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# The role of nucleon resonances in $\eta'$ production processes

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Part of a baryon spectroscopy program:

- $\gamma + N \rightarrow M + B$
- $\pi + N \rightarrow M + B$       ( $M = \pi, \eta, K, \eta', \rho, \omega, K^*, \phi$ )
- $N + N \rightarrow M + B + N$
- $\gamma + N \rightarrow M + M' + B$
- $\pi + N \rightarrow M + M' + B$

Collaboration:

Jülich

Athens/GA

Washington/DC

# Reaction Theory: $MN \rightarrow M'B$ reactions

## Jülich DCC model (TOPT):

$$T = V + VGT$$

C. Schütz et al., PRC49 '94; PRC57 '98

O. Krehl et al., PRC62 '00

A. M. Gasparyan et al., PRC68 '03

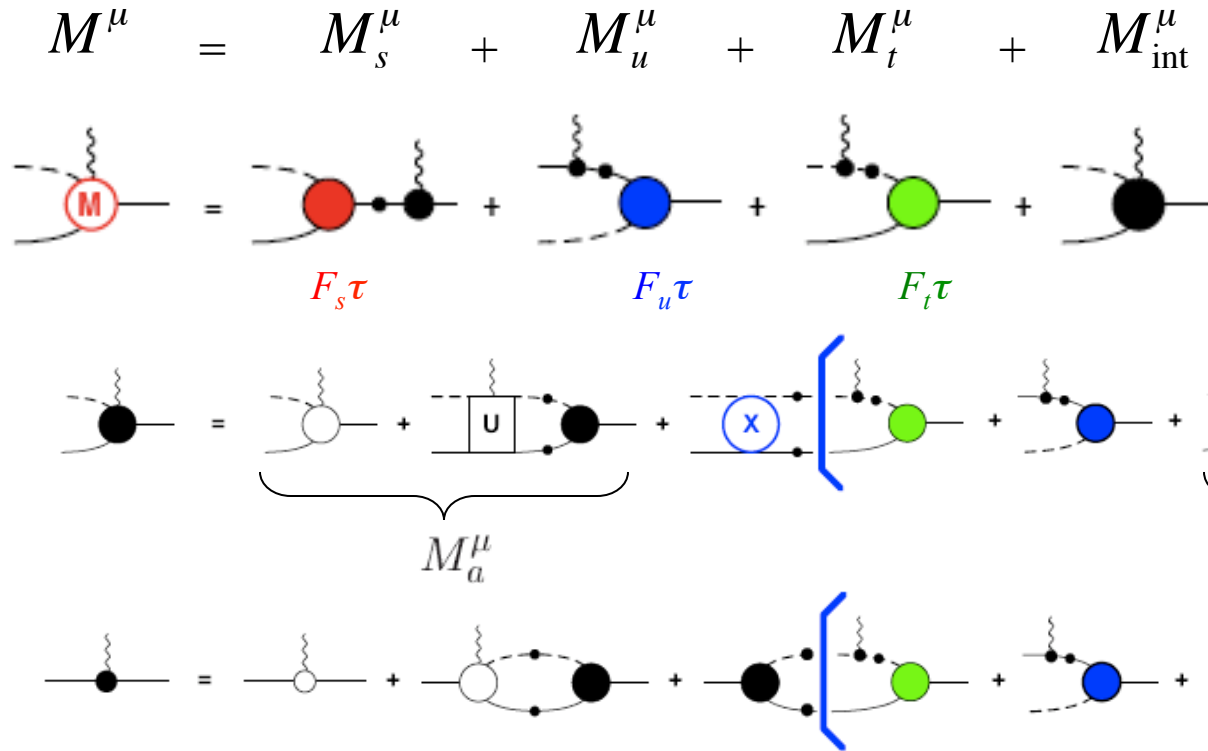
M. Döring et al., NPA829 '09; NPA851 '11

D. Rönchen et al., EPJA47'13

## Basic features:

- Coupled channels (so far):  $\pi N$ ,  $\eta N$ ,  $K\Lambda$ ,  $K\Sigma$ ,  $\pi\pi N$  [ $\sigma N$ ,  $\rho N$ ,  $\pi\Delta$ ]
- Analyticity
- 2-body unitarity & some requirements for 3-body unitarity
- Chiral Lagrangian of Wess and Zumino [PR163, '67; PR161, '88]
  - Hadron exchange provides the relevant dynamics
  - All partial waves are linked by u- and t-channel processes
  - Reaction channels are linked by SU(3) in the Lagrangian framework
  - Minimum # of explicit resonances needed due to the structured background

# Reaction Theory: $\gamma N \rightarrow MN$ (fully gauge invariant DCC)



H.Haberzettl, PRC56, 2041, '97  
 H.Haberzettl, et al., PRC74, '06  
 H.Haberzettl, et al., PRC83, '11  
 F. Huang, et al., PRC85, '12

approximated current:

$$M_{\text{int}}^\mu = M_c^\mu + TG \left( M_u^\mu + M_t^\mu + M_c^\mu \right)_T$$

generalized contact current

$$k_\mu M_c^\mu = -[F_s]e_i + [F_u]e_f + [F_t]e_\pi$$

generalized  
Ward-Takahashi identity

$$\Gamma^\mu = \Gamma_0^\mu + m_{KR}^\mu G[F\tau] + [F_0\tau]G \left( M_u^\mu + M_t^\mu + M_c^\mu \right)_L$$

# Experimental data: some of the issues

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- Scarcity of (2-body) hadronic reactions data:
  - apart from the  $\pi N$  elastic scattering, only  $\sim 3800$  data points  
( $\sim 24600$  in photoproduction)
  - existing data suffers from large uncertainties
  - many of them are incompatible with each other
  - about dozen total cross section data points for  $\eta'$  and  $\phi$ .

HADES at GSI:  $\pi N \rightarrow \omega N, \rho N$  reactions ( $W < 2.4$  GeV); no spin observables.

J-PARC:  $\pi N \rightarrow KY, \pi\pi N$

EIC at Jlab in  $\sim 2020$  ??

One of the major limitations for developing more accurate coupled-channels models.

Accurate data from CELSIUS, COSY on  $NN \rightarrow NBM$  ( $M = \pi, \eta, K, \eta', \omega, \phi$ ) help constrain model parameters.

# Experimental data: $\eta'$ production reactions

- $\gamma + N \rightarrow \eta' + N:$ 
 $d\sigma/d\Omega$  CLAS-'09, free proton  
 CBELSA/TAPS-'09, free proton  
 CBELSA/TAPS-'11, quasi-free neutron & proton  
  
 $\Sigma, E$  (being measured at CLAS & CBELSA/TAPS)
- $\pi + N \rightarrow \eta' + N:$ 
 $\sigma$  (~15 data points with large uncertainties),  
 '68 - '72, Baldini et al., *Total Cross-Sections for Reactions of High Energy Particles*, Landolt-Boernstein, edited by H. Schopper (Springer, Berlin, 1988), Vol. I/12a.
- $N + N \rightarrow \eta' + N + N:$ 
 $\sigma$  DISTO-'98,'00, proton  
 COSY11-'00,'04 proton  
 COSY11-'10, neutron (upper limit)

$d\sigma/d\Omega_{\eta'}$  DISTO-'00, proton  
 COSY11-'04 proton

$d\sigma/dM_{\eta' p}, d\sigma/dM_{pp}$  COSY11-'07 proton

# Approach to $\eta'/\phi$ production reactions:

Elastic scattering (below first inelastic threshold):

$$T = \underbrace{\left\{ N^X |F_K\rangle \frac{1}{S_K^{-1} + iN^X \frac{\Gamma}{2}} \langle F_K| N^X \right\}}_{T^P} + \underbrace{N^X K^{NP}}_{X (=T^{NP})} \quad (\text{full phase structure})$$

$$N^X = 1 - i\rho X = e^{i\delta^X} \cos \delta^X \quad (\text{Watson's factor})$$

$$|F_K\rangle = (1 + K^{NP} G^R) |F_0\rangle$$

$$S_K^{-1} = S_0^{-1} - \langle F_0| G^R |F_K\rangle = E - m_0 - \langle F_0| G^R |F_K\rangle$$

$$\frac{\Gamma}{2} = \langle F_K| \pi\delta(E - H_0) |F_K\rangle = \langle F_K| \rho |F_K\rangle$$

$\rho$  = phase-space factor

$$K^{NP} = V^{NP} + V^{NP} G^R K^{NP}$$

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$\rho =$  phase-space factor

$$K^{NP} = V^{NP} + V^{NP} G^R K^{NP}$$

Phenomenology:

$$T = \left\{ N^X g \frac{1}{E - m_B + iN^X \frac{\Gamma_B}{2}} g N^X \right\} + N^X K^{NP} \quad (\text{full phase structure kept})$$

Parameters: [  $g$ ,  $m_B$ ,  $\delta^X$  ]

# Approach to $\eta'/\phi$ production reactions:

Full coupled-channels amplitude:

$$T_{\alpha'\alpha} = \underbrace{\sum_{r'r} \left\{ N_{\alpha'}^X |\hat{F}_K\rangle_{\alpha',r'} \frac{1}{S_{K r'r}^{-1} + i \sum_{\beta} N_{\beta}^X \frac{\Gamma_{r'r\beta}}{2}} \langle \hat{F}_K |_{\alpha,r} N_{\alpha}^X \right\}}_{T^P} + \underbrace{N_{\alpha'}^X \hat{K}_{\alpha'\alpha}^{NP}}_{X (=T^{NP})}$$

$$N_{\alpha'}^X \equiv 1 - i\rho_{\alpha'} X_{\alpha'\alpha'} = \frac{1}{2} \left( \eta_{\alpha'}^X e^{i2\delta_{\alpha'}^X} + 1 \right) \xrightarrow{\eta_{\alpha'}^X=1} e^{i\delta_{\alpha'}^X} \cos \delta_{\alpha'}^X \quad (\text{Watson's factor})$$

$$|\hat{F}_K\rangle_{\alpha',r'} \equiv |F_K\rangle_{\alpha',r'} - i \sum_{\beta \neq \alpha'} \rho_{\beta} \hat{K}_{\alpha'\beta}^{NP} |F_K\rangle_{\beta,r'}$$

$$\frac{\Gamma_{r'r\beta}}{2} \equiv \langle F_K |_{\beta,r'} \rho_{\beta} |\hat{F}_K\rangle_{\beta,r}$$

$$\hat{K}_{\alpha'\alpha}^{NP} \equiv K_{\alpha'\alpha}^{NP} - i \sum_{\beta, \beta' \neq \alpha'} K_{\alpha'\beta'}^{NP} \left( (D^X)^{-1} \right)_{\beta'\beta} \rho_{\beta} K_{\beta\alpha}^{NP}$$

