Eta-nuclear binding
$\leftrightarrow$
low energy scattering parametres

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Background and motivation

”Bound” states of η quasibound, very short-lived

$\longrightarrow$ direct observation difficult or impossible.

Might be seen as bumps in decay channels like pionic final states.

Some optimism and lots of activity in studies of final state interactions (FSI), sensitive to close-by poles in the vicinity of thresholds (Watson-Migdal).

Mostly looked in strong energy dependence of $pd \rightarrow {}^3He\,\eta$.

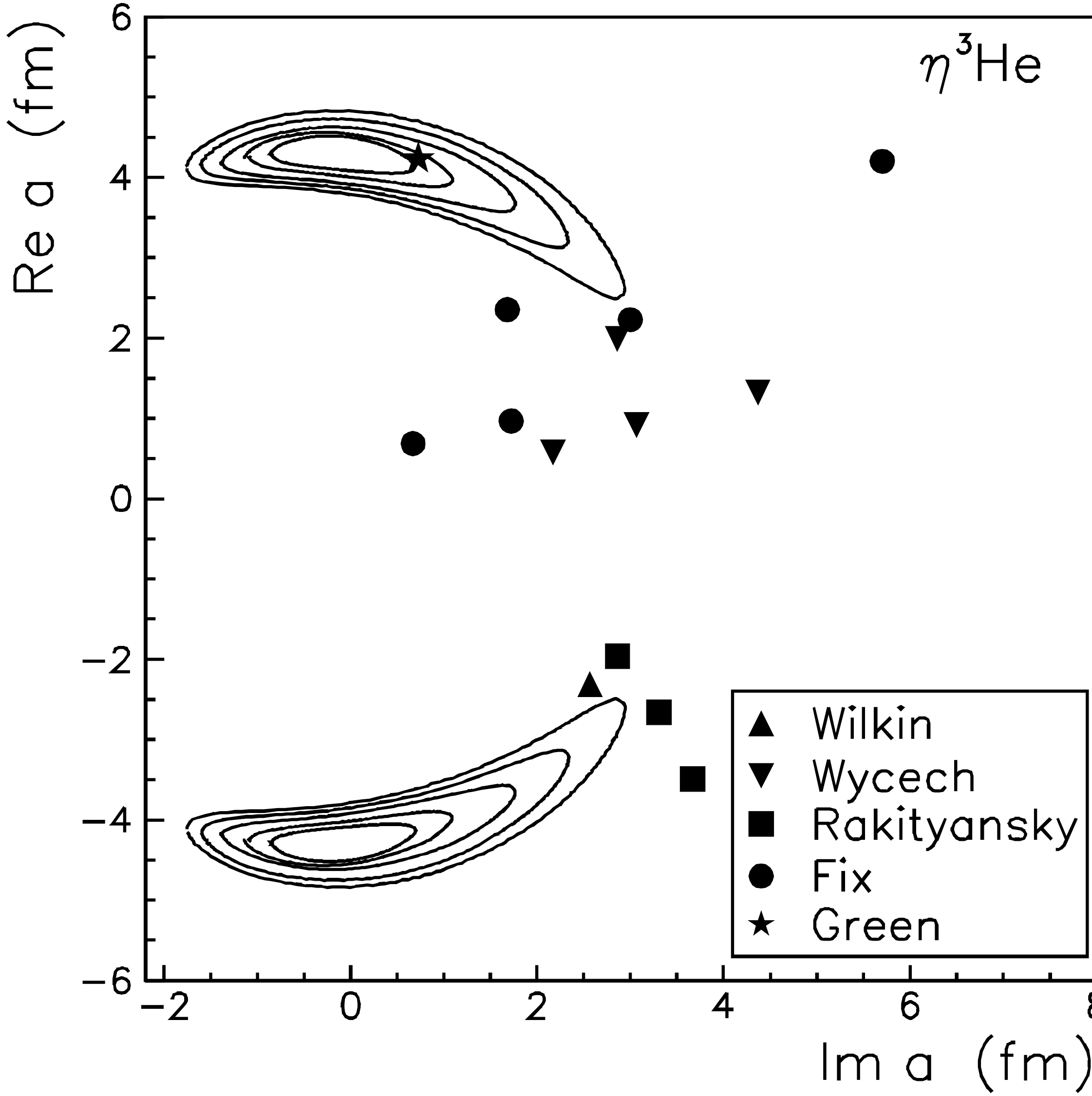
Hope: Energy dependence from a pole below threshold.

However, cross sections alone cannot yield the sign of the real part of the amplitude (tell-tale of a bound state)

AND

theories have been a big help…

Sibirtsev *et al.*
EPJA 22(2004)
495



Hope: Energy dependence from a pole below threshold.

However, cross sections alone cannot yield the sign of the real part of the amplitude (tell-tale of a bound state)

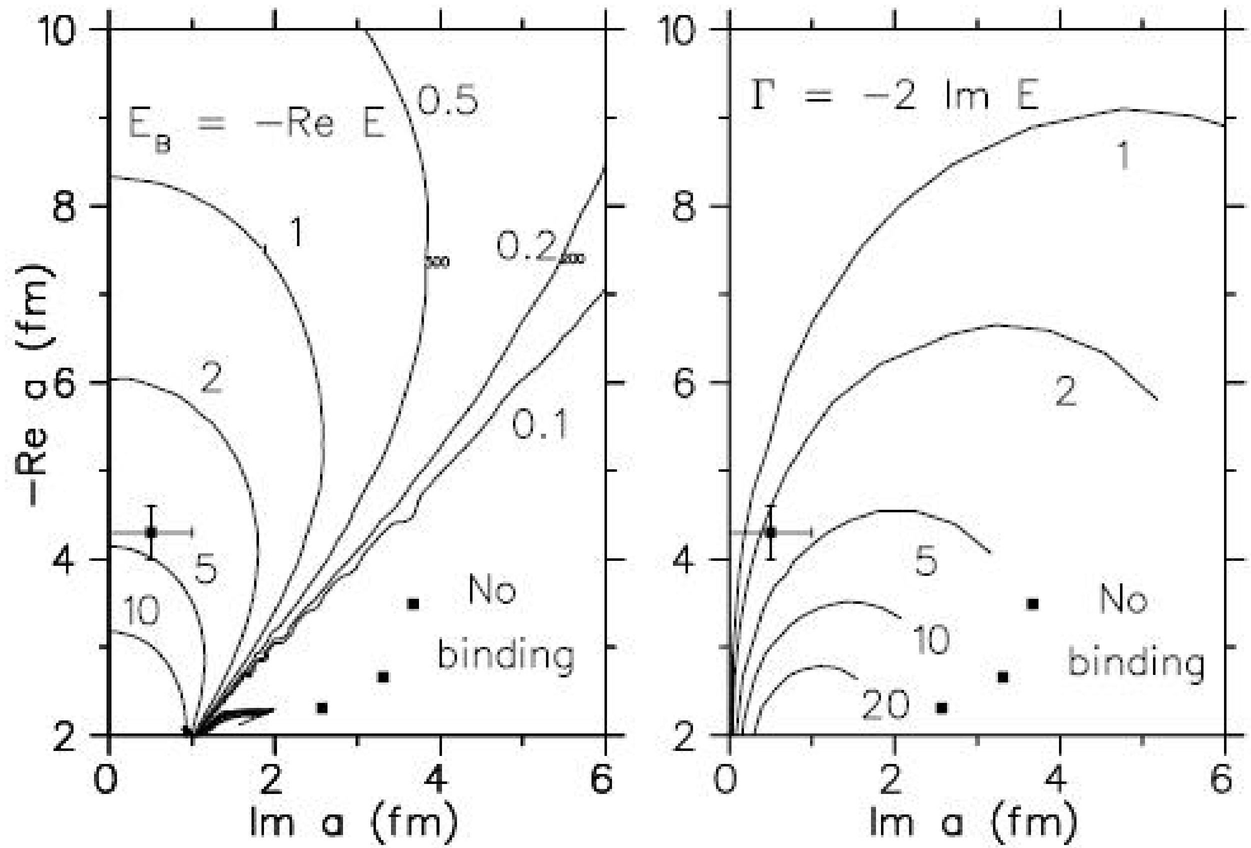
Nevertheless: proximity of the pole to threshold is related to the amplitude and its magnitude **if** a bound state exists.

