Phantoms and simple image reconstruction with TOF

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Plan of presentation

1. Phantoms
2. Reconstruction Methods
3. Image Metrics
4. Summary

Phantom

Phantom: a predetermined shape, made of geometric figures (ellipses, spheres, cylinders), represented by points (x, y, z) and their respective pairs of vectors (|v›,-|v›) of gamma quanta.

Phantom

Why we use phantoms:

→ to construct advanced geometrical figure,

→ to test our reconstruction methods in PET,

→ to say, how many detectors we need in PET,

→ to simulate influence of some parameters on work of PET .

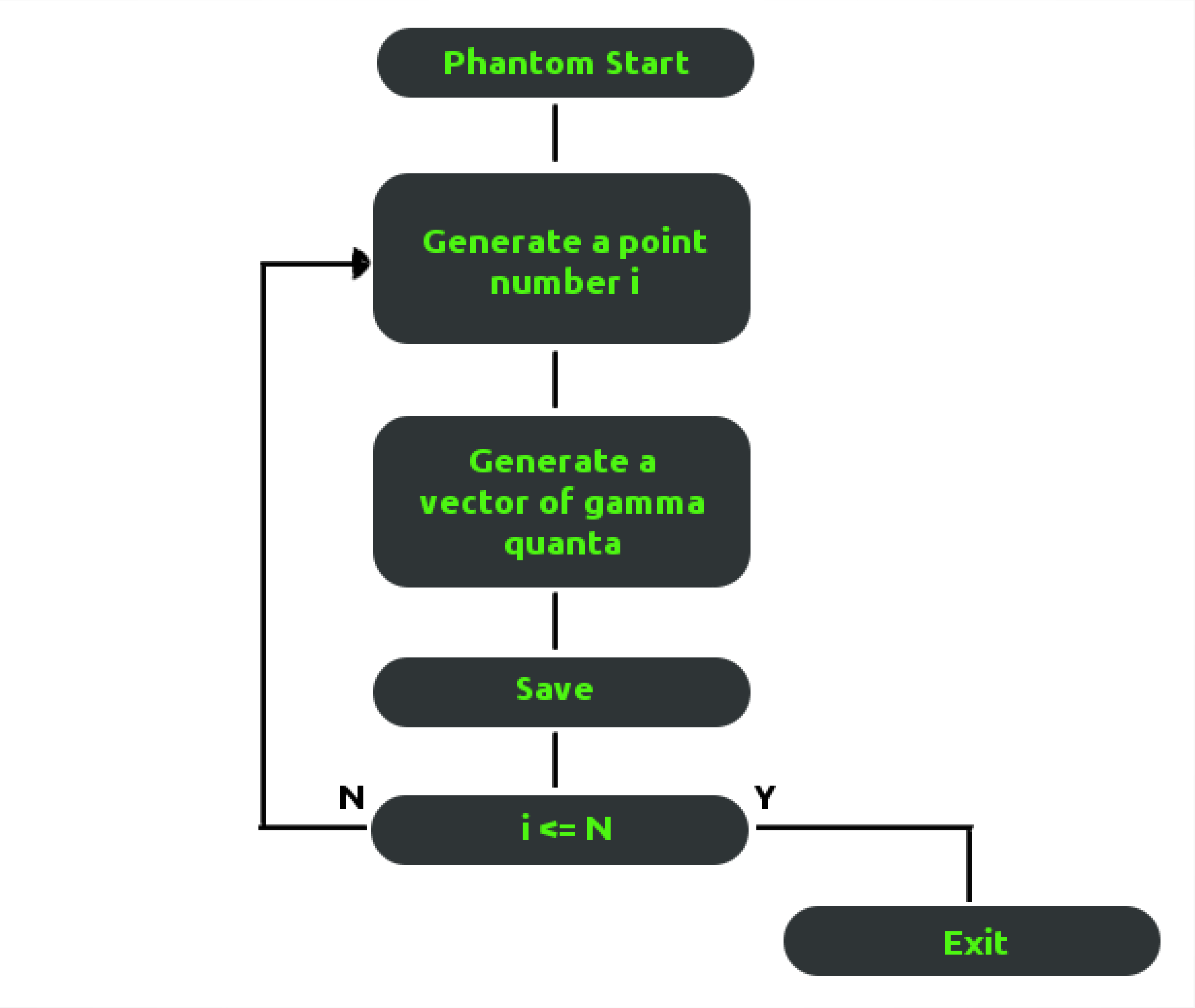
Phantom

Properties:

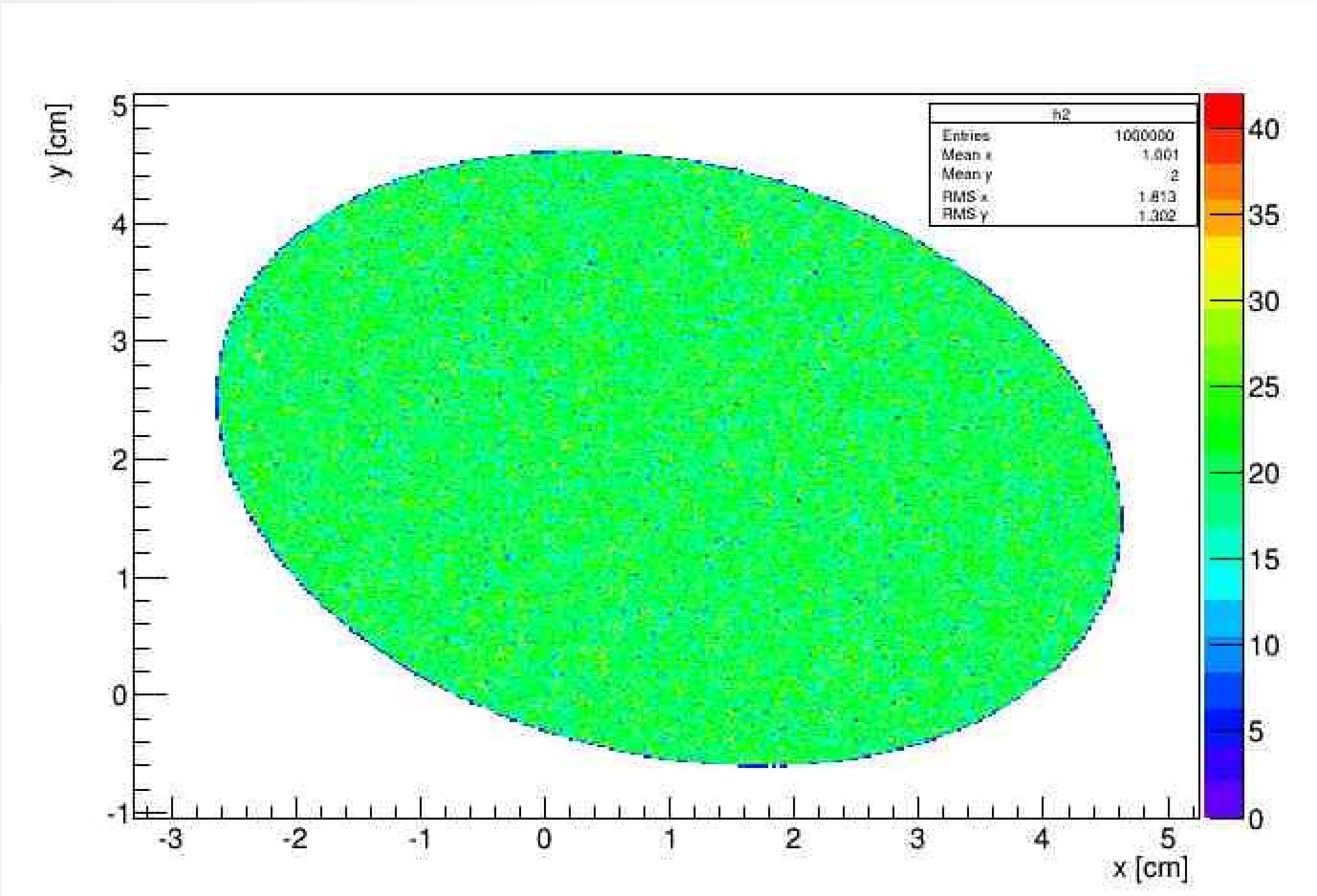
- uniform decay of emission's points
- uniform decay on sphere gamma-ray emission
- possibility to determine the relative probability of emission (for each figures)

```
Ellipse(double x = 0, double y = 0, double z=0, double a = 0, double b = 0, double angle = 0, double prob = 1)

Sphere(double x = 0, double y = 0, double z=0,double r=0,double prob=1)

Cylinder(double x = 0, double y = 0, double z=0,double a=0,double b=0,double h=0,double angle = 0, double prob=1)
```

