



Signal reconstruction in a long scintillator strips

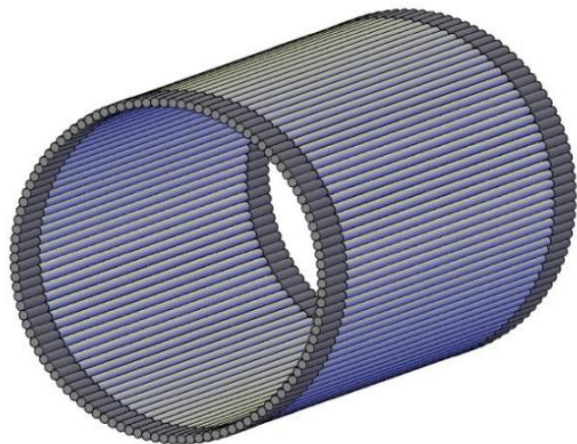
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Cracow, 21.09.2013



1. Introduction

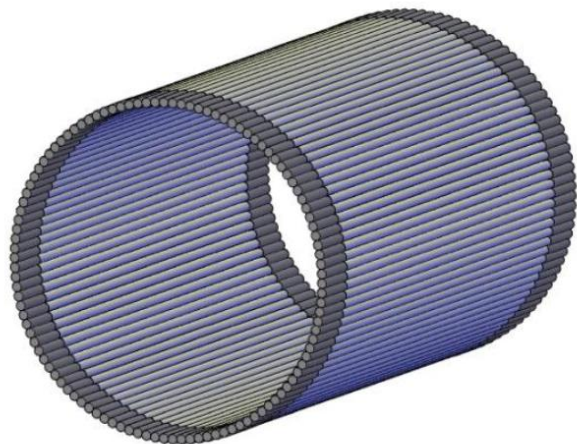


General idea of STRIP TOF-PET device

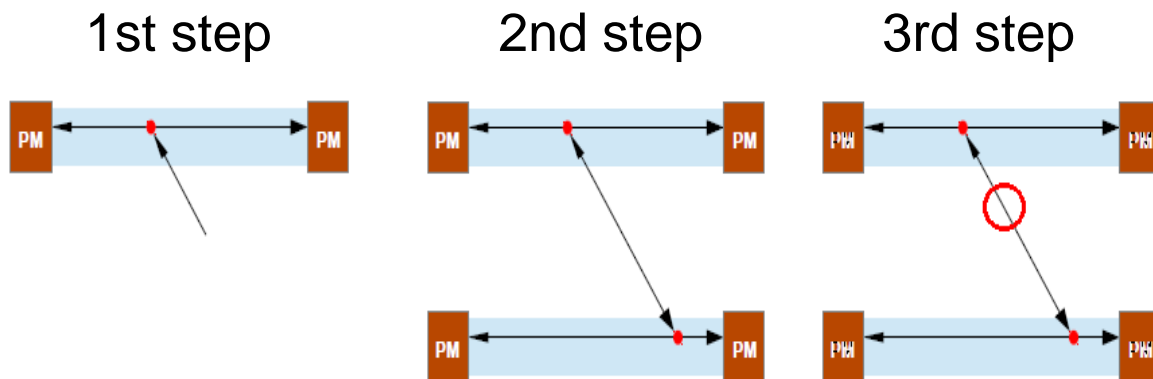
- In Time Of Flight PET (TOF-PET), the measurement of the difference in the arrival time to the two detectors is possible.
- In STRIP TOF-PET plastic scintillators form a ring. Signals propagate along the scintillator strip and are measured by two PMTS on its ends



1. Introduction



General idea of STRIP
TOF-PET device



- **1st step:** reconstruction of position where gamma particle hit the scintillator.
- **2nd step:** searching of the coincidence of hitting a two scintillators.
- **3rd step:** reconstruction a 3 dimensional position of positron annihilation.



2. Hit position reconstruction

Hit positions [cm]:

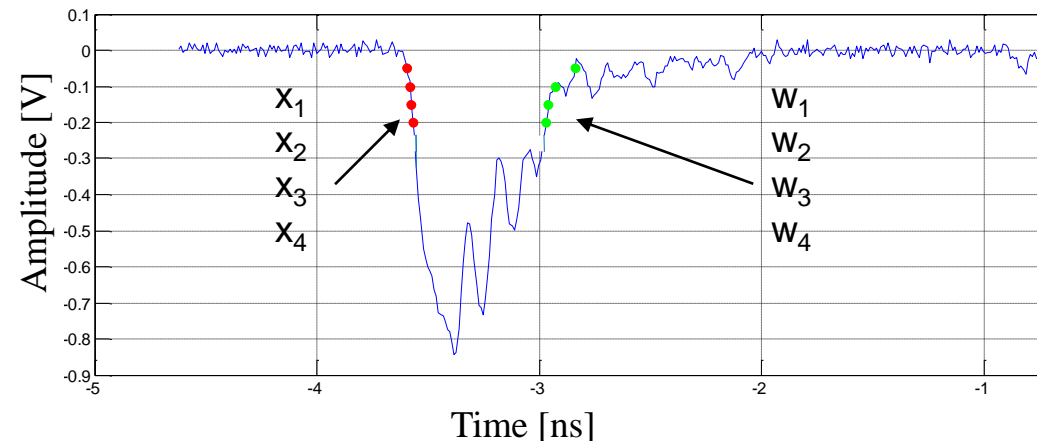
21.8 22.4 23.0 23.6 48.2 48.8 49.4 50.0 50.6



- The strip has a length of 30 cm,
- There were 49 places of irradiation along the strip with step **0.6 cm**.
- The position in the middle of the strip is marked as **36.2 cm**.
- About 10 000 of signal pairs was registered at each position.