Motivation of this work:
Connection of low energy scattering data to complex binding energies **if** there is binding.

Potential vs. binding and **potential** vs. scattering model dependent. However, direct connection between binding and low-energy scattering might be less model dependent (shape independence).

Studied earlier for pd → ^{3}He η (Sibirtsev *et al.,* PRC 70(2004)047001).

Binding and width for η in He-3 (PRC 70, 047001)



Extension to heavier nuclei necessary.

- Experimental activities at e.g. COSY.

Other considerations:

- Conditions of binding in terms of low-energy scattering parametres
- Is the scattering length enough or does the effective range play a role?

JAN & H. Machner, NPA 902(2013)40

Calculation

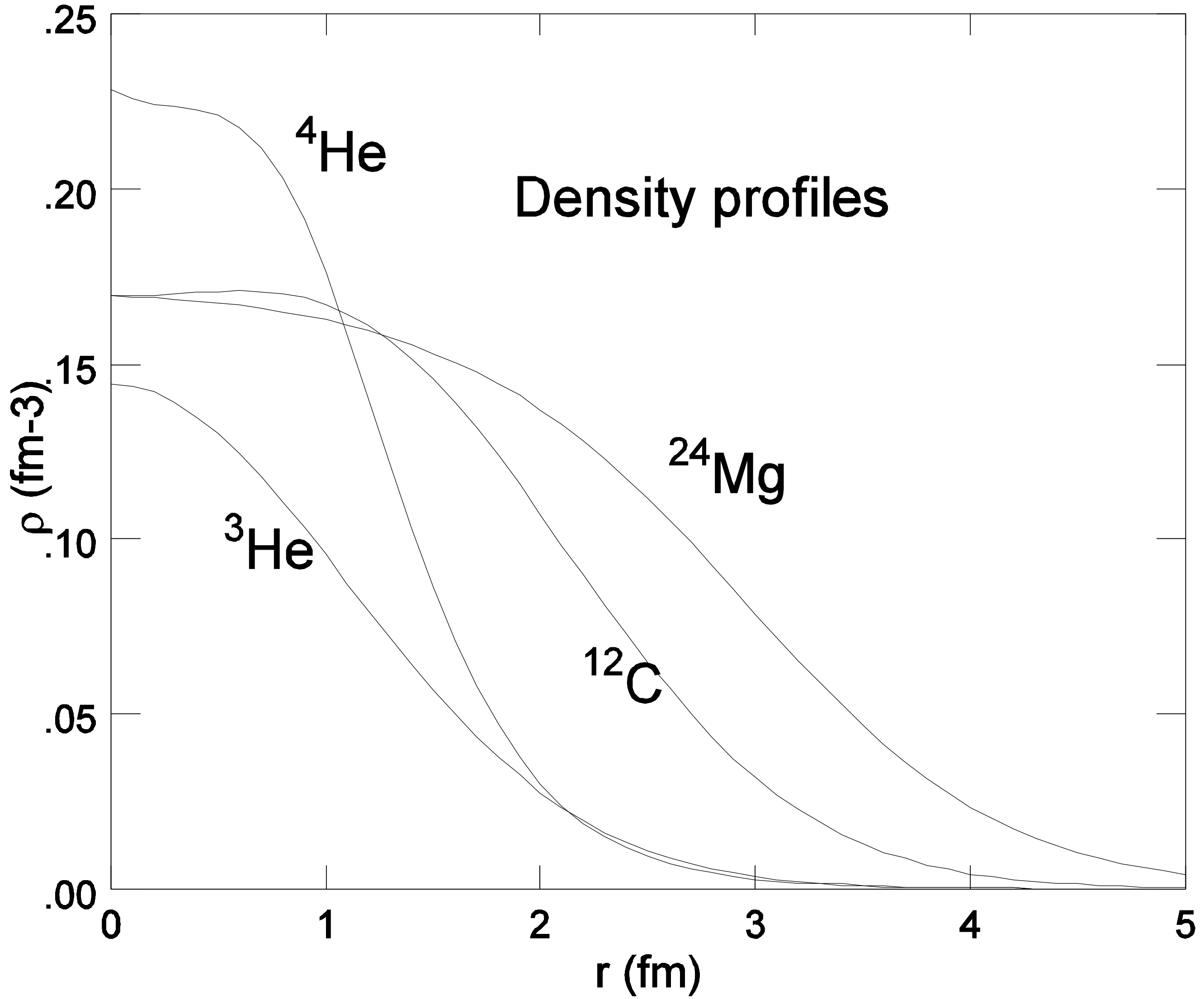
The scattering parametres are obtained from Schrödinger wave functions by fitting

$$q \cot \delta = 1/a + r_0 q^2 /2$$

(a<0 means a binding real potential).

The s-wave bound states and their energies are obtained by iterating the homogeneous Lippmann-Schwinger equation.

Finally complex binding energies presented as contours in the complex a-plane.