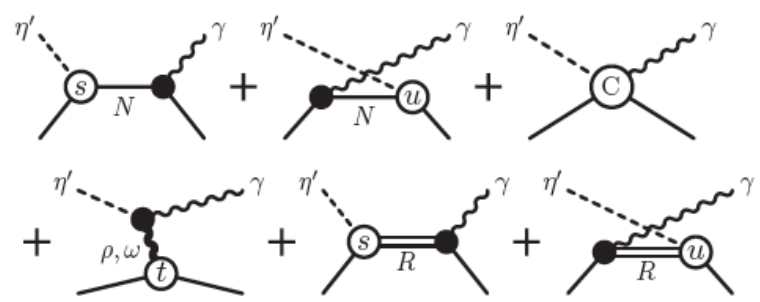
$$D_{\beta'\beta}^X \equiv \delta_{\beta'\beta} + i\rho_{\beta'} K_{\beta'\beta}^{NP}$$

$\rho_{\beta}$  = phase-space factor  
= 0 below  $\beta$  threshold

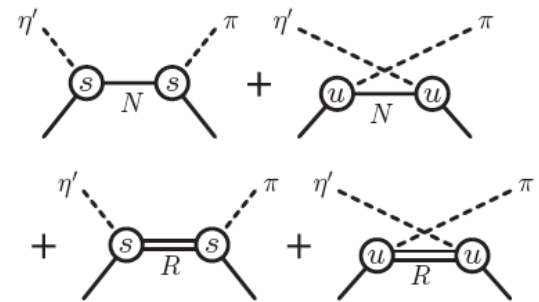


# Model for $\eta'$ productions: dynamical content

$\gamma + N \rightarrow \eta' + N$



$\pi + N \rightarrow \eta' + N$



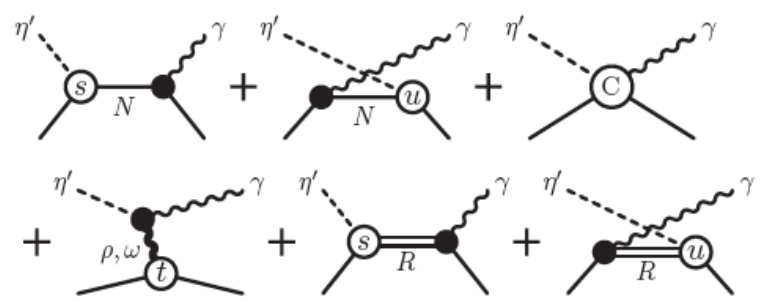
quasi-free: Fermi folding

$$T_{\eta'\alpha} \cong \sum_r \left\{ N_{\eta'}^X g_{r\eta'} \frac{1}{E - m_r + i \sum_{\beta} N_{\beta}^X \frac{\Gamma_{r\beta}}{2}} g_{r\alpha} N_{\alpha}^X \right\} + N_{\eta'}^X V_{\eta'\alpha}^{NP} \quad (\alpha = \gamma, \pi)$$

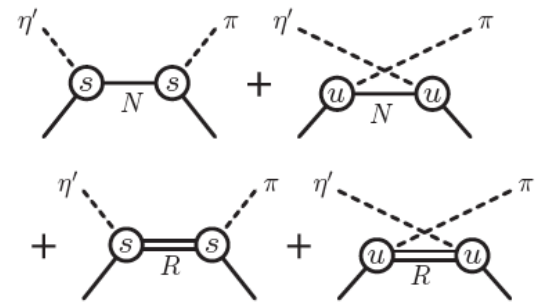
present calc.:  
 $\beta = M, \eta', \gamma$   
 $N_{\alpha}^X, N_{\beta}^X = 1$

# Model for $\eta'$ productions: effective Lagrangian approach

$\gamma + N \rightarrow \eta' + N$



$\pi + N \rightarrow \eta' + N$

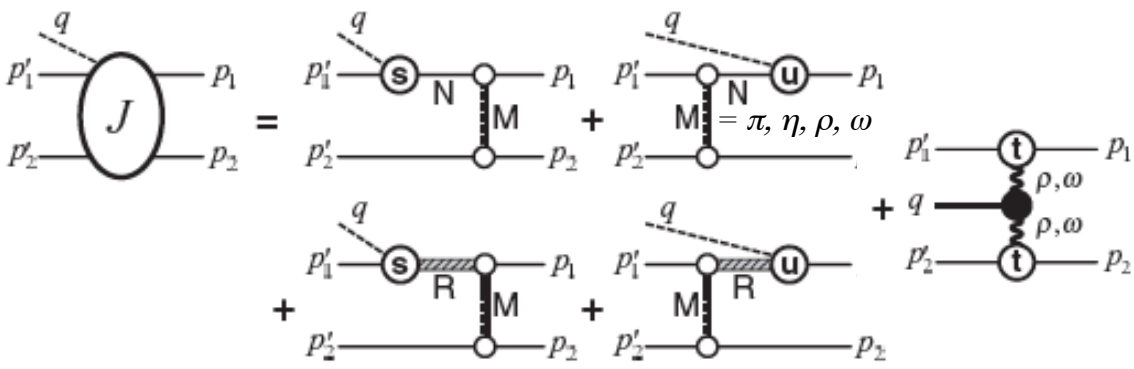


quasi-free: Fermi folding

$$T_{\eta'\alpha} \cong \sum_r \left\{ N_{\eta'}^X g_{r\eta'} \frac{1}{E - m_r + i \sum_{\beta} N_{\beta}^X \frac{\Gamma_{r\beta}}{2}} g_{r\alpha} N_{\alpha}^X \right\} + N_{\eta'}^X V_{\eta'\alpha}^{NP} \quad (\alpha = \gamma, \pi)$$

present calc.:  
 $\beta = M, \eta', \gamma$   
 $N_{\alpha}^X, N_{\beta}^X = 1$

$N + N \rightarrow \eta' + N + N$



DWBA:

$$M = (1 + T_f G_f) J (1 + G_i T_i)$$

# $\eta'$ production reactions: a combined analysis

[Huang, Haberzettl, Nakayama, PRC87'13]

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Res. considered: minimum number of spin-1/2 and -3/2 resonances to fit the existing data in photo- and hadron-reactions:

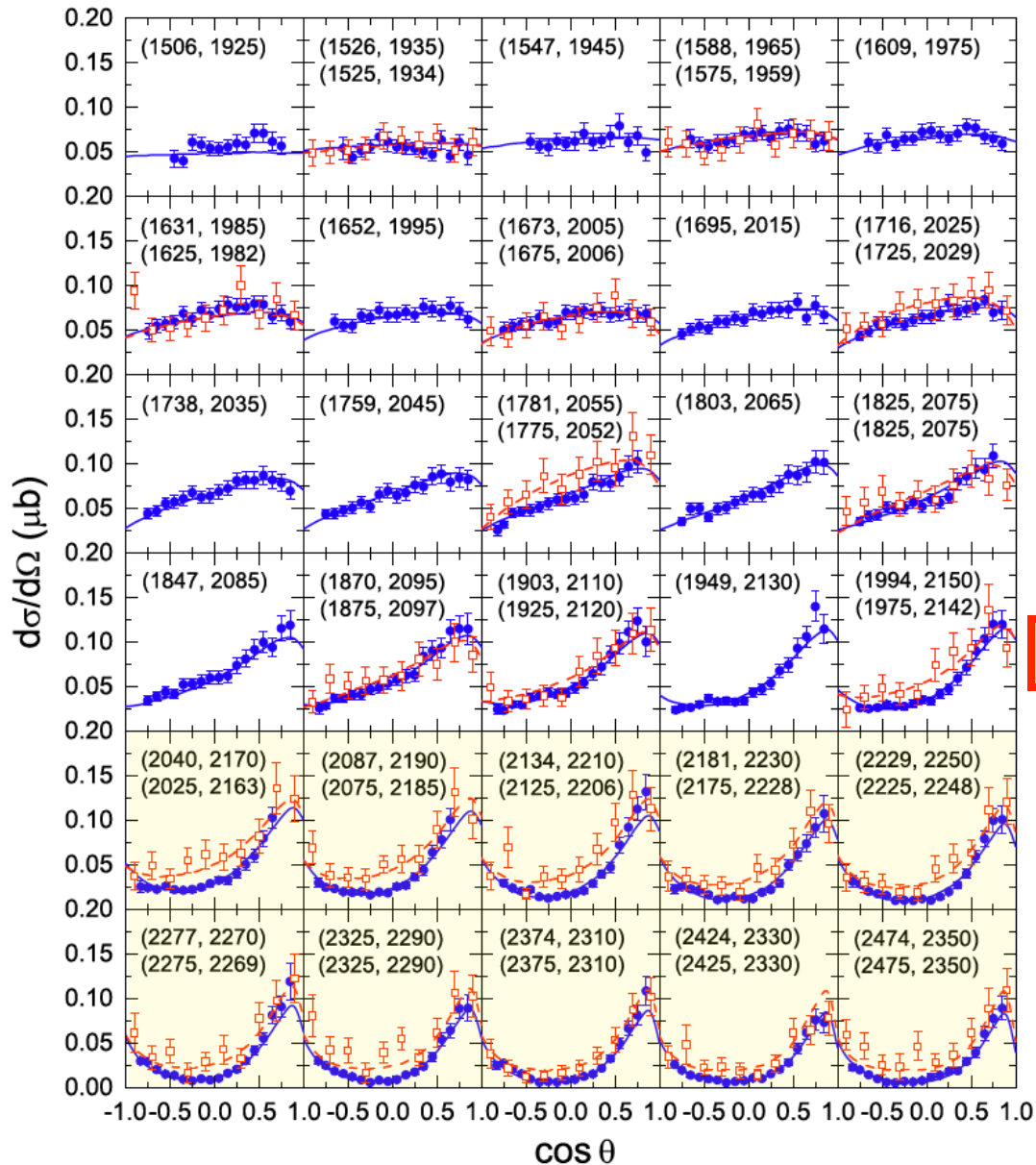
- above threshold:  $S_{11}(1925)$ ,  $P_{13}(2050)$ ,  $P_{11}(2130)$
- sub. threshold:  $P_{13}(1720)$  (required for  $d\sigma/dW$  in  $pp \rightarrow \eta'pp$ )

only photoreaction  $\rightarrow$  different sets of resonances are possible

Res. masses and widths: largely constrained by photoproduction

Role of higher-spin res. : requires spin-polarization observables ( $\Sigma$ ,  $T$ , etc)