2. Hit position reconstruction



1-st slope (red dots)

$$x_1 - y_1$$

$$x_1 - x_2$$

$$x_2 - y_2$$

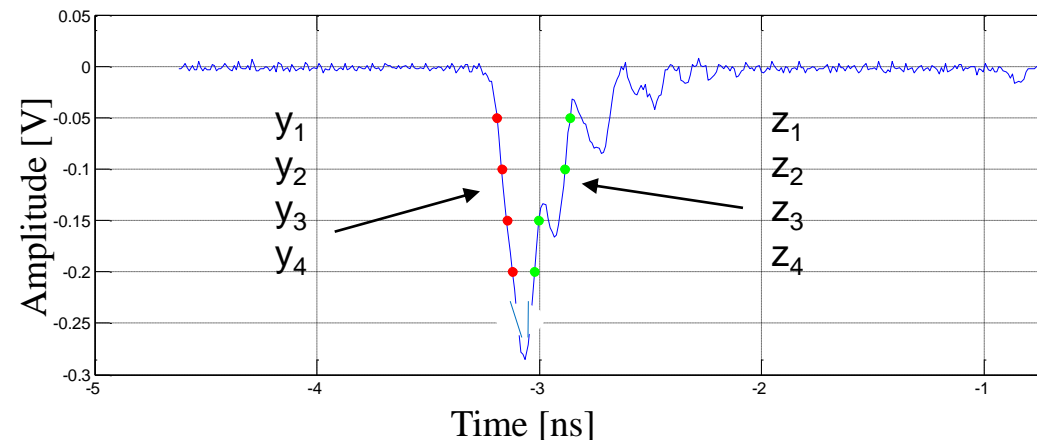
$$x_2 - x_3$$

...

...

$$x_N - y_N$$

$$x_{N-1} - x_N$$



2-nd slope (green dots)

$$x_1 - w_1$$

$$y_1 - z_1$$

$$x_2 - w_2$$

$$y_2 - z_2$$

...

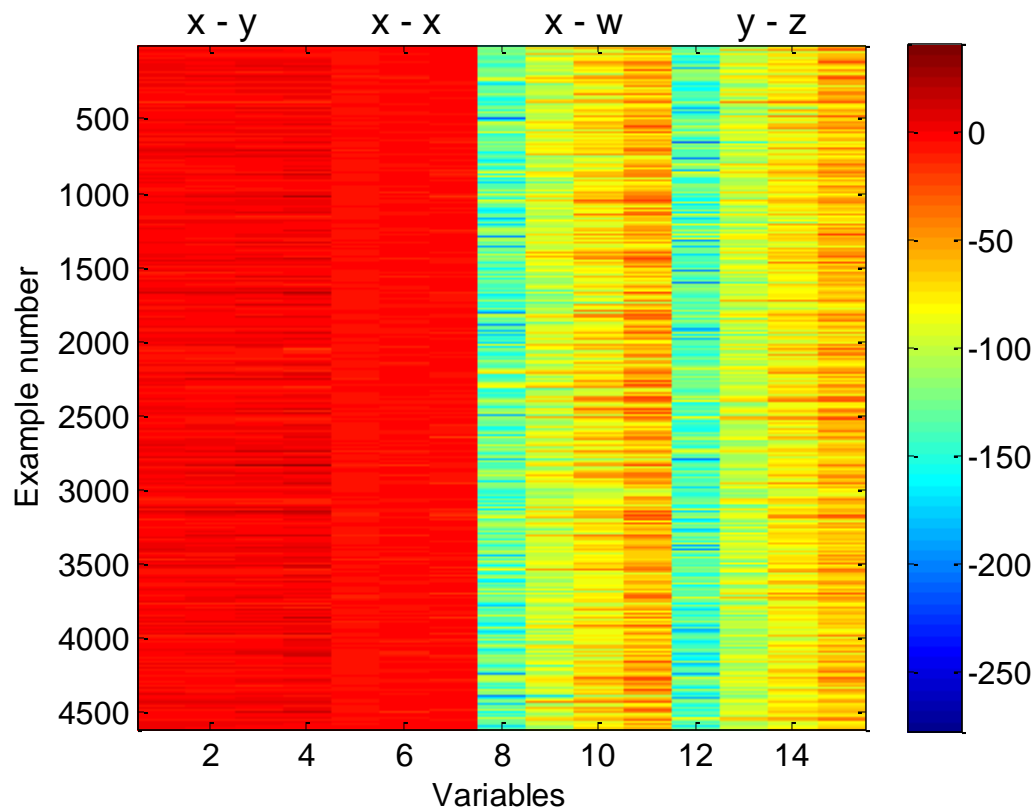
...

$$x_N - w_N$$

$$y_N - z_N$$

We have: **4** thresholds x **2** slopes x **2** signals = **16** measurements.
This gives **15** relative values (**variables**).

