In-medium $\bar{K}$ & $\eta$ mesons
Mesic Nuclei, JU Krakow, Sept. 2013

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- $\bar{K}N - \pi Y$ chiral dynamics and its consequences
- $\bar{K}$ nuclear few-body systems
- $\bar{K}$-nucleus potentials from $K^-$ atoms

- Quest for $\eta$ nuclear quasibound states
  E.Friedman, A.Gal, J.Mareš, PLB 725 (2013) 334
\bar{K}N − \pi Y  Chiral Dynamics
Strong subthreshold $K^-p$ attraction; $\Lambda(1405)$ physics
Consequences for kaonic atoms and $K^-$ nuclear quasibound states
$K^-$ absorption might be governed by out-of-model $K^-NN \rightarrow YN$
$K^-p$ subthreshold ambiguity

Two NLO chiral-model fits by Guo-Oller, PRC 87 (2013) 035202

- Fit I: one value of meson weak-decay constant $f = 125.7 \pm 1.1$ MeV.
- Fit II: separate fixed values for $f_\pi$, $f_K$, $f_\eta$.

Fit II will create problems when confronted with kaonic-atom data.
$K^- p \rightarrow \pi^\pm \Sigma^\mp$ reaction data fitted by LEC of NLO scheme for $\bar{K}N - \pi Y$ coupled channels ($Y = \Lambda, \Sigma$).


Large difference in cross sections $\Rightarrow$ Strong isospin dependence.

$I = 0$ coupled-channel amplitudes

Location of ‘resonances’: $\bar{K}N \approx 1420$ MeV, $\pi \Sigma \approx 1405$ MeV

Are there two distinct ‘Λ(1405)’ resonances?
$^\Lambda K$ nuclear few-body systems
Energy dependence in $\bar{K}$ nuclear few-body systems

- $\Lambda(1405)$ induces strong energy dependence of the scattering amplitudes $f_{\bar{K}N}(\sqrt{s})$ and the underlying effective single-channel input potentials $v_{\bar{K}N}(\sqrt{s})$.

- $s = (\sqrt{s_{th}} - B_K - B_N)^2 - (\vec{p}_K + \vec{p}_N)^2 \leq s_{th}$

- Expanding nonrelativistically near $\sqrt{s_{th}} \equiv m_K + m_N$:
  $$\delta \sqrt{s} = -\frac{B}{A} - \frac{A-1}{A} B_K - \xi_N \frac{A-1}{A} \langle T_{N:N} \rangle - \xi_K \left( \frac{A-1}{A} \right)^2 \langle T_K \rangle,$$
  $$\delta \sqrt{s} \equiv \sqrt{s} - \sqrt{s_{th}}, \quad B_K = -E_K, \quad \xi_{N(K)} \equiv \frac{m_{N(K)}}{(m_N+m_K)}.$$

- Self-consistency: output $\sqrt{s}$ from solving the Schroedinger equation identical with input $\sqrt{s}$. 
3– & 4–body $B$ & $\Gamma$ calculated self-consistently

![Graph showing $E_{g.s.}$ vs $K_{max}$ and $\Gamma$ vs $\delta \sqrt{s_{KN}}$]


- Variational calculation in hyperspherical basis controlled by $K_{max}$
- $\bar{K}N$ energy dependence [Hyodo–Weise, PRC 77 (2008) 035204]
  restraints $B$ & $\Gamma$ by treating $\delta \sqrt{s_{\bar{K}N}}$ self-consistently
- $B(4$-body$)$ small w.r.t. non-chiral estimates of over 100 MeV
• $\bar{K}NN$: is there an excited $I = 1/2$ quasibound state ($\bar{K}d$, dominantly $I_{NN} = 0$) on top of "$K^-pp$" g.s.?

• Bayar & Oset [NPA 881 (2012) 127]: YES, bound by about 9 MeV, from a peak in $|T_{\bar{K}NN}|^2$ calculated in a fixed-scatterer approximation to Faddeev equations.

• Shevchenko [NPA 890-1 (2012) 50]: UNLIKELY, judging from the $K^-d$ scattering length and effective range deduced from a $\bar{K}NN$ Faddeev calculation.