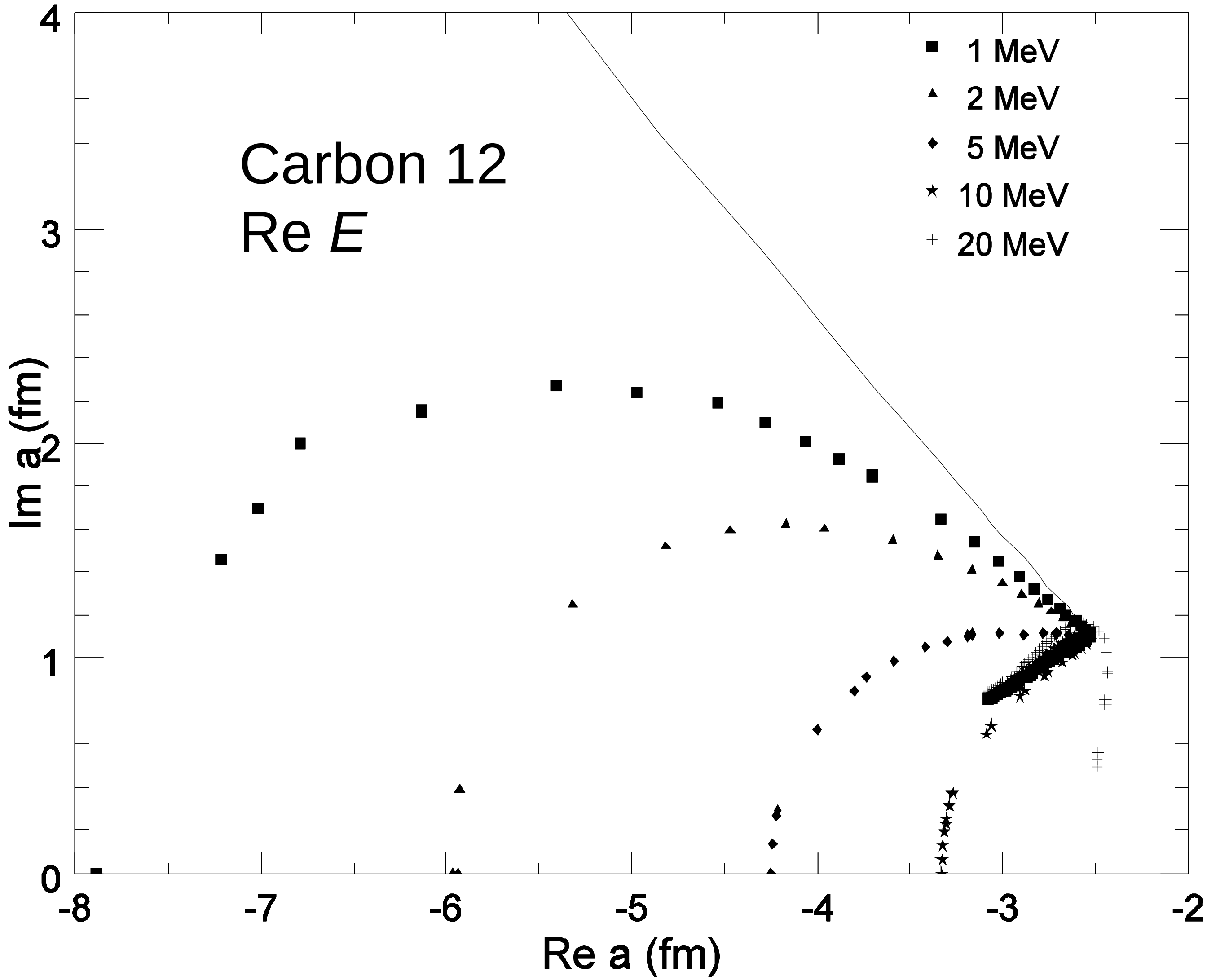

Density profiles
^{4}He
^{3}He
^{12}C
^{24}Mg
ρ (fm-3)
r (fm)
.25
.20
.15
.10
.05
.00
0
1
2
3
4
5

The profiles applied in a simple optical potential

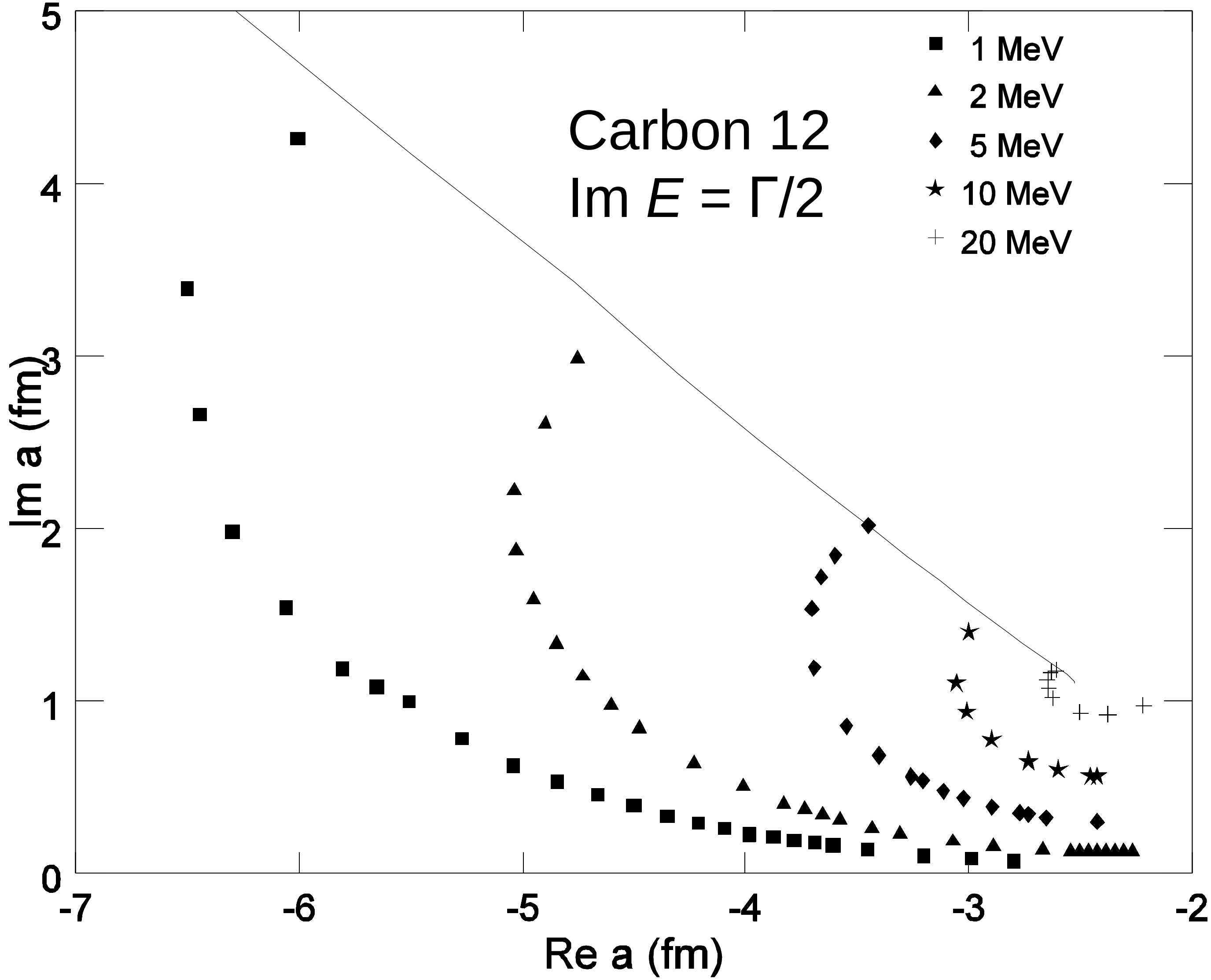
$$V_{opt} = -4\pi\,(V_R + i\,V_I)\,\rho(r)\,/\,(2\,\mu_{\eta N})$$

This is NOT meant to be a realistic potential model; used only to calculate BOTH scattering parametres and (complex) binding energies and **their** correlation with varying potential strength. This will not be sensitive on the actual potential shape.