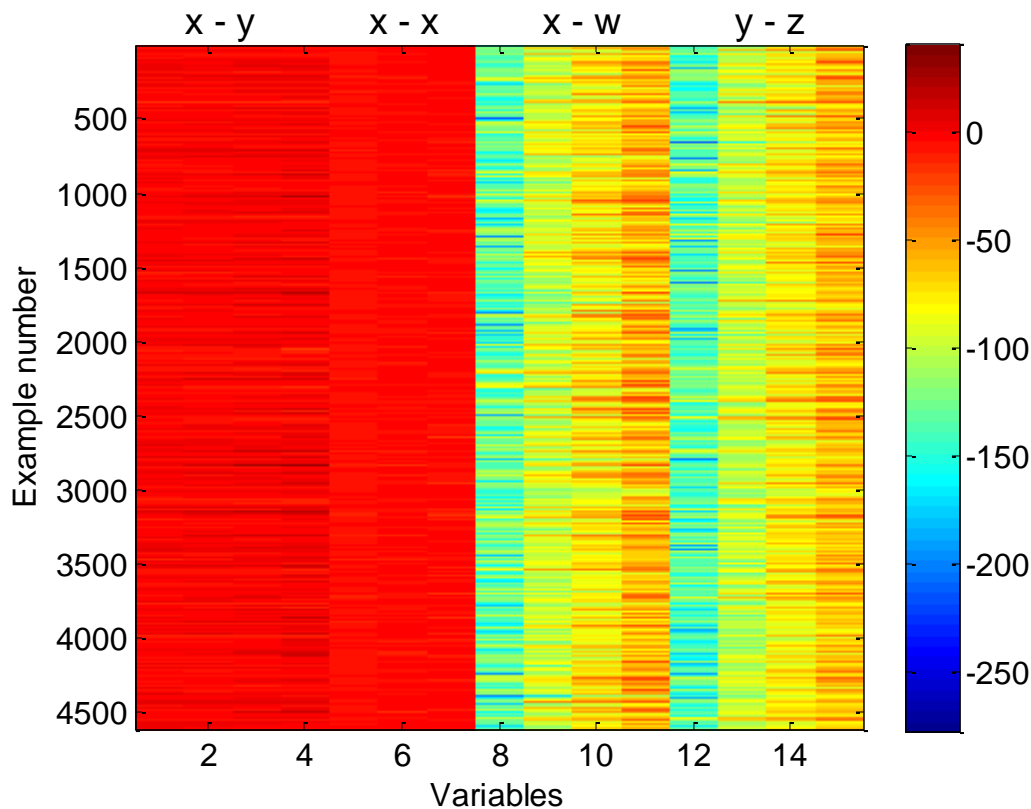
2. Hit position reconstruction



- Each row is a 15 dimensional vector containing the information about a position.
- Data's were registered near to the center position – similar shapes of two signals (see variables $x-w$ and $y-z$).
- It can be shown that the signals are described by **multidimensional normal distribution**.

Dataset registered at a given position of **36.8 cm**. Color on figure correspondes to time: **1 unit = 50 ps**

2. Hit position reconstruction



Multidimensional normal distribution is described by:

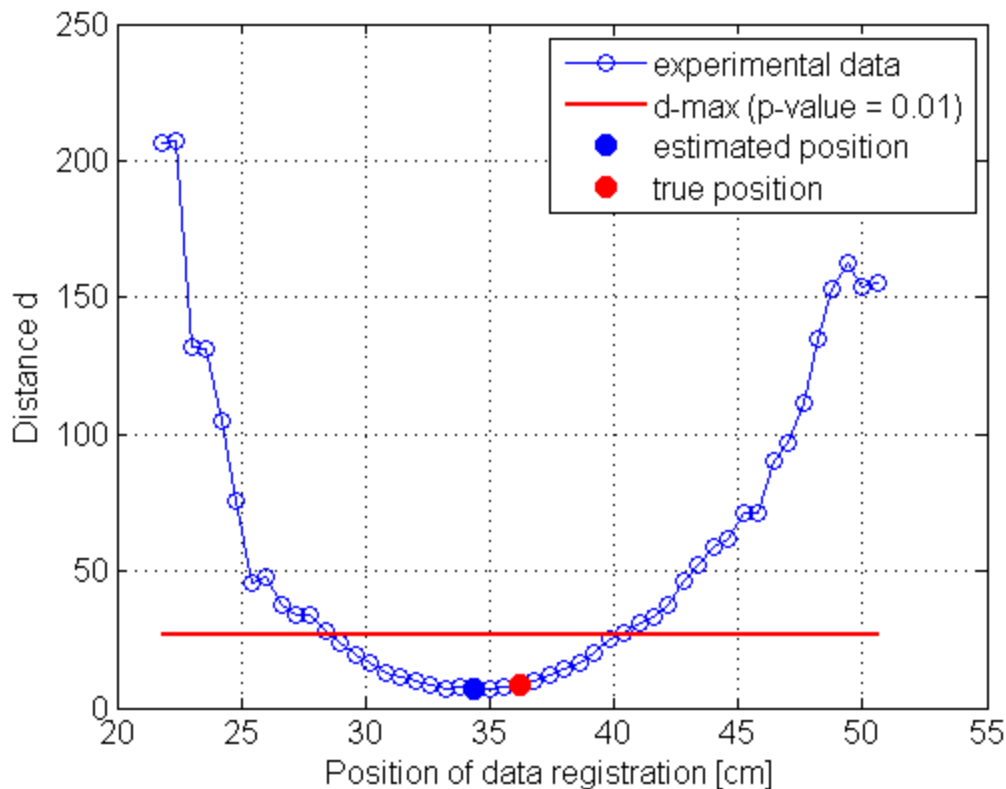
- covariance matrix (\mathbf{C}),
- mean value vector (\mathbf{m}).

