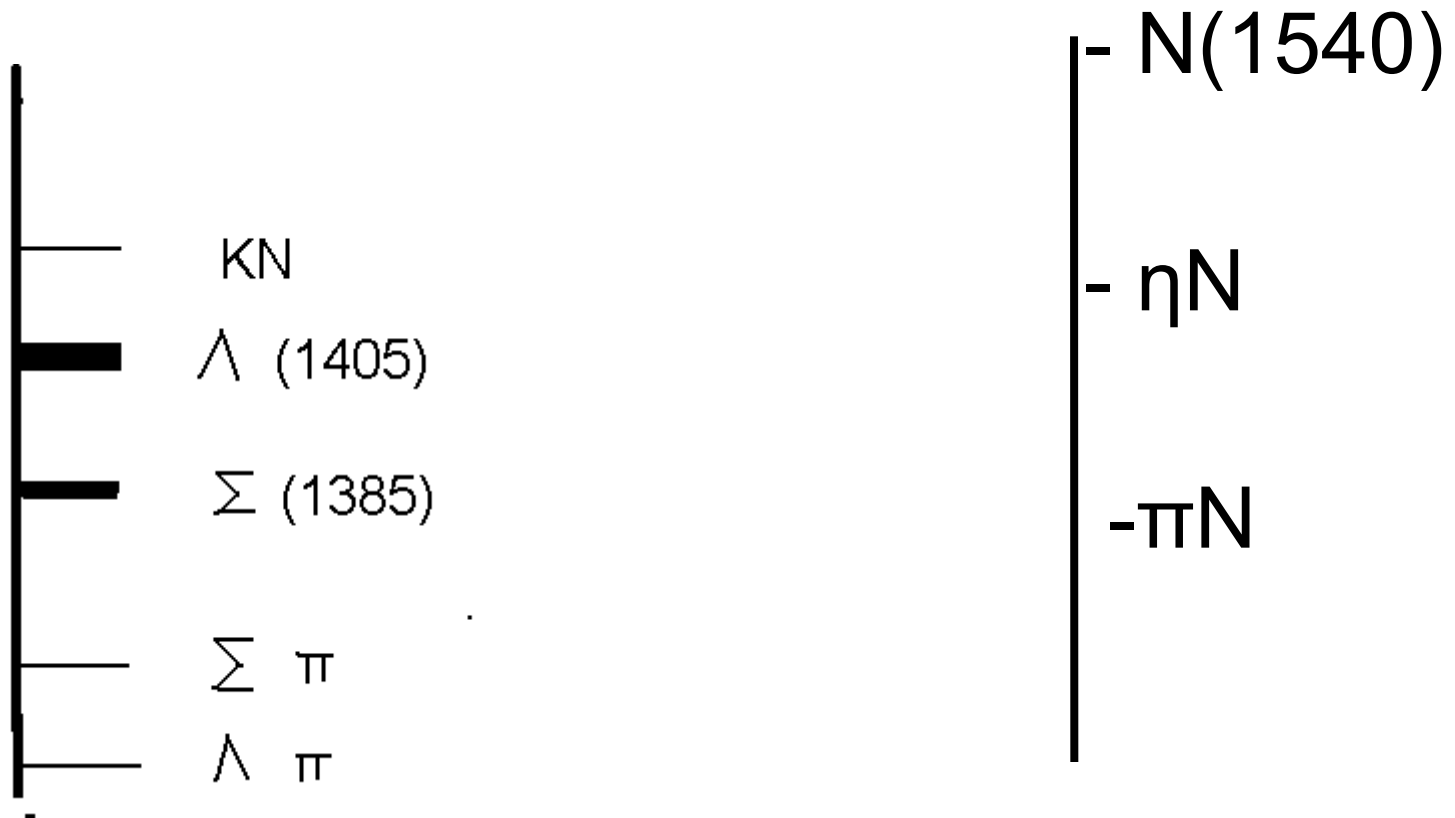


KAONIC-NUCLEI  
VS  
ETA-NUCLEI

Sławomir Wycech - Warsaw, IPJ

# Level repulsion rule : $KN$ , $\eta N$



# Normal prescription

- 1) Use scattering amplitude  
extrapolate below threshold

$$f(\eta N, E = -E_{\text{BINDING}} - E_{\text{RECOIL}})$$

- 2) Correct for medium
- 2) Use optical potential

$$V = 2\pi/M f(\eta N) \rho(r)$$

add  $\eta N N$  absorption

- 3) Solve for levels, widths

# Difficulties of the average field description of kaons in nuclei

- KN, KNN correlations ,  
short ranged NN repulsion
- KN interactions are energy dependent,  
far off-energy-shell

