Light η-mesic nuclei

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Second International Symposium on Mesic Nuclei, Cracow, 23/24 September 2013

There has been little new and compelling evidence for quasi-bound η-mesic nuclei. I will discuss here various approaches, especially in connection with light nuclei, in a rather semi-quantitative way to illustrate the underlying Physics. I will concentrate on real η production; the background is there under control and we have real data.

My talk will treat the following five topics:

1. Simple optical potentials.
2. Multiple scattering approach.
3. Relation between η production reactions and the decay widths of quasi-bound states.
4. The production of η^{7}Li and η^{7}Be final states.
5. Non-mesonic decays.

Simple potential approach

Liu & Haider started the whole bound η-mesic business through their estimates of binding within single-channel potential models, where $V_{\eta A} \propto f_{\eta N}\,\rho(r)$, with $\rho(r)$ being the nuclear density and $f_{\eta N}$ the η-nucleon elastic scattering amplitude. The major controversy is what to assume for $f_{\eta N}$. This has hidden the obvious truth that, because of the N*(1535) resonance, the scattering amplitude varies strongly with energy.

How does one choose the appropriate energy?

Gal and co-workers have tried to estimate the best energy to use in a more self-consistent way but this may not be sufficient because we do not know how the N*(1535) itself behaves when implanted in a nucleus. **Is it bound more or less than a nucleon?** Until we know that, there is going to be lots of uncertainty in the energy estimate.

Suppose we neglect the energy dependence and try to fit all the nuclei with the same complex scattering length input $a_{\eta N}$.

Even more *ambitious* (*i.e.*, foolhardy) let us apply this approach to nuclei as light as ^{3}He ! A VERY small imaginary part puts the $^3_\eta$He pole in a reasonable position. With $a_{\eta N}$ = (0.55+0.03i) fm

$$a_{\eta^3\mathrm{He}} = (-10.0 + 2.4i)\,\mathrm{fm},\ a_{\eta^4\mathrm{He}} = (-2.8 + 0.2i)\,\mathrm{fm},\ a_{\eta^7\mathrm{Li}} = (-2.7 + 0.19i)\,\mathrm{fm}.$$

$$Q_{\eta^3\mathrm{He}} = -(0.36 + 0.18i)\,\mathrm{MeV},\ \mathrm{Q}_{\eta^4\mathrm{He}} = -(5.0 + 0.7i)\,\mathrm{MeV},\ \mathrm{Q}_{\eta^7\mathrm{Li}} = -(5.3 + 0.8i)\,\mathrm{MeV}.$$

- Is there any reasonable model that can reproduce the tiny value of $|Q|$ without having a very small imaginary potential?
- In the ^{4}He case the η is quasi-bound.
- In a simple single-channel potential description, the binding to nuclear excited states should be similar to that of the ground state. Will cause problems when nuclear level spacing is smaller than the η-nuclear width.
- Can we neglect the coupling $\eta^{12}C \leftrightarrow \eta^{12}C^*(0^+)$?

Multiple scattering approach

The MAMI data on $\gamma^3He \to \eta^3He$ showed the well established FSI effect seen in $dp \to \eta^3He$ but the angular distributions were equally fascinating. Away from threshold the data are sharply forward peaked so as to minimise the momentum transfer between the initial and final ^3He.

At low energies there seems to be a lot of cross section in the backward hemisphere. Just as for the $dp \to \eta^3He$ case, this may be due to the interference of the *p*-wave with the rapidly changing phase of the *s*-wave pole.

Multiple scatterings are vital to generate the FSI but, at the necessarily large momentum transfers, one might expect multiple scatterings to induce deviations from the impulse approximation even away from threshold.

Hence we need a simple multiple scattering model.

Suppose the ηN interactions are of very short range and that the nucleons in ^{3}He are at vertices of a rigid equilateral triangle of side ℓ. In terms of the ηN amplitude f we find an η ^{3}He elastic scattering operator $\mathcal{F}$

$$\mathcal{F} = \frac{3f}{D}\left\{\left(1 - f\frac{e^{ik\ell}}{\ell}\right)S(\vec{k}\,',\vec{k}) + 2f\frac{e^{ik\ell}}{\ell}T(\vec{k}\,',\vec{k})\right\},$$

$$\text{where } D = \left(1 + f\frac{e^{ik\ell}}{\ell}\right)\left(1 - 2f\frac{e^{ik\ell}}{\ell}\right).$$

The form factors are expectations over the orientation

$$S(\vec{k}\,',\vec{k}) = \left\langle e^{-i(\vec{k}\,'\bullet\vec{r}_A - \vec{k}\bullet\vec{r}_A)}\right\rangle \text{ and } T(\vec{k}\,',\vec{k}) = \left\langle e^{-i(\vec{k}\,'\bullet\vec{r}_A - \vec{k}\bullet\vec{r}_B)}\right\rangle,$$

where $\vec{r}_A$ and $\vec{r}_B$ are two triangle vertices.