If the data are normally distributed we may calculate the statistics (distance) for a new vector \mathbf{u} :

$$d(\mathbf{u}) = (\mathbf{u} - \mathbf{m}) \cdot \mathbf{C}^{-1} \cdot (\mathbf{u} - \mathbf{m})^T$$

Dataset registered at a given position of **36.8 cm**. Color on figure corresponds to time: **1 unit = 50 ps**

2. Hit position reconstruction



Distances (d) calculated in each position. The estimated minimum (blue dot) is close to true minimum value (red dot).

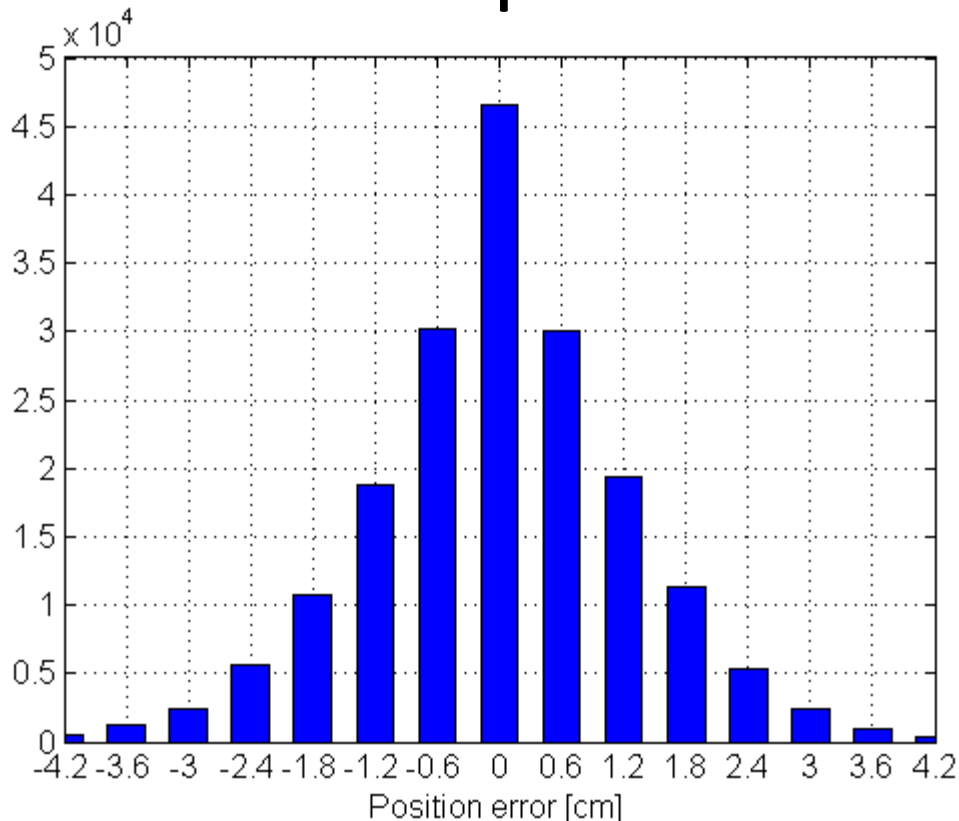
Example of signal \mathbf{u} classification:

- Calculate \mathbf{d}_i for $i = 1, 2, \dots, 49$

$$\mathbf{d}_i = (\mathbf{u} - \mathbf{m}_i) \cdot \mathbf{C}_i^{-1} \cdot (\mathbf{u} - \mathbf{m}_i)^T$$

- Indicate the index of minimal d value: the position of **34.4** cm is reconstructed based (true position is **36.2** cm and estimation error **1.8** cm).
- Additionally a preselected threshold **d -max** is compared with minimal d value to verify the significance of the measurement.

