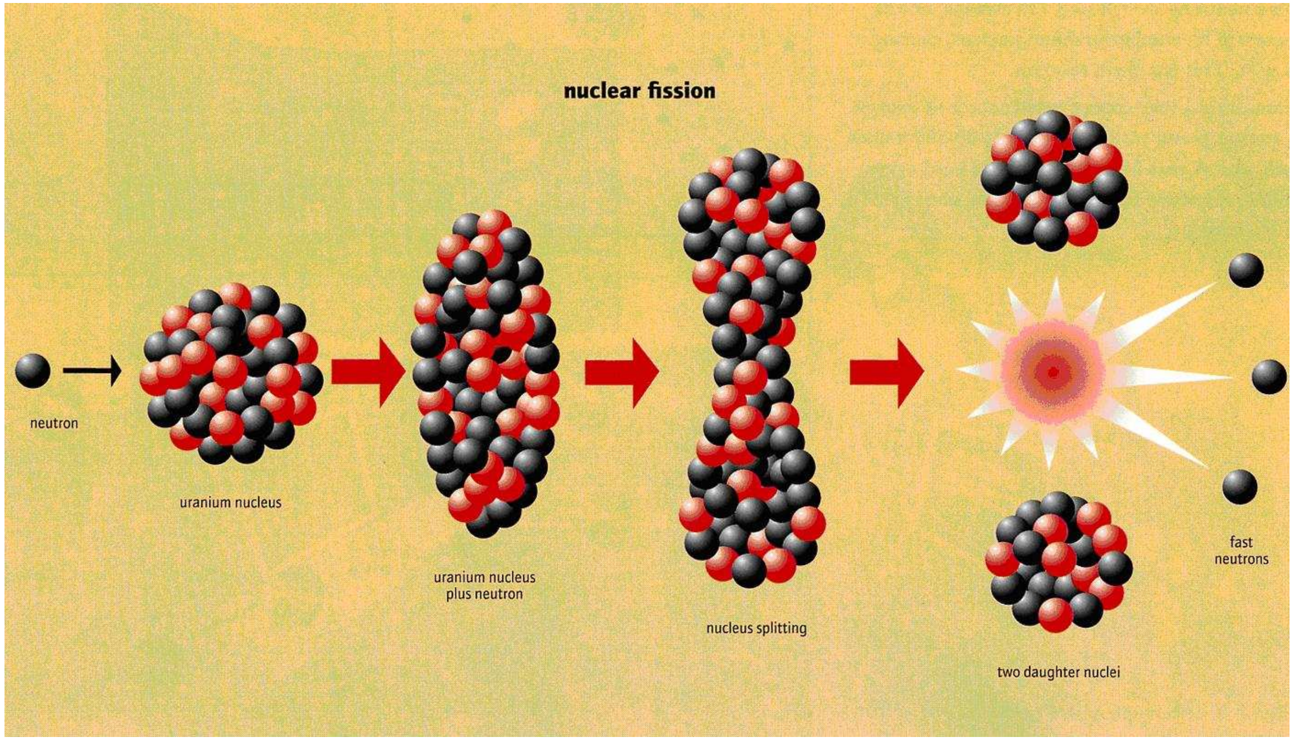
Nuclear fission within the Lublin-Strasbourg Drop model



Krzysztof Pomorski
Maria Curie Skłodowska University, Lublin, Poland

Symposium on applied nuclear physics and innovative technologies

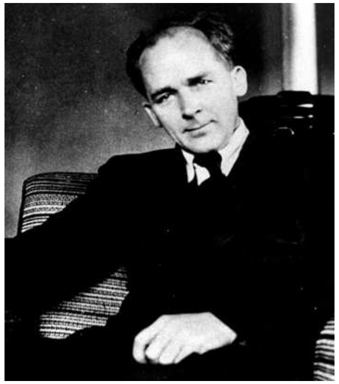
Jagiellonian University, Kraków, June 4th, 2013.

Program:

- Introduction,
- Macroscopic-microscopic model with the Lublin–Strasbourg Drop,
- Optimal in energy shapes of nuclear liquid drop,
- Modified Funny-Hills shape parametrisation versus spherical harmonics expansion,
- Potential energy surfaces and fission barriers,
- Topographical theorem of Świątecki,
- Masses of nuclei in saddle points and the fission barrier heights,
- Effect of the congruence and pairing energies on the barrier heights,
- Fission life-times within the Świątecki's phenomenological model from 1955 combined with the LSD
- Summary.

Discovery of Fission, Winter 1938/39

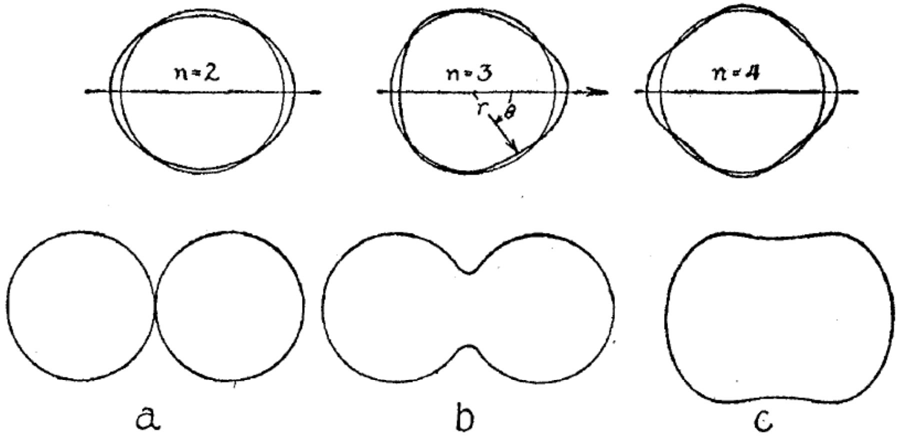
by Otto Hahn, Fritz Strassmann and Lisa Meitner and Otto Frisch

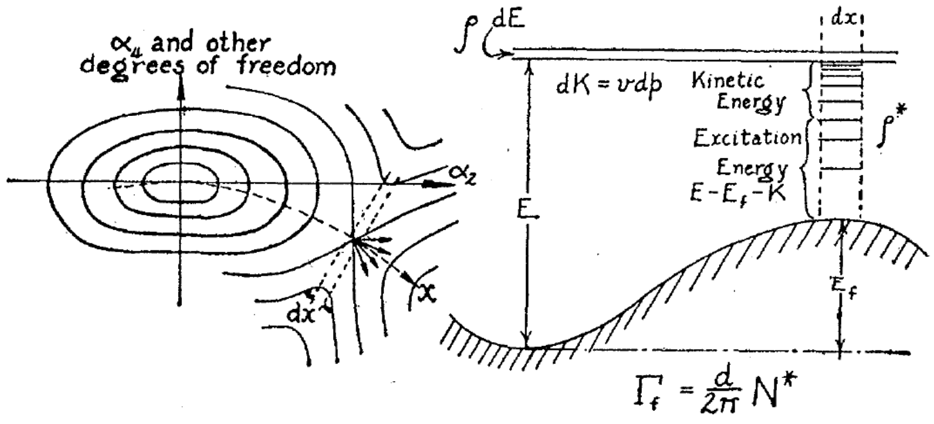



O. Hahn and F. Straßmann, Naturwiss. **27** (1939) 11
L. Meitner and O. R. Frisch, Nature **143** (1939) 239.

C. F. v. Weizsäcker, Z. Phys. **96** (1935) 431.
N. Bohr and A. Wheeler, Phys. Rev. **56** (1939) 426.

Bohr and Wheeler theory:





$$E_{LD} = a_{\rm vol} A + a_{\rm surf} A^{2/3} B_{\rm surf}(\{\alpha\}) + \frac{e^2 Z}{r_0 A^{1/3}} B_{\rm Coul}(\{\alpha\})$$

$$R(\theta) = r_0 A^{1/3} \left(1 + \sum_{l=0}^{\infty} \alpha_l P_l(\theta) \right) \quad ; \qquad x = \frac{E_{\rm Coul}(0)}{2E_{\rm surf}(0)} \approx \frac{Z^2}{50A} \text{MeV}$$

$$B_{\rm surf} = 1 + \frac{2}{5}\alpha_2^2 + \frac{5}{7}\alpha_3^2 + \ldots \quad ; \qquad B_{\rm Coul} = 1 - \frac{1}{5}\alpha_2^2 + \frac{10}{49}\alpha_3^2 + \ldots$$

Leptodermous expansion of the energy functional

The one-body density of the nucleus is given by the integral:

$$\rho = A \int \int \ldots \int \Psi^\star \Psi \, d^3 r_2 \cdots d^3 r_A \ ,$$

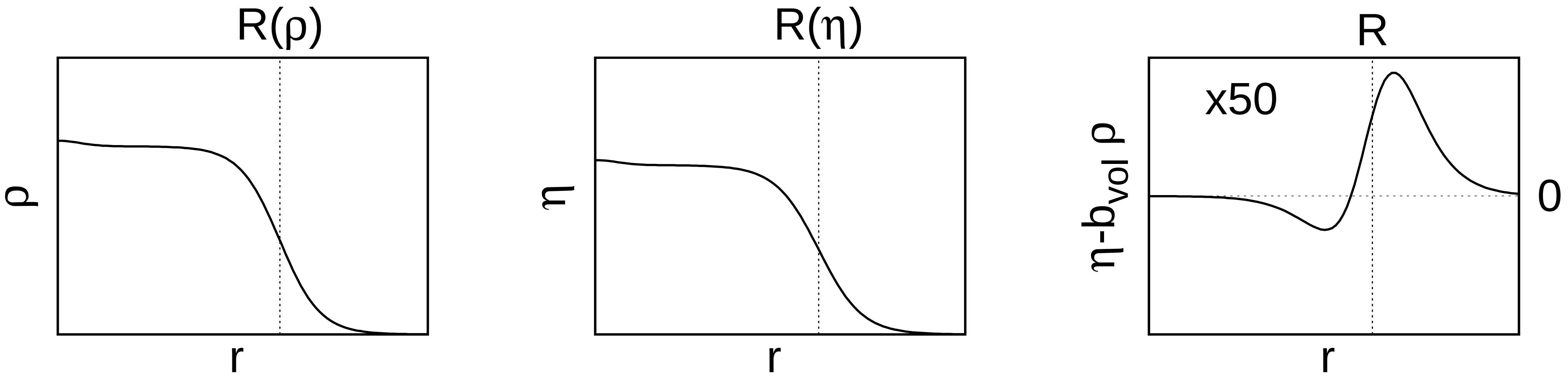
where $\Psi(\xi_1, \xi_2, \ldots, \xi_A)$ is the many-body wave function and $A = \int \rho d^3 r$.

Similarly one defines the energy density:

$$\eta = - \int \int \ldots \int \Psi^\star \hat{H} \Psi \, d^3 r_2 \cdots d^3 r_A \ .$$

The total energy of the nucleus is:

$$B = \int_V \eta \, dV = \int_V [\eta - b_{\mathrm{vol}}(\rho - \rho)] \, dV$$



This energy integral can be decomposed in the following way:

$$
\begin{aligned}
B &= b_{\mathrm{vol}} A + \int\limits_V (\eta - b_{\mathrm{vol}}\,\rho)\, dV \\
&= b_{\mathrm{vol}} A + \int\limits_\Sigma d\sigma \int\limits_{\{-\infty\}}^{\{\infty\}} (\eta - b_{\mathrm{vol}}\,\rho)\, dr_\perp \\
&= b_{\mathrm{vol}} A + \gamma^{(0)} \int\limits_\Sigma d\sigma + \gamma'_\kappa\, a \int_\Sigma \kappa\, d\sigma \\
&+ \frac{1}{2}\gamma''_{\kappa\kappa}\, a^2 \int\limits_\Sigma \kappa^2 d\sigma + \gamma'_\Gamma\, a^2 \int\limits_\Sigma \Gamma\, d\sigma + \ldots\ ,
\end{aligned}
$$

where κ and Γ are the first order and the second order (Gauss) curvatures respectively:

$$\kappa = \frac{1}{R_1} + \frac{1}{R_2} \quad \text{and} \quad \Gamma = \frac{1}{R_1 \cdot R_2}\,.$$

Here R_1 and R_2 are the local main radii of the surface.