D.Agassi & A.Gal, Ann.Phys.**75**(1973)56

In terms of partial waves

$$\mathcal{F}(\vec{k}',\vec{k}) = \sum_n (2n+1)\mathcal{F}_n(k)P_n(\cos\theta),$$

where θ is the scattering angle and

$$\mathcal{F}_n(k) = \frac{3f}{D}\left\{\left(1-f\frac{e^{ik\ell}}{\ell}\right)+2f\frac{e^{ik\ell}}{\ell}P_n\left(-½\right)\right\}\left[j_n(k\ell/\sqrt{3})\right]^2.$$

Model has many diseases – poles in all partial waves!

$$\mathcal{F}_0(k) = \frac{3f}{\left(1-2f\frac{e^{ik\ell}}{\ell}\right)}\left[j_0(k\ell/\sqrt{3})\right]^2,$$

$$\mathcal{F}_1(k) = \frac{3f}{\left(1+f\frac{e^{ik\ell}}{\ell}\right)}\left[j_1(k\ell/\sqrt{3})\right]^2.$$

The s-wave pole is at k_0=-(i/ℓ)ln(ℓ/2f). Since $\ell \approx 2.7$ fm, one needs a scattering length of the order of 0.7 fm to generate a virtual state pole within 1 MeV of threshold (taking intro account the different ηN and η^{3}He reduced masses). If $a_{\eta N}$=(0.6+0.3i) fm, the pole is at Q_0=(0.28+2.27i) MeV.

The p-wave pole is much further away at k_1=-(i/ℓ)ln(-ℓ/f).

To get a good electromagnetic form factor for ^{3}He, must smear the triangle with a weight function, giving $<\ell> \approx 2.7$ fm.

The γ^{3}He → η^{3}He photoproduction amplitude

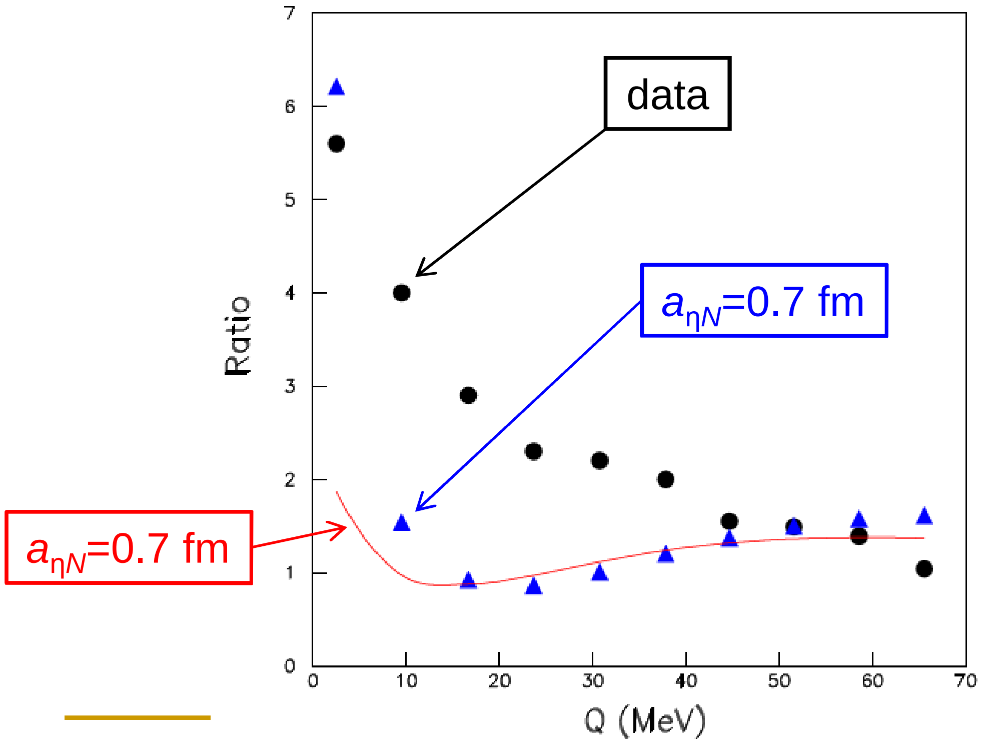
$$\mathcal{F}_n(k) = \frac{3 f_{\gamma^3\mathrm{He}\to\eta^3\mathrm{He}}}{D}\left\{\left(1 - f\,\frac{e^{ik\ell}}{\ell}\right) + 2f\,\frac{e^{ik\ell}}{\ell}P_n\left(-\tfrac{1}{2}\right)\right\} j_n(k\ell/\sqrt{3})\, j_n(k_\gamma \ell/\sqrt{3}),$$