2. Hit position reconstruction



Signal reconstruction based on minimum distance d method:

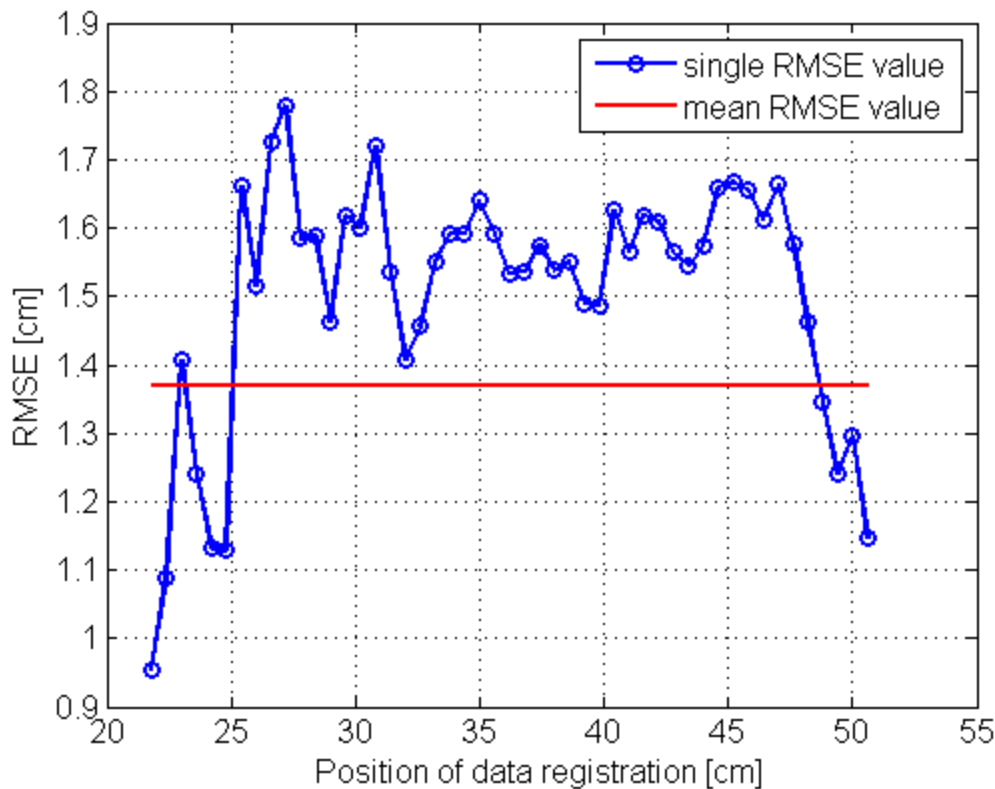
- Total number of data used in the study was about 200 000,
- Output resolution is given by input data points density and equal to **0.6 cm**,
- Estimated error: Root Mean Square Error is

Distribution of error ($p_T - p_E$):

p_T is a true position,
 p_E is a reconstructed position.

RMSE: ~1.28 [cm].

2. Hit position reconstruction

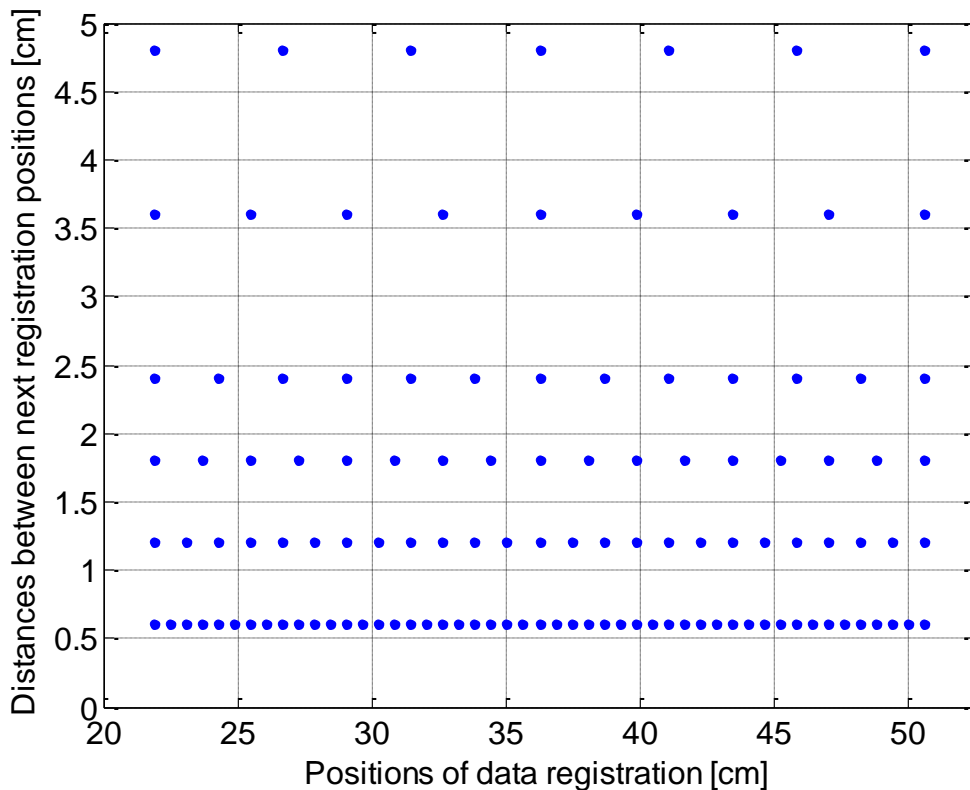


- The distribution of RMSE along the scintillator strip is symmetric.
- The smallest reconstruction error is obtained at both scintillator ends and is equal to c.a. 1 cm.
- The biggest reconstruction error is obtained in the middle of a scintillator.

Reconstruction errors (**RMSE [cm]**) as a function of a position of data registration [cm].



