List mode reconstruction in 2D strip PET with TOF

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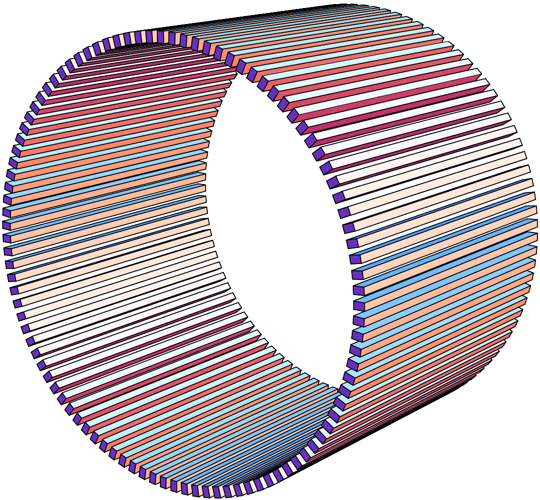
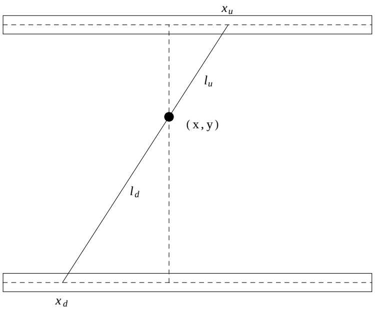
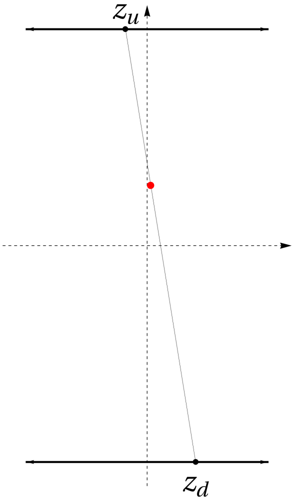


Detector geometry

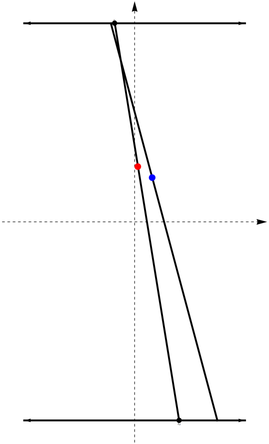


- Simple geometry - pair of scintillators at the distance R
- Each event is described by: $\widetilde{x_u}$, $\widetilde{x_d}$, $\widetilde{\Delta l}$

Detector geometry



Errors

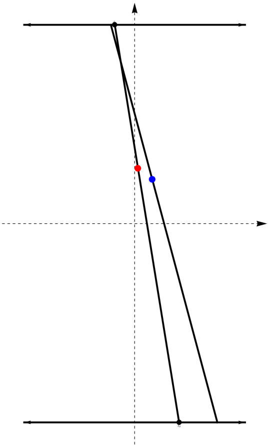


$$P(\widetilde{\mathbf{e}}|\mathbf{e}) = \frac{\det^{\frac{1}{2}} C(\mathbf{e})}{(2\pi)^{\frac{3}{2}}}\cdot$$

$$\exp\left(-\frac{1}{2}(\widetilde{\mathbf{e}}-\mathbf{e})^T C^{-1}(\mathbf{e})(\widetilde{\mathbf{e}}-\mathbf{e})\right).$$

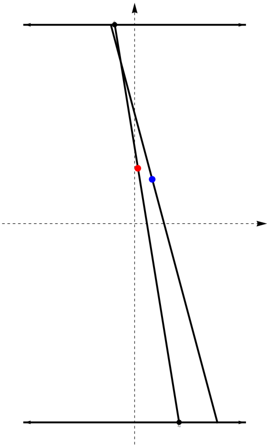
$$\widetilde{\mathbf{e}}-\mathbf{e} = \begin{pmatrix} \widetilde{z_u} - z_u \\ \widetilde{z_d} - z_d \\ \widetilde{\Delta l} - \Delta l \end{pmatrix}$$

Errors



$$C(\mathbf{e}) = \begin{pmatrix} \sigma_z^2 & 0 & \eta \\ 0 & \sigma_z^2 & \eta \\ \eta & \eta & \sigma_{\Delta l}^2 \end{pmatrix}$$

Kernel



$$P(\widetilde{\mathbf{e}}|y,z) = \int \mathrm{d}\theta\, P(\widetilde{\mathbf{e}}|y,z,\theta)$$

$$z_u = z + (R - y)\tan\theta$$
$$z_d = z - (R + y)\tan\theta$$
$$\Delta l = -2y\sqrt{1+\tan^2\theta}$$

Kernel

$$\vec{o} = \begin{pmatrix} -(\Delta y + \tilde{y} - R)\tan\tilde{\theta}\cos^{-2}\tilde{\theta} \\ -(\Delta y + \tilde{y} + R)\tan\tilde{\theta}\cos^{-2}\tilde{\theta} \\ -(\Delta y + \tilde{y})\cos^{-1}\tilde{\theta}\tilde{\theta}(1 + 2\tan^2\tilde{\theta}) \end{pmatrix}$$

$$\vec{a} = \begin{pmatrix} -(\Delta y + \tilde{y} - R)\cos^{-2}\tilde{\theta} \\ -(\Delta y + \tilde{y} + R)\cos^{-2}\tilde{\theta} \\ -(\Delta y + \tilde{y})\cos^{-1}\tan\tilde{\theta} \end{pmatrix}$$