where k_γ is the photon momentum. Does not allow for pion production followed by πN → ηN.

Results a bit disappointing. If $a_{\eta N}$=0.7 fm, the ratio of the predicted total cross section to single scattering is 6.2 at 2.5 MeV and 1.7 at 9.5 MeV, i.e., it falls off far too fast with *Q*.

If $a_{\eta N}$ = (0.476+0.279i) fm, the threshold enhancement is less than a factor of 2 but the $a_{\eta N}$ dependence is weak above 15 MeV.

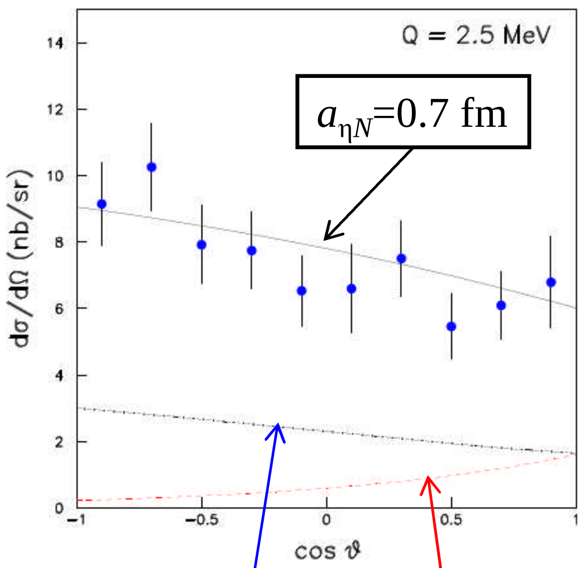
The predictions come far closer to the data at large values of *Q*.

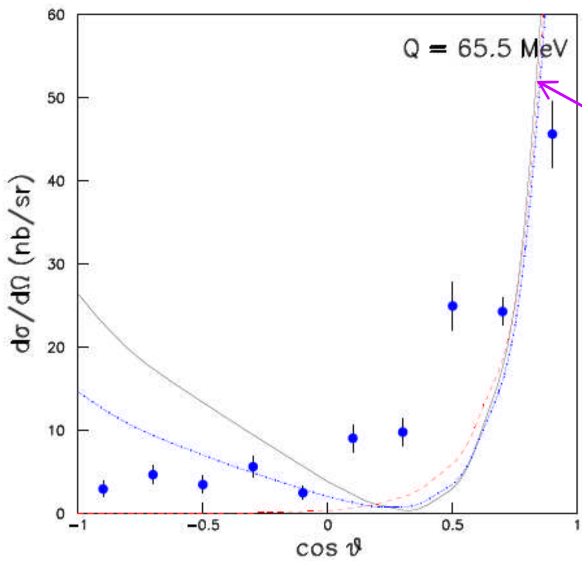


However, multiple scatterings do change the shapes of the angular distributions, even at large *Q*.

At low energy the multiple scattering can change the shape from the forward peaking of the impulse approximation to the gentle backward preference shown by the data. Even taking an input with $a_{\eta N}$ = (0.476+0.279i) fm, one gets the backward thrust but not the normalisation.

At high energies, both multiple scatterings give a backward peak. Getting too much cross section in the backward hemisphere is a universal problem.





Even with $a_{\eta N}$ = (0.476+0.279i) fm one sees very large deviations from the shape of the impulse approximation predictions even though the total cross section is changed by a factor of two.

If the interaction is so strong that the pole is at low $|Q|$, its effects remain too important at high values of Q and this leads to strange-looking angular distributions.