Spherical case:

The nuclear part of the total energy of a spherical nucleus can thus be written down as

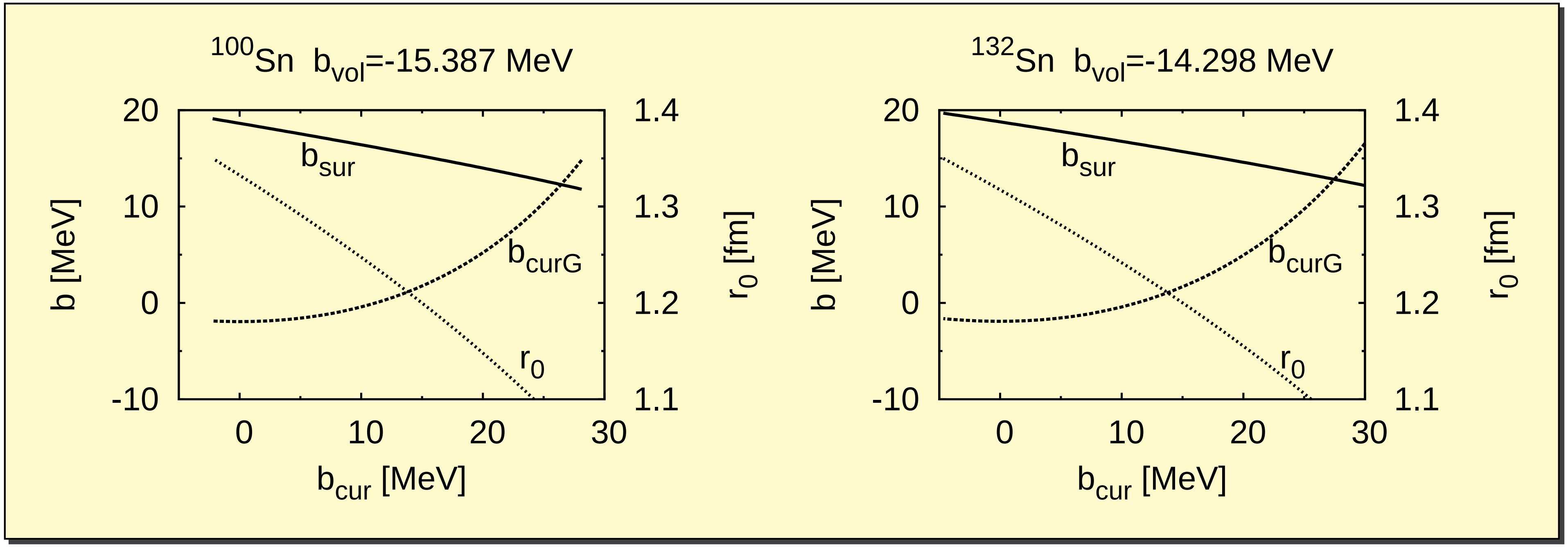
$$B = b_{\mathrm{vol}} A + \underbrace{4\pi R^2 \cdot \mathcal{I}_0}_{b_{\mathrm{surf}} A^{2/3}} + \underbrace{8\pi R \cdot (\mathcal{I}_1 - \mathcal{I}_0 R)}_{b_{\mathrm{cur}} A^{1/3}} + \underbrace{4\pi \cdot (\mathcal{I}_2 - 2R\mathcal{I}_1 + R^2 \mathcal{I}_0)}_{b_{\mathrm{curG}} A^0} \; ,$$

where

$$\mathcal{I}_n = \int_0^\infty (\eta - b_{\mathrm{vol}} \rho) \, r^n dr \; ,$$

Note, that the radius $R = r_0 A^{1/3}$ is not fixed here and in some reasonable limits can be treated as a free expansion parameter, while $\mathcal{I}_n$ are well defined by the both density distributions.

Leptodermous expansion of the ETF-Skyrme energy:



$$B = \int_{\mathrm{V}} \eta \, d^3\mathrm{r} = b_{\mathrm{vol}} A + b_{\mathrm{surf}} A^{2/3} + b_{\mathrm{curv}} A^{1/3} + b_{\mathrm{curG}} A^0 + \ldots$$

The magnitudes of the surface and curvature terms depend on the choice of the expansion radius!

Macroscopic – Microscopic Model*:

$$
\begin{aligned}
M(Z,N;\mathrm{def}) = {} & Z M_{\mathrm{H}} + N M_{\mathrm{n}} - b_{\mathrm{elec}}\, Z^{2.39} \\
& + b_{\mathrm{vol}}\ (1 - \kappa_{\mathrm{vol}}\ I^2\,)\ A \\
& + b_{\mathrm{surf}}\,(1 - \kappa_{\mathrm{surf}} I^2\,)\ A^{2/3} B_{\mathrm{surf}}(\mathrm{def}) \\
& + b_{\mathrm{cur}}\ (1 - \kappa_{\mathrm{cur}}\ I^2\,)\ A^{1/3} B_{\mathrm{cur}}(\mathrm{def}) \quad \longleftarrow \quad \text{new term} \\
& + \frac{3}{5}\,\frac{e^2 Z^2}{r_0^{ch} A^{1/3}}\, B_{\mathrm{Coul}}(\mathrm{def}) - \ C_4\, \frac{Z^2}{A} \\
& + E_{\mathrm{micr}}(Z,N;\mathrm{def}) + E_{\mathrm{cong}}(Z,N)
\end{aligned}
$$

where the microscopic energy is the sum of the shell and pairing energies:

$$
E_{\mathrm{micr}} = \delta E_{\mathrm{shell}} + \delta E_{\mathrm{pair}}
$$

and the congruence energy is†: $E_{\mathrm{cong}} = -10 \cdot e^{-4.2|I|}\mathrm{MeV}$

* W.D. Myers and W.J. Świątecki, Nucl. Phys. **81**, 1 1966.

† P. Möller, J.R. Nix, W.D. Myers, W.J. Świątecki, At. Data Nucl. Data Tab. **59**, 185 (1995).

Lublin Strasbourg Drop *

Fit to the 2766 experimental masses† with $Z \geq 8$ and $N \geq 8$:

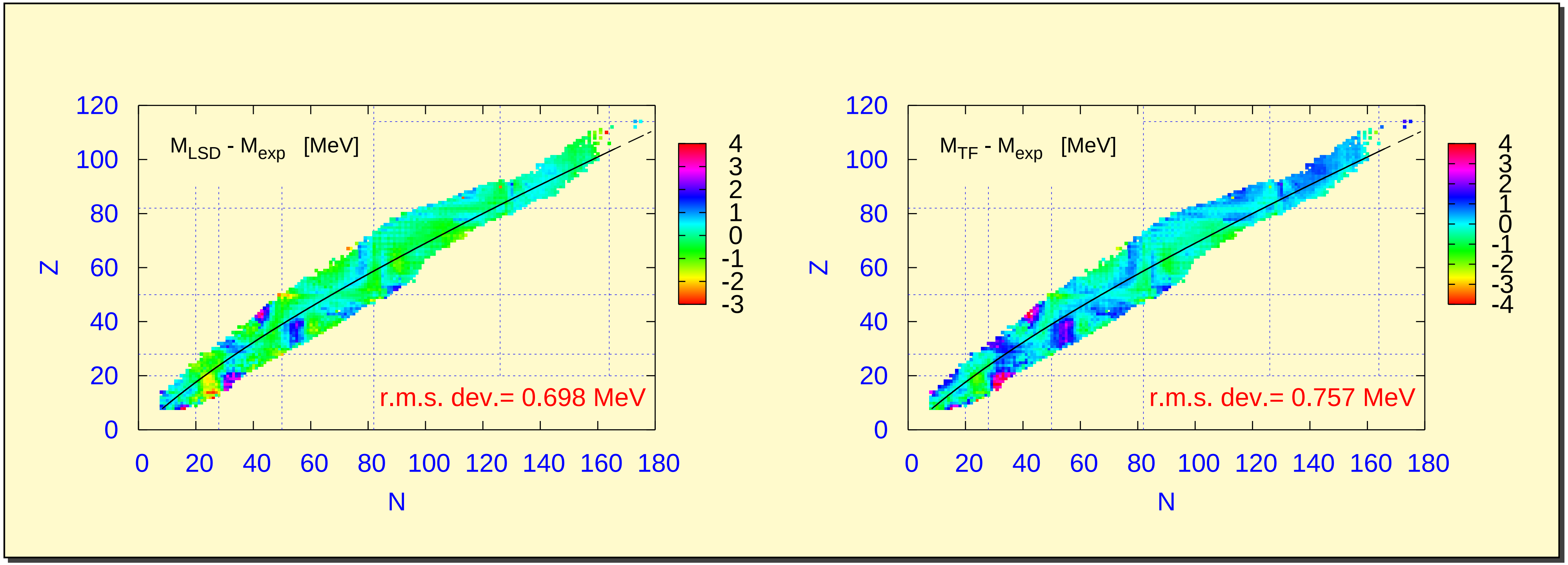
Term	Units	LDM	LSD
$b_{\rm vol}$	MeV	-15.8484	-15.4920
$\kappa_{\rm vol}$	-	1.8475	1.8601
$b_{\rm surf}$	MeV	19.3859	16.9707
$\kappa_{\rm surf}$	-	1.9830	2.2938
$b_{\rm cur}$	MeV	0	3.8602
$\kappa_{\rm cur}$	-	0	-2.3764
r_0	fm	1.18995	1.21725
C_4	MeV	1.1995	0.9181
δM	MeV	0.732	0.698
$\delta V_{B_{Z>70}}$	MeV	5.58	0.88

$M_{\rm H}$=7.289034 MeV; $M_{\rm n}$=8.071431 MeV; $b_{\rm elec}$=1.433 eV

*K. Pomorski , J. Dudek, Phys. Rev. **C67**, 044316 (2003).

†Chart of Nuclides by M.S. Antony, Strasbourg, 2002.

Masses of isotopes:



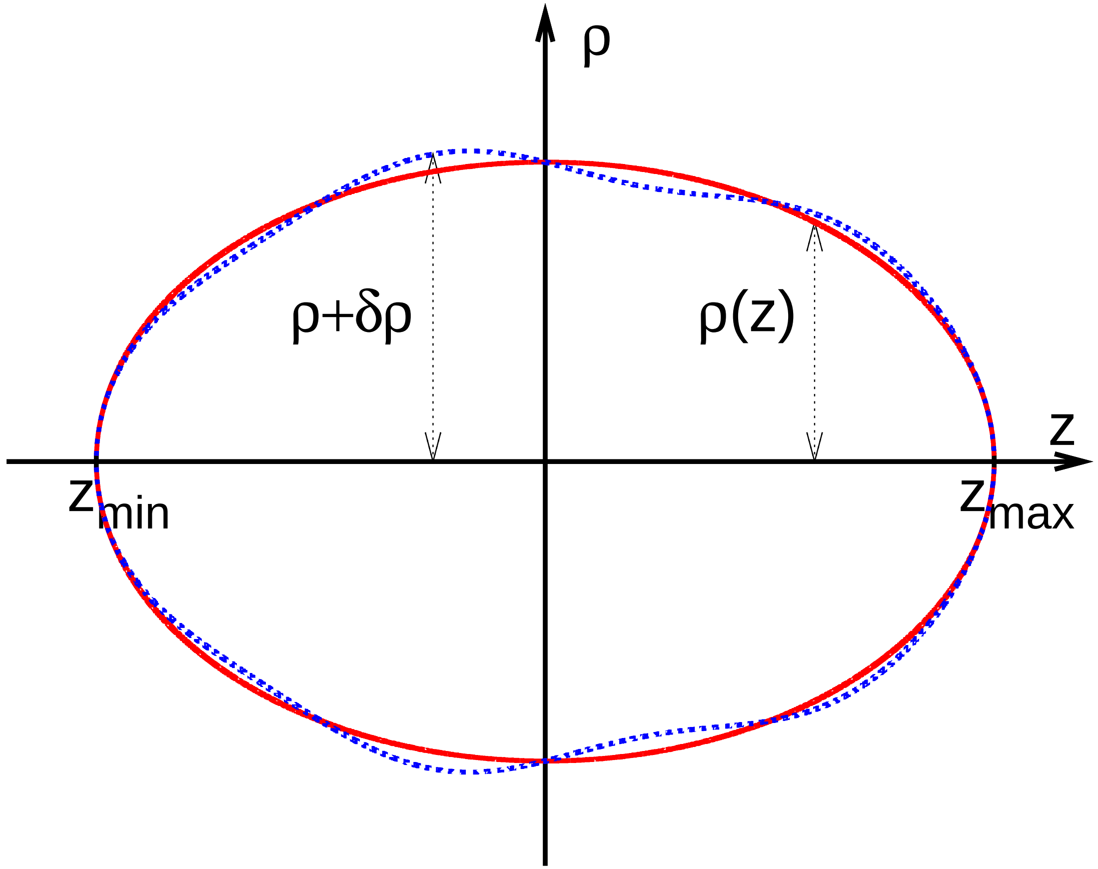
exp: The mass deviations are plotted for 2766 isotopes taken from the

. Chart of Nuclides by M.S. Antony, Strasbourg, 2002.

