National Centre for Nuclear Research Świerk

ŚWIERK COMPUTING CENTRE

Application of compressive sensing theory for the reconstruction of signals in polymer scintilators

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INNOVATIVE ECONOMY
NATIONAL COHESION STRATEGY

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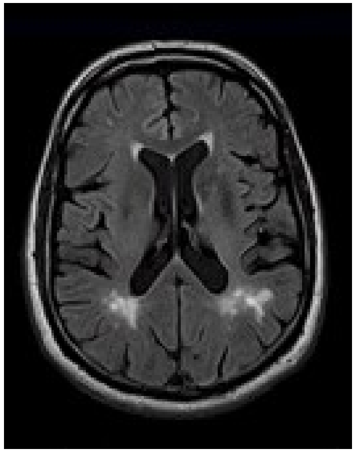
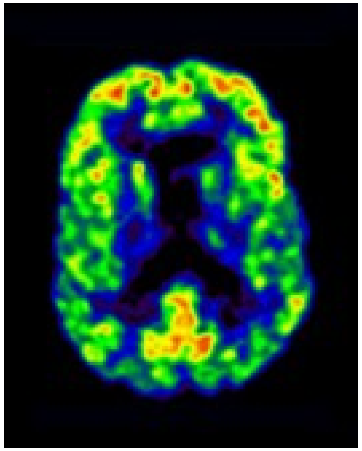
1. Introduction
Medical Imaging

- Medical imaging starts in **1895** with the discovery of X-rays by Roentgen
- Medical imaging is a routine and one of the most important part of a today's medicine
- Providing an objective diagnosis is possible (based not only on symptoms)
- Supporting the surgery with medical imaging

X-ray radiography



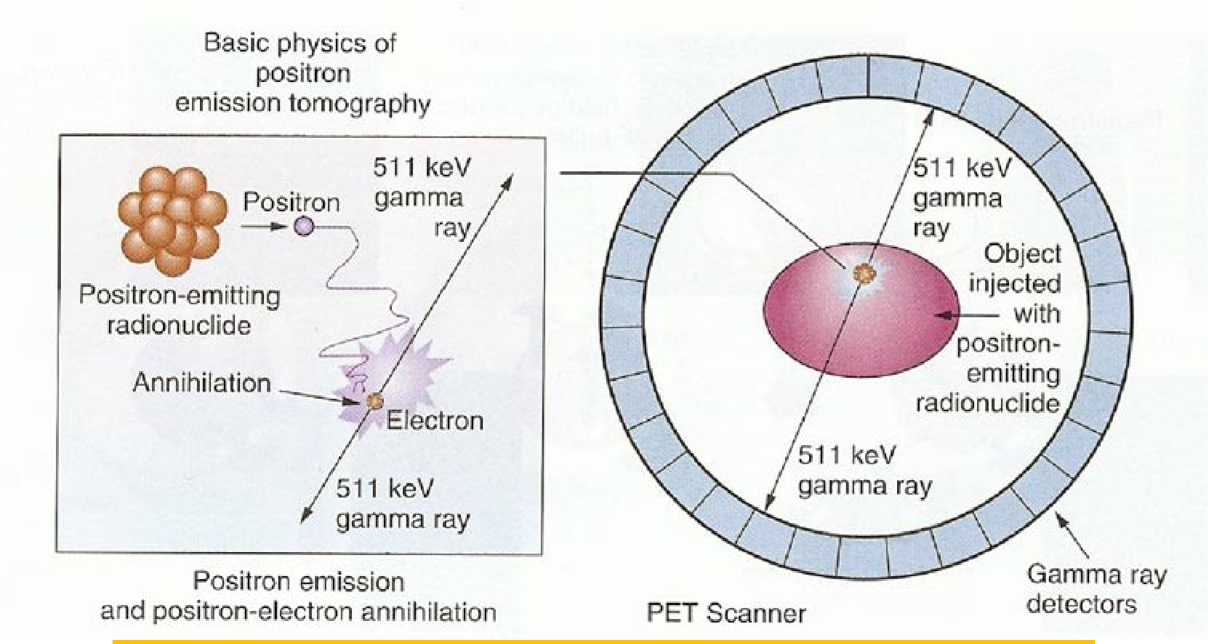
MRI image [1]



PET image [1]