• Barnea, Gal & Liverts do not find such a bound state below the $\Lambda^*N$ threshold at $B = 11.4$ MeV.
$K^{-}pp$ calculated binding energies & widths (in MeV)

<table>
<thead>
<tr>
<th></th>
<th>chiral, energy dependent</th>
<th>non-chiral, static calculations</th>
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</thead>
<tbody>
<tr>
<td>B</td>
<td>16</td>
<td>17–23</td>
</tr>
<tr>
<td>Γ</td>
<td>41</td>
<td>40–70</td>
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7. S. Wycech, A.M. Green, PRC 79 (2009) 014001 (including $p$ waves)

Robust binding & large widths; chiral models give weak binding
Yamazaki et al. PRL 104 (2010) 132502, DISTO data reanalysis at 2.85 GeV

Broad $K^-pp$ structure in $pp \rightarrow \Lambda p K^+$ at $\pi N\Sigma$ threshold

Forthcoming experiments: $pp \rightarrow (K^-pp) + K^+$ at GSI
$K^-^3He \rightarrow (K^-pp) + n$ (E15) & $\pi^+d \rightarrow (K^-pp) + K^+$ (E27) at J-PARC
RMF quasibound spectra calculated self-consistently (NLO30 ‘+ SE’)

D. Gazda, J. Mareš, NPA 881 (2012) 159

- NLO30 is a chirally motivated coupled channel separable model with in-medium versions [A. Cieplý, J. Smejkal, NPA 881 (2012) 115]
- $\Gamma_K$ due only to $K^-N \rightarrow \pi Y$ (no $K^-NN \rightarrow YN$) decay modes
- Self consistency: deep $K^-$ levels are narrower than shallow ones
What do $K^-$ atoms tell us?
$K^-_\text{atom}$ widths across the periodic table in model F (deep pot.)

Lowest $\chi^2$ phenom. model, $\chi^2 = 84$ per 65 data points,

Left: $K^-$-Ni 4f atomic wavefunction overlap with nuclear density for deep potential, revealing a nuclear $\ell = 3$ quasibound state.

Self-consistency requirement imposed in recent $K^-$ atom calculations [Cieplý-Friedman-Gal-Gazda-Mareš, PLB 702 (2011) 402]:

$$\sqrt{s_{K^-N}} \rightarrow E_{th} - B_N - B_K - \xi_N \frac{p_N^2}{2m_N} - \xi_K \frac{p_K^2}{2m_K}$$

$$\xi_N(K) = \frac{m_N(K)}{(m_N + m_K)} \quad \frac{p_K^2}{2m_K} \sim -V_{K^-} \approx 100 \text{ MeV}$$

$K^-$ is not at rest!

Friedman-Gal, NPA 899 (2013) 60

$K^-N$ subthreshold energy $vs$ nuclear density in $K^-$ atoms.

A dominant in-medium effect
Left: IHW free-space input $f_{K^-N}$  

- Subthreshold energy shift is applied self consistently to in-medium $1N$ amplitude plus $(2+\ldots)N$ phenomenological amplitude.  
- Multiple-scattering inclusion of in-medium correlations.  
- $K^-$-atom best-fit: $\chi^2/N_{\text{data}} = 118/65$  
  [Friedman-Gal, NPA 899 (2013) 60].
$K^-$ nuclear 1N (left) and 2N (right) absorptive potentials, both calculated in a chiral unitary approach [PRC 86 (2012) 065205] by Sekihara, Yamagata-Sekihara, Jido, Kanada-En’yo. Note: empirical 25% 2N:1N BR is reached at too high density.
$\eta$ nuclear quasibound states
\( f_{\eta N}(\sqrt{s}) \) from \( K \)-matrix & \( N^*(1535) \) chiral models

\begin{center}
\begin{tabular}{c c c c c c}
\hline
\( a_{\eta N} \) model dependence \\
\hline
\( a(\text{fm}) \) & M1 & M2 & GW & GR & CS \\
\hline
Re & 0.22 & 0.38 & 0.96 & 0.26 & 0.67 \\
Im & 0.24 & 0.20 & 0.26 & 0.24 & 0.20 \\
\hline
\end{tabular}
\end{center}

- Re \( a \) varies between 0.2 to 1.0 fm; Im \( a \) stable 0.2–0.3 fm.
- M1, M2, GW free-space models; GR, CS in-medium.
- In-medium: energy dependence, Pauli blocking, self-energies.
In-medium $\eta N$ amplitudes
Friedman-Gal-Mareš, PLB 725 (2013) 334
Cieplý-Friedman-Gal-Mareš, in preparation