Is this defect due to the neglect of explicit intermediate pion contributions? These are also neglected in potential models.

η-Mesic Nuclear Widths and Decay Rates

The search for "bound" η mesons through non-η decays has been very disappointing. Any $\pi^- pX$ final state coming from such a decay is likely to be a very small fraction of the non-η events. Hence, if there is an effect, it is likely to show up through an interference – it shouldn't then have a Breit-Wigner shape.

Is there any relation between the above threshold real η data and the below threshold signals?

Suppose ${}^3_\eta\mathrm{He}$ is a quasi-bound state whose width overlaps the threshold. Suppose further that any interference is washed out by experimental resolution.

$$\sigma(dp \to {}^3\mathrm{He}\eta) = \frac{2\pi}{p_d{}^2} \frac{\Gamma_{pd}\Gamma_{\eta\tau}/4}{\left[(Q-Q_R)^2 + \Gamma^2/4\right]} = \frac{2\pi\, p_\eta}{p_d{}^2} \frac{\Gamma_{pd}\gamma_{\eta\tau}/4}{\left[(Q-Q_R)^2 + \Gamma^2/4\right]},$$

where $\Gamma_{\eta\tau} = p_\eta \gamma_{\eta\tau}$.

The pole position (or its complex conjugate) $Q_0 = Q_R \pm i\Gamma/2$ may be determined from the energy dependence of the cross section and the product of the partial widths from its magnitude.
At $Q = 0$ the measured value is

$$\frac{p_d}{p_\eta}\sigma(dp \to {}^3\mathrm{He}\eta) = 4\pi \times 2.5\mu\mathrm{b},$$

Taking $|Q_0| = 1$ MeV and $p_d = 878$ MeV/c, one gets

$$\Gamma_{pd}\gamma_{\eta\tau} = 4.5 \times 10^{-5}\ \mathrm{MeV}.$$

But we need each of these widths separately!

Contribution of the pole to total elastic η^3He cross section:

$$\sigma(\eta^3\mathrm{He} \to \eta^3\mathrm{He}) = 2\pi \frac{[\gamma_{\eta\tau}]^2/4}{\left[(Q-Q_R)^2 + \Gamma^2/4\right]}.$$

Assuming a threshold enhancement of say a factor of ten compared to the single scattering estimate, one finds a threshold cross section of the order of 600 fm^2, which leads to $\gamma_{\eta\tau}$=0.1 and Γ_{pd}= 4.5×10^{-4} MeV. The total *dp* cross section passing through the pole ≈ 400 nb.

Same game for $dd \to {}^4\mathrm{He}\eta$ gives a total *dd* cross section of about 100 nb passing through the ${}^4_\eta\mathrm{He}$ pole. Of these perhaps 30 nb corresponds to $dd \to \pi^- pX$.

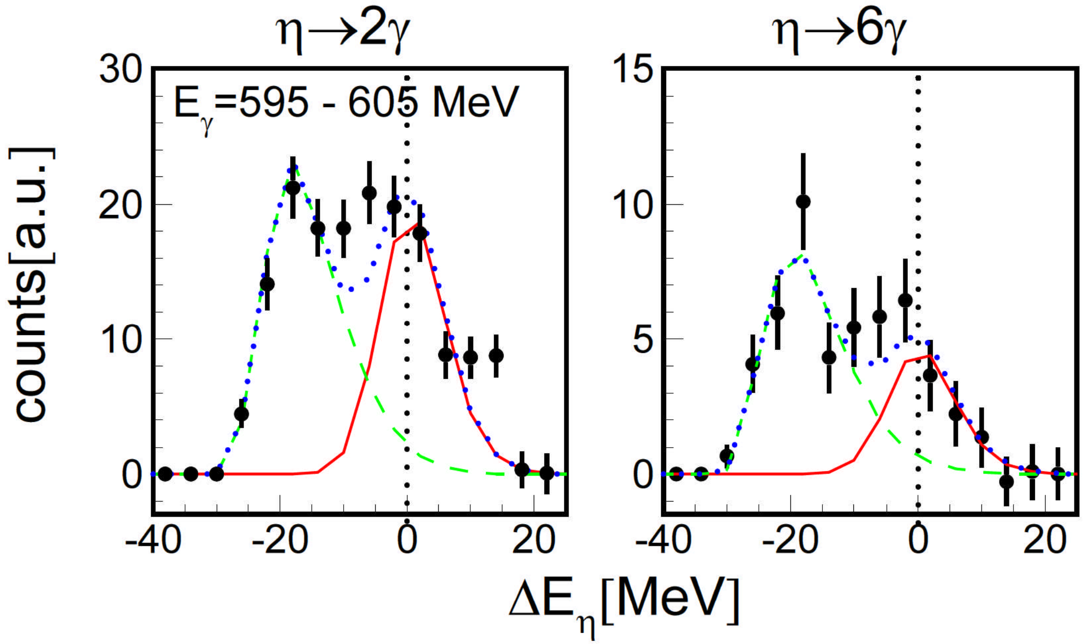
COSY-WASA find an upper limit of about 20 nb for the exclusive $\pi^- p\,{}^3\mathrm{He}$ final state.