# The questions discussed here

## CORRELATIONS

K-N

K-N-N

$\eta$ -N

$\eta$ -N-N

## EXPLICIT DECAY CHANNELS

$\Sigma$   $\pi$

$\pi$ N

# A variational method to find nuclear energies and widths

- Works for  $K$  – few – nucleons

An intuitive extensions to  $\eta$

# Two steps

1) K bound to N...N fixed nucleons

- correlated wave function  $\Phi_K( X , X_i )$   
 $= \sum_i \psi_i( X - X_i )$
- complex binding energy  $E( X_i )$
- contracting potential  $E( X_i ) - E( \infty )$

2) Variational wave for KN...N

$$F = \Phi_K( X , X_i ) \Theta_{N...N}(X_i )$$

# Nucleons Fixed at $X_i$

Brueckner, Foldy, Walecka

- multiple scattering equation for meson

$$\Phi = G_0 \sum_i V_i \Phi + \Phi_0$$

no incident wave  $\Phi_0 = 0$

separable  $V = v(u)v(u')\lambda$

- $K$  amplitudes at each nucleon

$$\varphi_i = \lambda \int du v(u) \Phi_K(X_i - u, X_i)$$

- multiple scattering  $\rightarrow$  eigenvalue equation

$$\varphi_i = \sum_{i \neq j} t_i G_{i,j}(k) \varphi_j$$

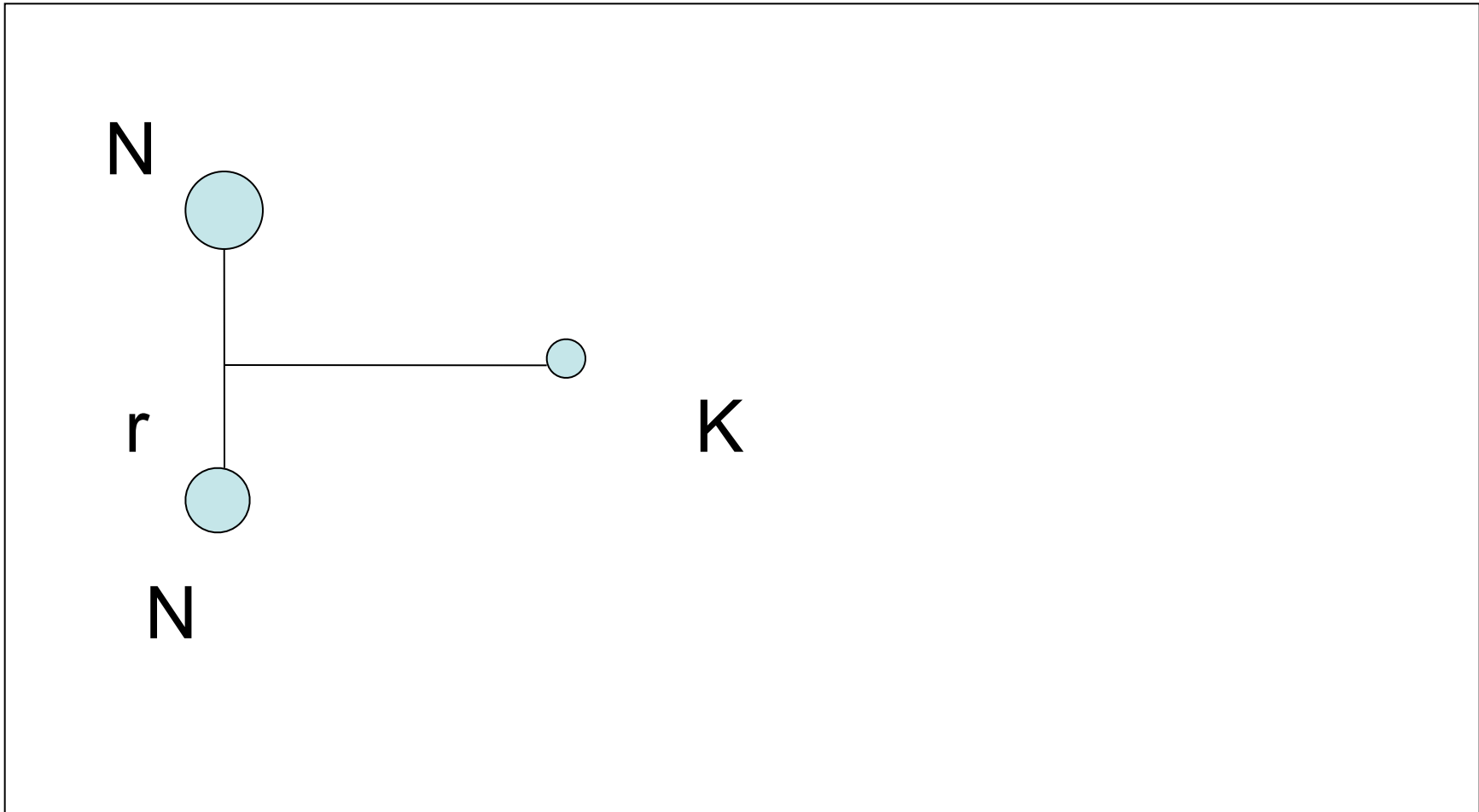
- $\rightarrow$  eigenvalue  $k$



# KNN

S-wave interactions  $\Lambda(1405)$

P-wave interactions  $\Sigma(1385)$



# Equations for amplitudes

$$\varphi_1 = t G_S \varphi_2$$

$$\varphi_2 = t G_S \varphi_1$$

$$G_S \sim \langle v | \exp(-k_I r + i k_R r) | v \rangle / r$$

$$\varphi_1 = \varphi_2 \quad \text{symmetric} = \text{total S wave}$$

$k(r)$  - complex eigenvalue

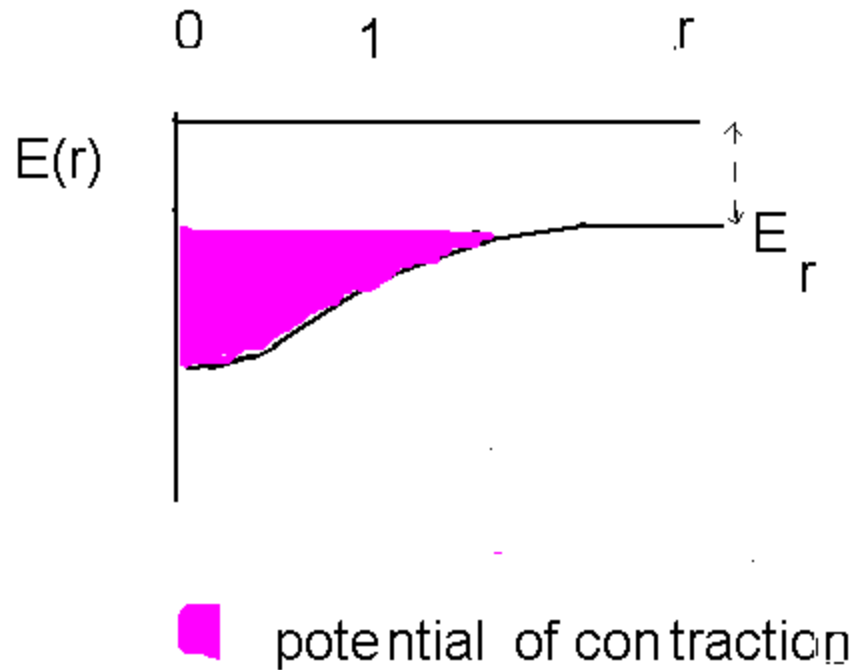
$$E(r) - i\Gamma/2 = k(r)^2 / 2 \mu_{KN}$$