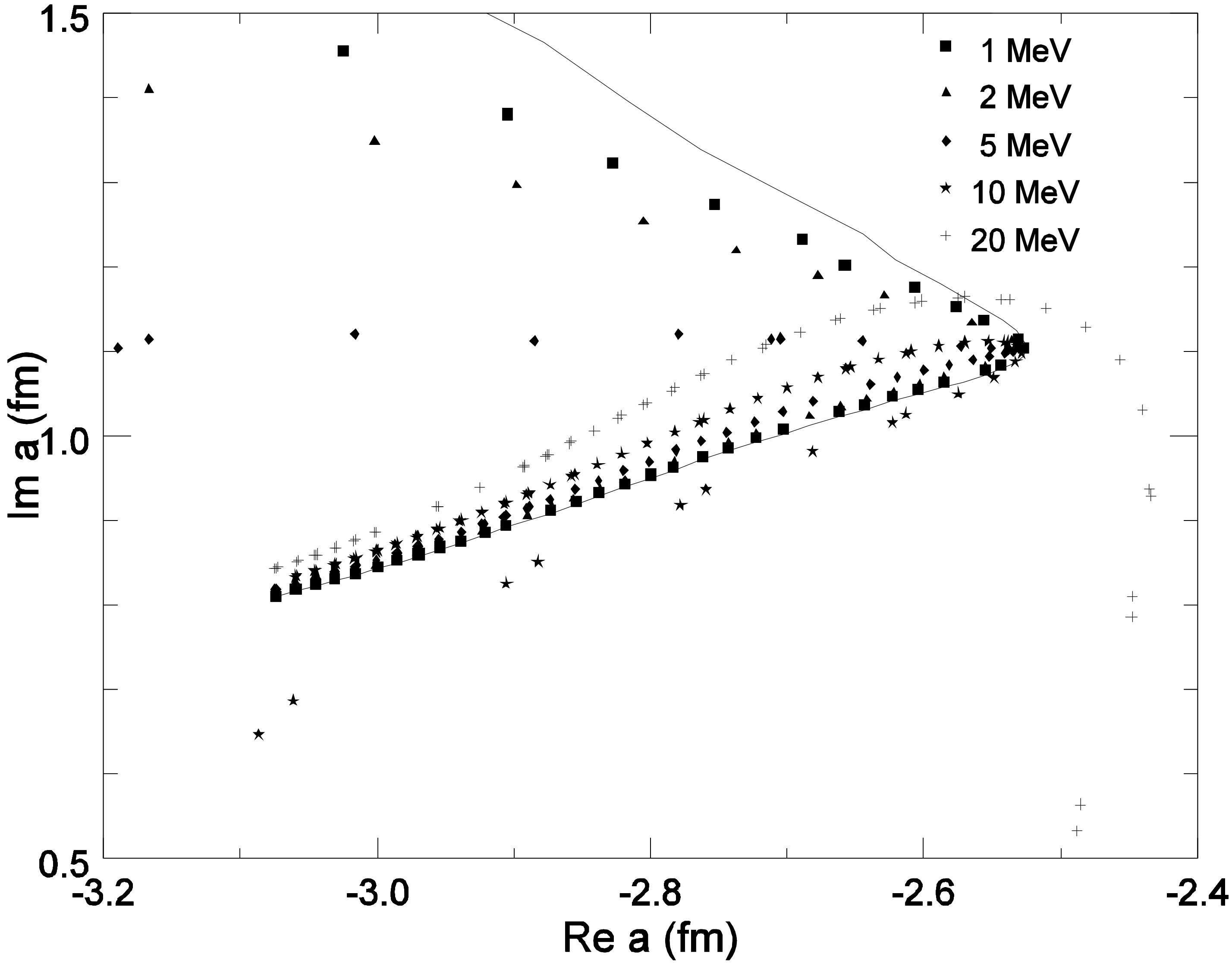
Scattering parametres NOT monotonous as functions of strength parametres – connection not trivial.



Carbon 12
Re E
1 MeV
2 MeV
5 MeV
10 MeV
20 MeV
Im a (fm)
Re a (fm)
4
3
2
1
0
-8
-7
-6
-5
-4
-3
-2