# $\eta'$ photoproduction: free proton (combined analysis)



blue : CLAS-'09  
[Williams et al., PRC80'09]

red : CBELSA/TAPS-'09  
[Crede et al., PRC80'09]

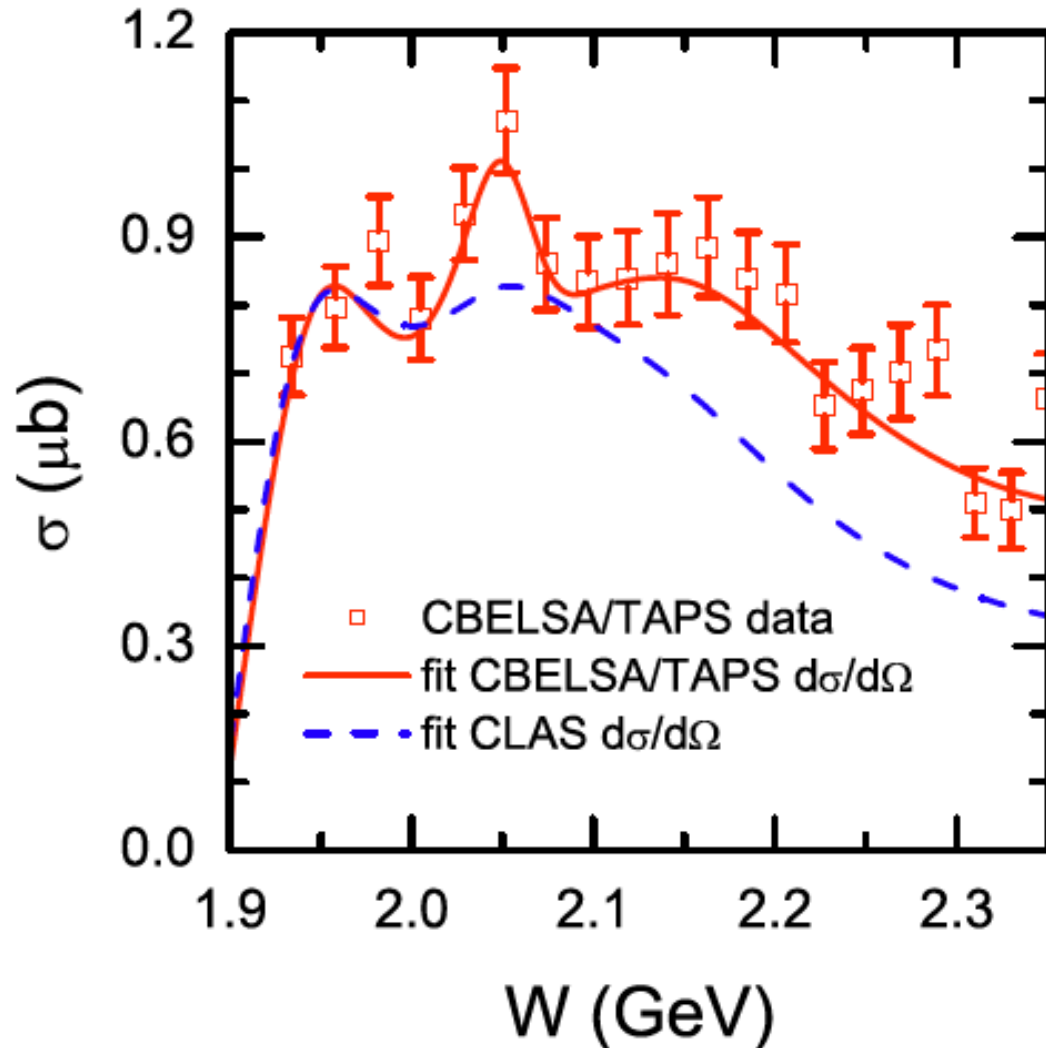
black : CLAS-'06  
[Dugger et al., PRL96'06]

blue & red curves : model results  
[Huang et al., PRC87'13]

{ $P_{13}(1720)$ ,  $S_{11}(1925)$ ,  $P_{13}(2050)$ ,  $P_{11}(2130)$ }

Discrepancy in the data:  
CLAS-'09 & CBELSA/TAPS-'09

# $\eta'$ photoproduction: free proton (combined analysis)

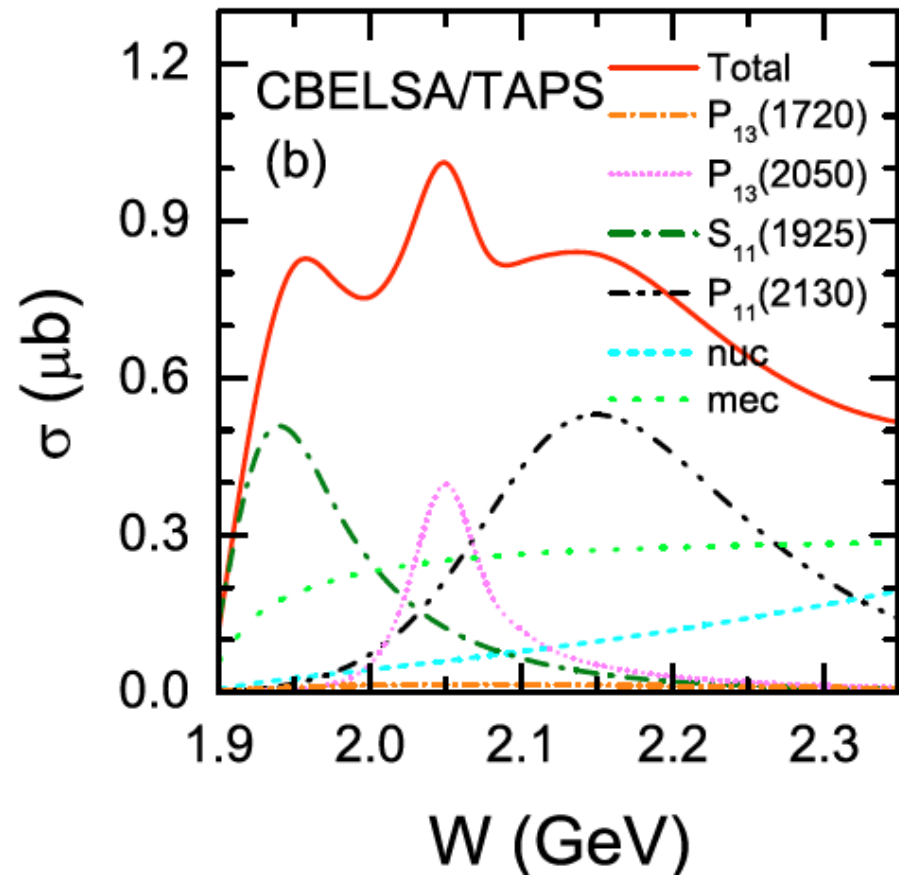
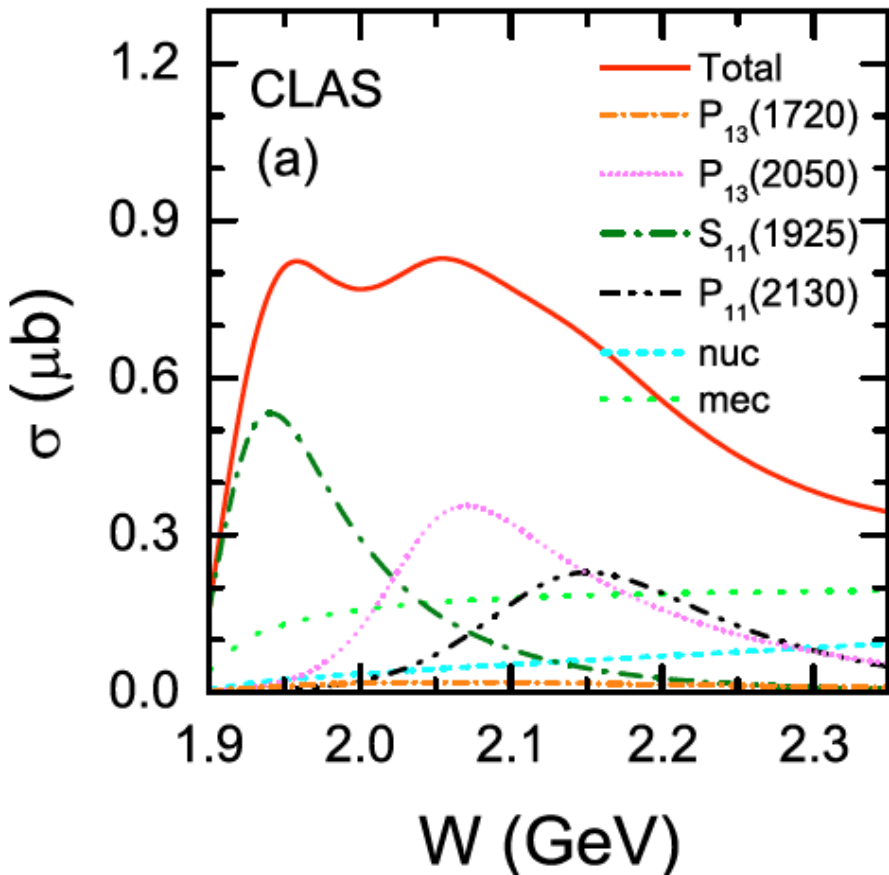