Asymptotic separation

$$KNN \rightarrow N + \Lambda(1405)$$

$$k(\infty)^2 / 2 \mu_{KN} = \text{binding of } \Lambda(1405)$$

# Contraction due to $\Lambda(1405)$



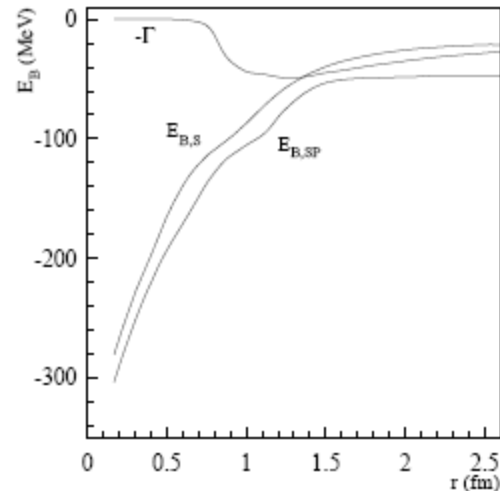
• K N N

$\Lambda(1405)$  N

# $E(r) - E(\infty) =$ contracting potential

KN parameters, A.Martin ,  $\Sigma$ - W.Cameron, O.Brown

- $E_{BS}$   $\Lambda(1405)$
- $E_{BSP}$   $\Lambda(1405) + \Sigma(1385)$



- KNN total S-wave
- NN even L

# Typical solutions

AM- Alan Martin    KWW-Brian Martin

KN –single channel ,

KN, $\Sigma\pi$ - two channel –collision broadening

solution	AM[19]			KWW[11]		
	$E_B$	$\Gamma$	$R_{rms}$	$E_B$	$\Gamma$	$R_{rms}$
$KN; S$	27	36	3.1	35.5	37	2.4
$KN, \Sigma\pi; S$	37	42	2.5	43.1	47	2.1
$KN; S, P$	49	36	3.7	49.7	36	3.3
$KN, \Sigma\pi; S, P$	52	37	2.9	56.5	39	2.3

# Extension to KNNN, KNNNN

Variational solutions : total S waves

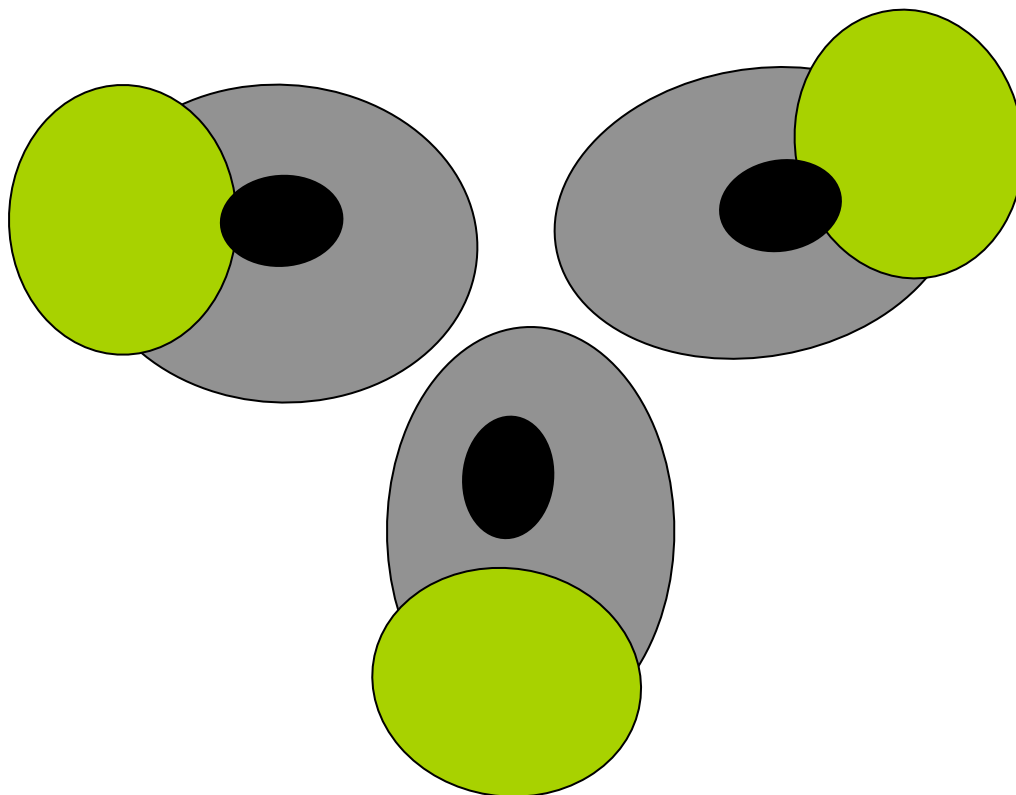
$$F = \Phi_K( X , X_i )_{\text{FIXED-NUCLEONS}} \cdot$$

$$\prod_{\text{NN pairs}} [ ( 1 - \exp(-r^2 \gamma^2) ) ( 1 - \exp(-r\lambda) ) / r ]$$