1. Introduction
PET

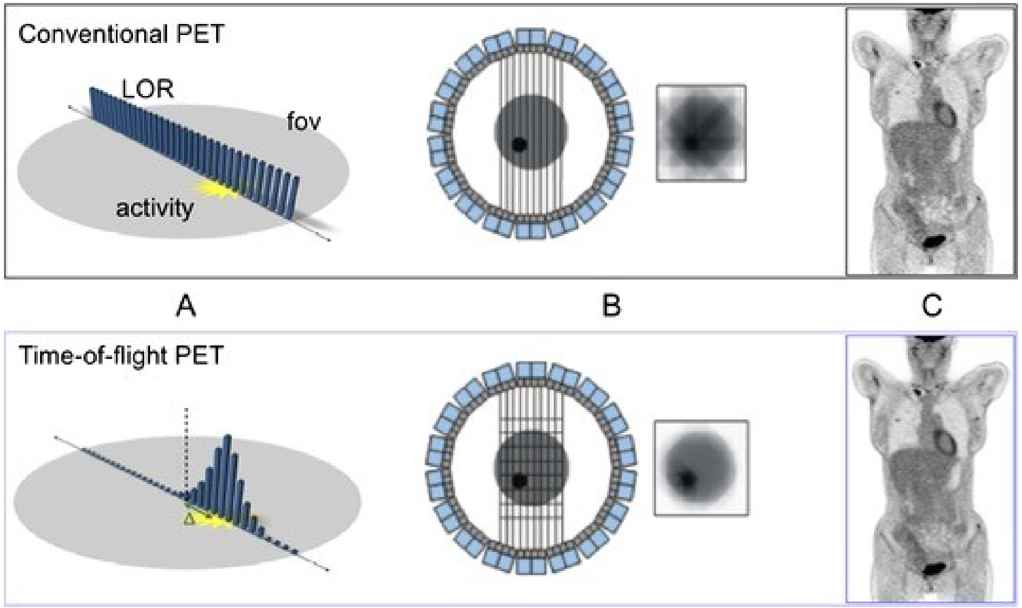
- Positron Emission Tomography (PET) is one of the most prominent imaging techniques
- Visualisation of functional processes in the body



Working principle of PET scanner [2]

1. Introduction
TOF PET

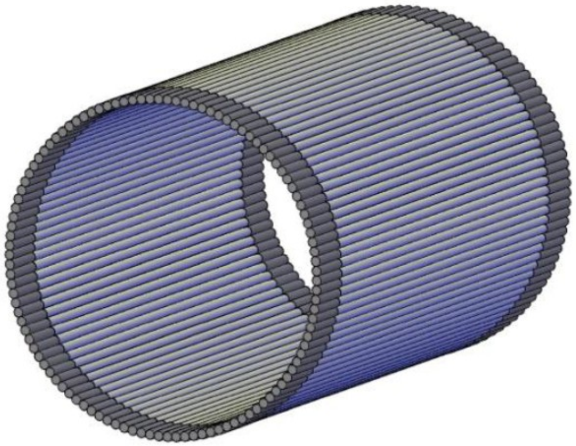
- Time of Flight (TOF) information improves the image reconstruction in PET
- In conventional PET, only a line through the patient's body is detected by two detectors
- In Time of Flight PET, the faster detectors enable the measurement of the difference in the arrival time to the two detectors



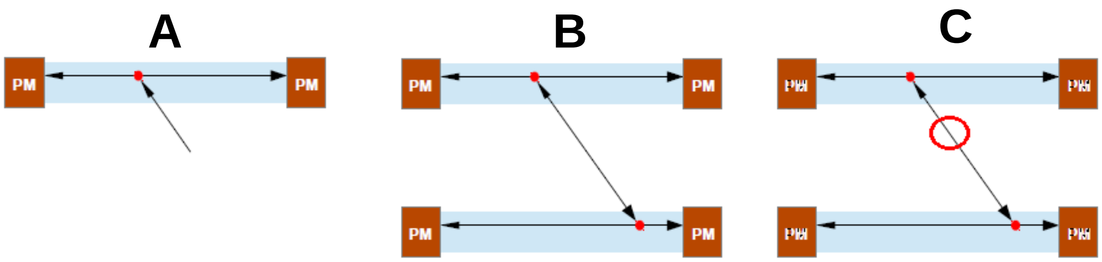
Time of Flight information [3]

1. Introduction
Striped TOF PET

- Detectors built from one plastic scintillator and two photmultipliers
- Double time-of-flight method