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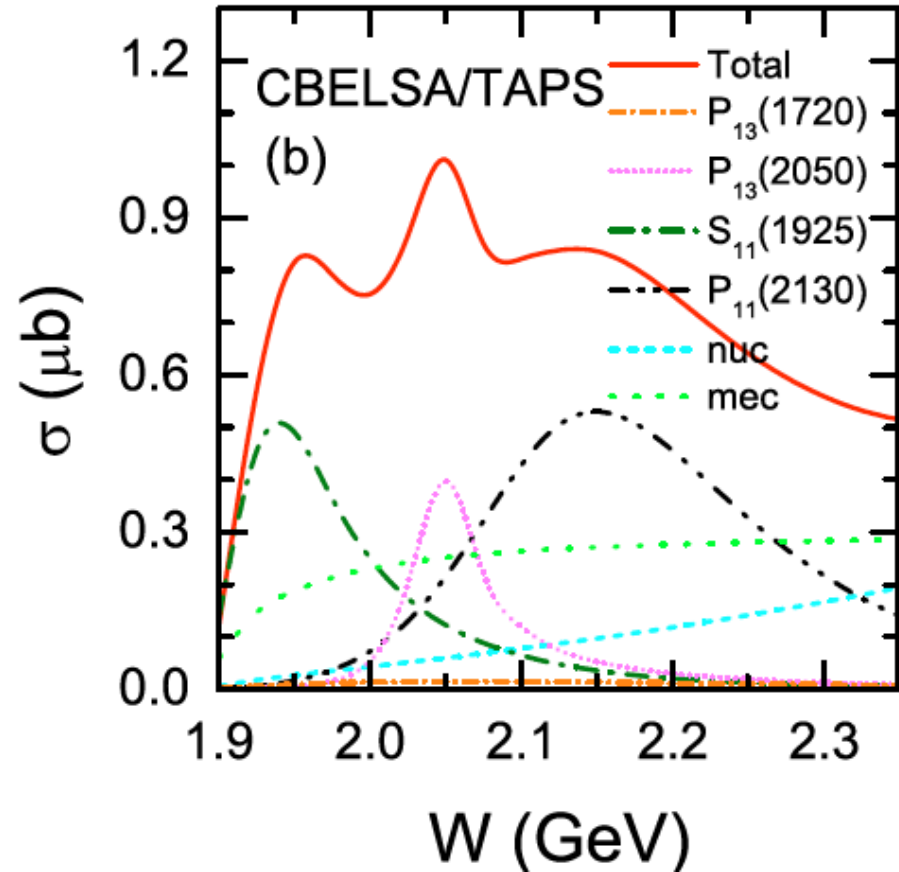
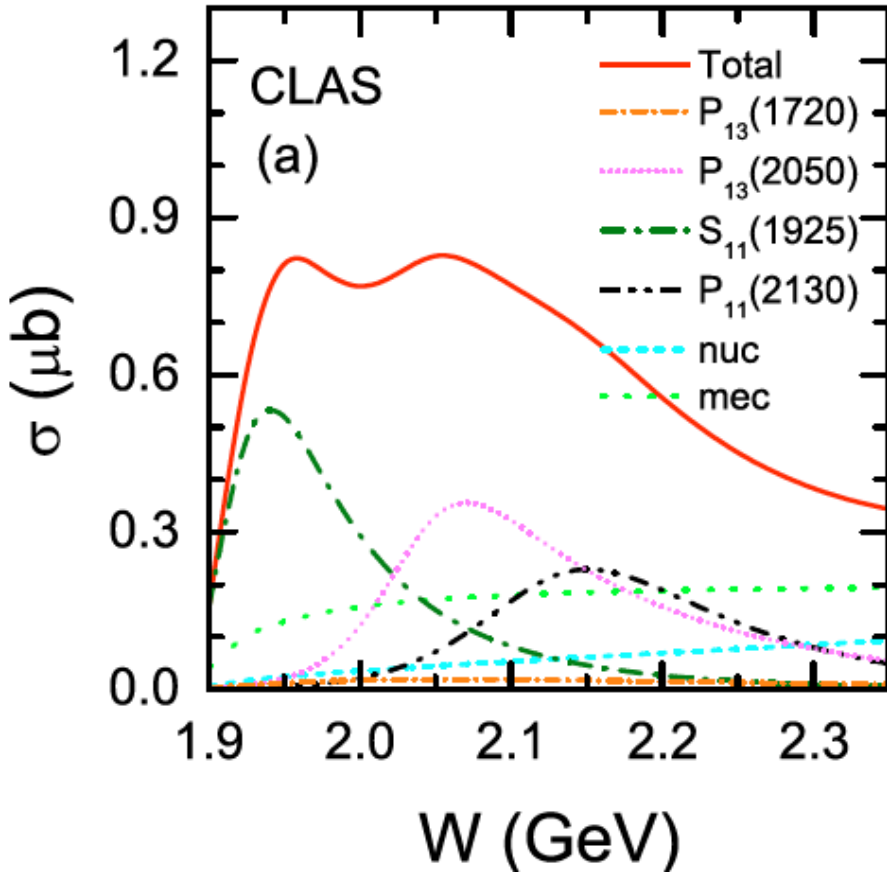
How would this affect the  
extracted resonance parameters?

# $\eta'$ photoproduction: dynamical content



$\{P_{13}(1720), S_{11}(1925), P_{13}(2050), P_{11}(2130)\}$

# $\eta'$ photoproduction: dynamical content



$\{P_{13}(1720), S_{11}(1925), P_{13}(2050), P_{11}(2130)\}$

PDG:  $P_{13}(1720)$ \*\*\*\*,  $S_{11}(1895)$ \*\* ,  $P_{13}(2040)$ \* ,  $P_{11}(2110)$ \*

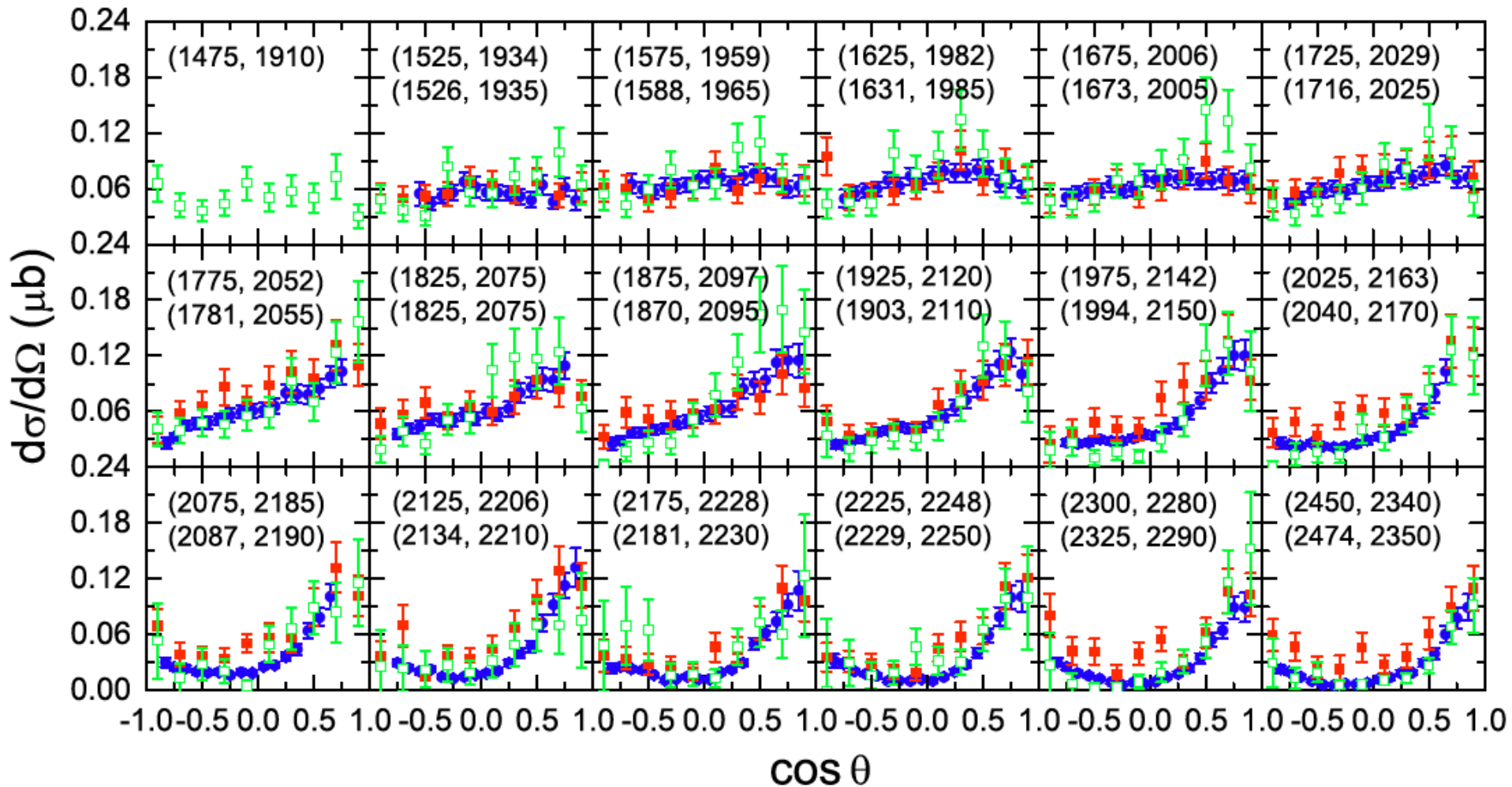
# $\eta'$ photoproduction: resonance parameters (combined analysis)

	free $p$		quasi-free $p$
	CLAS	CBELSA/TAPS	
$\chi^2/N$	0.65	0.53	0.77
$g_{NN\eta'}$	$1.00 \pm 0.06$	$1.17 \pm 0.31$	$1.00 \pm 0.24$
$\lambda_{NN\eta'}$	$0.53 \pm 0.06$	$0.44 \pm 0.22$	$0.64 \pm 0.24$
$\Lambda_v$ [MeV]	$1183 \pm 5$	$1244 \pm 35$	$1221 \pm 28$
$\hat{h}$	$3.89 \pm 0.18$	$5.37 \pm 1.57$	$4.27 \pm 0.89$
<b><math>P_{13}(1720)</math></b>			
$M_R$ [MeV]	<b>1720</b>	<b>1720</b>	<b>1720</b>
$\Gamma_R$ [MeV]	<b>200</b>	<b>200</b>	<b>200</b>
$\sqrt{\beta_{N\eta'}} A_{1/2}$ [ $10^{-3} \text{ GeV}^{-1/2}$ ]	$0.09 \pm 0.03$	$0.09 \pm 0.06$	$0.06 \pm 0.11$
$\sqrt{\beta_{N\eta'}} A_{3/2}$	$-0.16 \pm 0.05$	$-0.13 \pm 0.09$	$-0.03 \pm 0.06$
<b><math>P_{13}(2050)</math></b>			
$M_R$	$2050 \pm 4$	$2045 \pm 7$	<b>2048</b>
$\Gamma_R$	$140 \pm 10$	$52^{+184}_{-52}$	$51^{+241}_{-51}$
$\sqrt{\beta_{N\eta'}} A_{1/2}$	$-5.71 \pm 0.17$	$-2.02 \pm 0.26$	$-3.14 \pm 0.43$
$\sqrt{\beta_{N\eta'}} A_{3/2}$	$9.89 \pm 0.30$	$7.31 \pm 0.93$	$5.75 \pm 0.79$
<b><math>S_{11}(1925)</math></b>			
$M_R$	$1924 \pm 4$	$1926 \pm 10$	<b>1925</b>
$\Gamma_R$	$112 \pm 7$	$99 \pm 23$	$145 \pm 45$
$\lambda$	$1.00^{+0.00}_{-0.06}$	$1.00^{+0.00}_{-0.98}$	$1.00^{+0.00}_{-0.95}$
$\sqrt{\beta_{N\eta'}} A_{1/2}$	$-11.84 \pm 0.41$	$-11.07 \pm 1.43$	$-19.93 \pm 1.56$
<b><math>P_{11}(2130)</math></b>			
$M_R$	$2129 \pm 5$	$2123 \pm 23$	<b>2126</b>
$\Gamma_R$	$205 \pm 12$	$246 \pm 54$	$170 \pm 178$
$\lambda$	$1.00^{+0.00}_{-0.04}$	$1.00^{+0.00}_{-0.61}$	$1.00^{+0.00}_{-0.95}$
$\sqrt{\beta_{N\eta'}} A_{1/2}$	$-11.34 \pm 0.62$	$-18.80 \pm 0.90$	$-7.45 \pm 0.94$