$\Sigma(1385)$



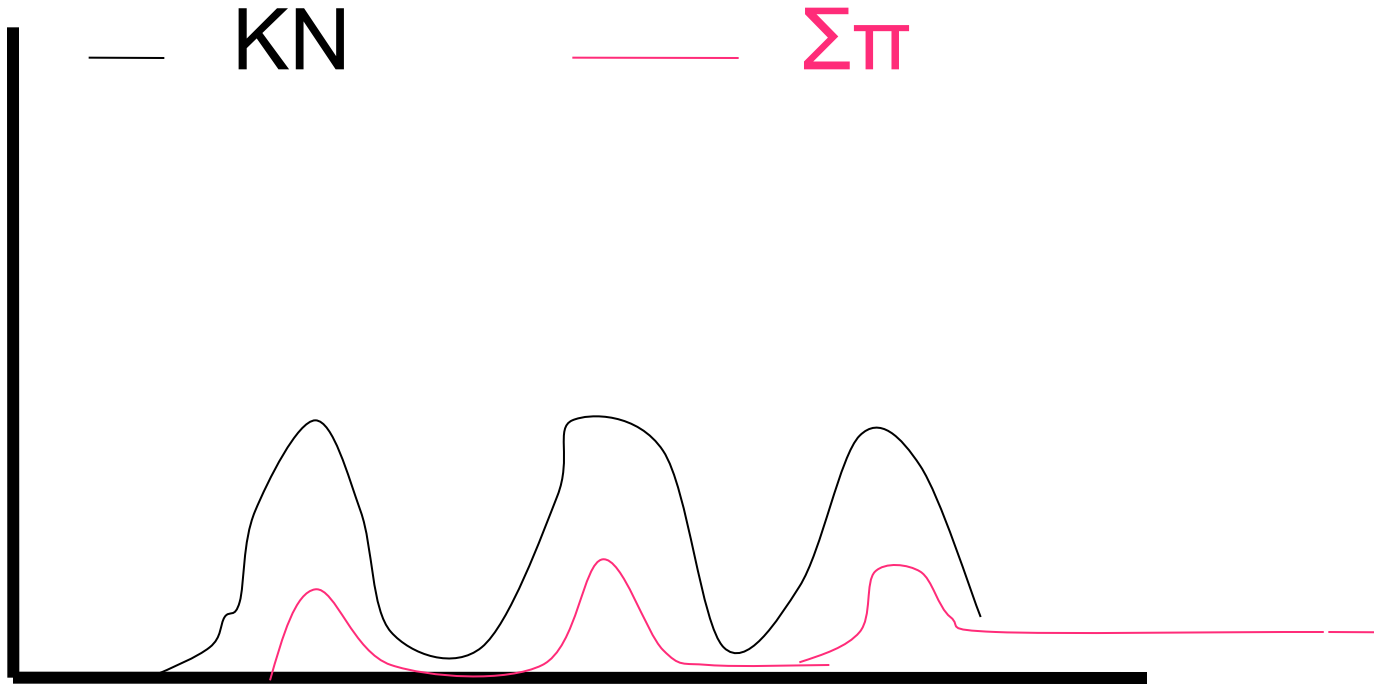
$\Lambda(1405)$





solution	AM[19]			KWW[11]		
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# Explicit decay channel