LSD: K. Pomorski, J. Dudek, Phys. Rev. **C67**, 044316 (2003).

TF: P. Möller, J.R. Nix, W.D. Myers, W.J. Świątecki, ADNDT **59**, 185 (1995).

Strutinsky's optimal shape procedure*:

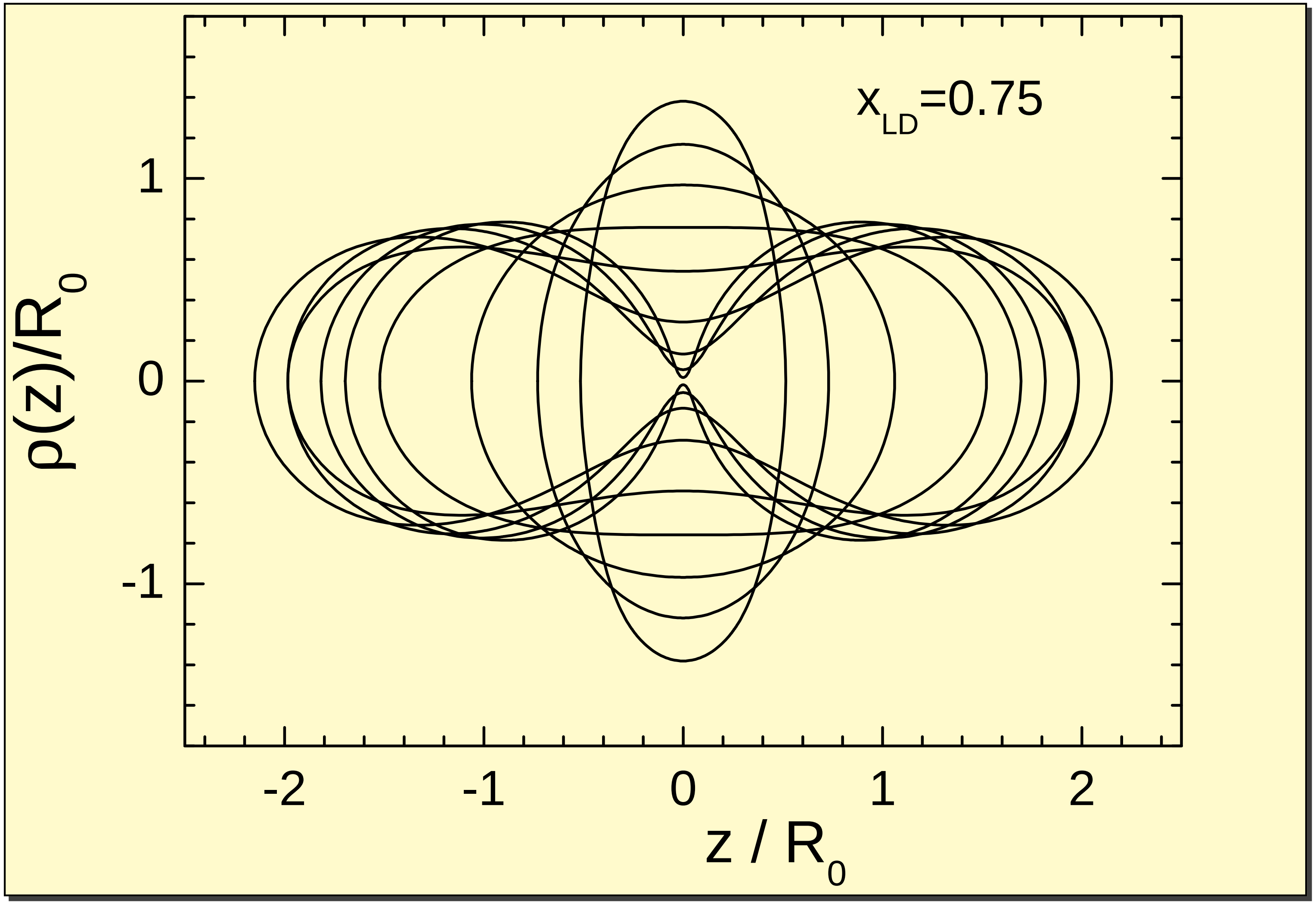


One looks here for the minimum of the LD energy with additional constraints for the volume of nucleus, its elongation, mass asymmetry and nonaxiality.
This variational problem leads to the Eular-Lagrange equations for $\rho(z)$.

* V. M. Strutinsky et al. Nucl. Phys. **46**, (1963) 659.

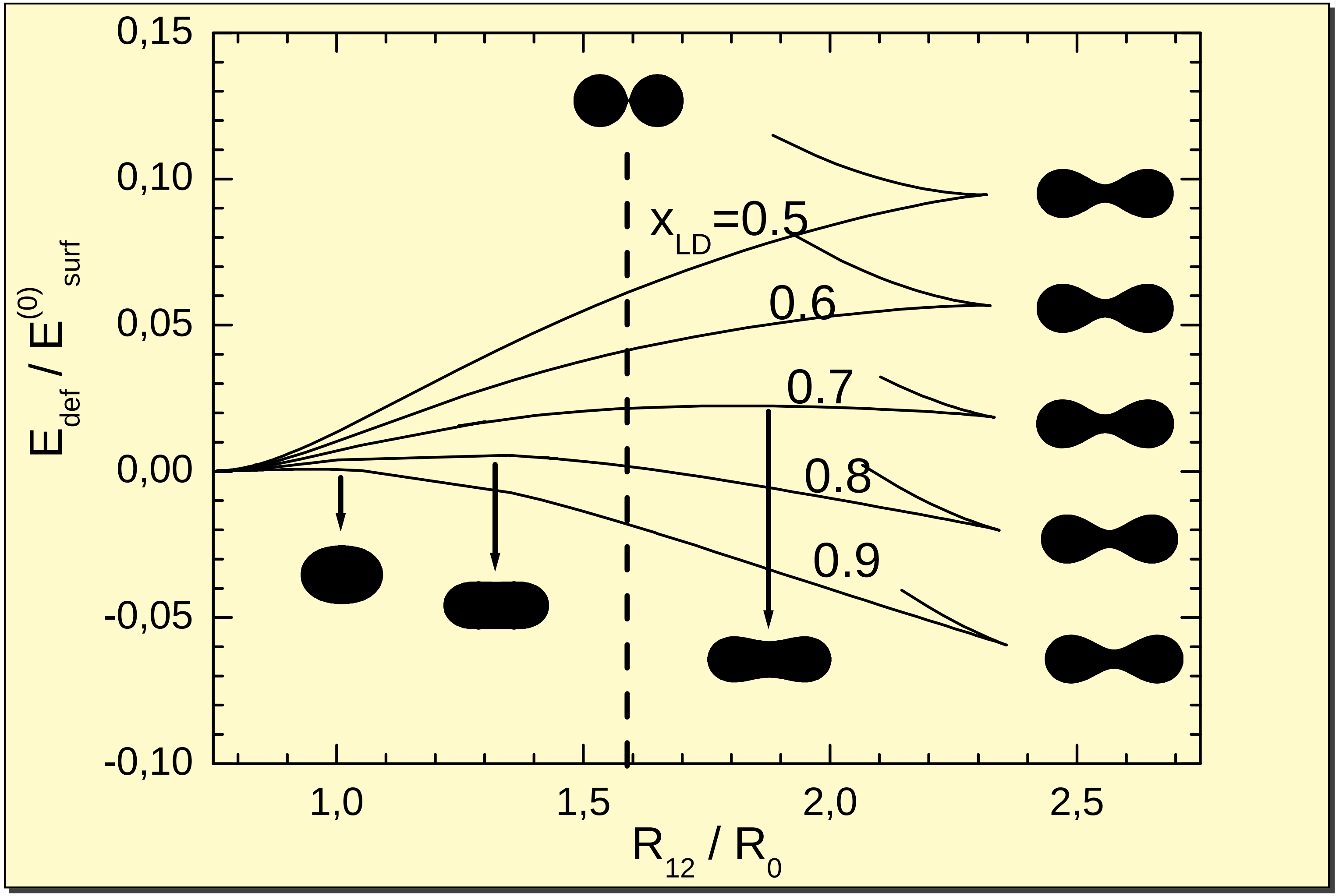
A. Ivanyuk and K. Pomorski, Phys. Rev. C **79**, (2009) 054327.

Optimal shapes for different elongations*:



*A. Ivanyuk and K. Pomorski, Phys. Rev. C **79**, (2009) 054327.

Optimal fission barriers:



A. Ivanyuk and K. Pomorski, Phys. Rev. C **79**, (2009) 054327.

Examples of shape parametrization:

- Original Funny-Hills parametrisation*:

$$\tilde{\rho}_s^2(z) = \begin{cases} R_0^2 c^2 \left(1-u^2\right)\left(A+\alpha u+Bu^2\right), & \text{for} \quad B \geq 0 \\ R_0^2 c^2 \left(1-u^2\right)\left(A+\alpha u\right)\exp\left(Bc^3u^2\right) & \text{for} \quad B \leq 0\,, \end{cases}$$

where $\mathbf{u} = (\mathbf{z}-\mathbf{z}_{\rm sh})/\mathbf{z}_0$, $\mathbf{z}_0 = \mathbf{c}\,\mathbf{R}_0$, and $z_{sh} = -c^3\alpha z_0/5$.

- Modified Funny-Hills (with Gaussian neck) parametrisation †:

$$\tilde{\rho}_s^2\,(z,\varphi) = \frac{R_0^2}{c\,f(a,B)}\,(1-u^2)\,(1-B\,e^{-w^2(u-\alpha)^2})\frac{\sqrt{1-\eta^2}}{1+\eta\cos(2\varphi)}\,.$$

The function $f(a,B,\alpha)$ ensures the volume conservation, while the deformation parameters: $\mathbf{c}, \mathbf{B}, \boldsymbol{\alpha}$ **and** $\boldsymbol{\eta}$, describe respectively: elongation, neck, mass-asymmetry, and nonaxiality of the nucleus.