# $\eta'$ photoproduction: data comparison (quasi-free proton)

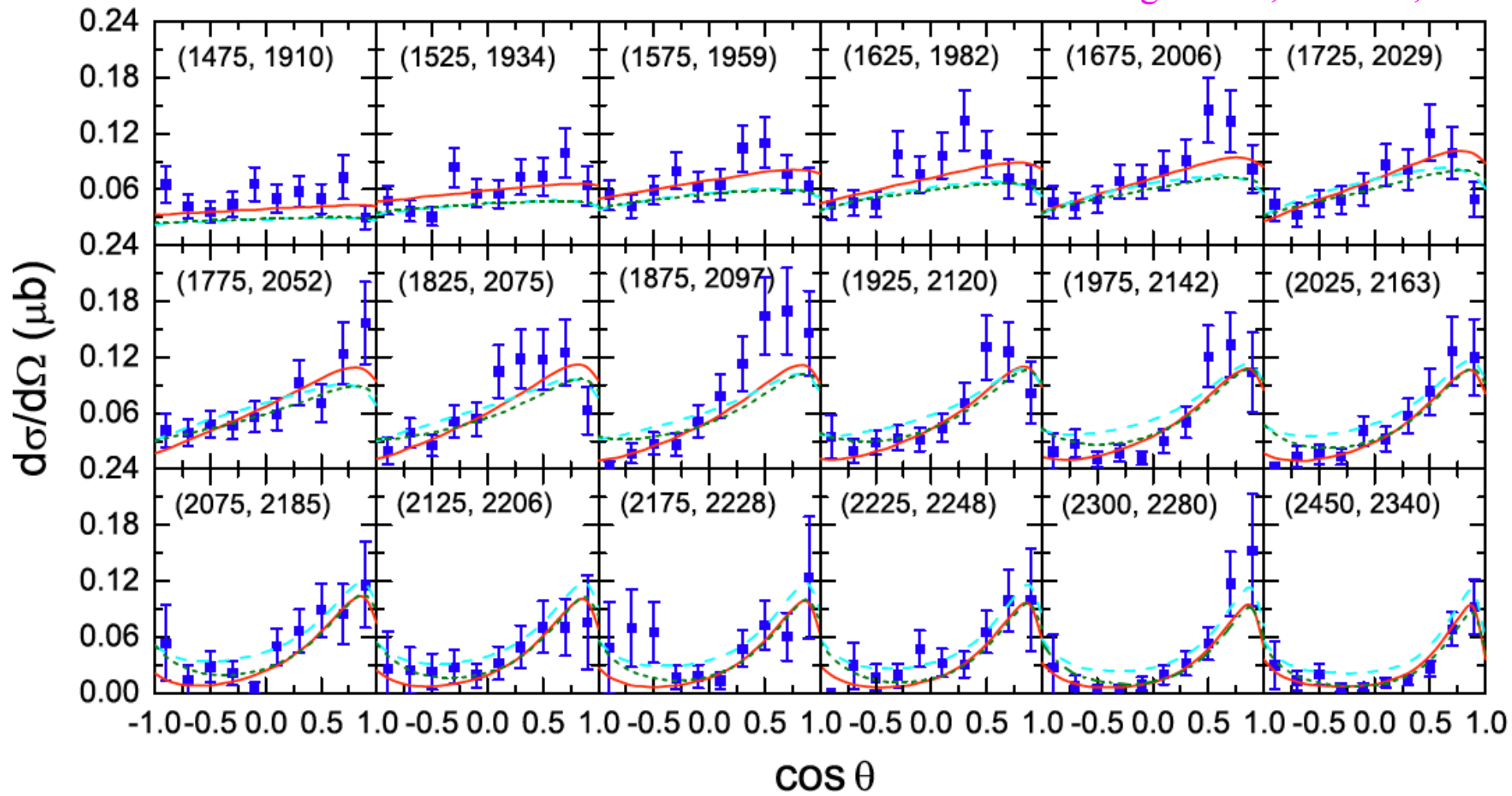
- Green : CBELSA/TAPS-'11 (quasi-free proton) [Jaegle et al., EPJA47,'11]
- Red : CBELSA/TAPS-'09 (free proton) [Crede et al., PRC80,'09]
- Blue : CLAS-'09 (free proton) [Williams et al., PRC80,'09]



# $\eta'$ photoproduction: quasi-free proton (combined analysis)

Red : CBELSA/TAPS-'11 quasi-free proton fit  
Cyan : CBELSA/TAPS-'09 free proton fit  
Green : CLAS-'09 free proton fit

data: Jaegle et al., EPJA47,'11

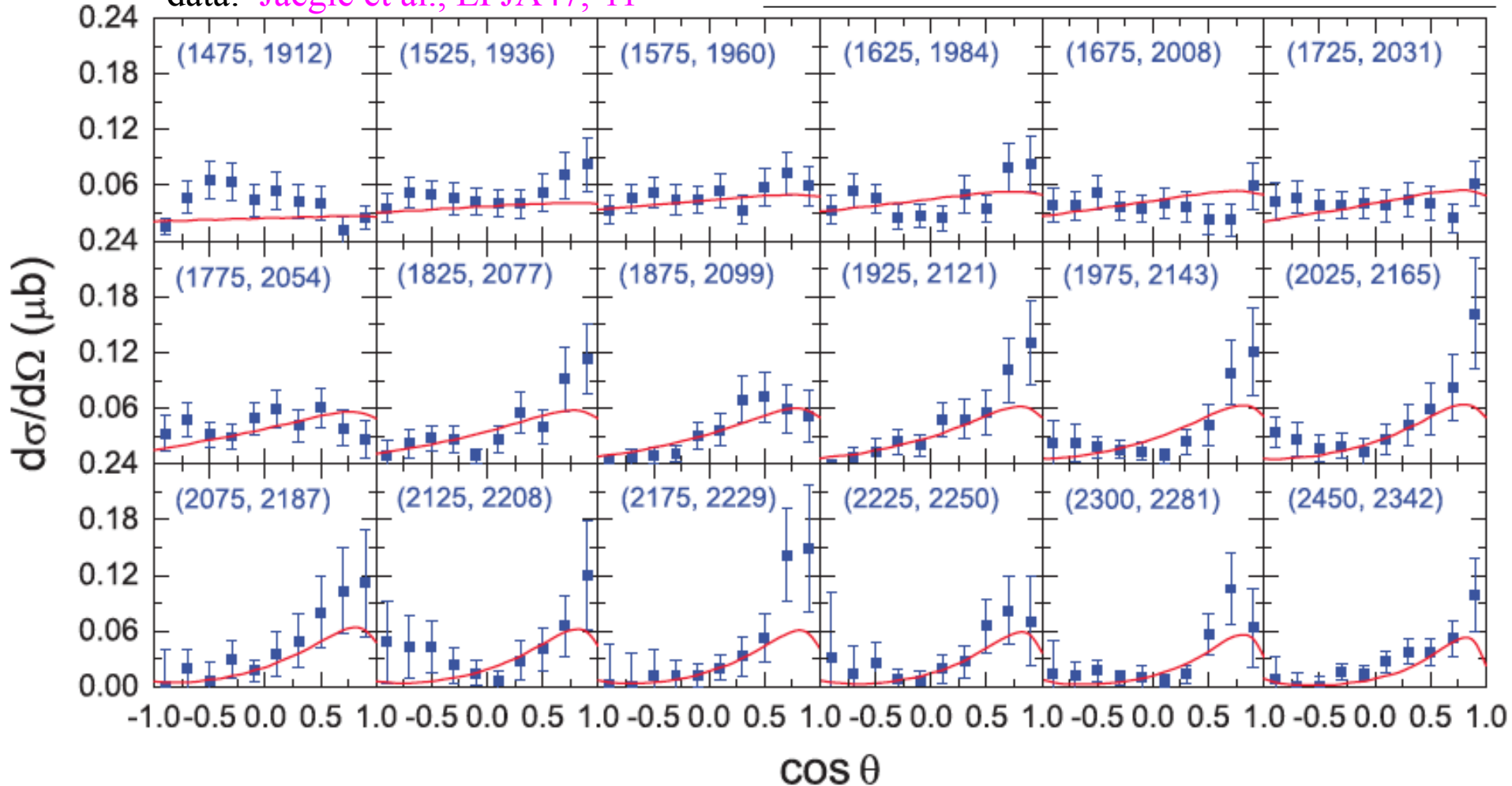


# $\eta'$ photoproduction: quasi-free neutron (combined analysis)

Red : CBELSA/TAPS-'11 quasi-free neutron fit

	$P_{13}(1720)$	$P_{13}(2050)$	$S_{11}(1925)$	$P_{11}(2130)$
$\sqrt{\beta_{N\eta'}} A_{1/2}$	0.04	0.94	15.54	7.60
$\sqrt{\beta_{N\eta'}} A_{3/2}$	-0.00	-1.64		
$\beta_{n\gamma} / \beta_{p\gamma}$	0.32	-0.09	-0.61	-3.06

data: Jaegle et al., EPJA47,'11



# *NN $\eta'$ coupling strength:*

Present analysis:  $g_{NN\eta'} \sim 1.0$  (cannot much larger)