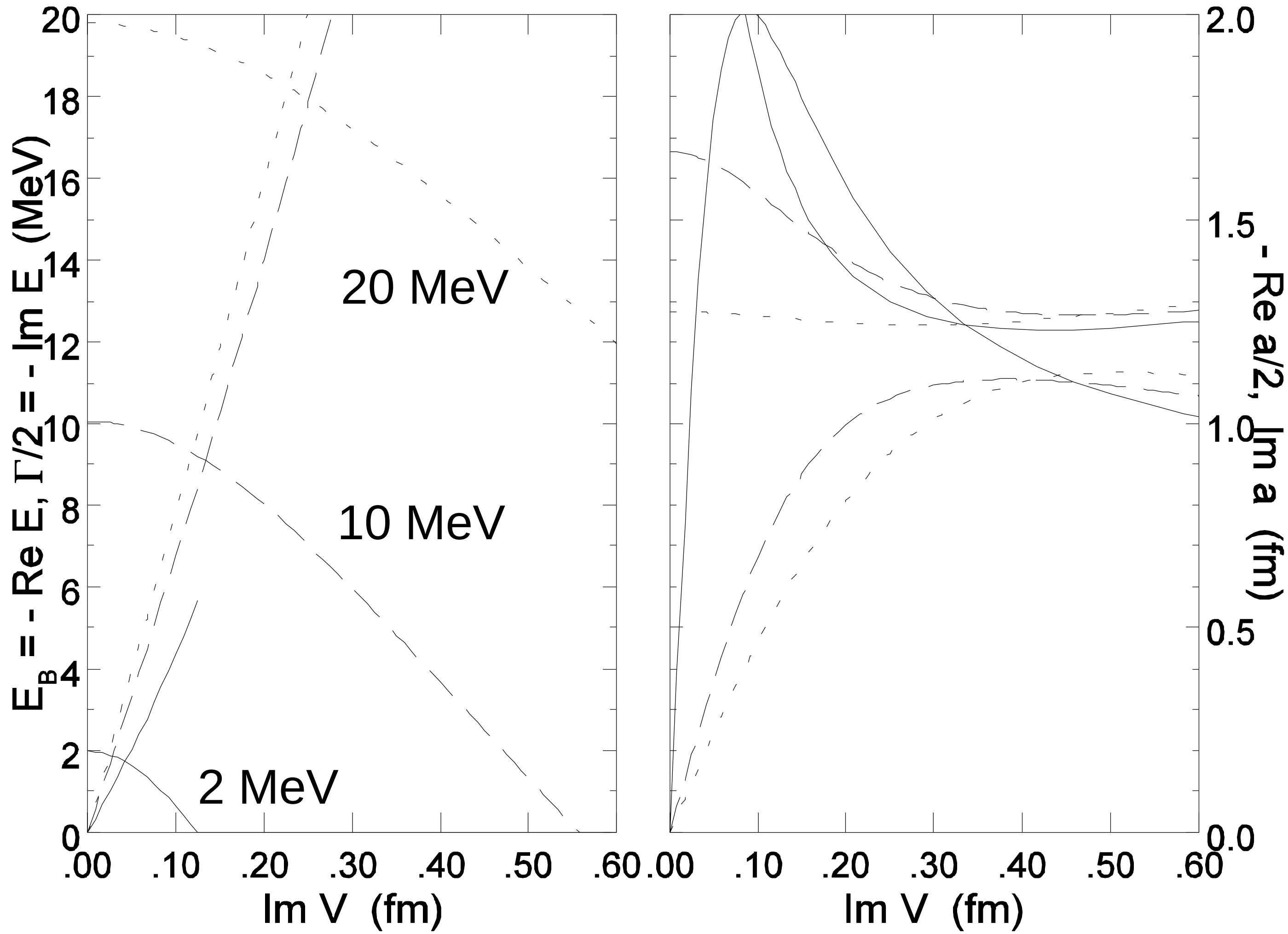


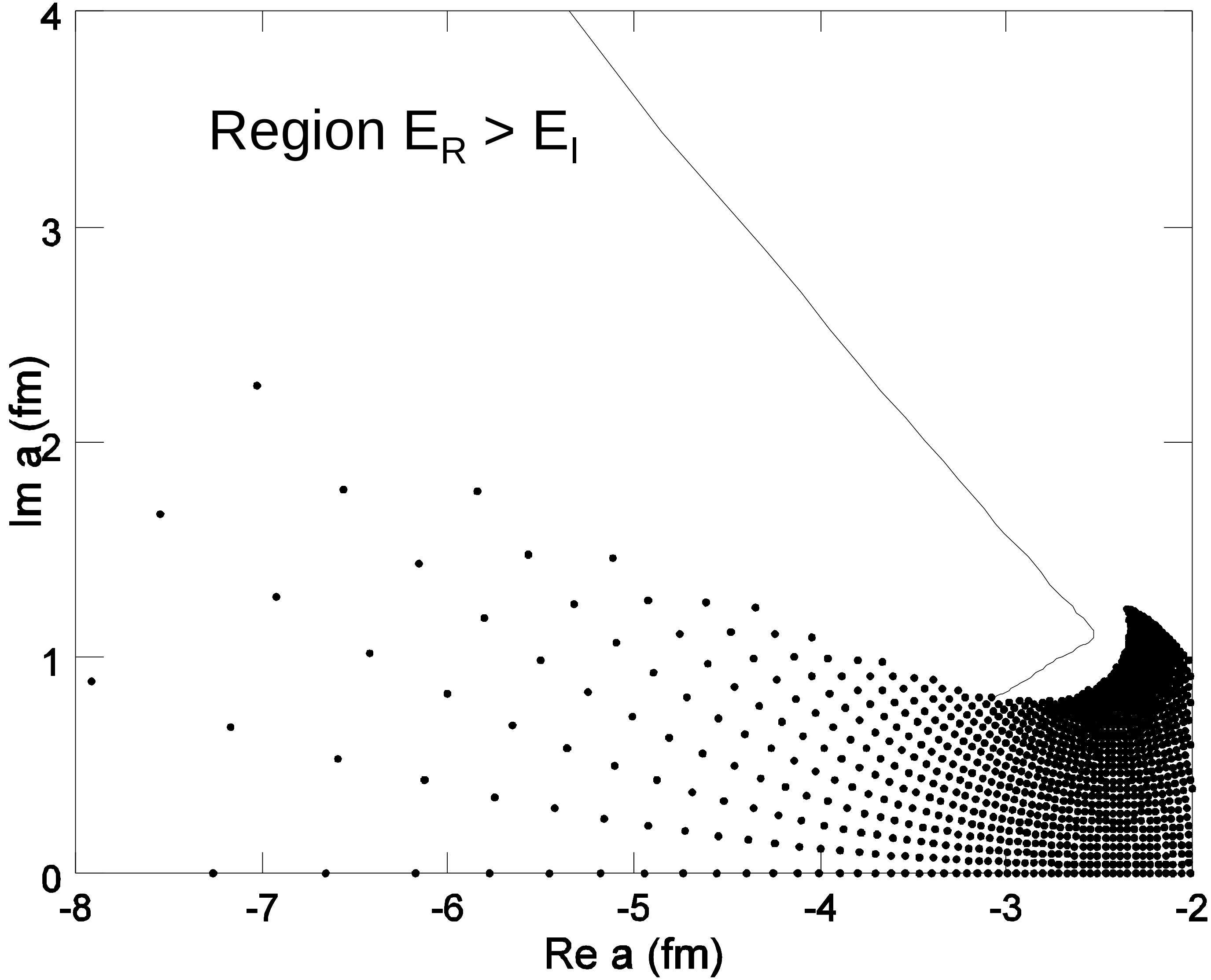
Carbon 12
Im E = Γ/2
1 MeV
2 MeV
5 MeV
10 MeV
20 MeV
Im a (fm)
Re a (fm)
0
1
2
3
4
5
-7
-6
-5
-4
-3
-2



