Image reconstruction in Strip Positron Emission Tomograph

Piotr Białas

Department of Game Technology

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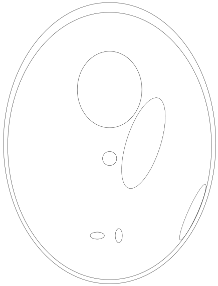
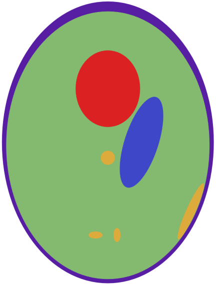
Outlook

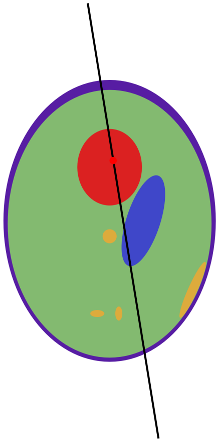
- Principles of PET imaging
- Strip PET



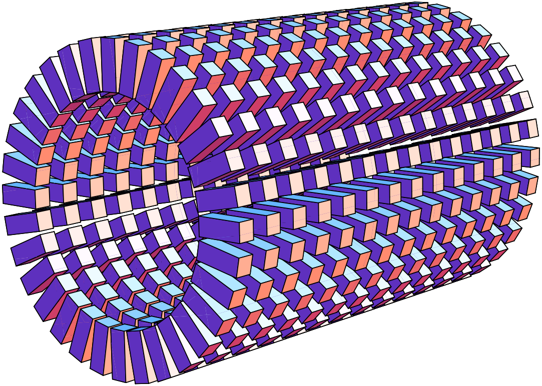
PET is designed for imaging *metabolism*.

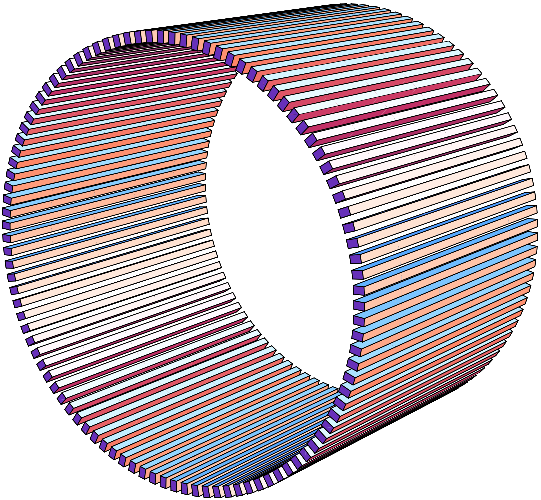


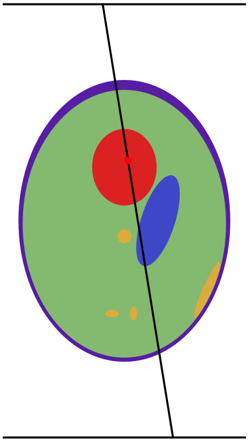
Patient is injected with radioactive tracer/marker (sugar) which collects in regions of high metabolism.



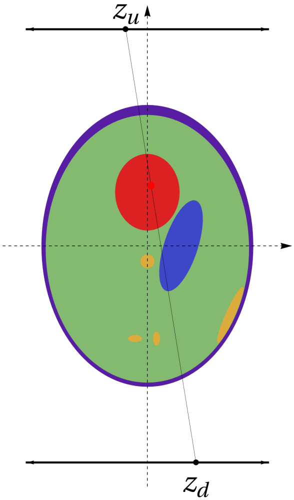
- Tracer decays emitting *positrons*.
- Positrons anihilate with electrons, emitting two γ quanta.



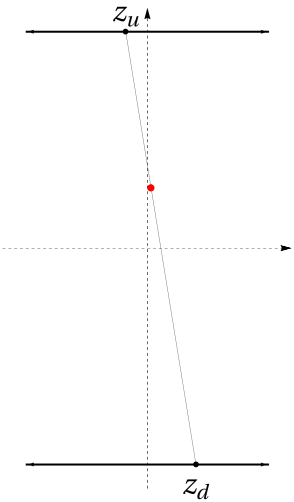


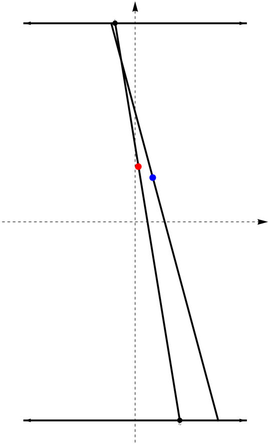


γ are absorbed by bars of scintillators.



Scintillators emit light captured by the photomultipliers at the ends.