# Two channels

Two amplitudes at each nucleon

$$\chi_i = \begin{vmatrix} \varphi_i(KN) \\ \varphi_i(\pi\Sigma) \end{vmatrix}$$

- multiple scattering equation

$$\chi_i = \sum_{i \neq j} T_{i,j}(k) \chi_j$$

- $\rightarrow$  eigenvalue  $k$

# Induced shift , induced width

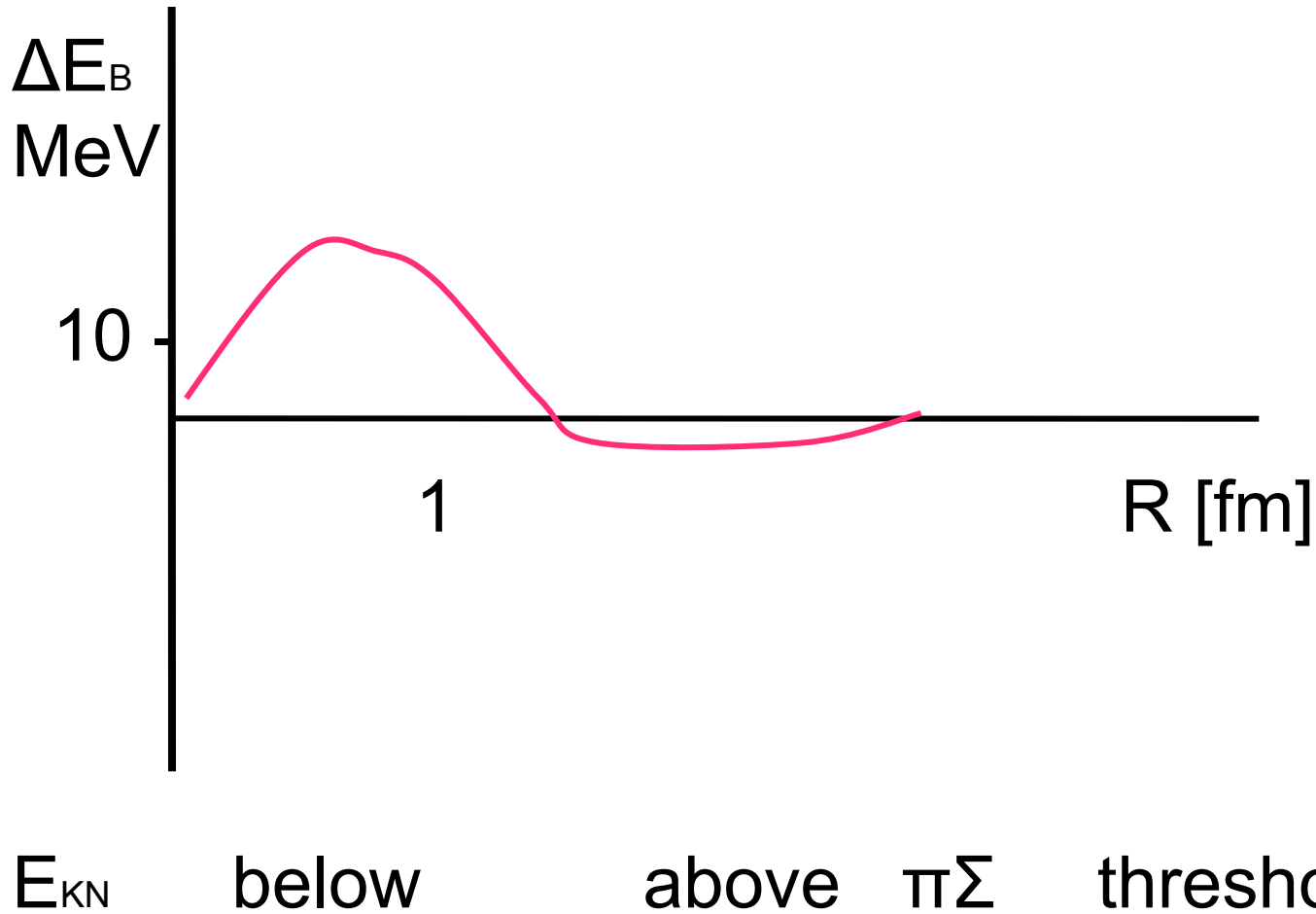
Two nucleons at distance  $r$ , zero range force

$$T_{\alpha\beta} = \gamma_{\alpha} \gamma_{\beta} / (E - E + i \Gamma/2)$$

$$\Delta E = - (\gamma_{\pi})^2 \cos(q_{\pi}r) / r$$

$$\Delta \Gamma = (\gamma_{\pi})^2 \sin(q_{\pi}r) / (q_{\pi}r)$$

# Extra binding of KNN MeV due to multiple scattering in decay channel



KNN ,  $T = 1/2$  „deep state”