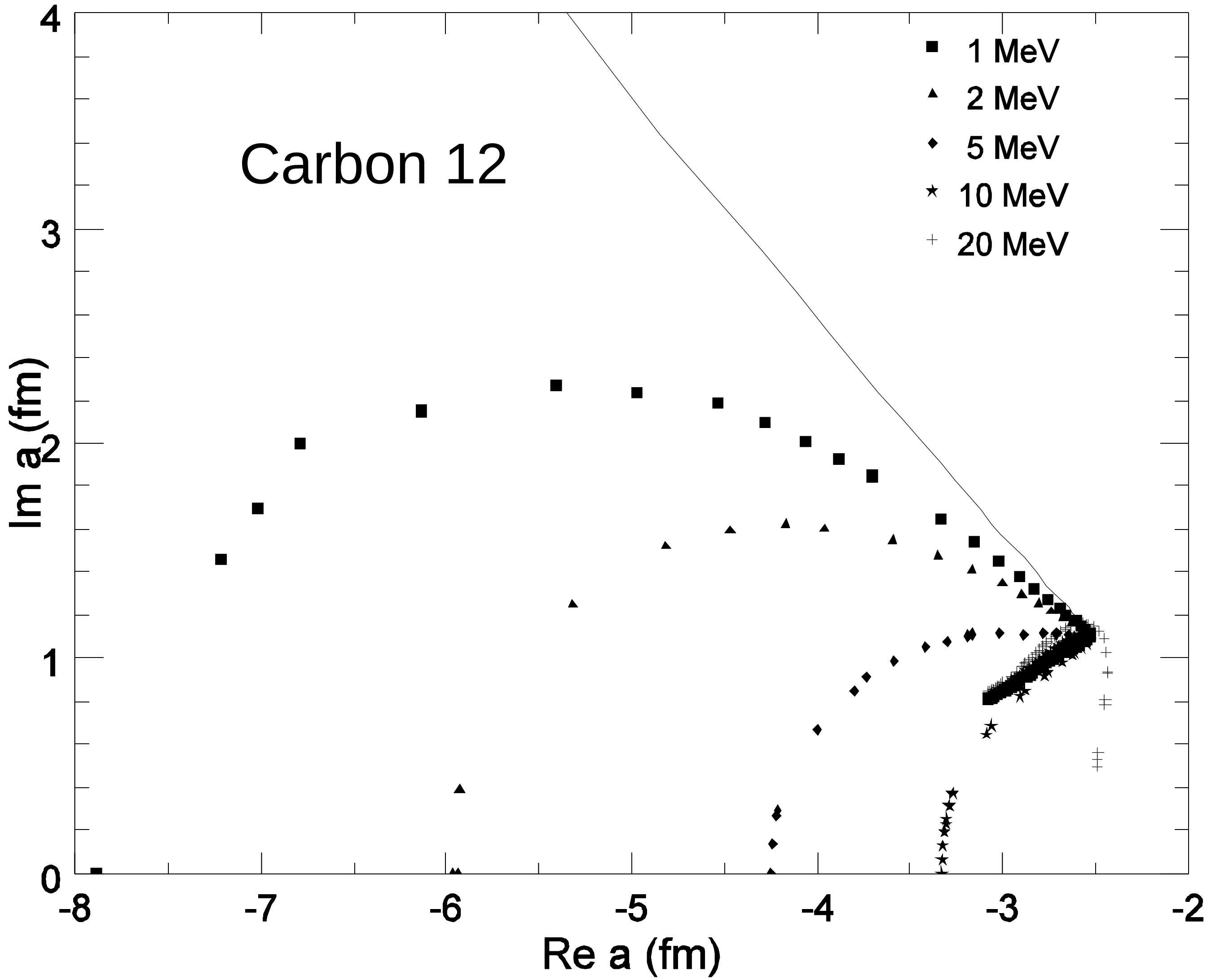
1.5
1.0
0.5
-3.2
-3.0
-2.8
-2.6
-2.4
Re a (fm)
Im a (fm)
1 MeV
2 MeV
5 MeV
10 MeV
20 MeV

Strange looking nonlinearities and accumulations may be understood with the relation to Im *V* effects