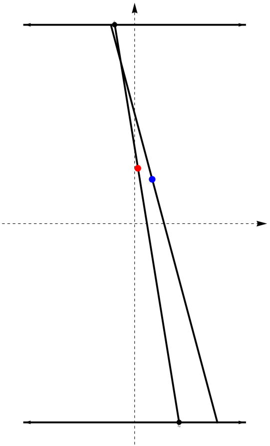

z_u
z_d



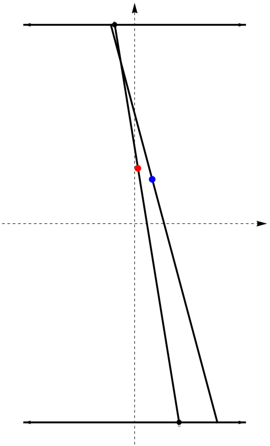
$$P(\tilde{\mathbf{e}}|\mathbf{e}) = \frac{\det^{\frac{1}{2}} C(\mathbf{e})}{(2\pi)^{\frac{3}{2}}} \cdot$$

$$\exp\left(-\frac{1}{2}(\tilde{\mathbf{e}} - \mathbf{e})^T C^{-1}(\mathbf{e})(\tilde{\mathbf{e}} - \mathbf{e})\right).$$

$$\tilde{\mathbf{e}} - \mathbf{e} = \begin{pmatrix} \widetilde{z_u} - z_u \\ \widetilde{z_d} - z_d \\ \widetilde{\Delta l} - \Delta l \end{pmatrix}$$



$$C(\mathbf{e}) = \begin{pmatrix} \sigma_z^2 & 0 & \eta \\ 0 & \sigma_z^2 & \eta \\ \eta & \eta & \sigma_{\Delta l}^2 \end{pmatrix}$$



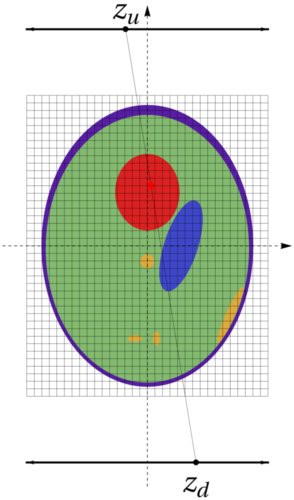
$$P(\tilde{\mathbf{e}}|(y,z)) = \int \mathrm{d}\theta\, P(\tilde{\mathbf{e}}|\mathbf{e}(y,z,\theta))$$

$$z_u = z + (R - y)\tan\theta$$
$$z_d = z - (R + y)\tan\theta$$
$$\Delta l = -2y\sqrt{1+\tan^2\theta}$$

PET image reconstruction is a statistical process.

$$\boxed{\max_{\rho>0} P(\{\tilde{\mathbf{e}}_j\}|\rho)}$$

$$\boxed{\max_{\rho>0} \log P(\{\tilde{\mathbf{e}}_j\}|\rho)}$$

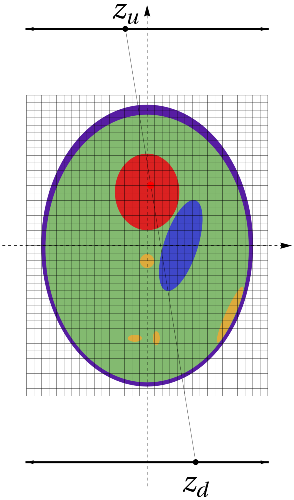


$$P(\{\tilde{\mathbf{e}}_j\}|\rho) = \prod_j \left(\sum_p P(\tilde{\mathbf{e}}_j|p) P(p|\rho) \right)$$

$$P(p|\rho) = \frac{s(p)\rho(p)}{\sum_p s(p)\rho(p)}$$

$$P(\tilde{\mathbf{e}}_j|p) = \int_{x,y\in p} P(\tilde{\mathbf{e}}_j|x,y)$$

$$\approx vol(p) P(\tilde{\mathbf{e}}_j|center(p))$$



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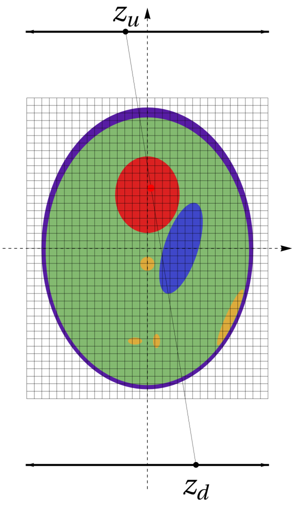

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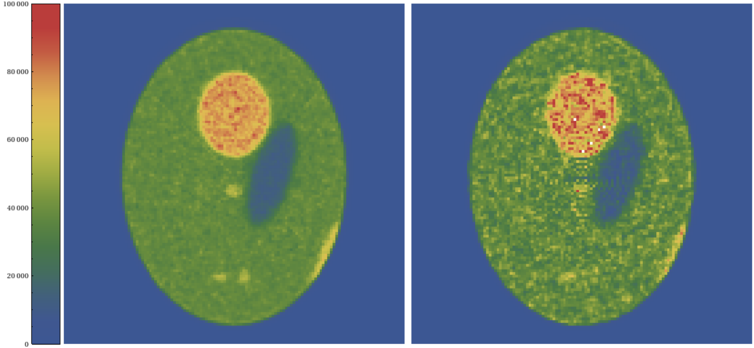
EM algorithm

$$\rho(p)^{(t+1)} = \sum_{j=1}^{J} \frac{P(\tilde{\mathbf{e}}_j|p)\rho(p)^{(t)}}{T\sum_{p'=1}^{M} P(\tilde{\mathbf{e}}_j|p')s(p')\rho(p')^{(t)}}.$$


100 000
80 000
60 000
40 000
20 000
0

Simple but not easy

- Time resolution has to be fantastic.
- $P(\tilde{\mathbf{e}}|p)$ has to be known analytically.
- Scintilators are not lines but bars.
- We need

$$\#events \times \#contributingpixels \approx \#events \times 1000$$

  operations in each iteration
- 3D
- Positrons do not anihilate imediately,
- nor at rest.