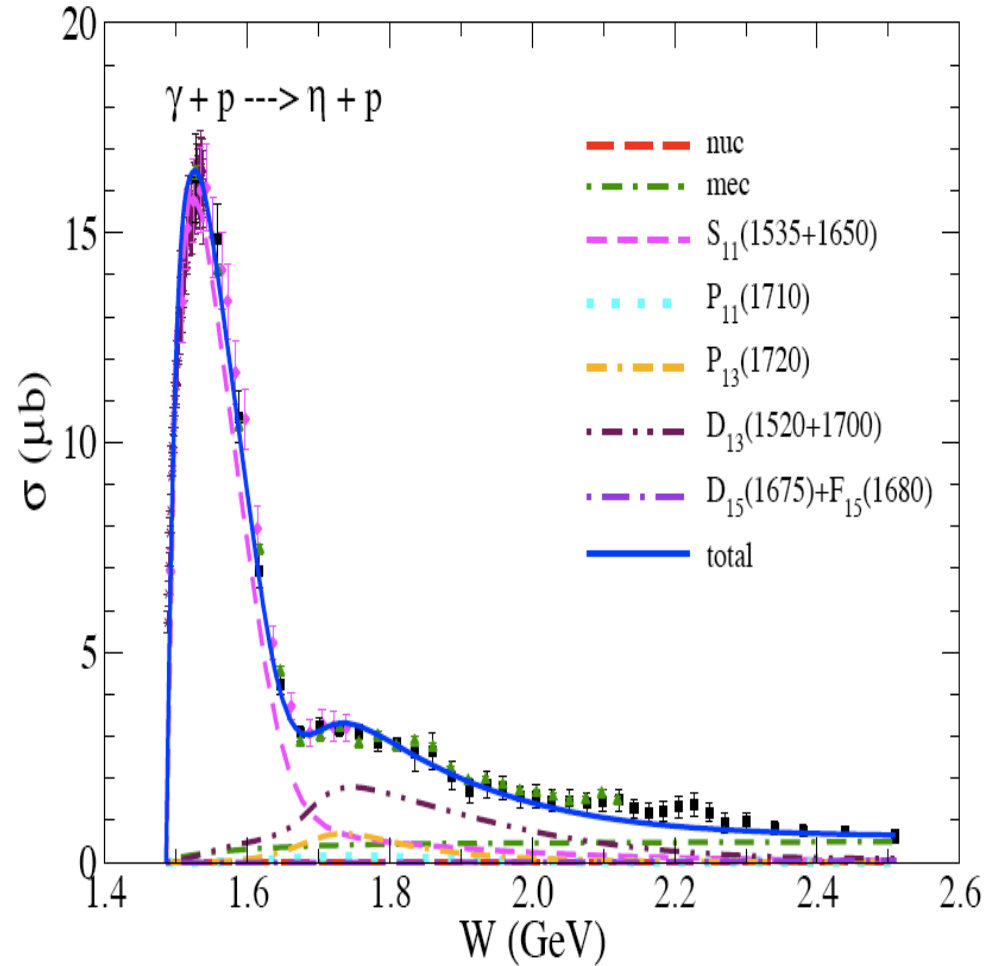
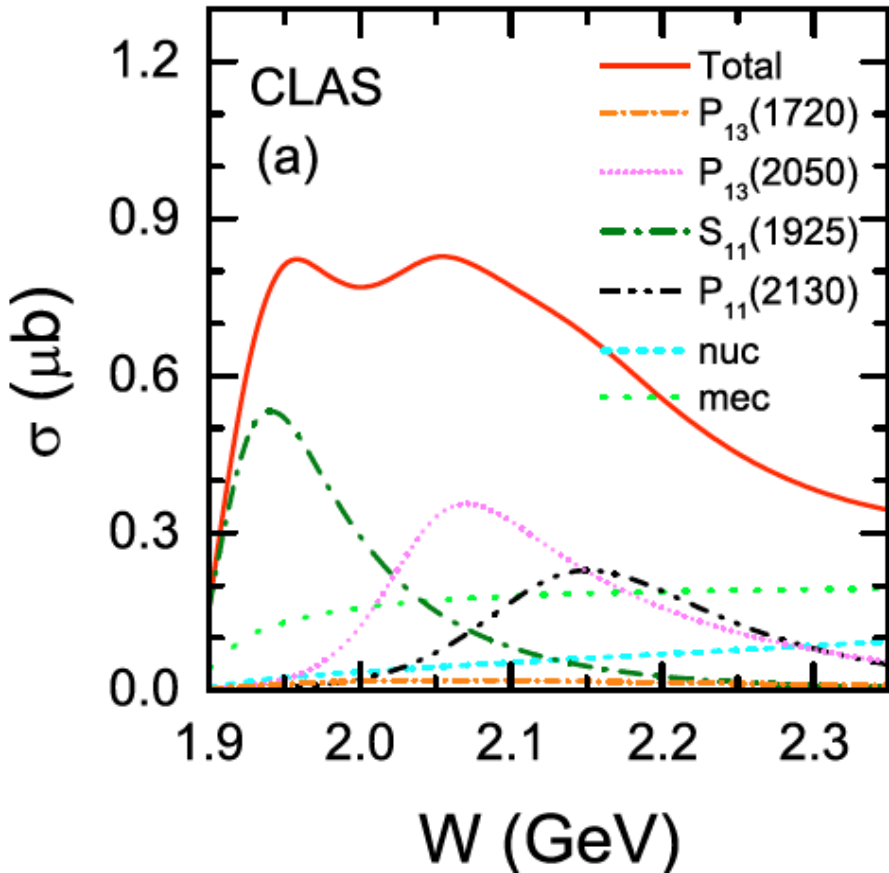
Particular interest in connection to the “nucleon-spin crisis” (EMC collaboration, PLB206, '88). NN $\eta'$  coupling constant is related to the flavor-singlet axial charge  $G_A$  through the U(1) Goldberger-Treiman relation:

$$2m_N G_A(0) \cong \underbrace{\sqrt{2N_F} F_\pi g_{NN\eta'}(0)}_{\text{quark contribution to the proton "spin"}} + \underbrace{F_\pi^2 m_{\eta'}^2 g_{NNG}(0)}_{\text{gluon contribution to the proton "spin"}}$$

$G_A(0) \approx 0.16 \pm 0.10$   
(SMC collaboration, PRD56, '97)

Shore & Veneziano, NPB381, '92

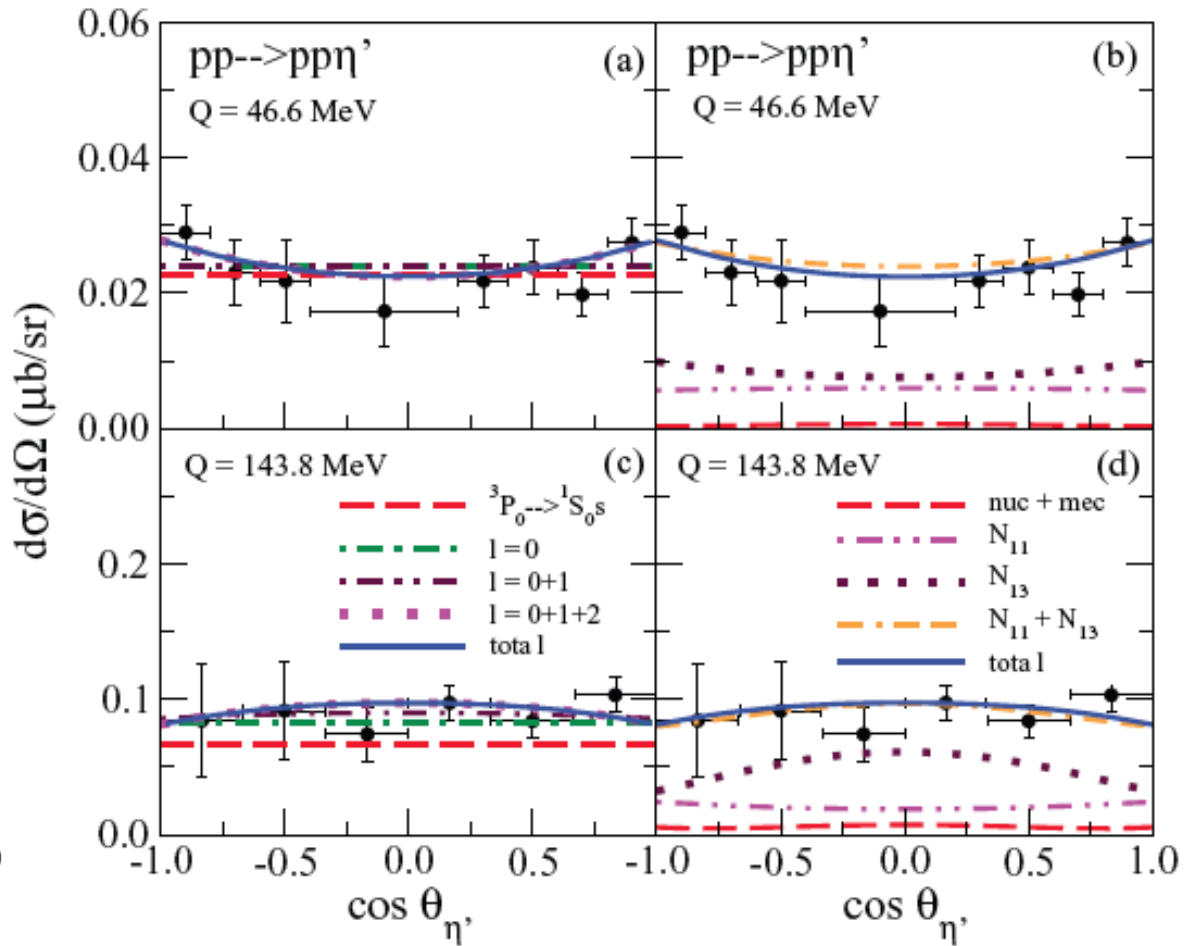
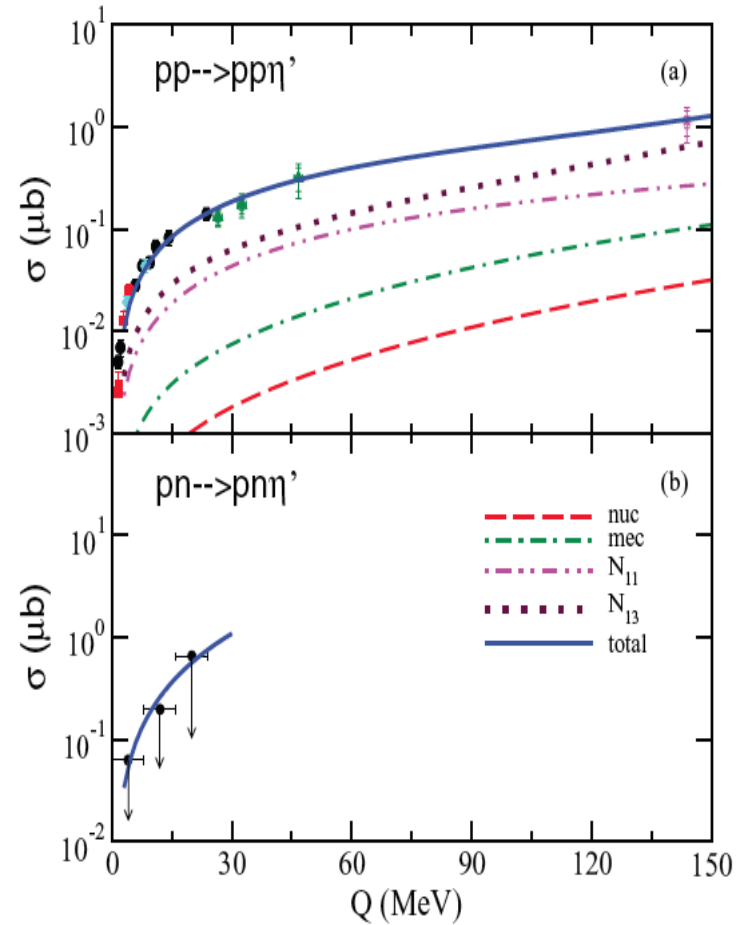
# $\eta'$ photoproduction: comparison with $\eta$ photoproduction