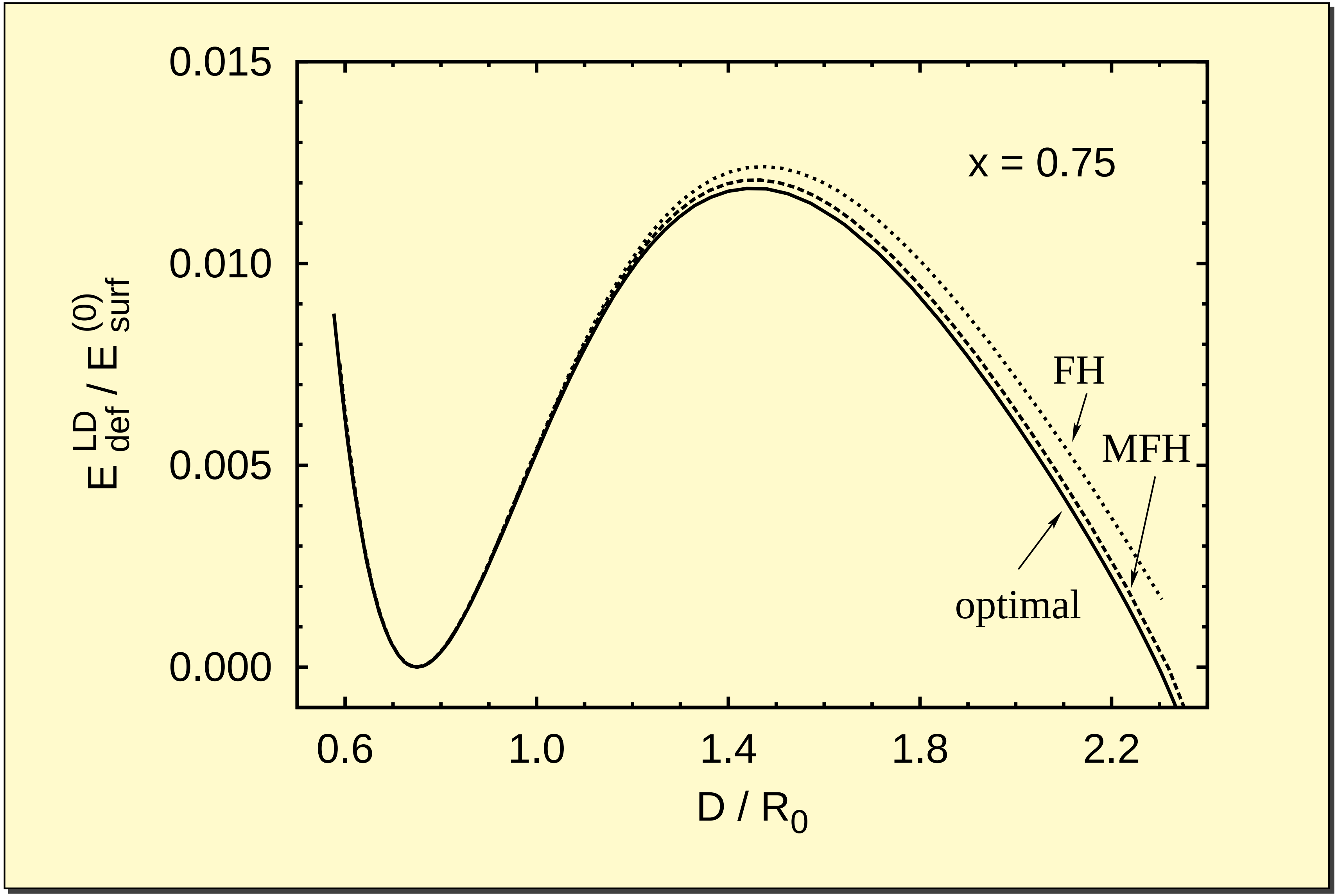
- Expansion in spherical harmonics:

$$R(\theta) = R_0 \sum_{\lambda=0}^{\lambda_{max}} \beta_\lambda P_\lambda[\cos(\theta)]\,.$$

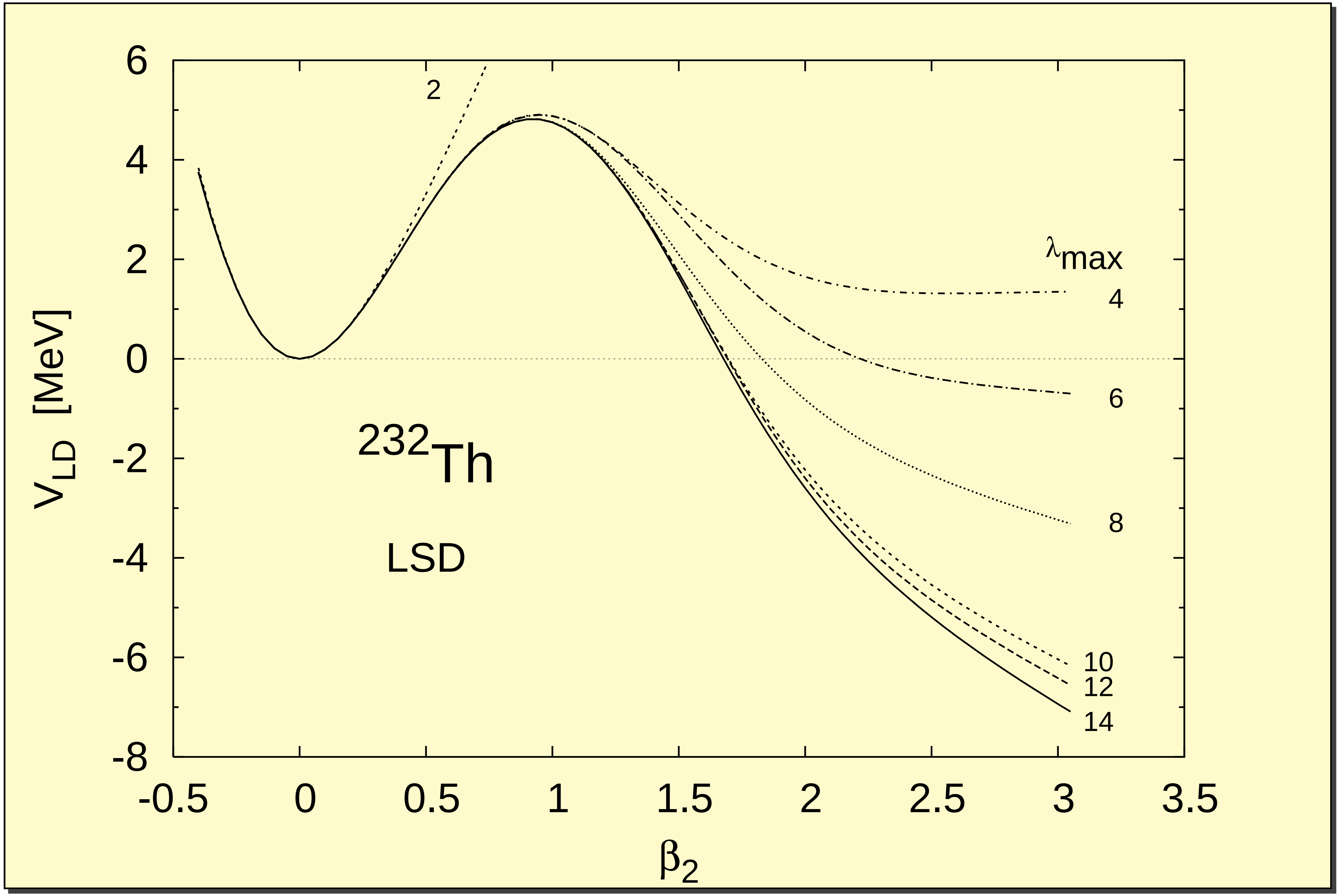
*M. Brack et al., Rev. Mod. Phys. **44** (1972) 320

†J. Bartel, F. Ivanyuk and K. Pomorski, Int. J. Mod. Phys **E19** (2010) 601.

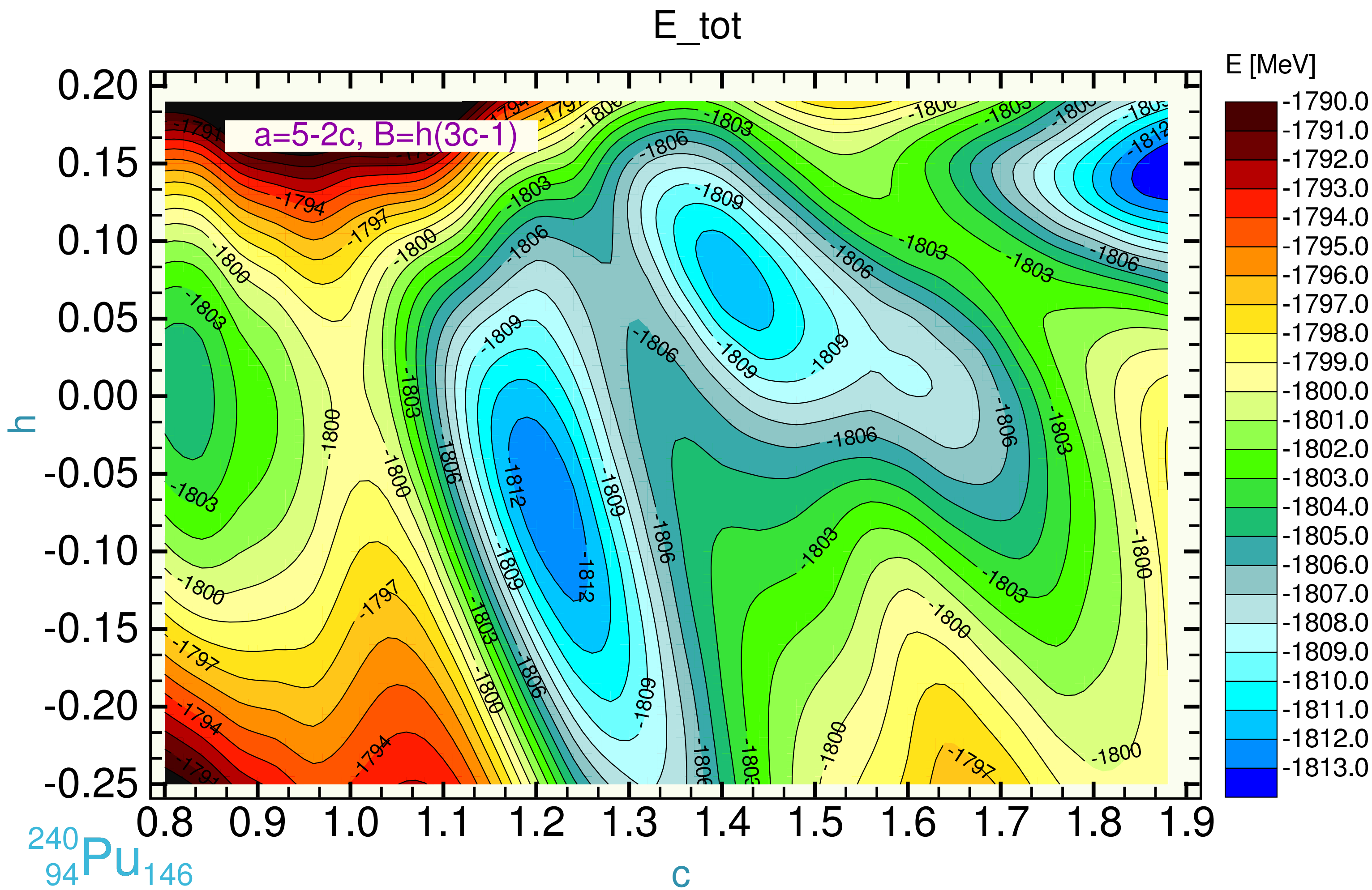
Optimal versus FH and MFH barriers:



Fission barriers in the spherical harmonics expansion:



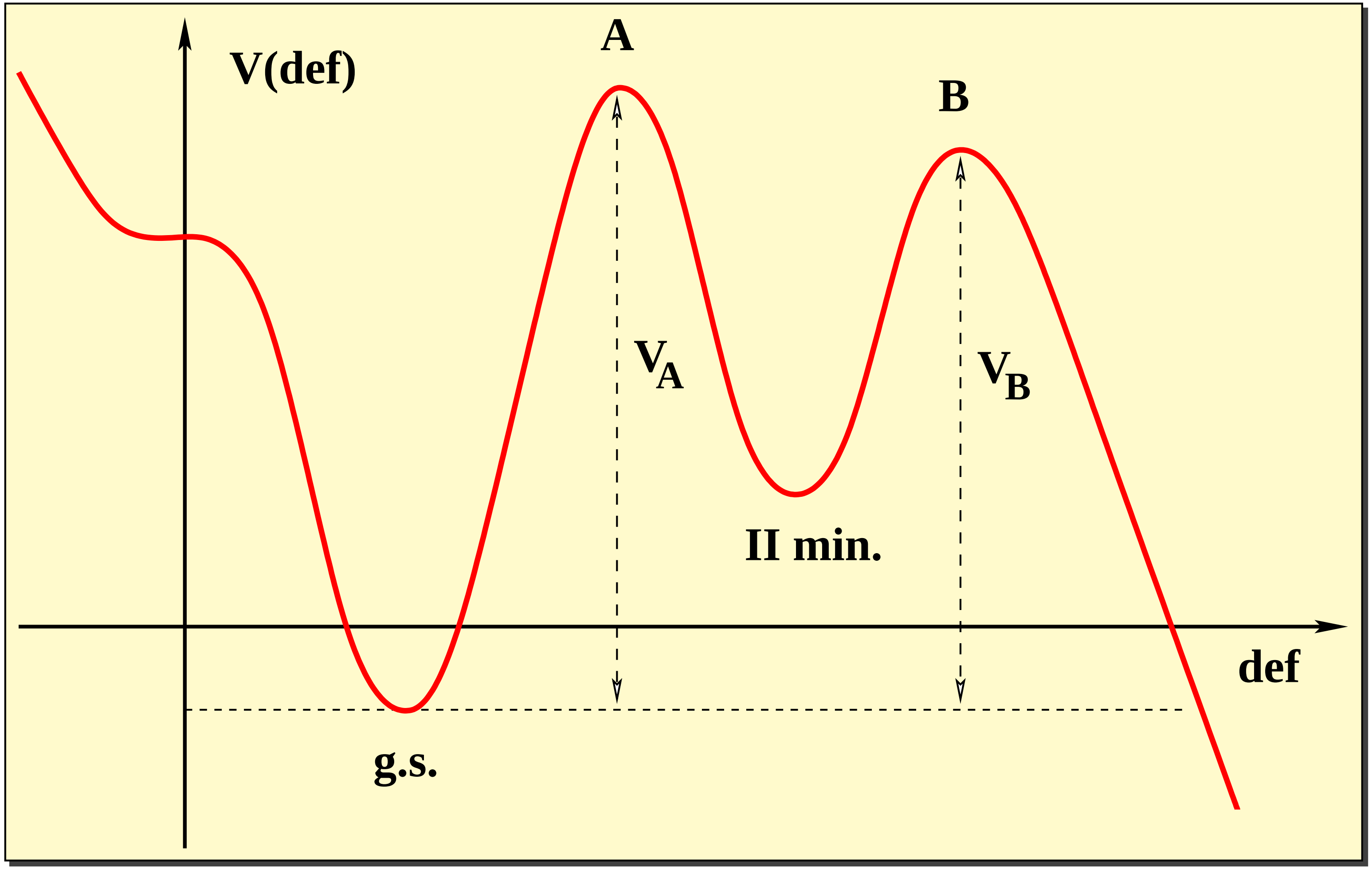
PES of ^{240}Pu on the (c,h) plane for α=0, η=0:



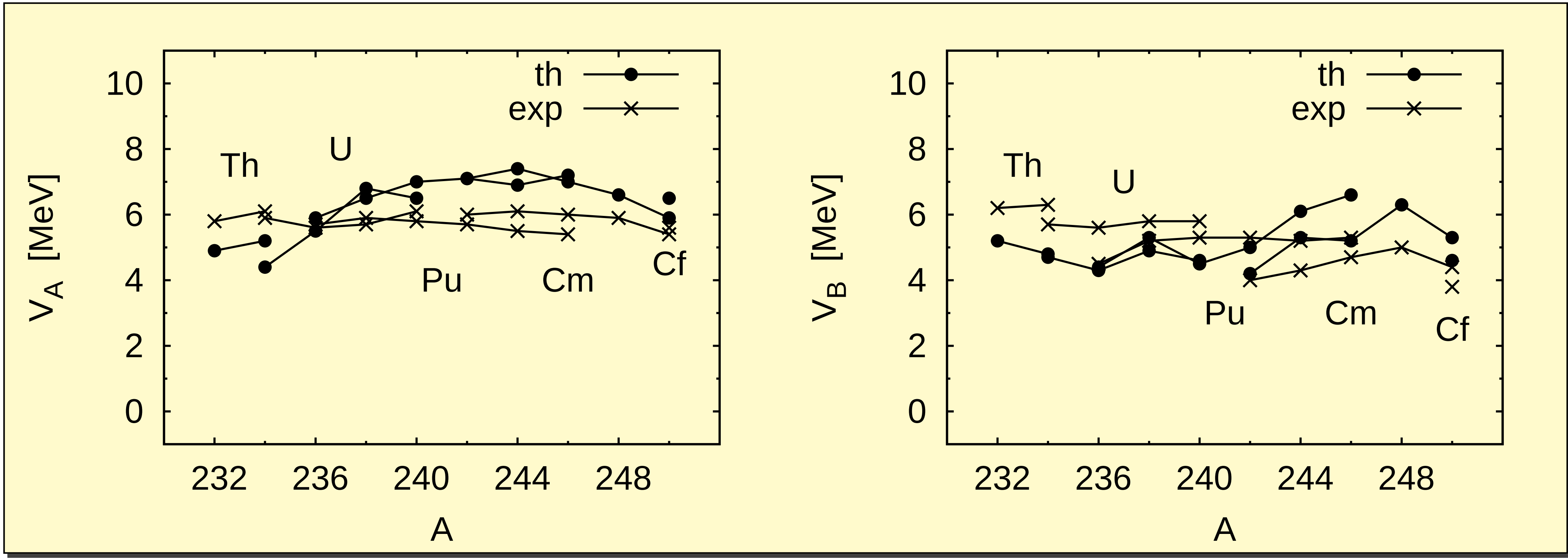
LSD with shell and pairing corrections are obtained with the Yukawa-folded single-particle potential.

A. Dobrowolski, K. Pomorski, J. Bartel, Phys. Rev. **C75**(2007) 024613.

Schematic plot of a fission barrier



LSD + YF barrier heights*



$$V_{\rm A} = M(Z, A; \mathbf{def_A}) - M(Z, A; \mathbf{def_{gs}})$$

and

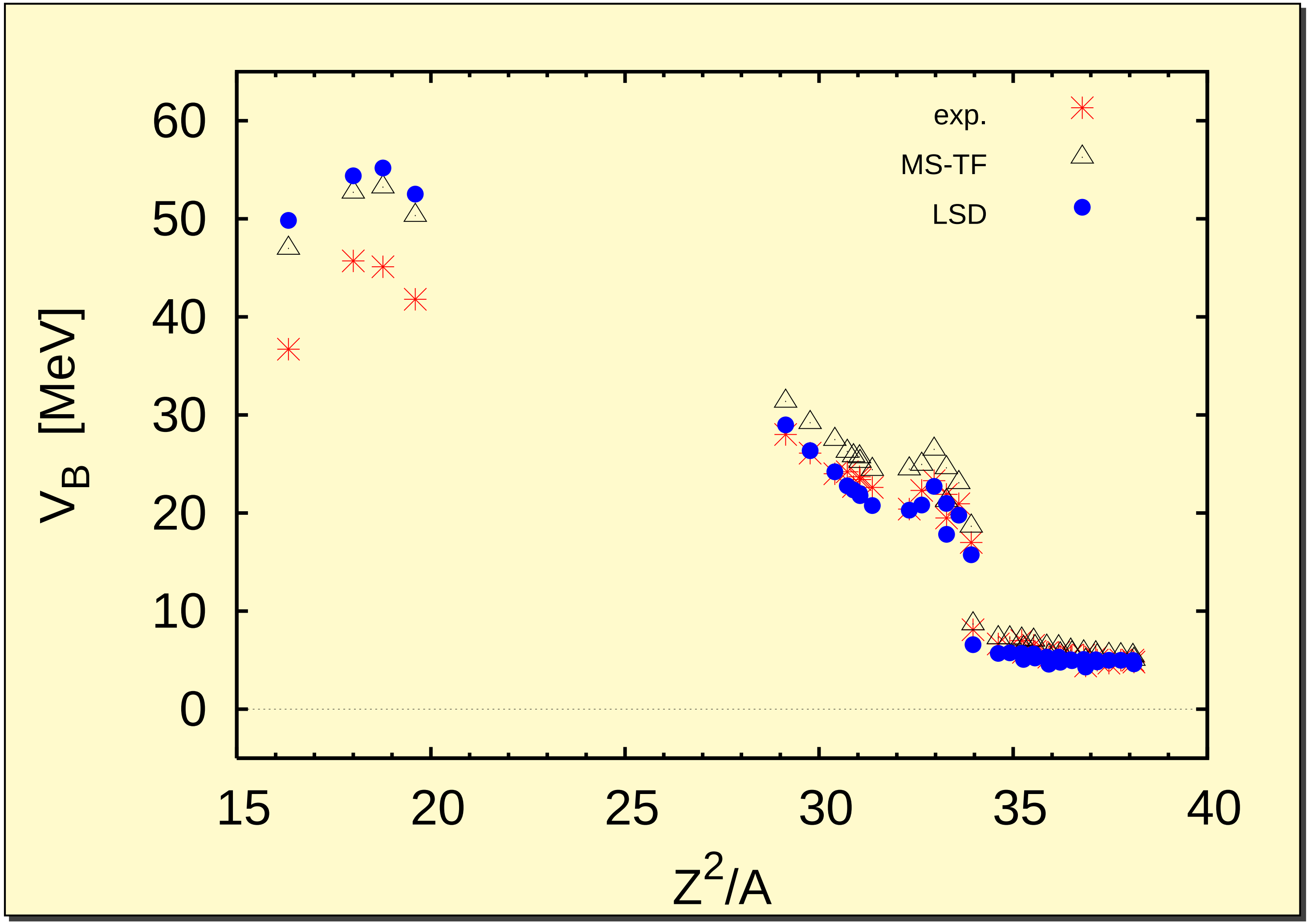
$$V_{\rm B} = M(Z, A; \mathbf{def_B}) - M(Z, A; \mathbf{def_{gs}})$$

where $\mathbf{def_{gs}}$, $\mathbf{def_A}$ and $\mathbf{def_B}$ correspond to the ground-state, first and second saddle deformation, respectively.

* A. Dobrowolski, K. Pomorski, J. Bartel, Phys. Rev. **C75** (2007) 024613.


Topographical theorem of Swiatecki

LSD and TF fission barrier heights



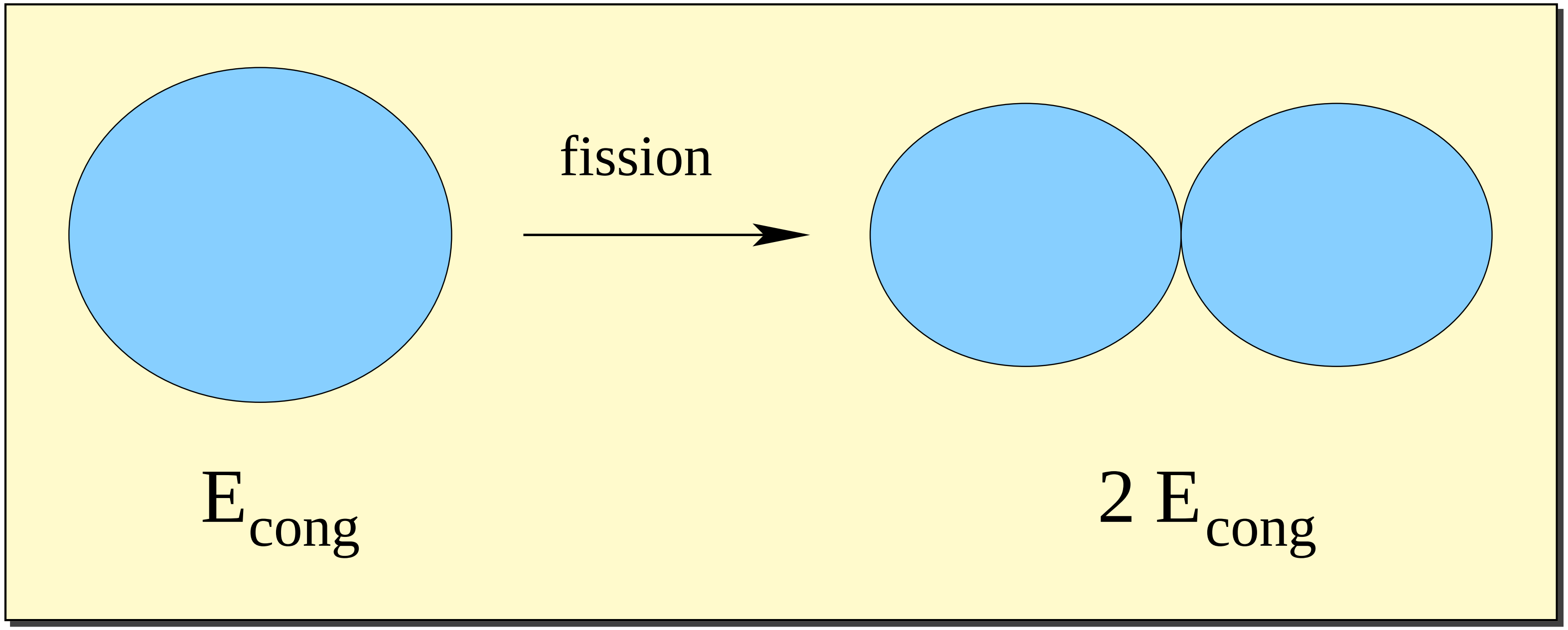
Deformation dependent congruence energy term is not included here!

