INTERNATIONAL PHD PROJECTS IN APPLIED NUCLEAR PHYSICS AND INNOVATIVE TECHNOLOGIES
This project is supported by the Foundation for Polish Science – MPD program, co-financed by the European Union within the European Regional Development Fund

The two-nucleon and three-nucleon system in three dimensions

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23 września 2014

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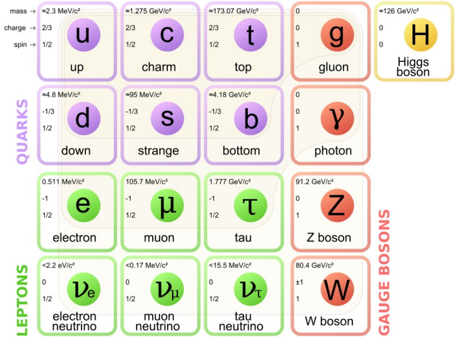
THE STANDARD MODEL OF PARTICLE PHYSICS



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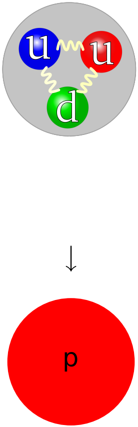
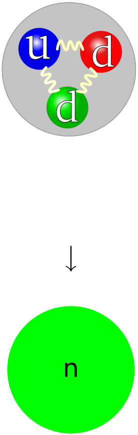


- In order to gain a better understanding of the theory the low energy sector needs to be studied
- The everyday world is composed mainly from low energy protons and neutrons
- Effective, coulomb like, models of nuclear interactions are introduced

OUR GOAL: Create tools to verify modern nuclear potentials constructed from effective field theory.



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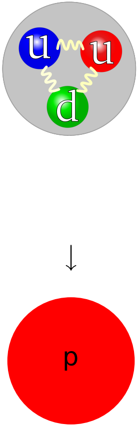

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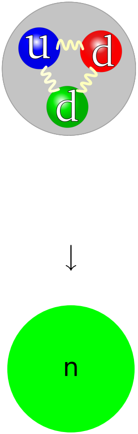

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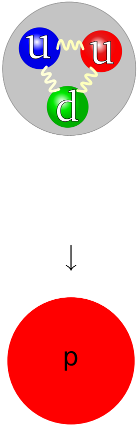

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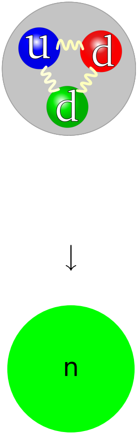

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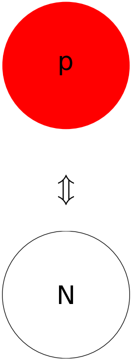
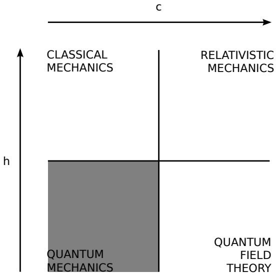


INPUT

- 2N, 3N interactions
- Constructed from QCD



OUTPUT

- Precise predictions
- Experimentally verifiable

"SIMPLE" DESCRIPTION

$| N\rangle = | \text{mom.}\rangle \otimes | \text{spin}\rangle \otimes | \text{isospin}\rangle$
$| p\rangle = | \boldsymbol{p}\rangle \otimes |\uparrow,\downarrow\rangle \otimes |\uparrow\rangle$
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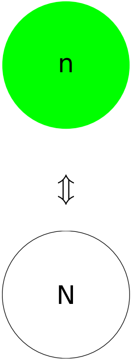
STARTING POINT

- Lippmann-Schwinger
- Schrödinger, Faddeev-Yakubowsky

TWO APPROACHES

- PWD - partial waves
- 3D DOF (+ operator forms based on symmetry constraints)



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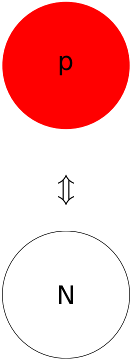

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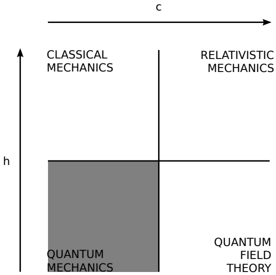

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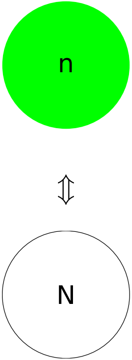

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- Two body potential:
  - $V(1,2) + V(2,3) + V(3,1)$
  - $V(i,j)$ function of the degrees of freedom of particles i, j
  - solar system (gravity)
  - classical electrostatic interactions
- Three body potential:
  - $V(1,2) + V(2,3) + V(3,1) + V(3;1,2) + V(1;2,3) + V(2;3,1)$
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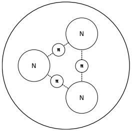

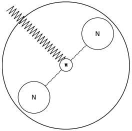

PRECISE CALCULATIONS

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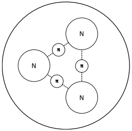