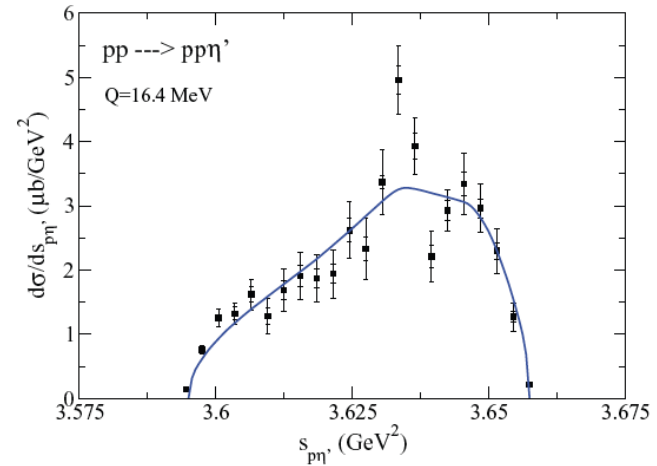
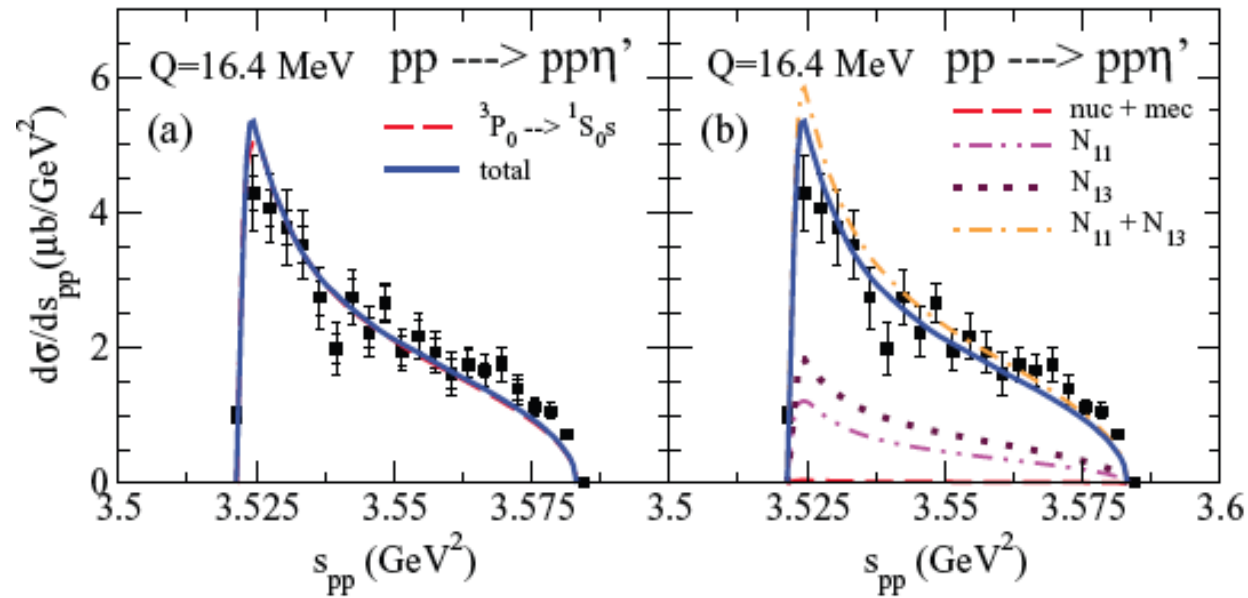
$\{P_{13}(1720), S_{11}(1925), P_{13}(2050), P_{11}(2130)\}$

PDG:  $P_{13}(1720)$ \*\*\*\*,  $S_{11}(1895)$ \*\* ,  $P_{13}(2040)$ \* ,  $P_{11}(2110)$ \*

# $\eta'$ hadroproduction: $NN \rightarrow \eta' NN$ (combined analysis)

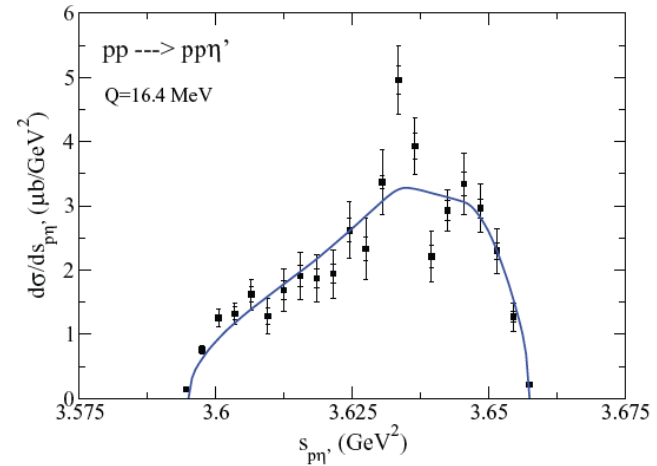
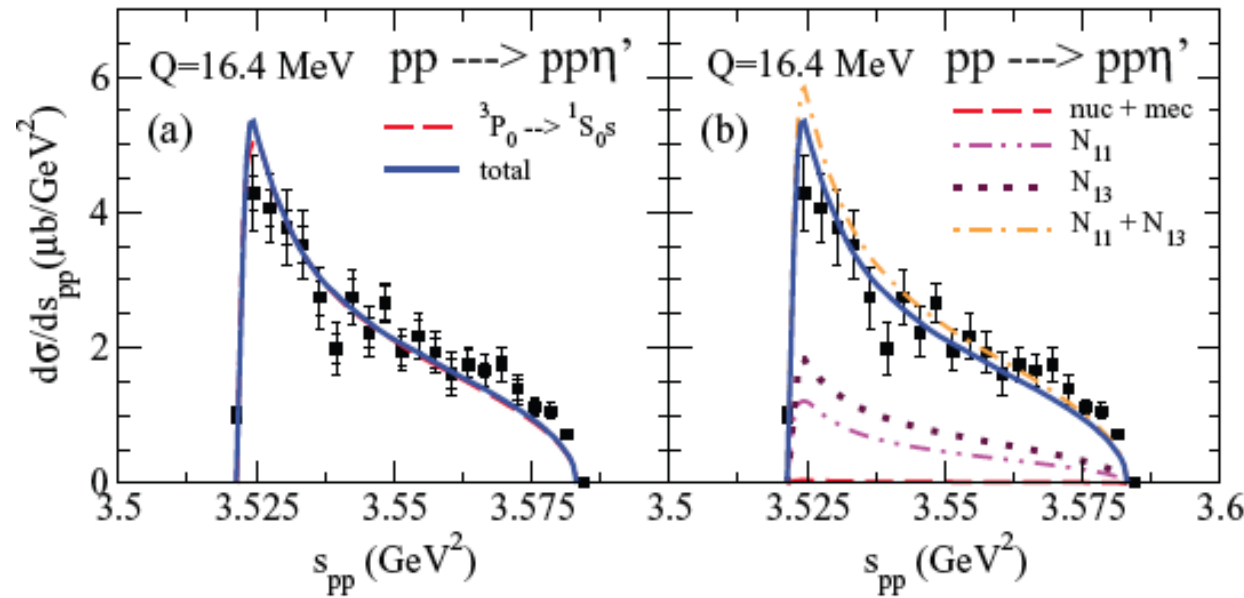


# $\eta'$ hadroproduction: $NN \rightarrow \eta' NN$ (combined analysis)

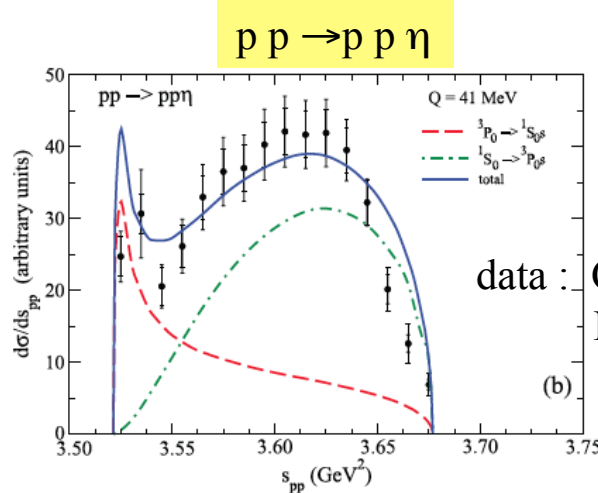
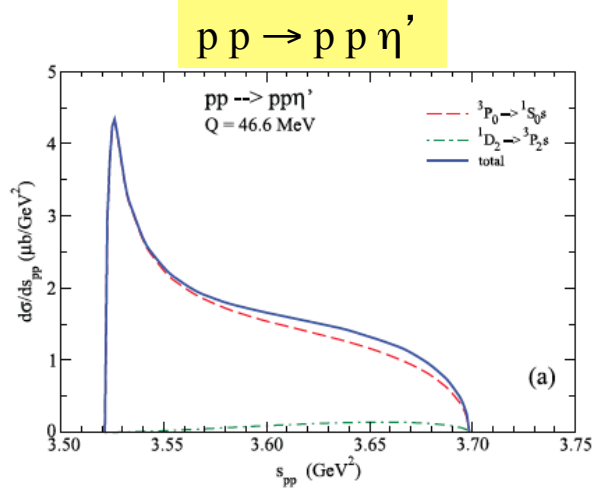