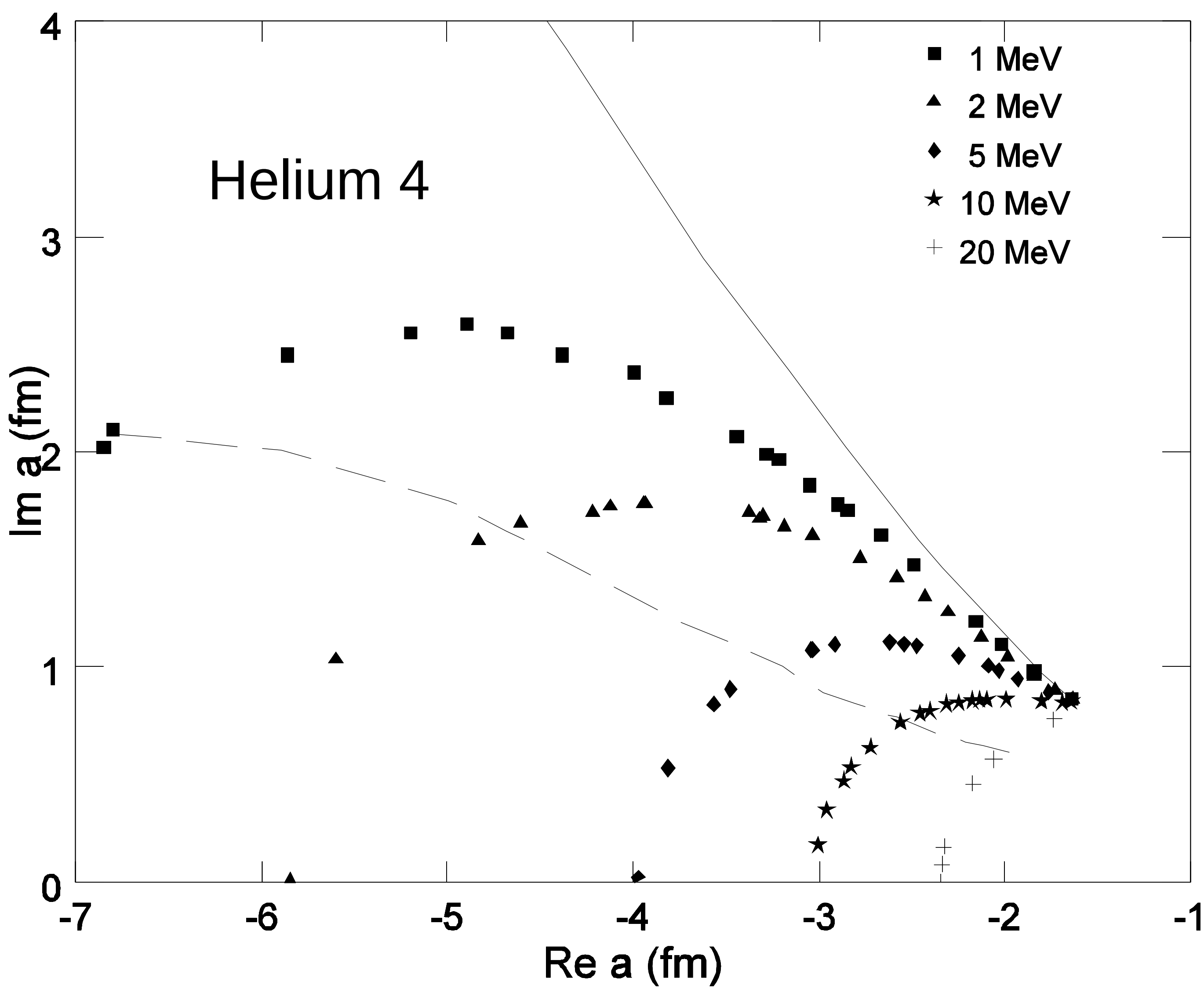




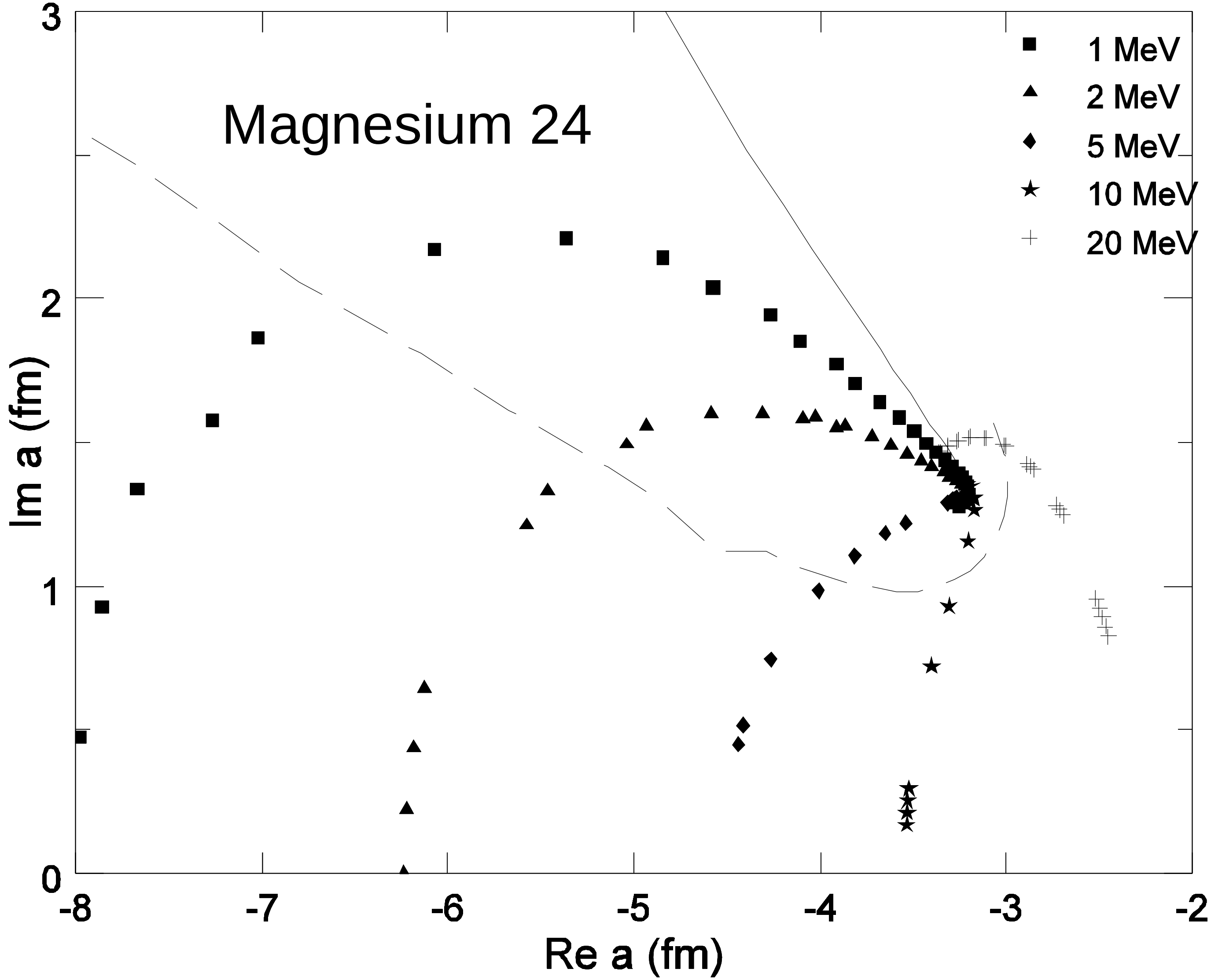
Region $E_R > E_I$
Im a (fm)
Re a (fm)
4
3
2
1
0
-8
-7
-6
-5
-4
-3
-2



Carbon 12
1 MeV
2 MeV
5 MeV
10 MeV
20 MeV
Im a (fm)
Re a (fm)
4
3
2
1
0
-8
-7
-6
-5
-4
-3
-2



Helium 4
1 MeV
2 MeV
5 MeV
10 MeV
20 MeV
Im a (fm)
Re a (fm)
4
3
2
1
0
-7
-6
-5
-4
-3
-2
-1


Magnesium 24
1 MeV
2 MeV
5 MeV
10 MeV
20 MeV
Im a (fm)
Re a (fm)
3
2
1
0
-8
-7
-6
-5
-4
-3
-2

Real and weak binding limit same for all nuclei (1 MeV about 8 fm)
Variations arise for larger E, smaller a – moderate shape dependence from r_0 (q^2 term)

For our fitting parametrization a is a function of binding energy (and r_0) – note, potential strength depends on E_B. Simple relation constrains:

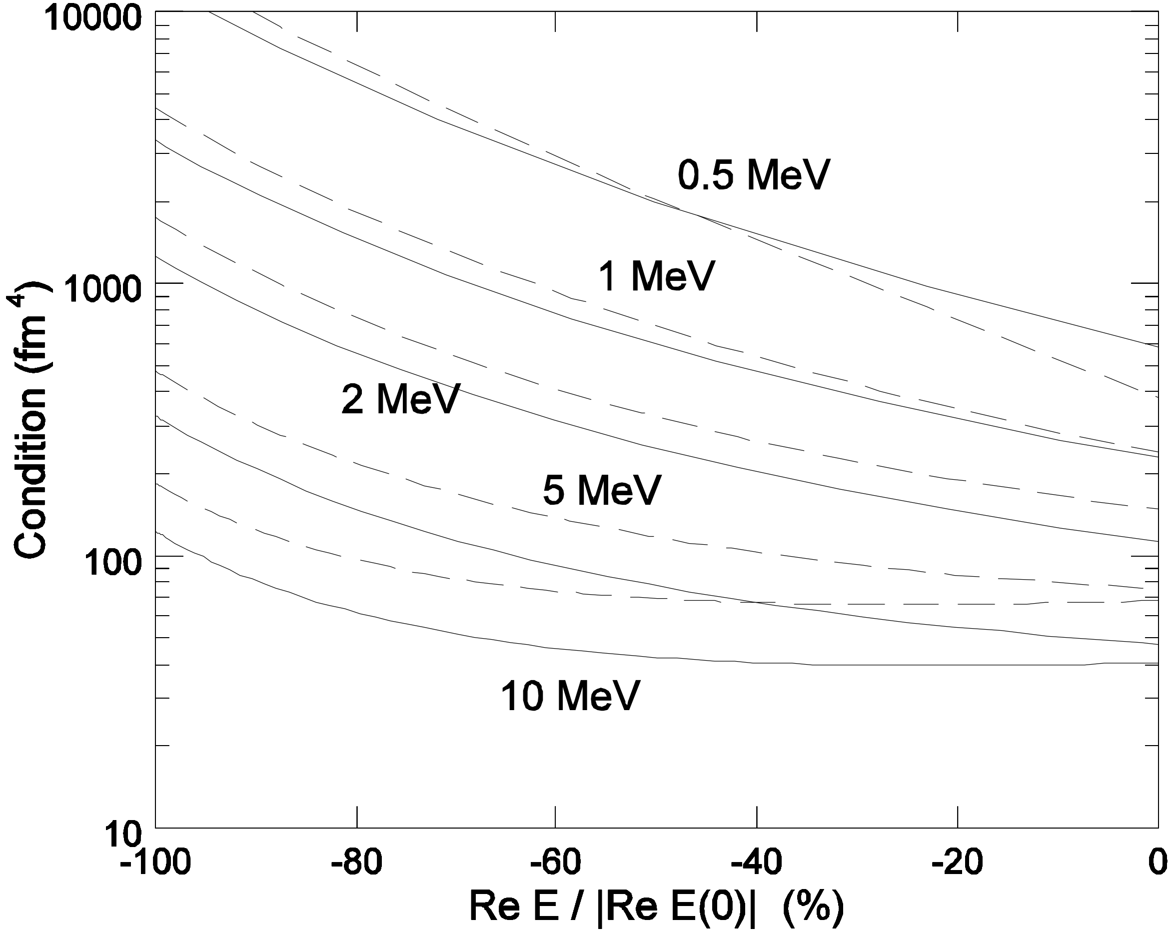
$$1/a \approx -(-2\mu E / \hbar^2)^{1/2} - r_0\, \mu\, E / \hbar^2$$