Data above and below the η threshold are NOT mutually independent.

The production of η^7Li and η^7Be final states

New data on $\gamma^7\text{Li} \rightarrow \eta^7\text{Li}$ and older data on $p^6\text{Li} \rightarrow \eta^7\text{Be}$.

A=7 nuclei have a L=1 ground state doublet and a L=3 excited state doublet at $E_x \approx 5$ MeV, both with relatively small spin-orbit splitting. Analysis of the $p^6\text{Li} \rightarrow \eta^7\text{Be}$ data in a cluster model suggests L=3 final states dominate.



Situation is far from clear for $\gamma^7\text{Li} \rightarrow \eta^7\text{Li}$ because there could be a lot of strength in the 5 MeV region of excitation energies.

If impulse approximation is *reasonable* then form factors suggest L=3 final states should be at least as large as L=1.

Impulse approximation for γ^7Li $\rightarrow \eta^7$Li in a cluster model

In a ^{3}H-^{4}He model, form factor at momentum transfer *q*

$$F_7(q) = \left[2F_4(q)G(3q/7) + F_3(q)G(4q/7)\right],$$

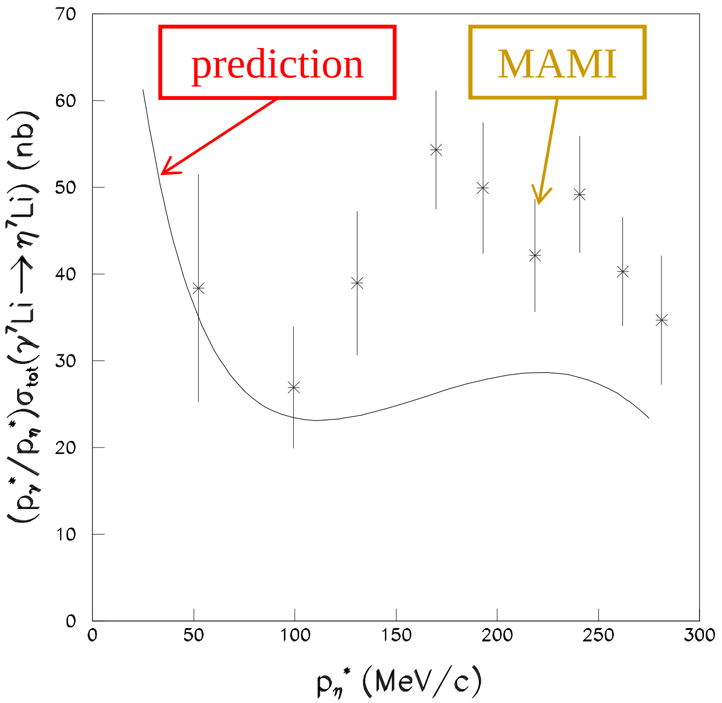
where $F_3(q)$ and $F_4(q)$ are the form factors for ^{3}H and ^{4}He and $G(q)$ reflects the relative motion of the two clusters.

$G(q)$ obtained by fits to data including the ½$^-$ level – neglect spin effects!

Squares of the amplitudes for the photoproduction of the η meson from the ^{7}Li and ^{3}He targets related by

$$\frac{k_\gamma^{(7)}}{k_\eta^{(7)}}\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}(\gamma\,{}^7\mathrm{Li}\rightarrow\eta\,{}^7\mathrm{Li}) = \frac{k_\gamma^{(3)}}{k_\eta^{(3)}}\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}(\gamma\,{}^3\mathrm{He}\rightarrow\eta\,{}^3\mathrm{He})\times\left[G(4q/7)\right]^2.$$

Assume relation is valid for the same η momentum in the laboratory frame of the inverse reaction.