K. Pomorski, J. Dudek: Phys. Rev. **C67**, 044316 (2003).

W. D. Myers, W. J. Świątecki, Nucl.Phys. **A601**, 141 (1996). $\longleftarrow$ The Thomas-Fermi model

Deformation dependent congruence energy:

$$E_{\rm cong} = -10 \cdot e^{-4.2|I|} \;\; \text{MeV}$$

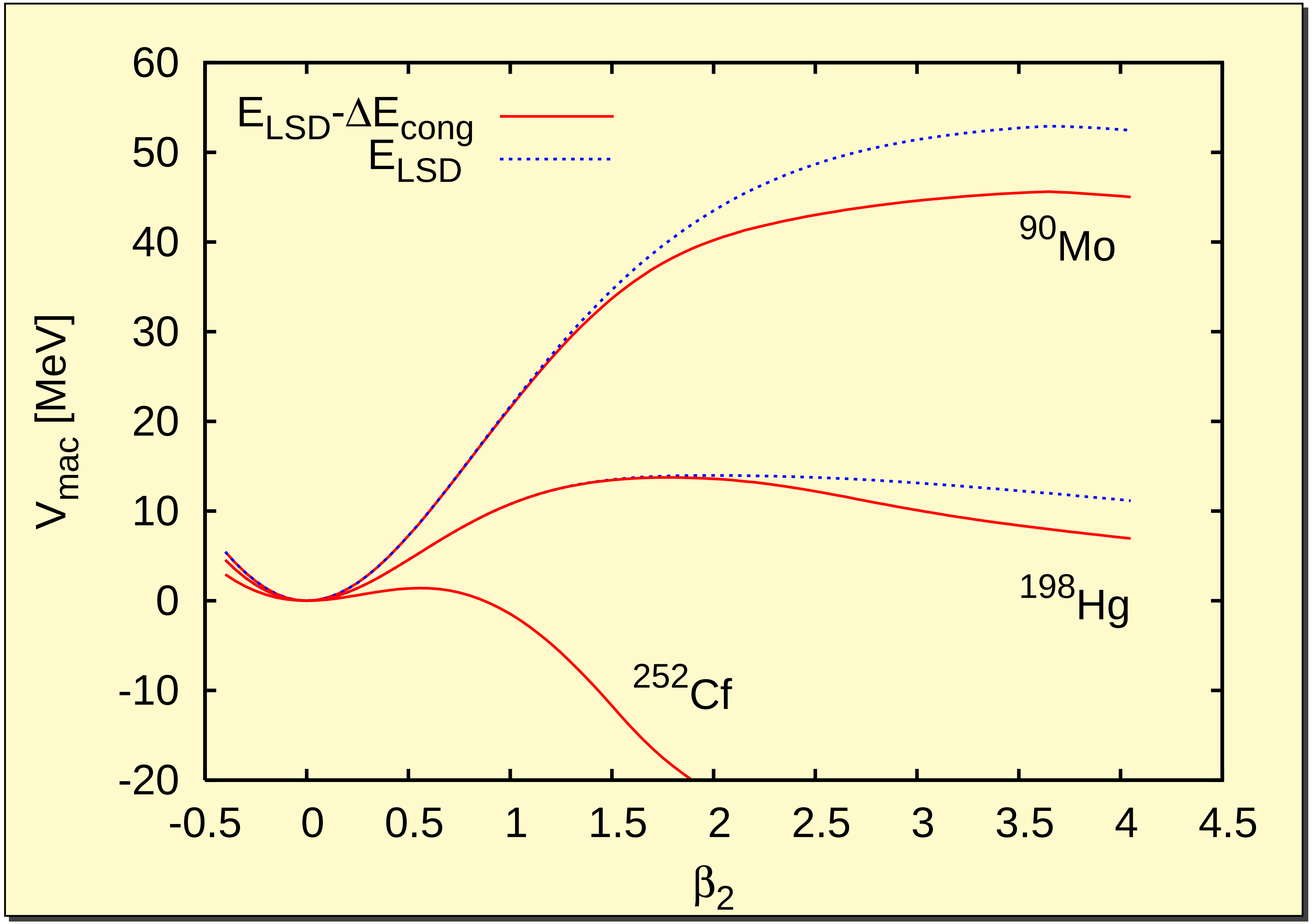


$$V_{\rm macr}(\text{def}) = E_{\rm LSD}(\text{def}) + \frac{E_{\rm cong}}{1 + e^{\frac{r_{neck}-a}{b}}}$$

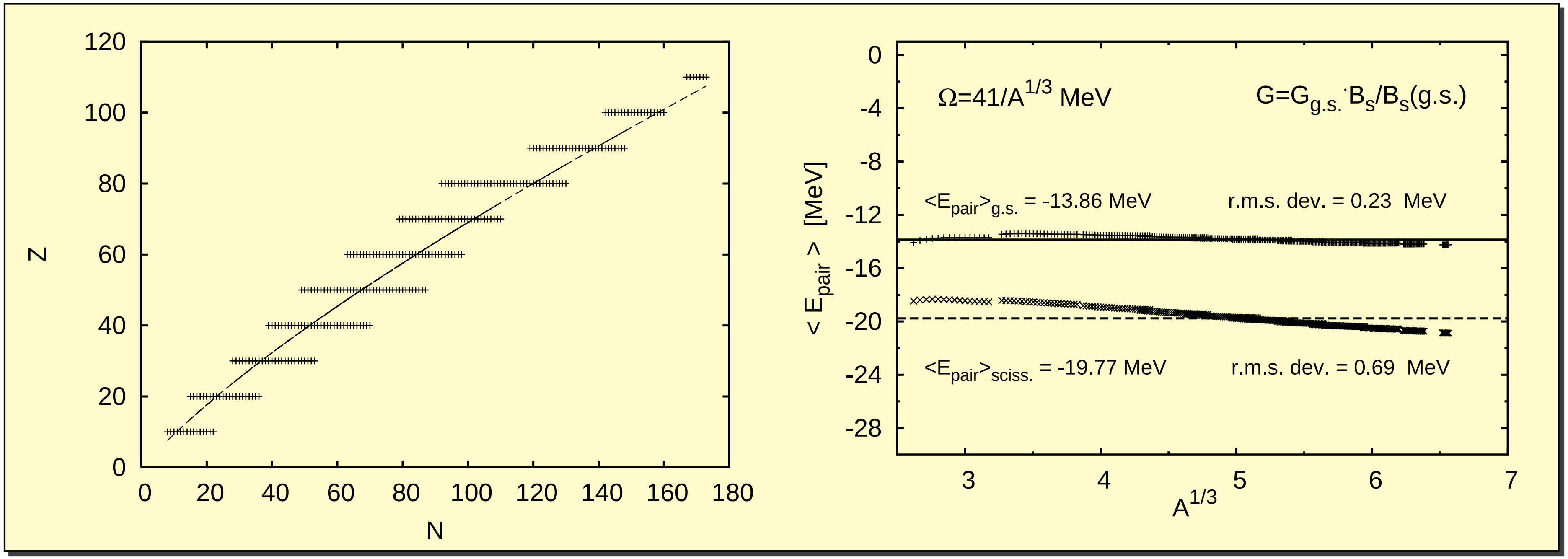
with a=2 fm and b=0.3 fm.

W. D. Myers, W.J. Świątecki, Nucl.Phys. **A612** (1997) 249.

Effect of the congruence energy on fission barriers



Average pairing energy*:



The pairing strength was evaluated using the average experimental gaps

$$\bar{\Delta}^{(p)}_{\rm exp} = \frac{4.8\, B_s}{Z^{1/3}} {\rm MeV} \;; \qquad \bar{\Delta}^{(n)}_{\rm exp} = \frac{4.8\, B_s}{N^{1/3}} {\rm MeV}$$

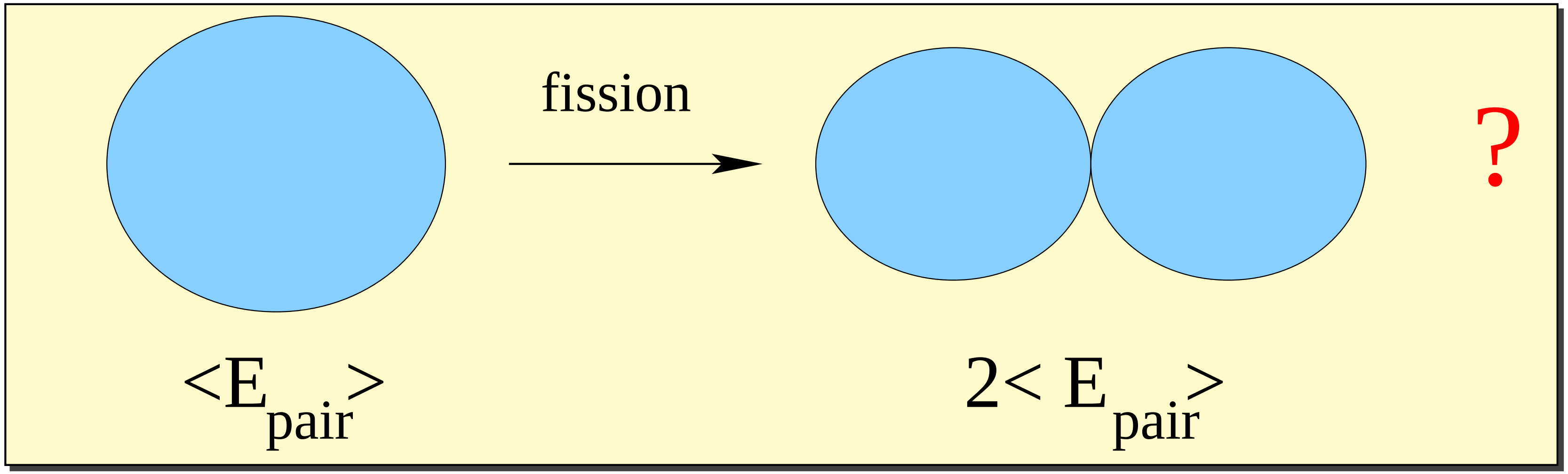
taken from Ref. {P. Möller and J.R. Nix, Nucl. Phys. A536 (1992) 61 }.

*K. Pomorski, F. Ivanyuk, Int. Journ. Mod. Phys. **E18** (2009) 900.

Average pairing energy is A almost independent !!