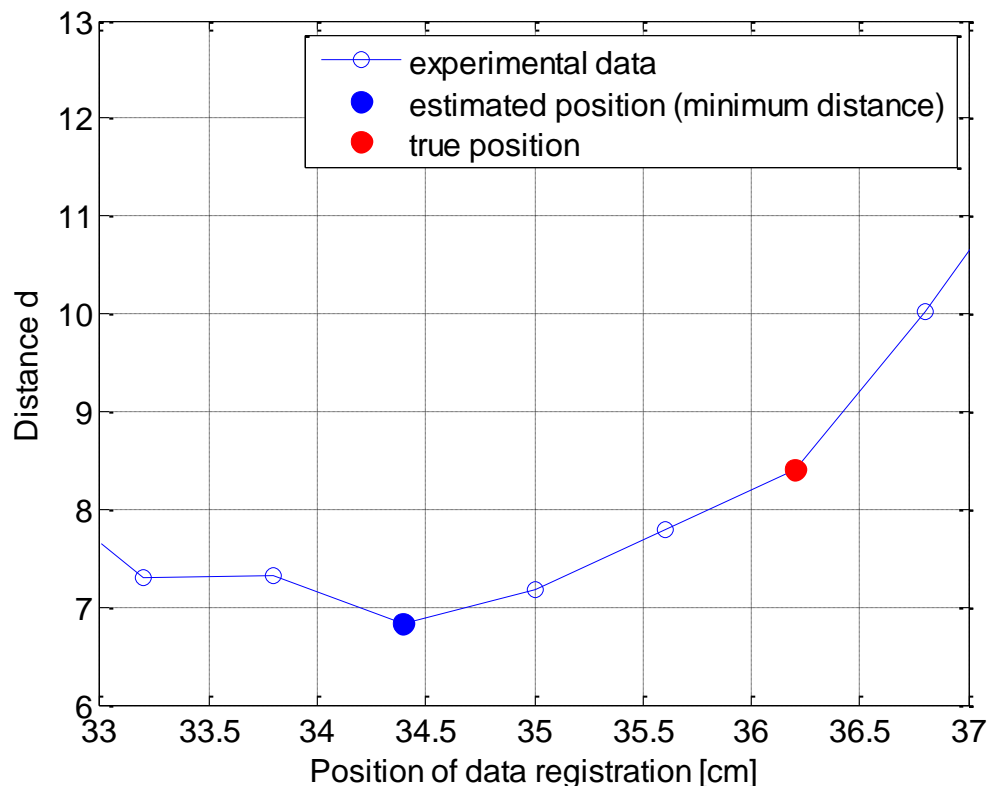
3. Resolution tests



- The interesting question is: how „dense” we should sample the signals along the strip ?
- Does the **0.6 cm** is small enough to ensure good condition for reconstruction ?
- With the data registered with step 0.6 cm we can perform the test with **larger distances (i.e. an integer multiple of 0.6)**.

Selected datasets for a given resolution tests from **0.6 cm** to **4.8 cm** (some values where skipped).

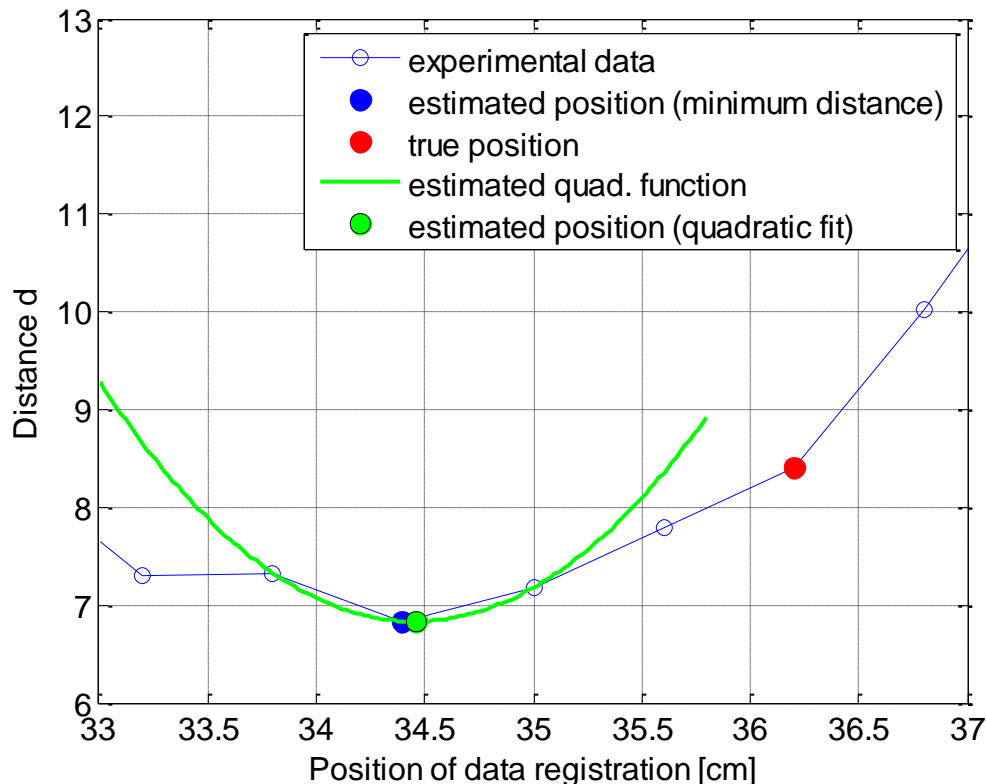
3. Resolution tests



Distances (d) calculated in each position. In **minimum distance** method the output values are from **discrete set**.