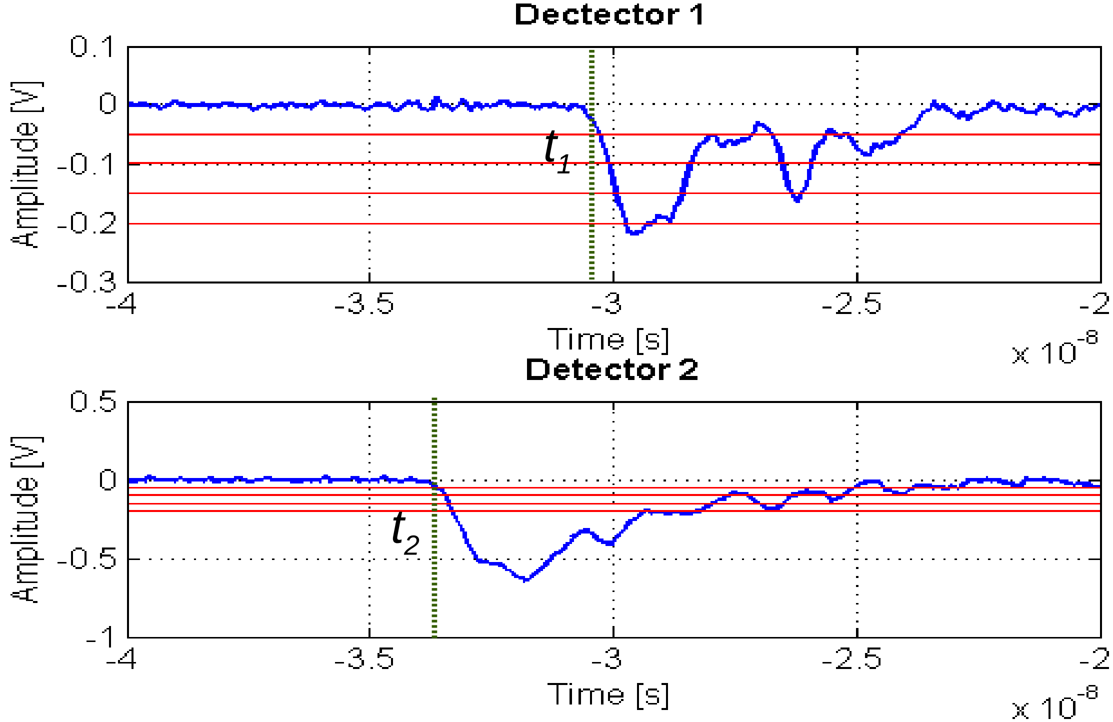


Striped TOF PET



Time of Flight method fo Striped TOF PET

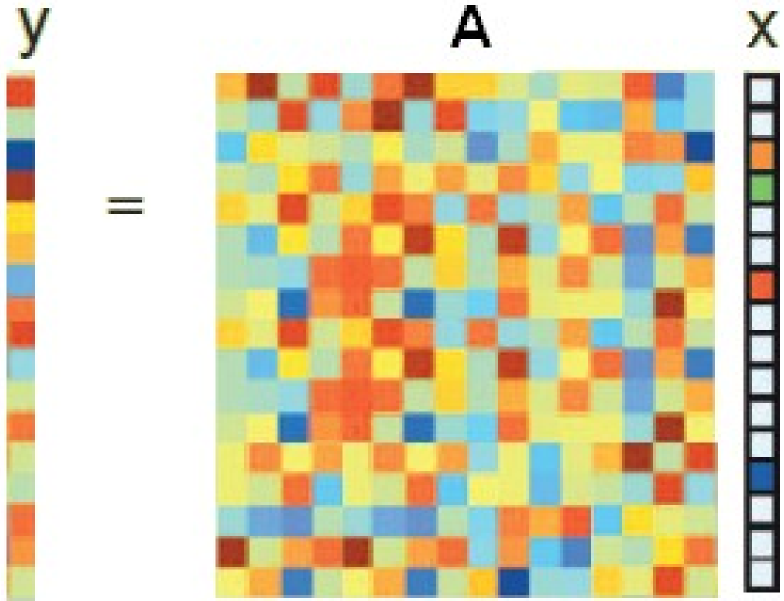
1. Introduction
Striped TOF PET



- Constant multi-level discriminator is used to sample a time domain signal
- The goal of this work was to investigate the possibilities of reconstruction of the original signal with a small number of measurements based on Compressive Sensing theory