One learns little from the angular distributions because both data and model are so forward peaked.

Both data and predictions include effects from exciting the ½⁻ level.

Rapid rise in the predictions at low momentum is due to the strong FSI in the η^3He input. One would need much more precise data near the threshold to identify (or not) any FSI effects for η^7Li.

There is clearly room for contributions from the $7/2^-$ and $5/2^-$ levels but this calculation is only semi-quantitative. Need consistent microscopic cluster approach.

Non-mesonic decays

Gal will mention the possibility of a contribution to an η-nucleus width from absorption of the meson on two nucleons in the nucleus.

Old fits to even older data show that

$$\frac{p_\eta^{\mathrm{lab}}}{m_\eta}\sigma_{\mathrm{tot}}\left(\eta N \to \pi N\right) \approx 22\ \mathrm{mb},$$

where p_η^{lab} is the η momentum in the frame where the nucleon is at rest.

The production of η mesons much stronger in *np* than *pp*.

$$\frac{p_\eta^{\mathrm{lab}}}{m_\eta}\sigma_{\mathrm{tot}}\left(\eta d \to np\right) \approx 1.7\ \mathrm{mb}.$$

Hence single-nucleon absorption on the deuteron leading to a πp pair is about 25 times stronger than two-nucleon.

How does this change in a nucleus?

Detailed Monte Carlo variational calculations have estimated the number R_{Ad} of quasi-deuterons in light nuclei.

A	^3He	^4He	^6Li	^7Li	^{16}O
R_{Ad}	2.0	4.7	6.3	7.2	18.8

The expectation value of any short-ranged operator that is large only in the $(T,S) = (0,1)$ state should scale as R_{Ad}.

To a first approximation $R_{Ad} \propto A$ so that non-mesonic absorption is likely to be less than say 5% for light η-mesic nuclei. Hence one can ignore it when estimating widths.

Could non-mesonic decays be a useful tool for investigating η-mesic states? Central value of proton spectator momenta in $dp \rightarrow {}^3_\eta He \rightarrow p_{sp} np$ is 440 MeV/c, which is far from the central value of 1570 MeV/c coming from deuteron break-up.

This may help to beat the background.

Is it possible to detect the decay of an η-meson while it is orbiting a nucleus?

Total η width is about 1.3 keV, of which 39% corresponds to 2γ decay. The ${}^{3}_{\eta}He$ width is less than 500 keV. Hence, if this is a quasi-bound system, about one in a thousand should decay through 2γ emission. The 6γ branch will be slightly less.
Small but clean!

The natural decay width of the η' is much larger, 226 keV, but only about 2% of these go via 2γ emission.
Even IF an η' is bound to a nucleus, the situation there is likely to be no more promising for detecting it through radiative decays (depending up the width of the state).

CONCLUSIONS

- Data on $\vec{d}p \to \eta\,^3\mathrm{He}$ and $\gamma\,^3\mathrm{He} \to \eta\,^3\mathrm{He}$ show a pole for $|Q| < 1$ MeV that must arise from η^3He dynamics.
- Radically different behaviour seen between η^3He and η^4He can be described with an effective potential but with a very small imaginary part.
- It is very hard in a simple multiple scattering scheme to get good low energy behaviour in $\gamma^3\mathrm{He} \to \eta^3\mathrm{He}$ without wiping out the forward peaking at higher energies.
- The $\gamma^7\mathrm{Li} \to \eta^7\mathrm{Li}_{gs}$ total cross section can be described in impulse approximation in a cluster model in terms of the $\gamma^3\mathrm{He} \to \eta^3\mathrm{He}$ data. But it is likely that the experimental data include contributions from nuclear excited levels.

- The η-meson is as likely to bind to an excited nuclear level as strongly as to the ground state.
- Below-threshold decays of η-mesic states into say $\pi^- pX$ are NOT completely independent of η production data, though there is some model dependence in establishing the link.
- Non-mesonic decay is likely to be less than 5% of single-nucleon emission.
- We are awaiting new precise data on the differential and total cross sections for $np \to \eta d$ near threshold that should tie down better the ηd scattering length and hence the pole position.
- More questions have been asked than answers given!

Thanks and Goodbye!

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Though I realise that I am talking at an institute that will ever be associated with Nicolaus Copernicus, we must all agree that Physics is universal.

Hence let me finish by showing a picture taken in August of the house where Johannes Kepler died in Regensburg.