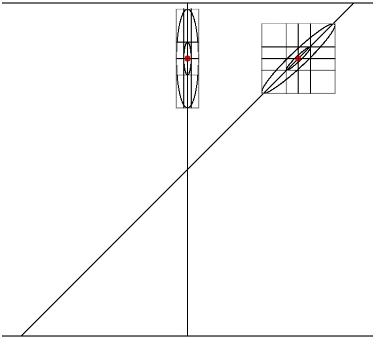
$$\vec{b} = \begin{pmatrix} \Delta x - \Delta y\tan\tilde{\theta} \\ \Delta x - \Delta y\tan\tilde{\theta} \\ -2\Delta y\cos^{-1}\tilde{\theta} \end{pmatrix}$$

Kernel

We're approximating $P(\widetilde{e}|i)$ by formula:

$$p(\widetilde{\mathbf{e}}|(x,y)) \approx \frac{\det^{\frac{1}{2}} C^{-1}}{2\pi\sqrt{\vec{a}C^{-1}\vec{a} + 2\vec{o}C^{-1}\vec{b}}} \exp\left(-\frac{1}{2}\left(\vec{b}C^{-1}\vec{b} - \frac{\left(\vec{b}C^{-1}\vec{a}\right)^2}{\vec{a}C^{-1}\vec{a} + 2\vec{o}C^{-1}\vec{b}}\right)\right).$$

Kernel



$$\vec{b}C^{-1}\vec{b} = 9$$

Maximal likelihood

PET image reconstruction is a statistical process.

$$\boxed{\max_{\rho>0} \log P(\{\widetilde{\mathbf{e}}_j\}|\rho)}$$

Maximal likelihood

$$P(\{\widetilde{\mathbf{e}}_j\}|\rho) = \prod_j \left(\sum_p P(\widetilde{\mathbf{e}}_j|p) P(p|\rho) \right)$$

$$P(p|\rho) = \frac{s(p)\rho(p)}{\sum_p s(p)\rho(p)}$$

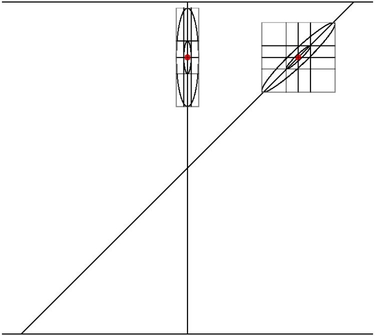
MLEM algorithm

$$\rho(p)^{(t+1)} = \sum_{j=1}^{J} \frac{P(\widetilde{\mathbf{e}}_j|p)\rho(p)^{(t)}}{\sum_{p'=1}^{M} P(\widetilde{\mathbf{e}}_j|p')s(p')\rho(p')^{(t)}}.$$

H.H. Berret, T. White, and L. Parra, J. Opt. Soc. Am. A. Opt. Image Sci. Vis., **14(11)**, 2914–2923 (1997).

L. Parra and H.H. Berret, IEEE Trans. Med. Imaging, **17**, 228–235 (1998).

Kernel



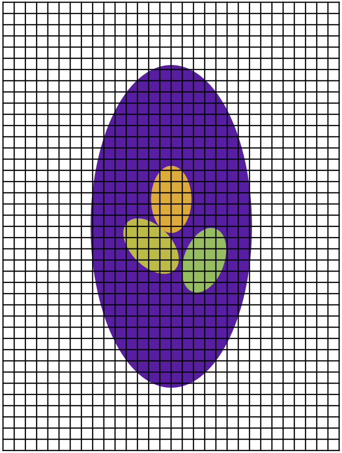
$$\vec{b}C^{-1}\vec{b} = 9$$

$$r_z \approx 3\frac{\sigma_z}{\sqrt{2}} \approx 21 mm$$

$$r_{\Delta l} \approx 3\frac{\sigma_{\Delta l}}{2} \approx 90 mm$$

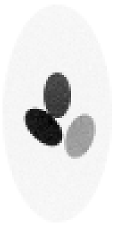
$$\approx 237 pixels, \quad \approx 1333 voxels$$

Simulation

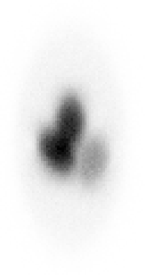


$\sigma_x \approx 10mm$
$\sigma_{\Delta l} \approx 63mm$
Number of iterations: 10
Number of events $\approx 40 * 10^6$
Time per 10^6 events on Intel Core i7-4770K using OpenMP: 2.25s

Reconstruction

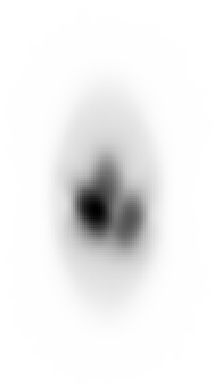


Phantom without errors



TOF reconstruction

Reconstruction



Iteration 1

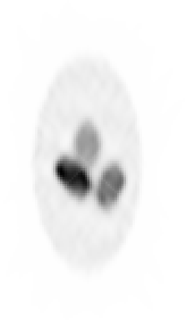
Iteration 2

Reconstruction

Iteration 5

Iteration 10

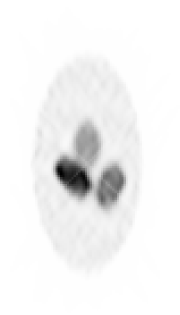
Reconstruction



Iteration 20

Iteration 30

Reconstruction



Iteration 40

Iteration 50