2. Compressive Sensing

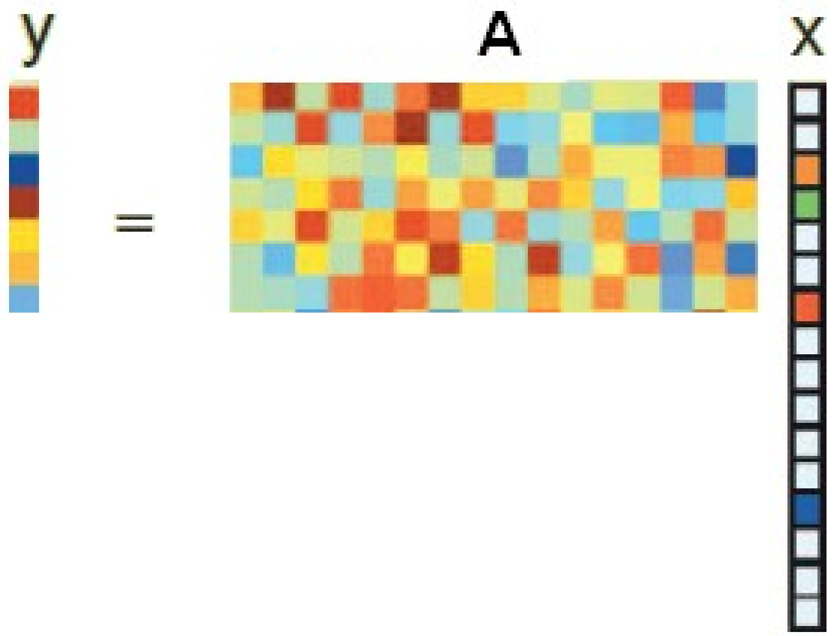
- The discrete signal measured by an osciloscope is denoted by ***y***
- The ***K*–sparse** representation, denoted by ***x***, contains only ***K*** non-zero coefficients (where ***K* << *M***)
- The relation between ***x*** and ***y*** is depicted by the transform matrix ***A***



$$x \in R^M$$
$$y \in R^M$$
$$y = A \cdot x$$

2. Compressive Sensing

- The reconstruction of ***x*** does not require the full information about ***y***
- In general it can be shown that the ***N*** random measurements of ***y*** are required, where ***N*** > ***2K*** and ***K*** is the sparsity of the ***x***



$$x \in R^M$$
$$y \in R^N \qquad N < M$$
$$y = A \cdot x$$

2. Compressive Sensing

Optimization problem:

- l_0 minimization problem

$$\hat{x} = arg\left(min\|x\|_0\right) \quad \wedge \quad \mathrm{y} = \mathrm{A} \cdot x$$

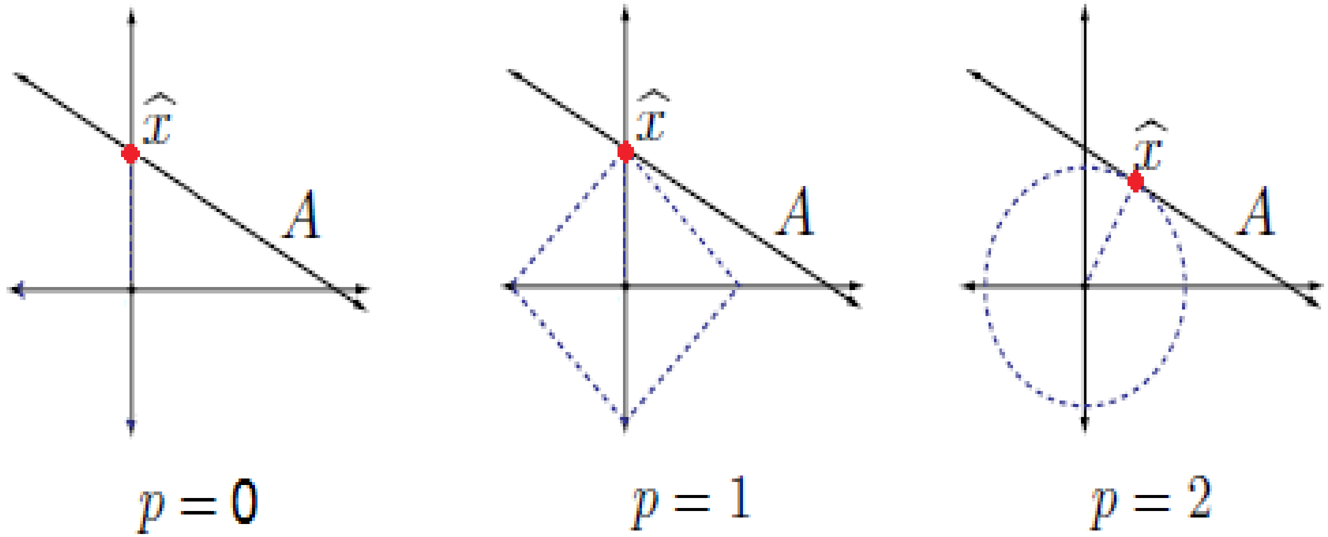
$$\|x\|_0 = K\,, \quad \text{K - nr of non-zero elements of x}$$

