B	Harmonic well	$\alpha=1$, $a=2.45$ fm
^{11}B	Harmonic well	$\alpha=1$, $a=2.42$ fm
^{12}C	Harmonic well	$\alpha=4/3$, $a=2.53$ fm
^{16}O	Harmonic well	$\alpha=1.6$, $a=2.75$ fm
^{26}Mg	Two-parameter Fermi	$c=3.050$ fm, $z=0.524$ fm
^{40}Ca	Two-parameter Fermi	$c=3.510$ fm, $z=0.563$ fm
^{90}Zr	Three-parameter Gaussian	$c=4.500$ fm, $z=2.530$ fm, $w=0.20$ fm
^{208}Pb	Two-parameter Fermi	$c=6.624$ fm, $z=0.549$ fm

[a]Reference [40] for $A=3-16$ and Ref. [41] for the rest of the nuclei.

ETA-MESIC NUCLEUS AND PION DCX

(Phys. Rev. C 36, 1636 (1987))

- Eta-mesic nucleus can affect high-energy pion double-charge-exchange (DCX) reactions

- For $T_\pi > 400$ MeV, η production channel is open in most nuclei

- The DCX reaction can proceed via $\pi^+ \to \pi^0 \to \pi^-$ or $\pi^+ \to \eta \to \pi^-$

- π^0 is in the continuum; η can either be in the continuum or in a nuclear bound state

- Calculated differential cross sections for the reaction $^{14}C(\pi^+, \pi^-)^{14}N$ as a function of T_π at momentum transfer $q = 0$ and 200 MeV/c

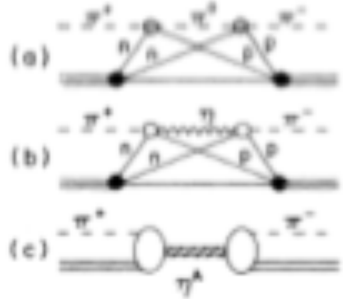


FIG. 1. Reaction diagrams of $U_{\pi\eta}$: (a) the $\pi^+ \to \pi^0 \to \pi^-$ amplitude; (b) the $\pi^+ \to \eta \to \pi^-$ amplitude due to unbound η; (c) the $\pi^+ \to \eta \to \pi^-$ amplitudes due to bound η. The shaded, open, and solid circles denote, respectively, the $t_{\pi N \to \pi N}$ and $t_{\pi N \to \eta N}$ matrices, and the nuclear vertices. The shaded multiple lines denote the η-mesic nucleus ${}_{\eta}A$.

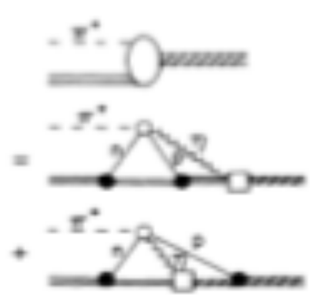
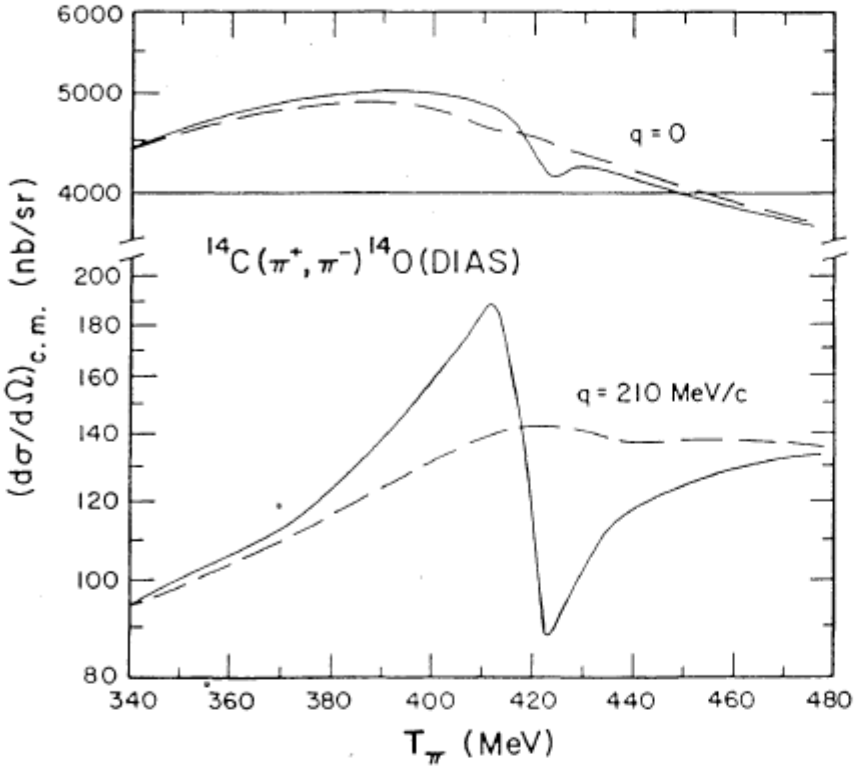


FIG. 2. Formation process of an η-mesic compound nucleus. The decay process [right half of Fig. 1(c)] has a similar structure.



6000
5000
4000
q = 0
$^{14}C(\pi^+,\pi^-)^{14}O$(DIAS)
200
180
160
140
120
100
80
q = 210 MeV/c
$(d\sigma/d\Omega)_{c.m.}$ (nb/sr)
340
360
380
400
420
440
460
480
T_π (MeV)

Observations:

- Dashed line: non-resonant background processes alone (π^0 is in the continuum)
- Contribution of continuum η is very small
- Solid line: Includes non-resonant terms, η-mesic nucleus and their interference
- Cross section varies rapidly over an energy range of 10 MeV, (reflecting the width 11.1 MeV of $^{14}N_\eta$) with a peak at 410 MeV
- For smaller q, background becomes much larger and the resonance feature decreases
- Experiment at LAMPF: Johnson et al. Phys. Rev. **C 47**, 2571 (1993) $^{18}O(\pi^+, \pi^-)^{18}Ne$
- "The measured excitation function shows some evidence for structure near the η production threshold.... statistical precision of the data is not sufficient to allow more than a qualitative characterization of the effect"

Summary and Outlook

- All theoretical models point to the existence of η-mesic nucleus and it is a consequence of the attraction between η and the nucleon at low energies

- Our calculations predict η-mesic nuclei with mass number $A > 10$

- Other models predict the existence of $^{3,4}\mathrm{He}_\eta$

- Nevertheless, existence of η-mesic nuclei will provide an opportunity to studying nuclei having an excitation energy of 540 MeV

- Our FA calculations with $\Delta = 0, 30$ MeV correspond, respectively, to the full off-shell and on-shell results

- s-wave ηN scattering length $a_{\eta N}$ varies greatly from model to model

- Since formation of η-mesic nuclei depends on $a_{\eta N}$, its value needs to be sorted out

- Our calculations do not support a large scattering length, even at the expense of increasing the coupling constant $g_{\eta NN^*}$

- Modifications and improvements to our model will have to wait till the experimental detection of η-mesic nucleus

- Experiments: BNL and LAMPF (inconclusive), Julich (promising)