- The **minimum distance** method: qualifying measurement to one of the predefined sets.
- The output value is from a **discrete set** $\{\dots 33.8, \mathbf{34.4}, 35, 35.6, \mathbf{36.2}, 36.8, \dots\}$ - signals registration positions with step of **0.6 cm**.
- For testing the required signals registration resolution the method that produce **continuous output value** is needed.

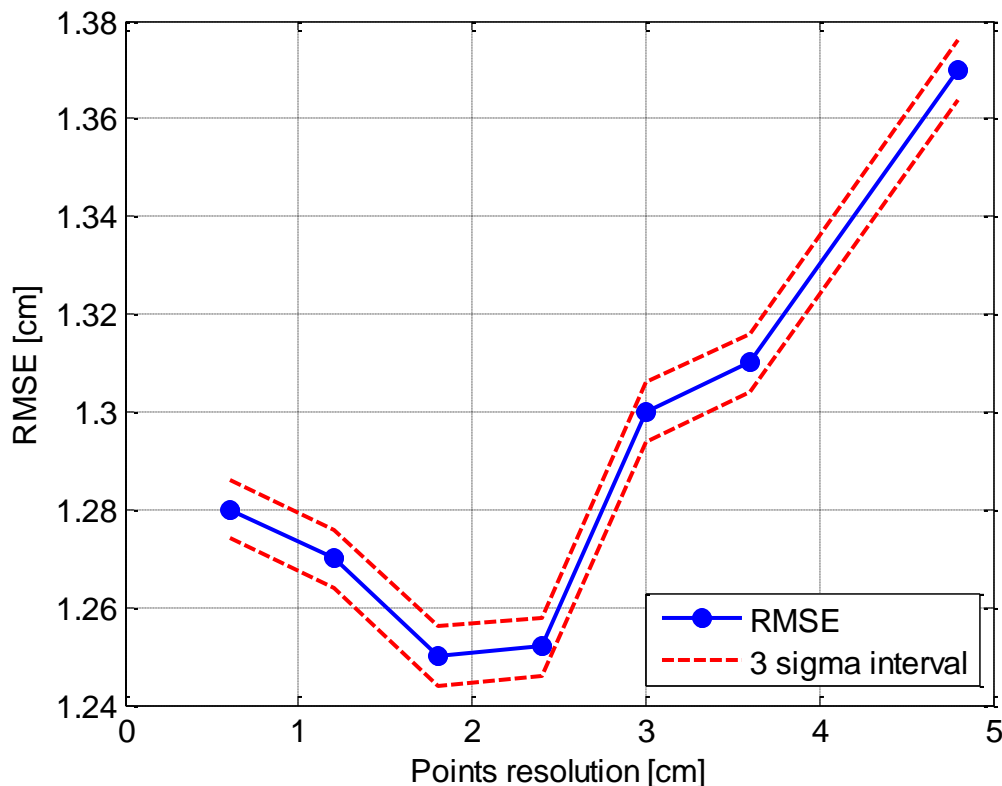
3. Resolution tests



Distances (d) calculated in each position. In **quadratic fit** method the output values are from **continuous set**.

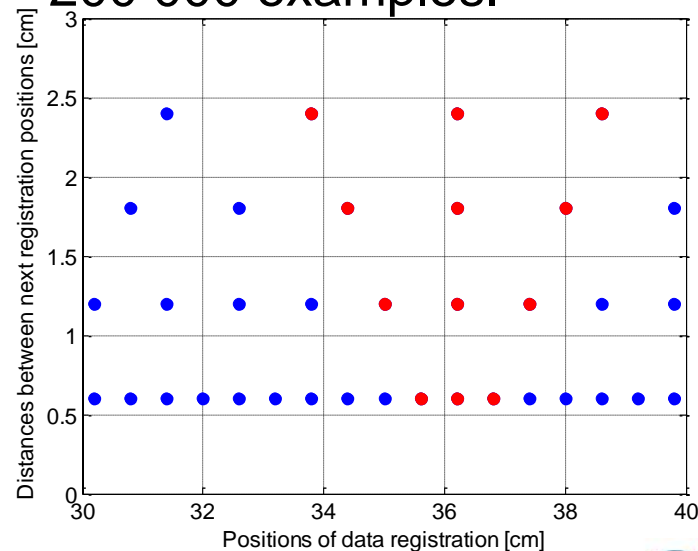
- Since the relation between distance d and position of registration is quasi-quadratic, the locally **quadratic fit** is performed.
- To estimate the parameters of parabola at least **three** points are needed (the number of points included can be experimentally tested).
- In this study only two adjacent points placed on the left and right of the minimum, together with the minimum, are considered.