- l_1 minimization problem

$$\hat{x} = arg\left(min\|x\|_1\right) \quad \wedge \quad \mathrm{y} = \mathrm{A} \cdot x$$

$$\|x\|_1 = \sum_i^M |x_i|\,, \quad \text{M - size of vector x}$$

2. Compressive Sensing

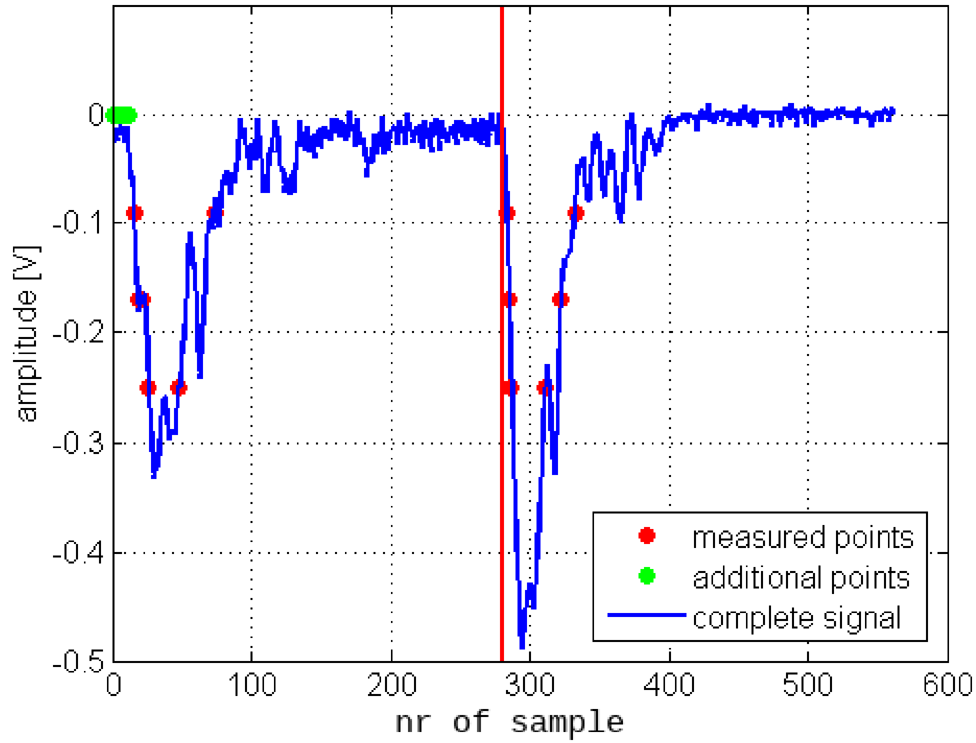


- Searching for the sparse solution of ***y = Ax*** with different ***p*** norms
- Minimazing of ***p*** norm is adequate to finding the first intersect of ***p*** sphere with line ***y = Ax*** when ***p*** sphere grows from (0, 0)
- It can be shown that for ***p <= 1*** sparse solution might be found

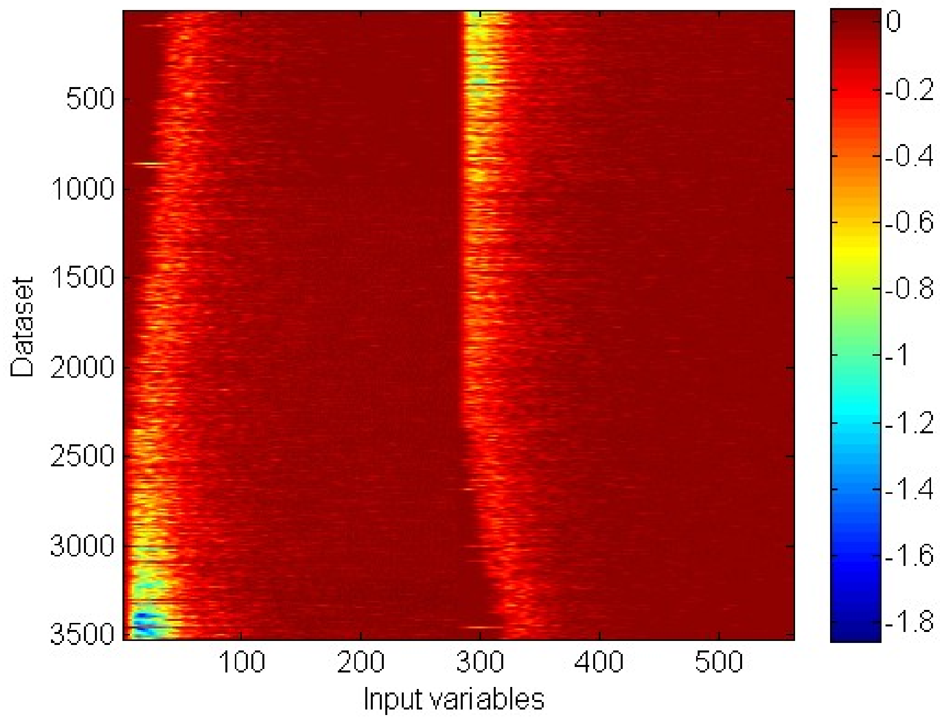
3. Signal representation

- Single data ***y*** is a concatenation of a two measured signals (place marked in red)
- This approach enable to take into account the time difference (additional points marked in green)
- The registration starts after one of the signals reach the amplitude of -0.05 V (in the example a second one)
- Data sampling: 10 GHz (100 ps between samples)
- Data dimension: M = 560