Data: COSY11,  
P. Klaja et al., PLB684'07

# $\eta'$ hadroproduction: $NN \rightarrow \eta' NN$ (combined analysis)



Data: COSY11,  
P. Klaja et al., PLB684'07



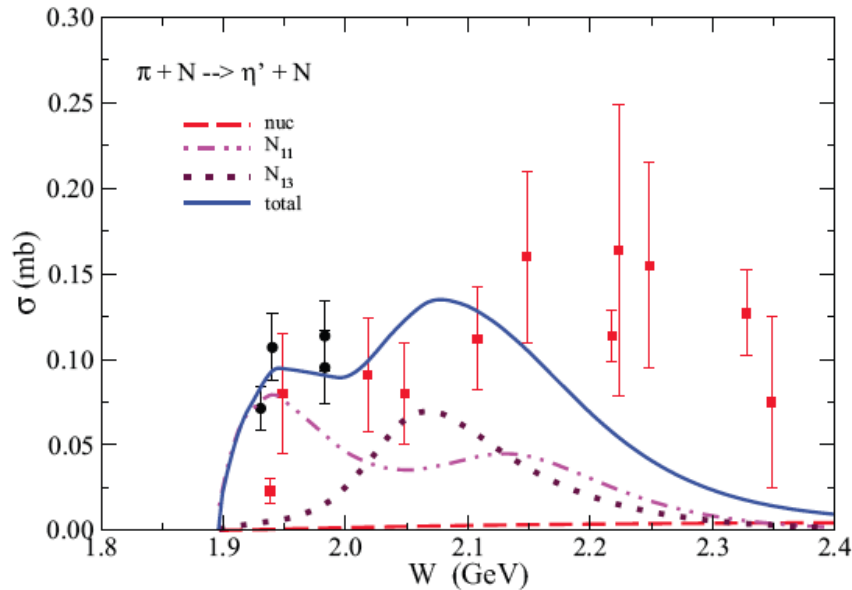
data : COSY-TOF,  
M Abdel-Bary et al., EPJA16'03



# $\eta'$ hadroproduction: $NN \rightarrow \eta' NN$ (combined analysis)

Parameters	$S_{11}(1925)$	$P_{11}(2130)$	$P_{13}(1720)$	$P_{13}(2050)$
$M_R$ (MeV)	1924	2129	<b>1720</b>	2050
$\Gamma_R$ (MeV)	112	205	<b>200</b>	140
$\beta_{N\eta'}$ (%)	6	3	0.09	2
$\beta_{N\pi}$ (%)	22	25	[11 ± 3] 16	25
$\beta_{N\eta}$ (%)	4	[61 ± 60] 0.5	[4.0 ± 1.0] 9	0.03
$\beta_{N\rho}$ (%)	22	62	[70-85] 75	37
$\beta_{N\omega}$ (%)	47	13	2	36
$(g_{RN\eta'}, \lambda)$	(0.68, 1.00)	(1.77, 1.00)	(1.20, -)	(1.38, -)
$(g_{RN\pi}, \lambda)$	(-0.36, 1.00)	(-1.28, 1.00)	(-0.17, -)	(-0.12, -)
$(g_{RN\eta}, \lambda)$	(-0.28, 0.81)	(-0.35, 0.34)	(-1.50, -)	(-0.04, -)
$(g_{RN\rho}^{(1)}, g_{RN\rho}^{(2)}, g_{RN\rho}^{(3)})$	(-2.42, 0.04, -)	(2.58, -0.14, -)	(-23.63, 54.09, 16.72)	(0.50, 9.10, 28.66)
$(g_{RN\omega}^{(1)}, g_{RN\omega}^{(2)}, g_{RN\omega}^{(3)})$	(1.02, -1.70, -)	(2.47, 0.53, -)	(-27.64, 138.87, -318.85)	(-3.19, -16.75, -36.39)

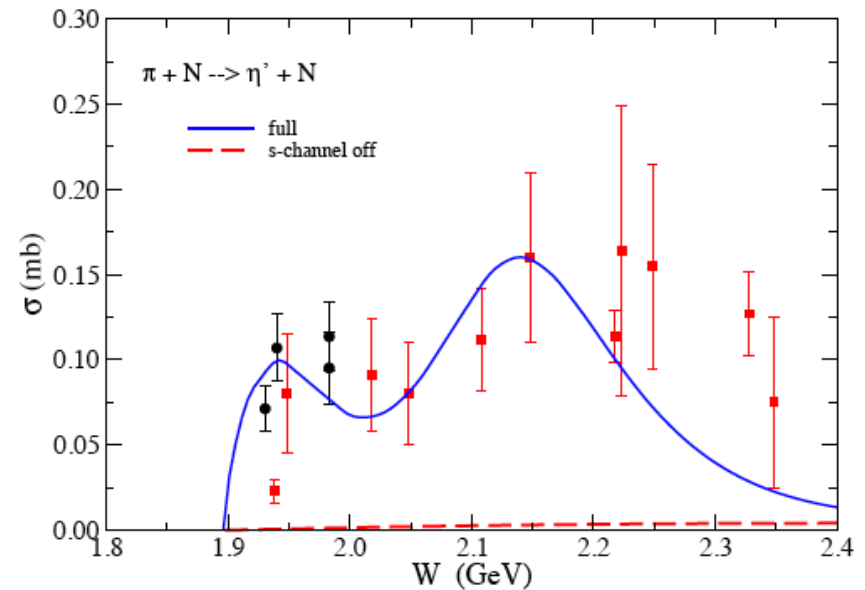
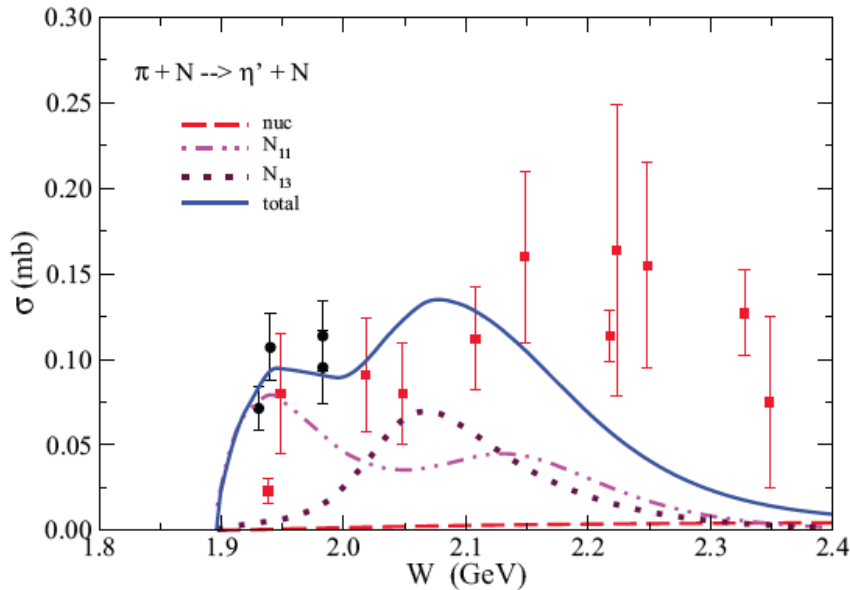
# $\eta'$ hadroproduction: $\pi N \rightarrow \eta' N$ (combined analysis)



## Double-bump structure:

- interference between  $S_{11}(1925)$  and  $P_{13}(2050)$
- remains to be confirmed
- if corroborated, rules out the sub-threshold resonance-dominance assumption of  $S_{11}(1535)$  [Xu and Cao, PRC78'08]  
[motivated by its strong coupling to  $\eta$  and  $\eta'$   
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$$T = T^P(\text{s-channel}) + T^{\text{NP}}(=X)$$

$$X = N^X_\eta, K^{\text{NP}} \sim V^{\text{NP}},$$

$$|X| \ll |T^P| \quad (\text{consistent with small FSI, } N^X_\eta = 1 - i\rho_\eta, X_{\eta, \eta'} \sim 1)$$

# $\eta'N$ scattering length: *a rough estimate*

Present phenomenology:

$$a_{\eta'N} \sim (0.017 + i 0.005) \text{ fm}$$

Oset & Ramos, PLB704, 334(2011)

$\alpha$	$a_{\eta p}$ [fm]	$ a_{\eta p} $ [fm]	$a_{\eta' p}$ [fm]	$ a_{\eta' p} $ [fm]
-0.126	$0.272 + i0.246$	0.367	$0.073 + i0.019$	0.075
0.204	$0.247 + i0.233$	0.340	$-0.072 + i0.020$	0.075
-0.193	$0.276 + i0.248$	0.371	$0.098 + i0.020$	0.1
0.256	$0.241 + i0.231$	0.334	$-0.098 + i0.020$	0.1
-0.333	$0.282 + i0.251$	0.378	$0.149 + i0.020$	0.15
0.352	$0.228 + i0.225$	0.320	$-0.149 + i0.021$	0.15

Full DCC: [D. Roenchen et al., EJPA 47, 44(2013)]

$$\begin{aligned} a_{\eta N} &= (0.49 + i 0.24) \text{ fm} \quad (\text{model A}) \\ &= (0.55 + i 0.24) \text{ fm} \quad (\text{model B}) \end{aligned}$$

# Summary:

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- Phenomenological approach to two-body reactions keeping the full phase structure of the reaction amplitude developed.
- Combined analysis of photo- and hadro-reactions constrain the model parameters:
  - $\gamma + N \rightarrow \eta' + N$  (free and quasi-free)
  - $N + N \rightarrow \eta' + N + N$ 
    - unique set of minimal number of spin-1/2 and -3/2 resonances obtained  
 $\{P_{13}(1720), S_{11}(1925), P_{13}(2050), P_{11}(2130)\}$
    - results consistent with very small FSI
- Role of FSI : spin-polarization observables can help to learn more (sensitive to the relative phase).
- Role of higher-spin resonances: spin-polarization observables required.
- Same approach to other meson production reactions where the hadronic data are scarce to fix the FSI : e.g.,  $\phi$  production.

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*The End*