(For binding $|E| < 10$ MeV % level for real part)

Importance of effective range ≠ 0 (2nd term)

Also condition for $E_R < 0$ requires

$\mathrm{Re}[a^3 (a^* - r_0^*)] > 0$ (for $r_0 = 0$ $|\mathrm{Re}\, a| > \mathrm{Im}\, a$)



Anything for potentials? For their strengths?

Speculations:
For He-4 strength V_R = 0.33, V_I = 0 gives a barely bound state, so about all standard Re $a_{\eta N}$ 0.4 – 0.7 fm would bind. Then C-12 would be bound by 8 MeV and Mg-24 by 14 MeV.
Barely bound state for all would need
$V_R = 0.86 \times A^{-0.7}$.

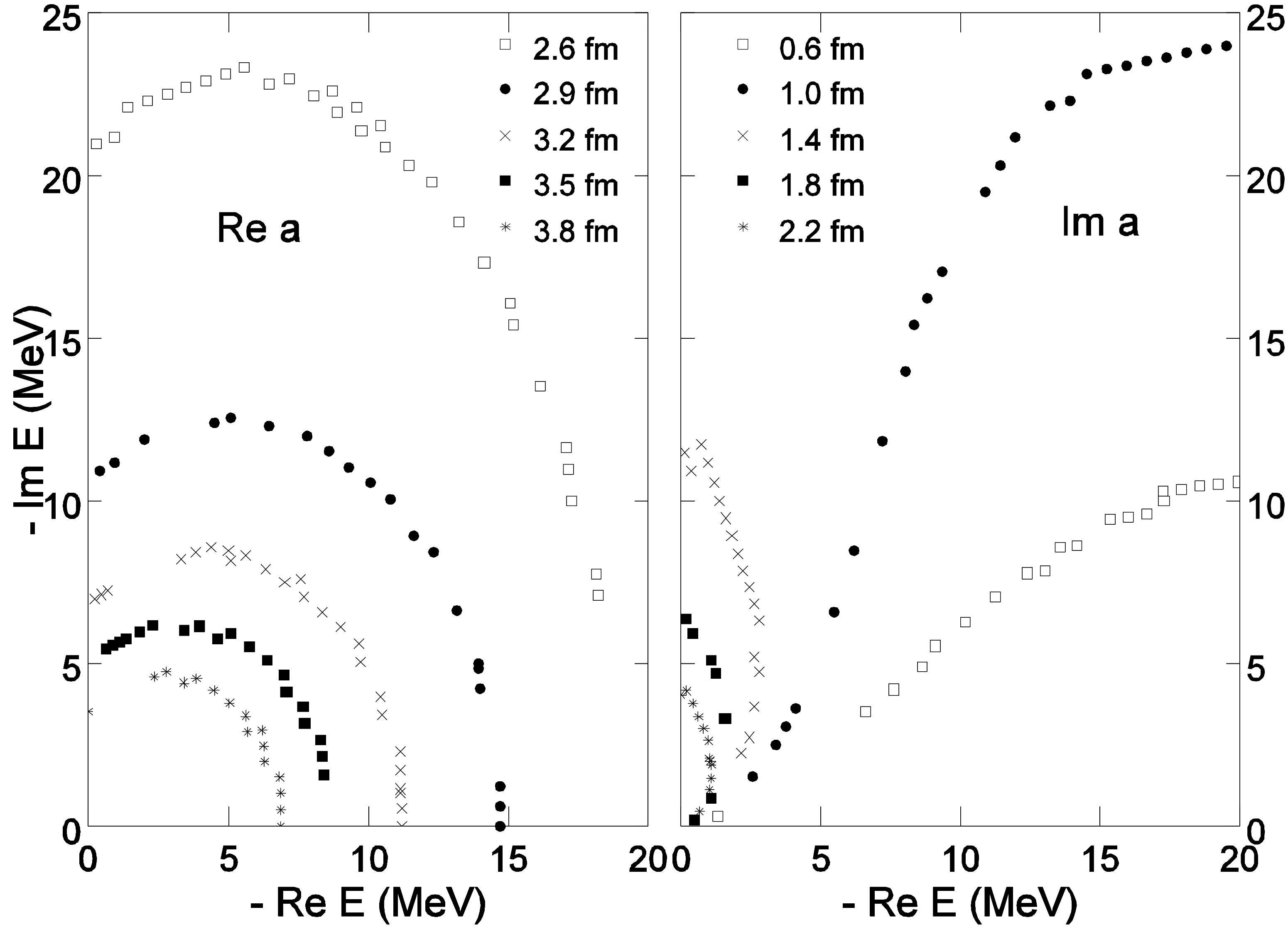
However $a_{\eta N}$ complex: assume $V_I = V_R/2$.
Then V_R = 0.47 fm would bind He-4 and Re E of C-12 and Mg-24 would be 15 and 22 MeV (Im E ≈ same) and strength for bare binding
$V_R = 1.25 \times A^{-0.7}$ fm (~ $1/R^2$ of square well).

Conclusions

Connection between scattering parametres and binding energies through a potential nonlinear and rather complicated in the complex case – still numerically meaningful.

Constraints between scattering and binding quite strong and very model independent – range from He to Mg rather similar.
May be useful for planning experiments.

Simple algebraic relation for minor binding energies.



Re a
2.6 fm
2.9 fm
3.2 fm
3.5 fm
3.8 fm
Im a
0.6 fm
1.0 fm
1.4 fm
1.8 fm
2.2 fm
- Im E (MeV)
- Re E (MeV)
- Re E (MeV)
0
5
10
15
20
25