- KG equation and self-energies:
  \[ [\nabla^2 + \tilde{\omega}_\eta^2 - m_\eta^2 - \Pi_\eta(\omega_\eta, \rho) ] \psi = 0 \]
  \[ \tilde{\omega}_\eta = \omega_\eta - i\Gamma_\eta/2, \quad \omega_\eta = m_\eta - B_\eta \]
  \[ \Pi_\eta(\omega_\eta, \rho) \equiv 2\omega_\eta V_\eta = -4\pi \frac{\sqrt{s}}{m_N} f_{\eta N}(\sqrt{s}, \rho) \rho \]

- Pauli blocking (Waas-Rho-Weise NPA 617 (1997) 449):
  \[ f_{\eta N}^{\text{WRW}}(\sqrt{s}, \rho) = \frac{f_{\eta N}(\sqrt{s})}{1 + \xi(\rho)(\sqrt{s}/m_N)f_{\eta N}(\sqrt{s})\rho}, \quad \xi(\rho) = \frac{9\pi}{4p_F^2} \]

- $N^*(1535) \Rightarrow$ energy dependent $f_{\eta N}(\sqrt{s})$.
  In medium $\Rightarrow$ go subthreshold: \[ \delta \sqrt{s} = \sqrt{s} - \sqrt{s_{\text{th}}} \]
  \[ \delta \sqrt{s} \approx -B_N \frac{\rho}{\rho_0} - \xi_N B_\eta \frac{\rho}{\rho_0} - \xi_N T_N(\frac{\rho}{\rho_0})^{2/3} + \xi_\eta \text{Re } V_\eta(\sqrt{s}, \rho) \]
  
  Self-consistency relationship between $\delta \sqrt{s}$ & $\rho$
\( \delta \sqrt{s} \) vs. \( \rho \) for \( 1s_\eta \) bound state in Ca using in-medium \( f_{\eta N} \)

- 40–60 MeV subthreshold energy shifts at nuclear matter density \( \rho_0 \), larger than shifting down by \( B_\eta \) (GR) or by 30 MeV (Haider-Liu)

- Larger Re \( a_{\eta N} \) \( \Rightarrow \) larger \( \delta \sqrt{s} = E - E_{th} \)
Model dependence I

Binding energy and width of $1s_{\eta}$ bound states across the periodic table using WRW Pauli-blocked $f_{\eta N}$

- Larger Re $a_{\eta N} \Rightarrow$ larger $B_{\eta}$
- Widths are unrelated to Im $a_{\eta N}$
Sensitivity of calculated $B_{1s_\eta}$ and $\Gamma_{1s_\eta}$ to version of self-consistency

- $\delta \sqrt{s}$ recipe reduces both $B_{1s_\eta}$ and $\Gamma_{1s_\eta}$ w.r.t. $-B_{1s_\eta}$ recipe
- GR’s widths are too large to resolve $\eta$ bound states

Why $\Gamma_\eta(\text{GR}) \gg \Gamma_\eta(\text{CS})$ for similar Im $a_{\eta N}$?
Energy dependence of free-space & in-medium amplitudes

- Subthreshold Re $f_{\eta N}$ similar in both in-medium models in spite of large free-space difference at threshold
- Subthreshold Im $f_{\eta N}$ differ widely, which explains the huge difference between $\Gamma_{\eta}(\text{GR})$ and $\Gamma_{\eta}(\text{CS})$
more theoretical work is needed to figure out what makes subthreshold values of $\text{Im } f_{\eta N}$ sufficiently small to generate small widths.
Summary

- Large widths, $\Gamma_{\bar{K}} > 50$ MeV, expected for single-$\bar{K}$ quasibound nuclear states. Focus on light systems. Searches for $K^-pp$ are underway in GSI and J-PARC.

- Major issues: (i) how deep is $\bar{K}$ nuclear spectrum? (ii) how big is $\Gamma(\bar{K}NN \rightarrow YN)$ w.r.t. $\Gamma(\bar{K}N \rightarrow \pi Y)$?

- Subthreshold behavior of $f_{\eta N}$ is crucial in studies of $\eta$-nuclear bound states to decide whether (i) such states exist, (ii) can they be resolved (widths?), and (iii) which nuclear targets and reactions to try?

Thanks to my collaborators N. Barnea, A. Cieplý, E. Friedman, D. Gazda, J. Mareš