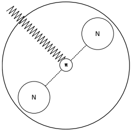

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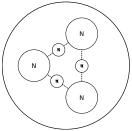

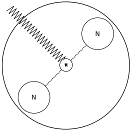

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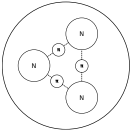

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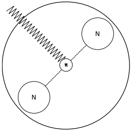

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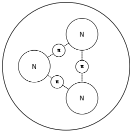

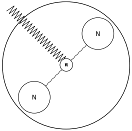

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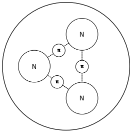

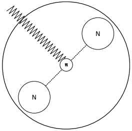

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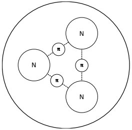

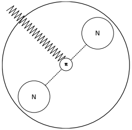

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Break-up channels in muon capture on $^3\mathrm{He}$
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Phys. Rev. C 90(2):024001 — 2014

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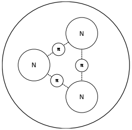

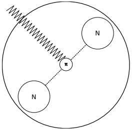

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accepted for publication in Jounal of Physics G : JPhysG-100014.R1 — 2014

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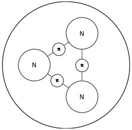
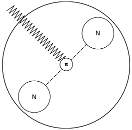
  - Flexible calculation scheme, the possibility to quickly test new models
  - planned - 4N forces, . . . , new models





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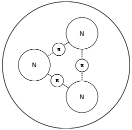

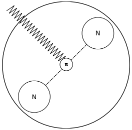

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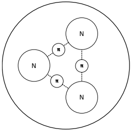
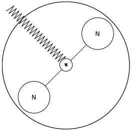
FOUNDATIONS





THE FOUNDATIONS OF OUR CALCULATIONS

- 2N, 3N potentials:
  - traditional "realistic" nuclear interactions
  - chiral effective field theories inspired by QCD
- Electro-weak currents:
  - derived using the potentials
  - single nucleon and two nucleon operators

SYMBOLIC PROGRAMMING

CHALLENGE

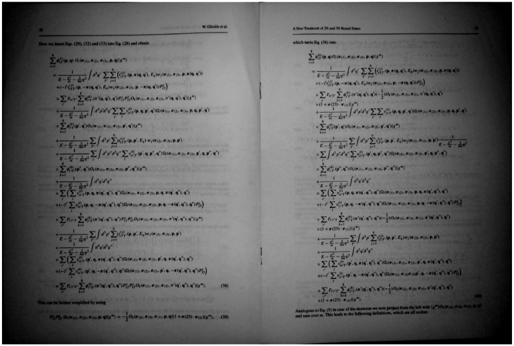
- Solve the fundamental equations, without simplifications
- The expressions look good on paper . . . until you try to use a specific (2N,3N) force or (EM, weak) current model
- A very complicated numerical realization of seemingly simple matrix elements, for example:

$$\langle \Psi_{^3\mathrm{H}} \mid j_{\mathrm{weak}} \mid \Psi_{^3\mathrm{He}} \rangle$$

SYMBOLIC PROGRAMMING

CHELLANGE

- Automatic code production is necessary:



Few-Body Syst (2010) 47:25-38

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- Automatic code production is necessary:
  - the numerical realization requires the creation of $\propto$ 10000 lines of code implementing eg. potentials, currents etc.
  - carrying out this work by hand, for a single model, would take ≈ 1 PhD
  - fixing bugs in the implementation is another $\approx 3, 4$ years :(
  - currently, with our newly developed tools, this work takes only a couple of hours :)

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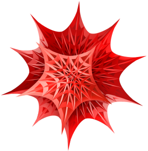

SOLUTION

- Development of tools for dealing with 2N and 3N systems with the use of symbolic programming (Mathematica®):
  - the abbility to quickly adapt the calculations to new models of 2N, 3N forces, currents
  - a FORTRAN implementation is created automaticaly, the resulting code is ready to be compiled and linked to our code
  - we have a very universal tool but are only begining to discover it's possibilities

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NUMERICAL REALISATION

CHALLENGE

- We use symmetry considerations to limit te form of operators involved in the calculations. This leads to a reduction of the size of the numerical work.
- Some calculations are still, numerically, very heavy (3N bound state, the continuum of scattering states)
- Mainly large eigen-problems ($\propto 1000000 \times 1000000$)

NUMERICAL REALISATION

SOLUTION

- Krylov methods, Arnoldi iterations:
  - the possibility to turn the 1000000×1000000 dimensional problem to a smaller $\approx 100 \times 100$ problem
  - the iterations in our algorithms are still very demanding numerically

- We use large computing clusters. Access through LENPIC:
  - JUROPA, Jülich Supercomputing Center, Germany
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MY PLANS FOR THE NEAREST FUTURE:

- Further use of our tools
  - Extend calculations to include the description of 3N scattering
  - Electro-weak processes
  - Extend current calculations to ^3He bound state with the Coulomb interaction
- Optimize the 3D calculations
- Test new models

THANK YOU FOR YOUR ATTENTION