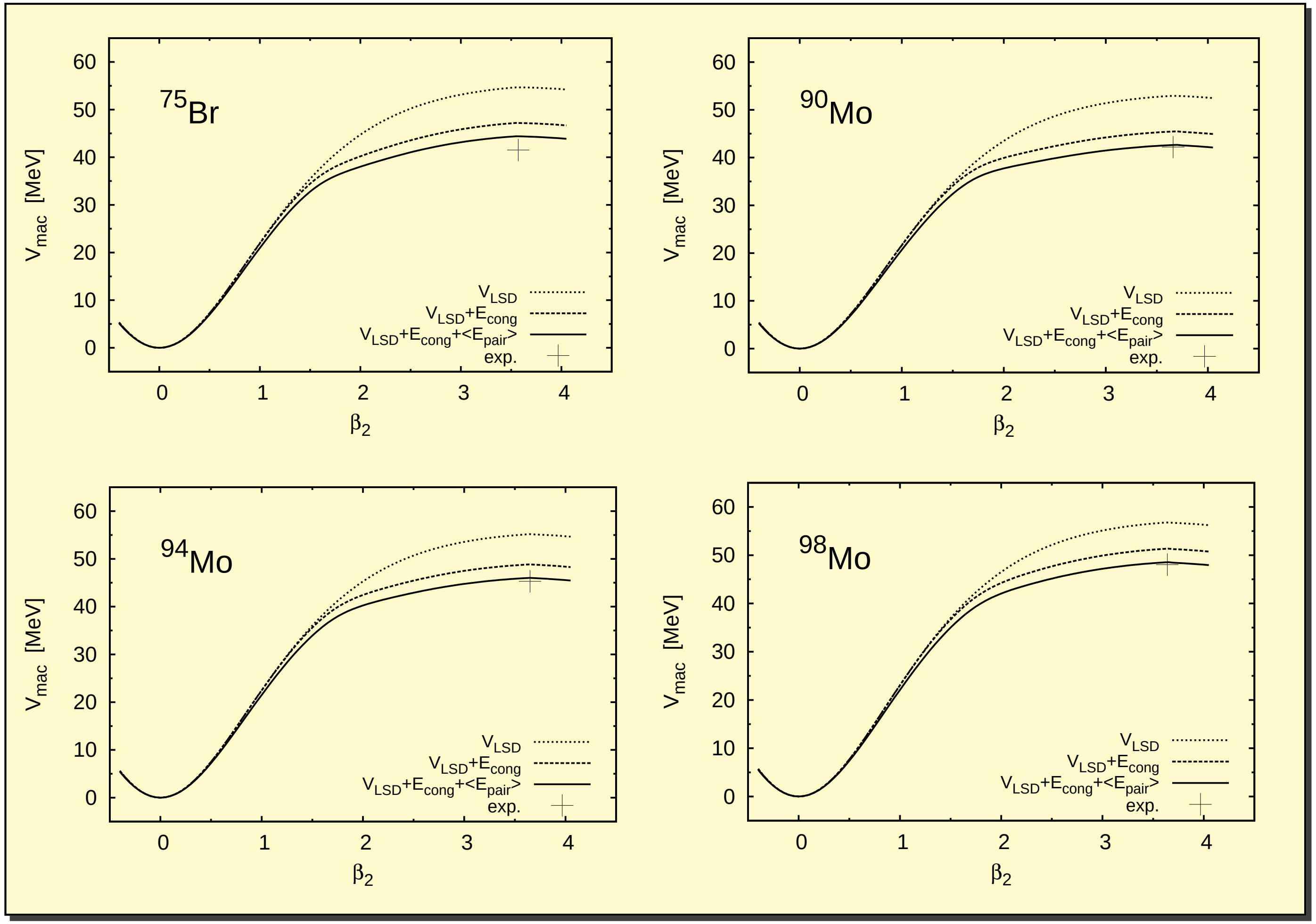
- What does it mean?

- What will happen with the pairing energy when nucleus fission into two fragments?



- Should the pairing strength depend on deformation of fissioning nucleus?

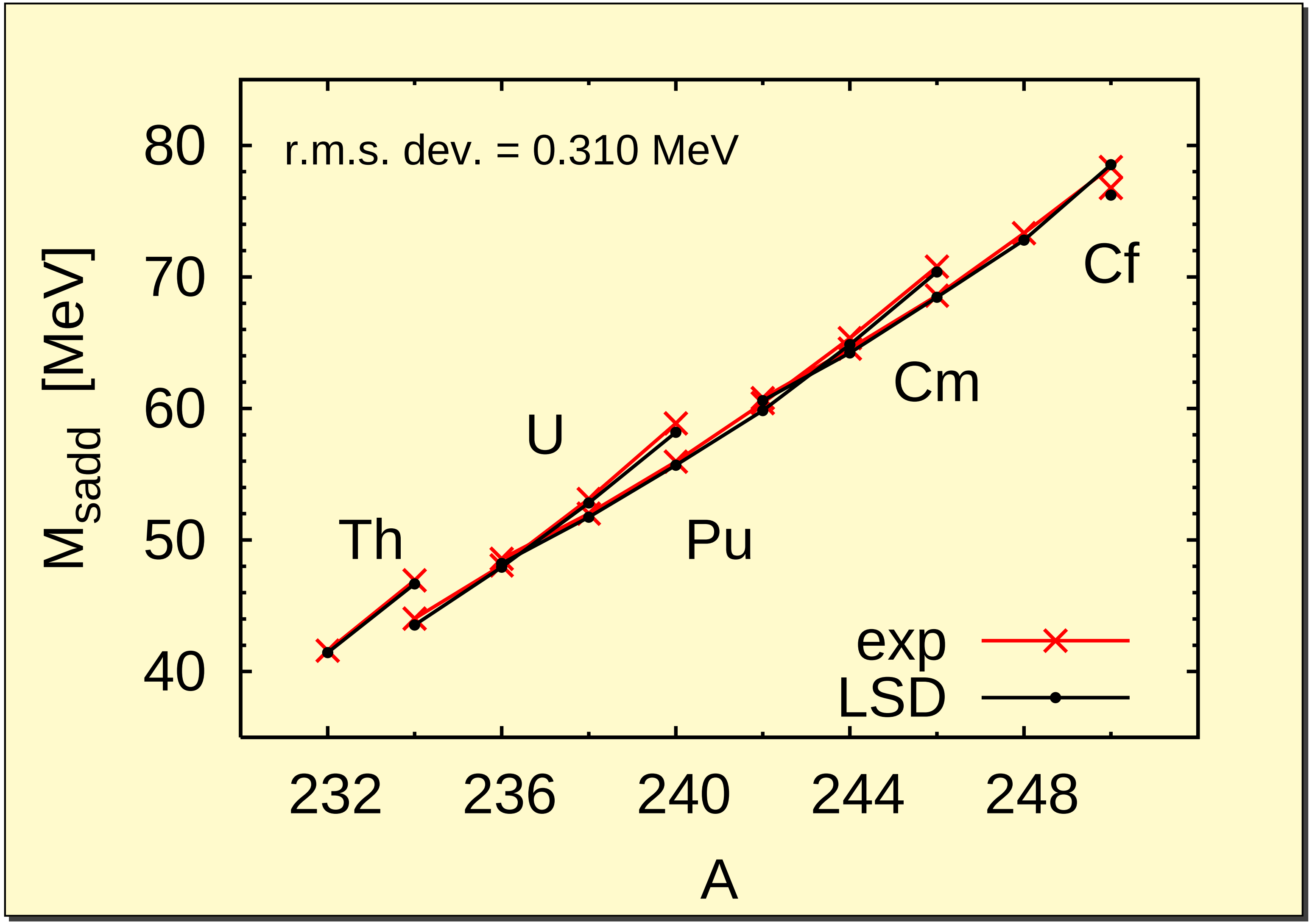
Effect of the congruence and the average pairing*:



$$G = G_0 \cdot B_{\mathrm{surf}}$$

*K. Pomorski, F. Ivanyuk, Int. Journ. Mod. Phys. **E18** (2009) 900.

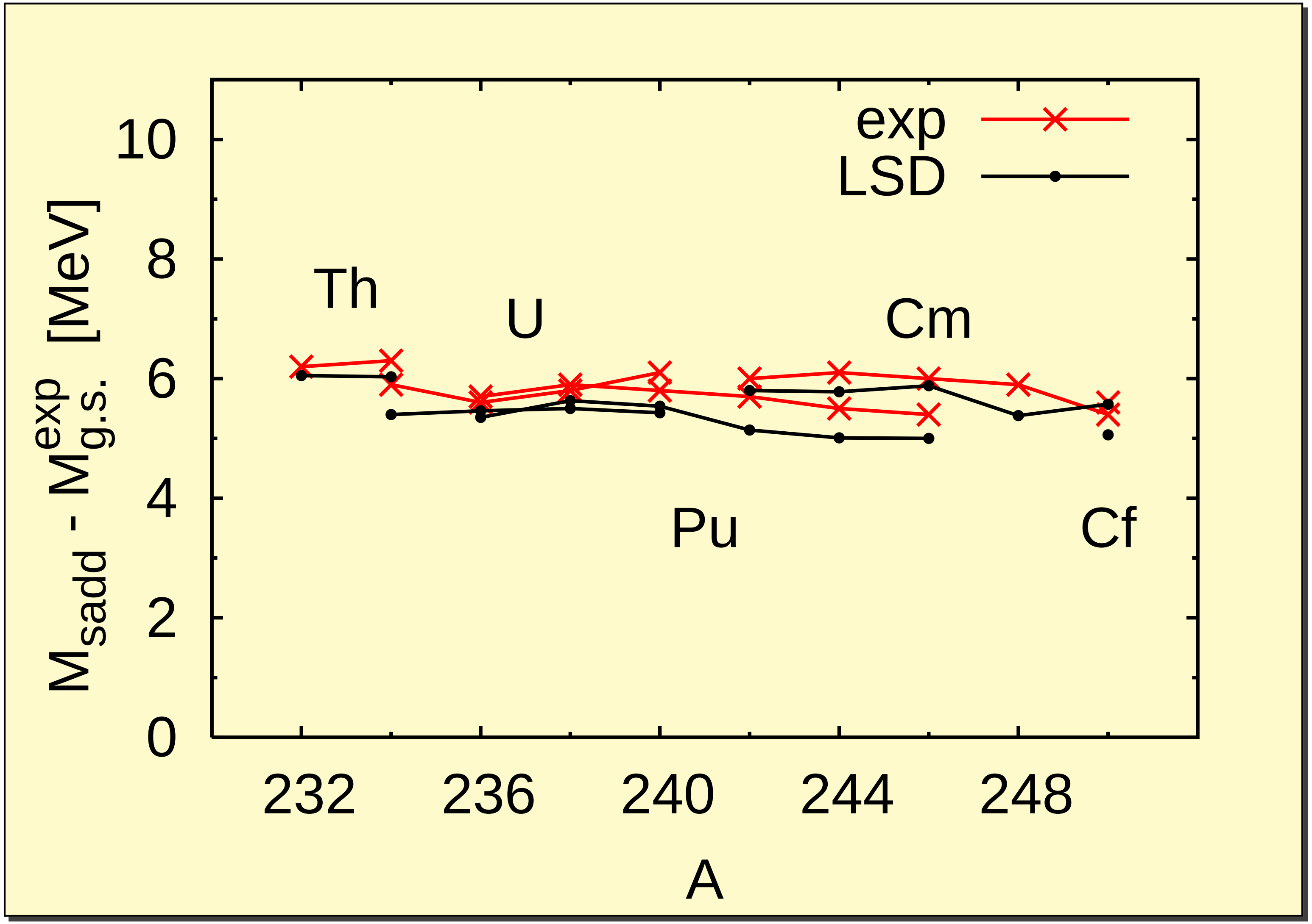
Pure LSD saddle point masses of heavy nuclei:



W. D. Myers, W.J. Świątecki, Nucl.Phys. **A612** (1997) 249. ← **Topographical theorem**

J. Bartel, A. Dobrowolski, and K. Pomorski, IJMP **E16**, 459 (2007)

LSD barriers according to the topographical theorem



Systematics of Spontaneous Fission Half-Lives

W. J. SWIATECKI

Institute for Mechanics and Mathematical Physics and The Gustaf Werner Institute for Nuclear Chemistry, Uppsala, Sweden

(Received July 18, 1955)

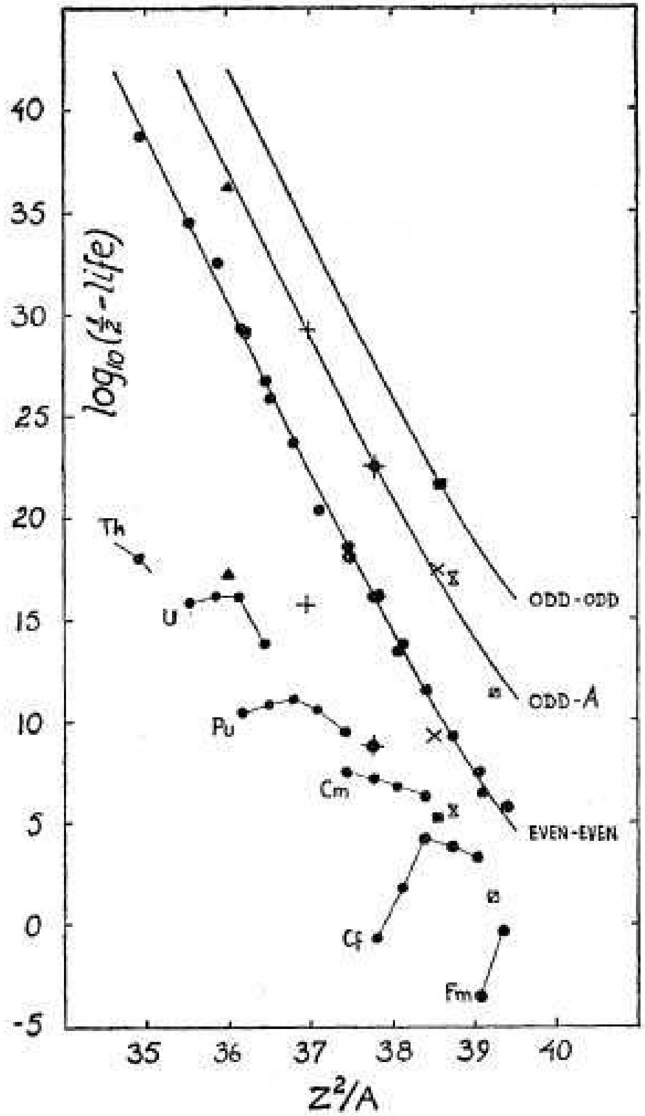
Phys. Rev. 100 (1955) 937.

$$\delta M = M - M_{\rm ref},$$

$$M_{\rm ref} = 1000A - 8.3557A + 19.120A^{\frac{2}{3}} + 0.76278Z^2/A^{\frac{1}{3}} + 25.444(N-Z)^2/A + 0.420(N-Z) \text{ millimass units.} \quad (1)$$

TABLE I. Values of $\log_{10}$(half-life).