$E_B \sim 80 \text{ MeV}$  :  $\Gamma \sim 70 \text{ MeV}$

Effects of explicit decay channel

$\Delta\Gamma$  : 10 -30 MeV

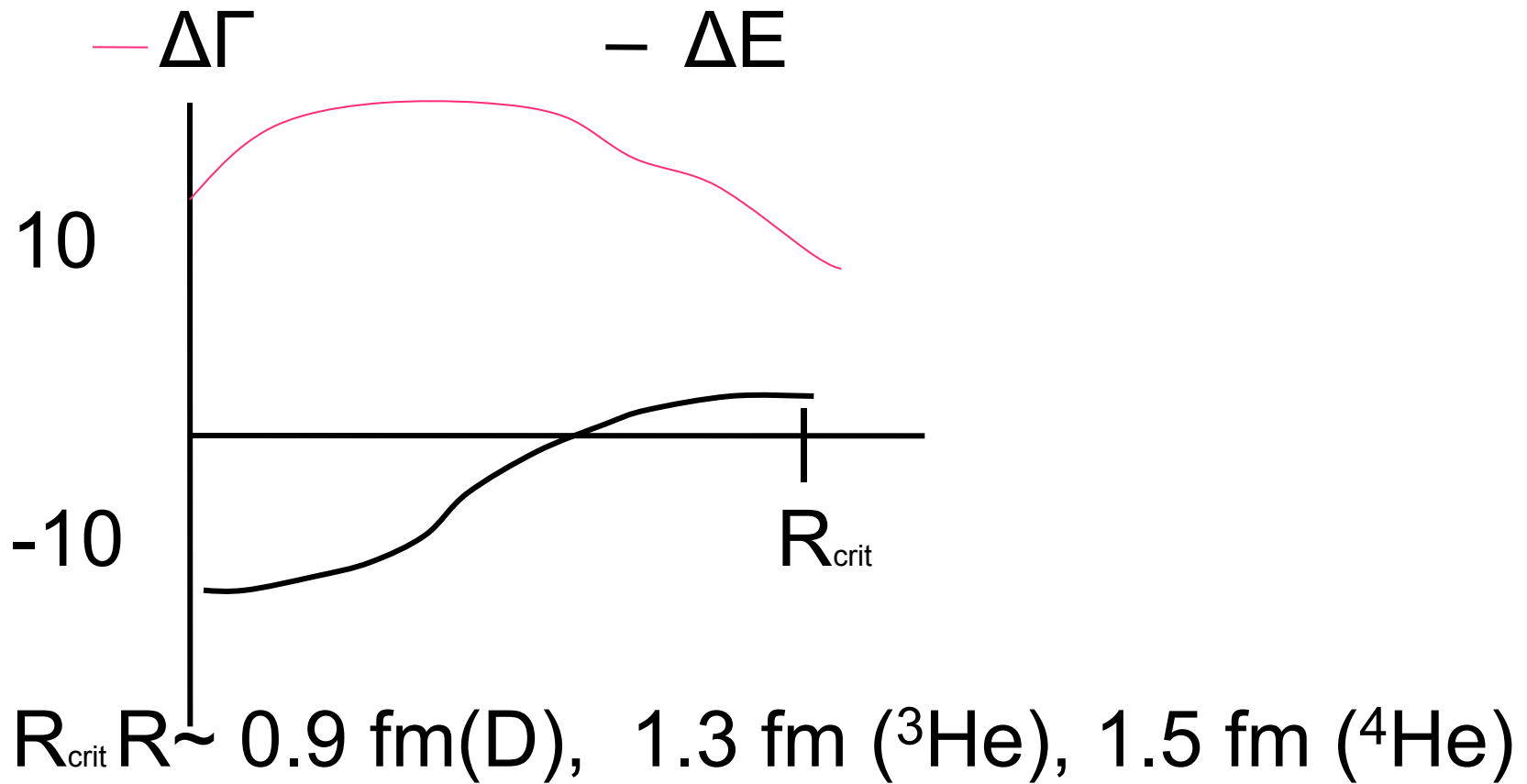
$\Delta E$  : 10 -20 MeV

Similar results with Faddeev method

Shevchenko, Gal, Mares, Revai

Expectations for  $\eta$  +few nucleons ?

# Decay channel in $\eta$ - nuclei



# Applicability of this method

Is limited to central part of nuclei

BUT

multiple scattering in other channels enhances widths , probably adds attraction

This is not  $\rho(r)^2$  effect

This is a boundary condition and interaction in decay channels

(textbook effect in many channel systems)



# Summary

Two channel description

$\eta N$

$\pi N$

of  $\eta$ -nuclear states is  
recommended, if someone finds  
a method to do it.