3. Resolution tests

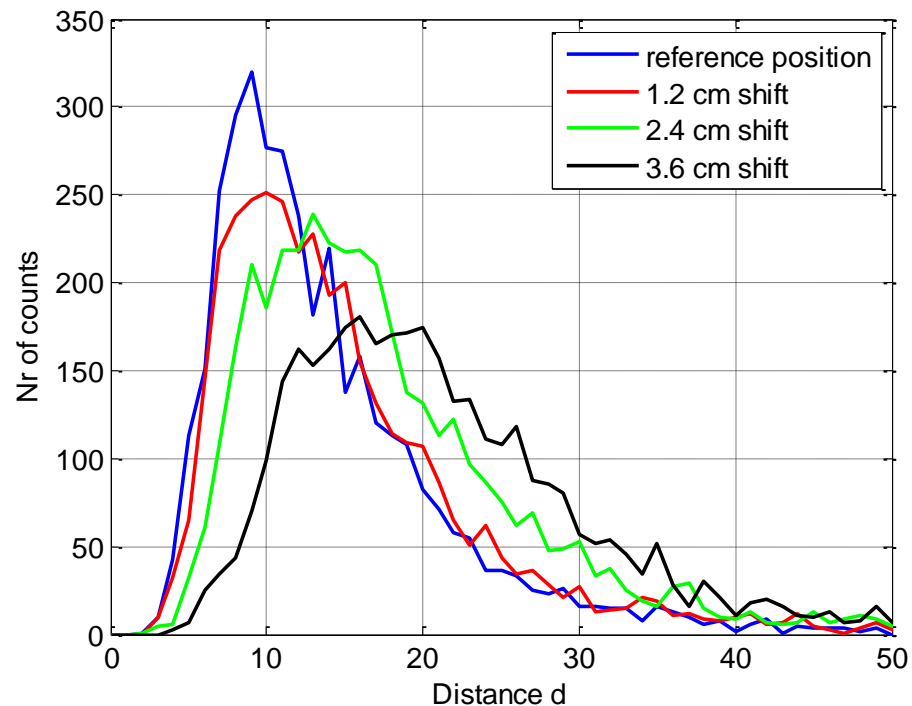
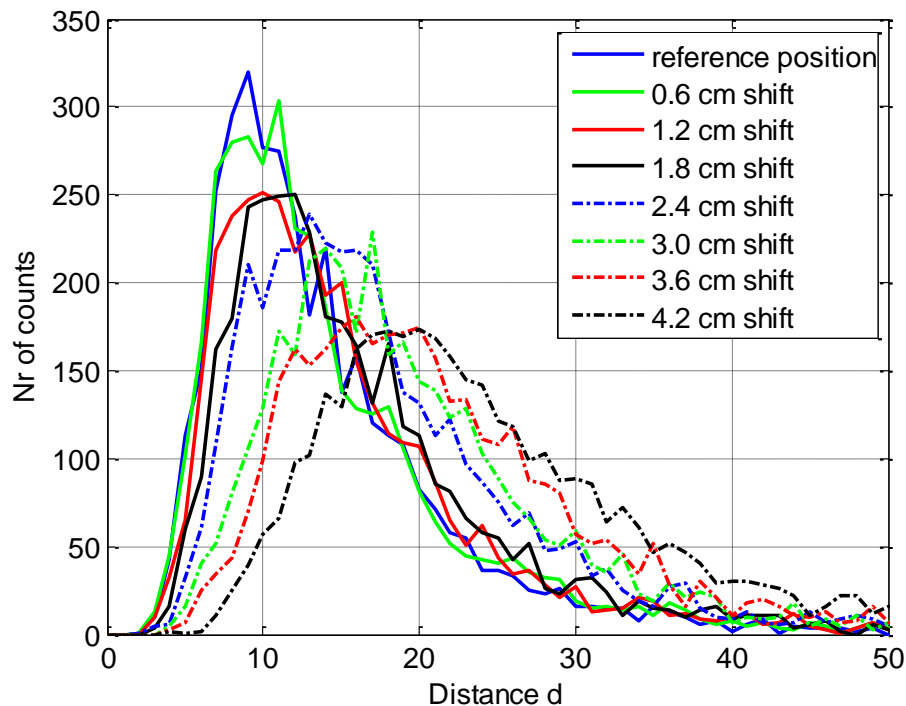


Reconstruction errors (**RMSE [cm]**) for different resolutions in the range from **0.6 cm to 4.8 cm**.

- The experimental results show that smallest **RMSE** has been obtained for **1.8 cm** and is equal c.a. **1.25 cm**.
- The standard deviation of RMSE is c.a. **0.002 cm** for 200 000 examples.



3. Resolution tests



- The phenomenon of the **distributions overlap** is presented.
- The data were taken from the center of the strip (**reference position**) and qualified to adjacent datasets: 0.0 cm (the same position), 0.6 cm, 1.2 cm, and so on.



4. Summary

- Simple reconstruction method based on datasets taken from all the positions along the 30 cm long scintillator strip was introduced. The reconstruction error is c.a. **1.28 cm**.
- With the data registered with step 0.6 cm the test for larger distances (1.2 cm, 1.8 cm, 2.4 cm, 3.6 cm, 4.8 cm) was provided. The presented results show that smallest RMSE has been achieved for 1.8 cm.
- Limitations of the method:
 - ✓ Sampling the signal with only **4** amplitude thresholds,
 - ✓ No additional information, like charge of the signals for instance, is taken into account.