  N = 21 (12+9)

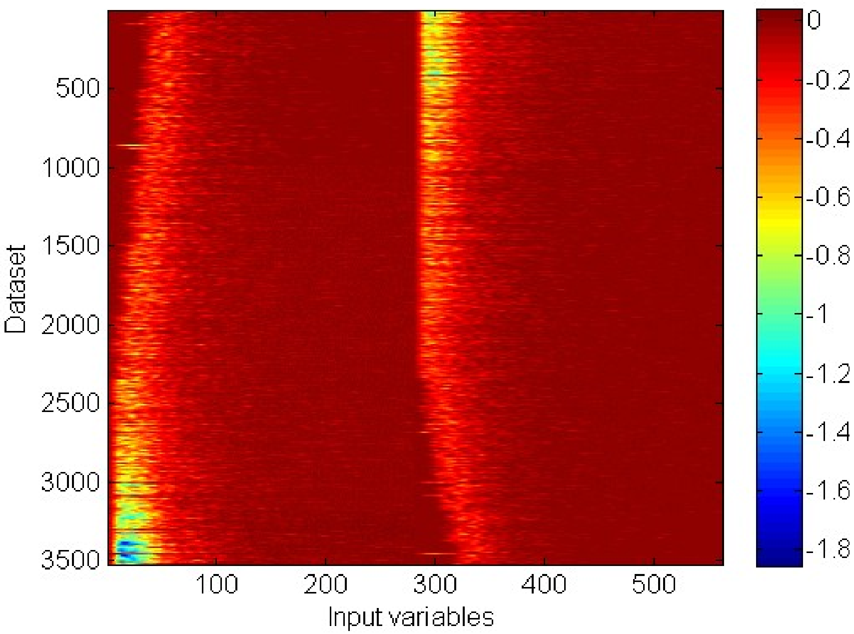


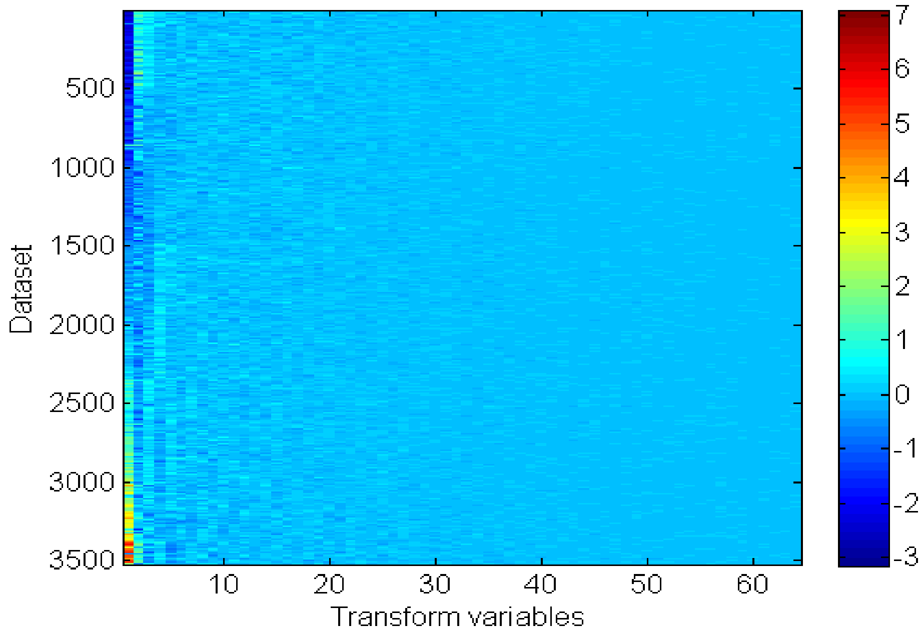
3. Signal representation



- The complete data set used in this study with about 3500 signals
- The signals ***y*** are present in the rows
- The amplitude of a signals is indicated by a color in the image
- High correlation of the signals ***y*** in the data set