# Input

- $V(N,N)$  Argonne V18

R.B.Wiringa

$V(KN)$  : 5-channel  $K$  matrix for  $S$  wave,

A.Martin , B.Martin –M.Sakitt

2-channel  $\Sigma(1385)$  for  $P$  wave

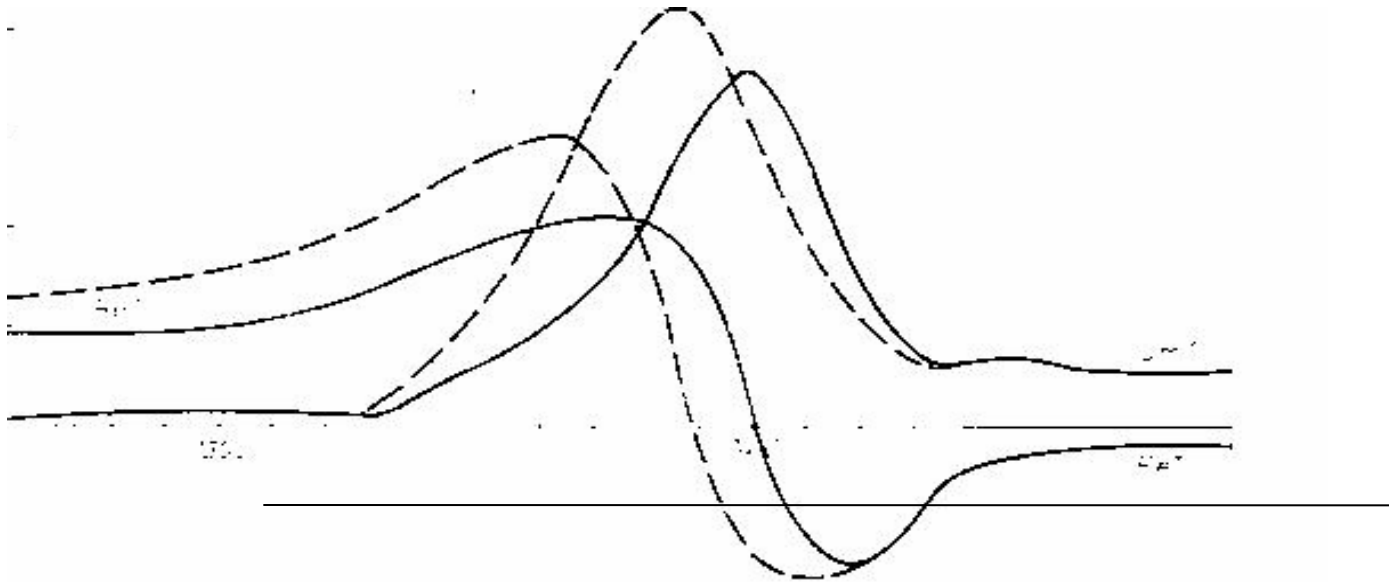
O.Brown, W.Cameron

# Strong dependence on $\Lambda$ resonance position at 1405

solution KWW*			
	$E_B$	$\Gamma$	$R_{rms}$
$KN; S$	50	51	2.05
$KN, \Sigma\pi; S$	71	85	1.81
$KN; S, P$	65	43	2.09
$KN, \Sigma\pi; S, P$	78	60	1.88

# Main uncertainty-position of $\Lambda(1405)$

in potential model



Other –chiral models. Low  $\Lambda(1405)$  mass ,  
second pole due to  $\Sigma\pi$ , [Munich, Valencia]

- KNNN S wave

	$E_B$	$\Gamma$	$E_B$	$\Gamma$
$S$	103	29	142	25
$S + P$	119	23	153	21

- KNNNN S wave

	$E_B$	$\Gamma$	$E_B$	$\Gamma$
$S$	121	25	170	10
$S + P$	136	20	172	10

# New branch, states bound by $\Sigma(1385)$

- KNN  $J(NN)=2, L(NN) = 1$

	$I_{NNK}$	$I_{NN}$	$E_B [MeV]$	$\Gamma [MeV]$	$R_{rms} [fm]$
$K^- nn$	3/2	1	48.5	36	4.9