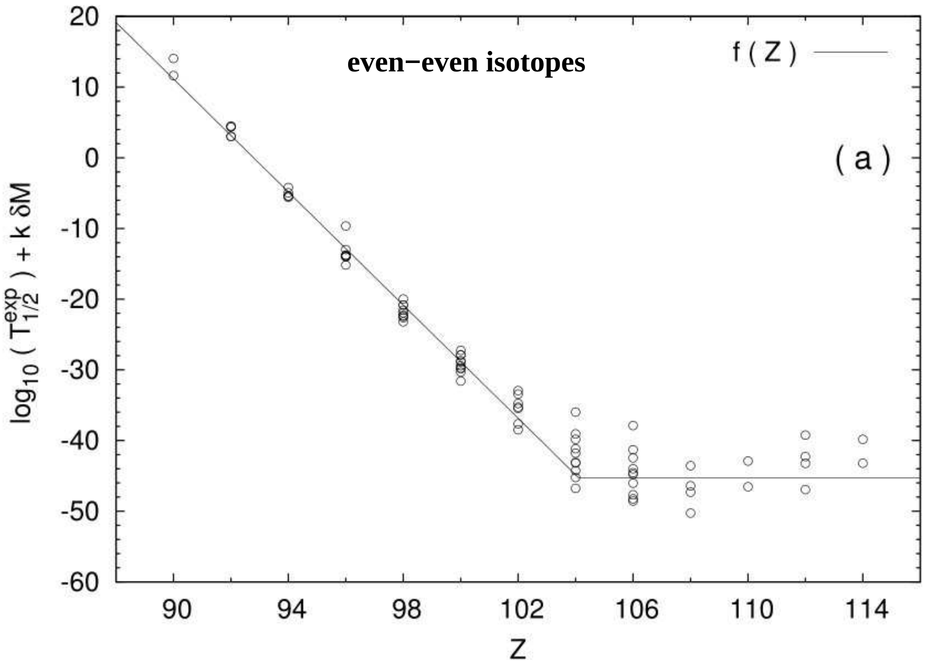
Nucleus	Experimental[a]	Formula (3)	Nucleus	Experimental[a]	Formula (3)
Even-even nuclei			Even-even nuclei		
Th 230	≳7.18	19.39	Cf 246	3.32	3.27
232	18.15	18.84	248	3.85	3.92
U 232	13.90	13.56	250	4.18	4.24
234	16.30	15.98	252	1.82	1.60
236	16.30	15.21	254	−0.70	−1.02
238	15.90	15.52	Fm 254	−0.30	−0.85
Pu 236	9.54	9.66	256	−3.52	−3.02
238	10.69	11.57	Odd-A nuclei		
240	11.08	11.09	U 235	17.26?	18.02
242	10.86	11.22	Pu 239	15.74	15.42
244	10.40	10.13	Bk 249	8.78	8.67
Cm 240	6.28	6.27	Cf 249	9.18	8.65
242	6.86	7.27	E 253	5.48	4.38
244	7.15	7.09	Fm 255	1.30	2.79
246	7.48	7.88	Odd-odd nuclei		
			E 254	5.18	5.17

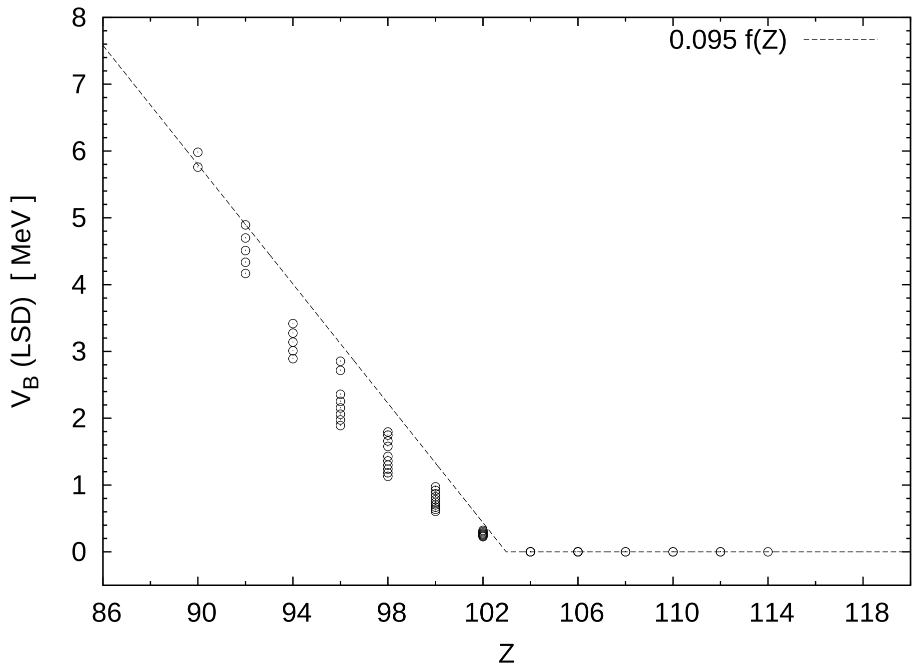


The result can be stated in the form of an empirical formula for half-lives; e.g., for even-even nuclei,

$$\tau_{ee} = f(Z^2/A) - k\delta M, \quad (2)$$

Świątecki prescription 58 years after



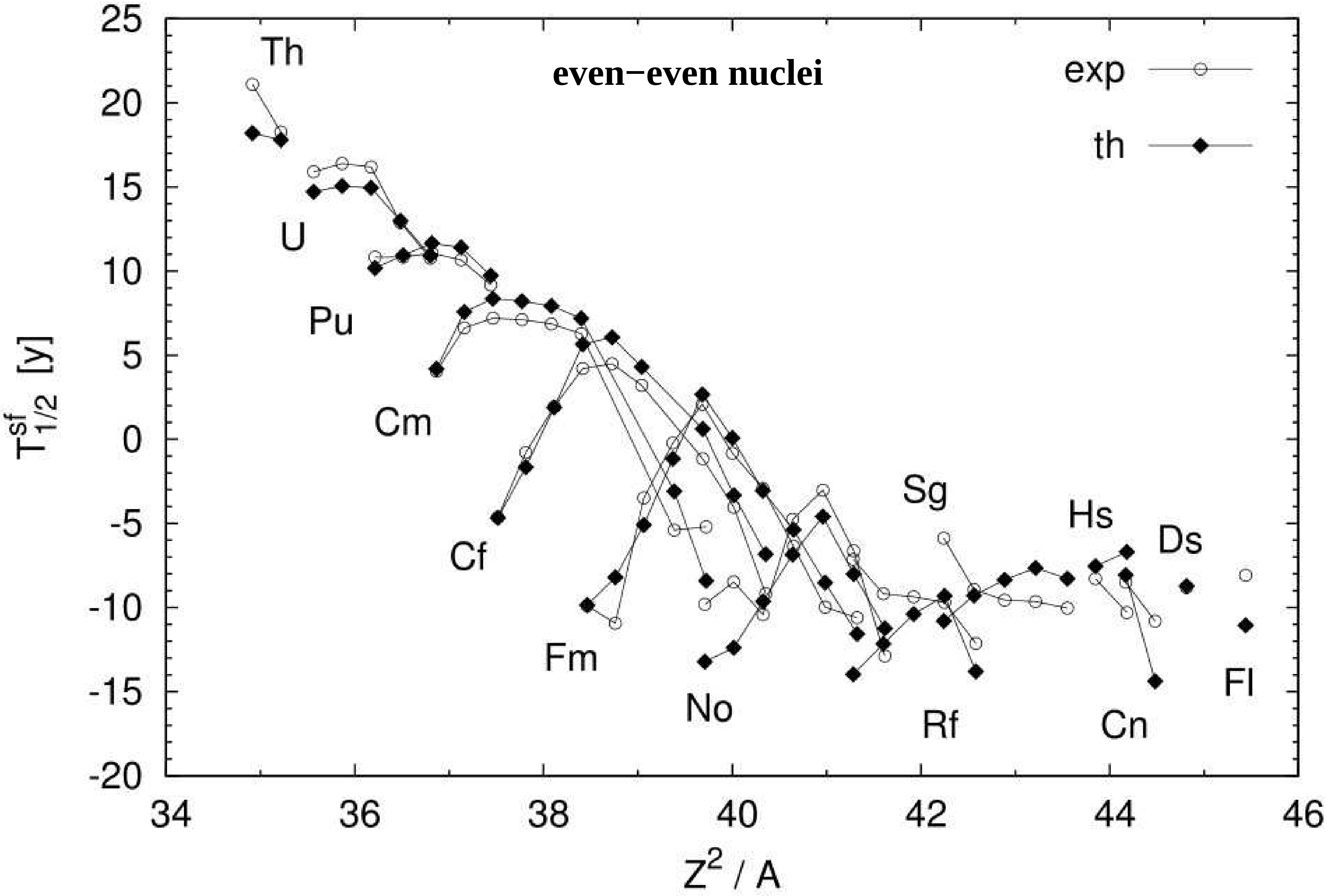


Experimental masses and $T_{1/2}^{SF}$ of all isotopes are taken from the NUDAT depository. The LSD model is used to estimate the macroscopic part of the binding energy. The data are plotted for $k = 7.5$.

The logarithm of the spontaneous fission half-lives is given by:

$$\log(T_{sf}) = f(Z) + k \cdot (M_{exp} - M_{LSD})$$

Świątecki prescription 58 years after *



*K. Pomorski and A. Zdeb, to be published

Summary:

- **L**ublin **S**trasburg **D**rop describes well masses of the known isotopes both in the ground state and saddle points.

- The **M**odified **F**unny-**H**ill parametrisation approximates well the optimal in energy shapes of nuclear liquid drops.

- Inclusion of the deformation dependent congruence (Wigner) energy and taking the pairing strength proportional to the surface area improves significantly the estimates of the barrier heights of the light nuclei.

- Topographical theorem of Świątecki approximates very well the saddle point masses and fission barrier heights.

- Old phenomenological formula of Świątecki combined with the LSD masses reproduces well the fission life-times of heavy nuclei.

Welcome to 20th Nuclear Physics Workshop in Kazimierz Dolny

My collaborators:

- Johann Bartel – IPHC and University in Strasbourg
- Artur Dobrowolski – UMCS, Lublin
- Jerzy Dudek – IPHC and University in Strasbourg
- Fedir Ivaniuk – KINR, Kiev
- Bożena Nerlo-Pomorska – UMCS, Lublin
- Michał Warda – UMCS, Lublin
- Anna Zdeb – UMCS, Lublin

If you like to learn more, read e.g.