3. Signal representation



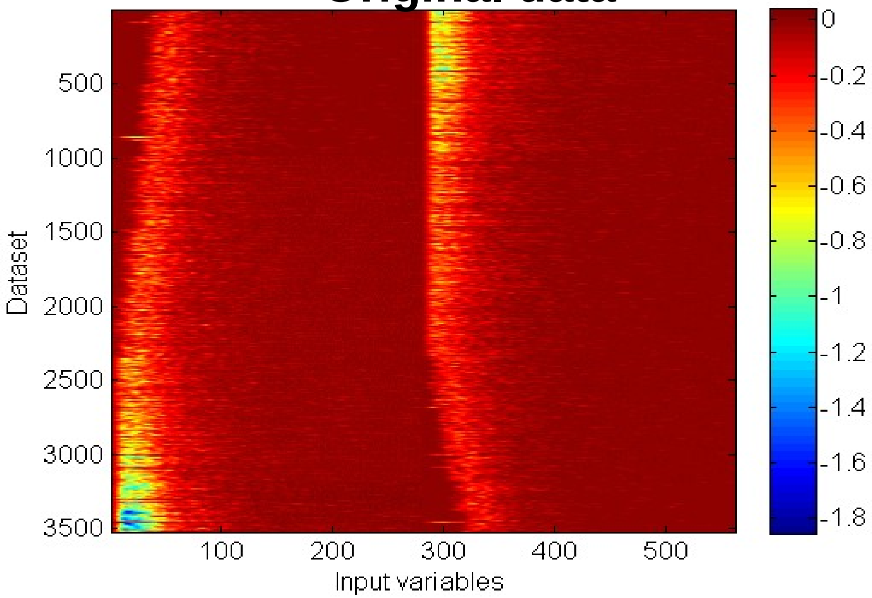


- Decorrelation of the data set with Singular Value Decomposition (SVD)
- Signal ***x***, in the new SVD space, is sparse: the information is concentrated in a first few coefficients (right image)
- The number of signal ***y*** measurements is greater than the sparsity of ***x***

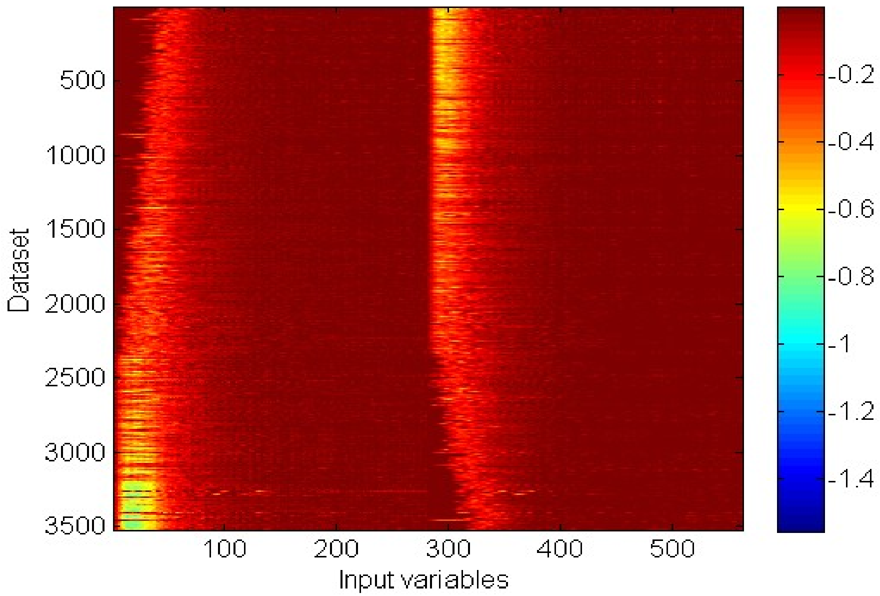
4. Results

- Original, measured signals y are stored in the matrix shown on the left
- Estimation of y based on measurements is 2 step process:
  - Step 1: reconstruction of ***x*** based on ***N*** random measurements of ***y***
  - Step 2: estimation of ***y*** based on ***x*** (presented on the right)
- The mean RMSE (between original and reconstructed signal) is c.a. 0.05 V

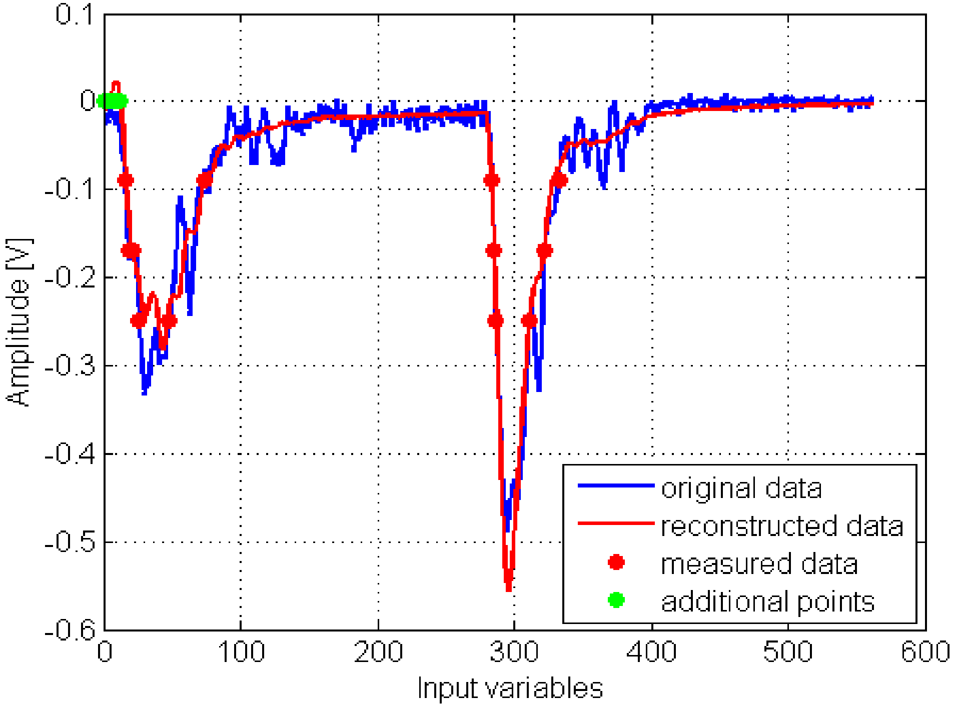
Original data



Reconstructed data



4. Results



N – nr of measurements

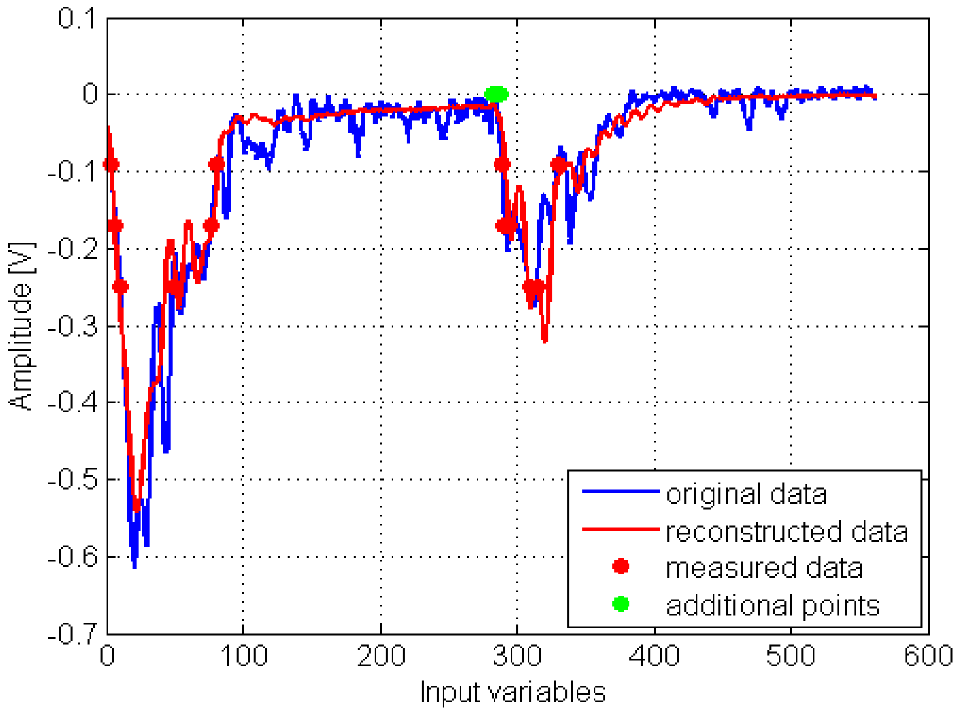
M – length of vector

Example 1:

N = 21 (12 + 9)

M = 562

4. Results



N – nr of measurements

M – length of vector

Example 2:

N = 18 (12 + 6)

M = 562

5. Conclusions

- It is possible to believably reconstruct full time series of the voltage signal using just several measurements
- Reconstruction of signals from the photomultipliers using presented scheme is fast. It should be stressed that this is the case of using a SVD transform and for another transformation it could last much longer
- The analysis of shapes of reconstructed signals may by used to reduce random coincidences and the background
- The conception of the *compressive sensing* is very general and it may be used in solving other problems, where it is possible to find the sparse representation for the original representation of the data

References

[1] http://www.gch.org/Services/Imaging/PET-CT.aspx

[2] http://www.cellsighttech.com/technology/pet.html

[3] http://www.springerimages.com/Images/RSS/1-10.1007_s13244-011-0069-4-4

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Thank you for your attention

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