Phantom generation

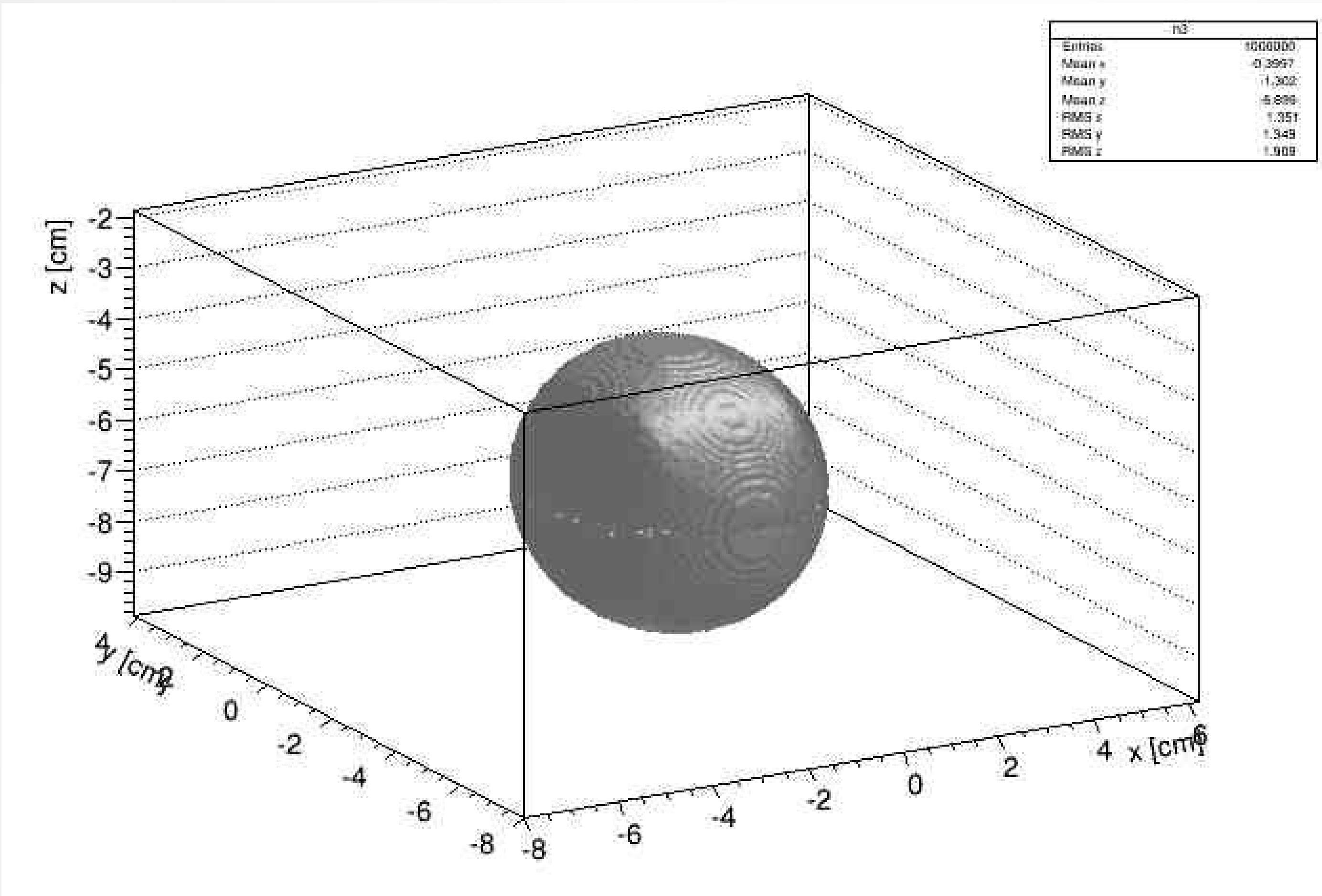


Phantom: Ellipse



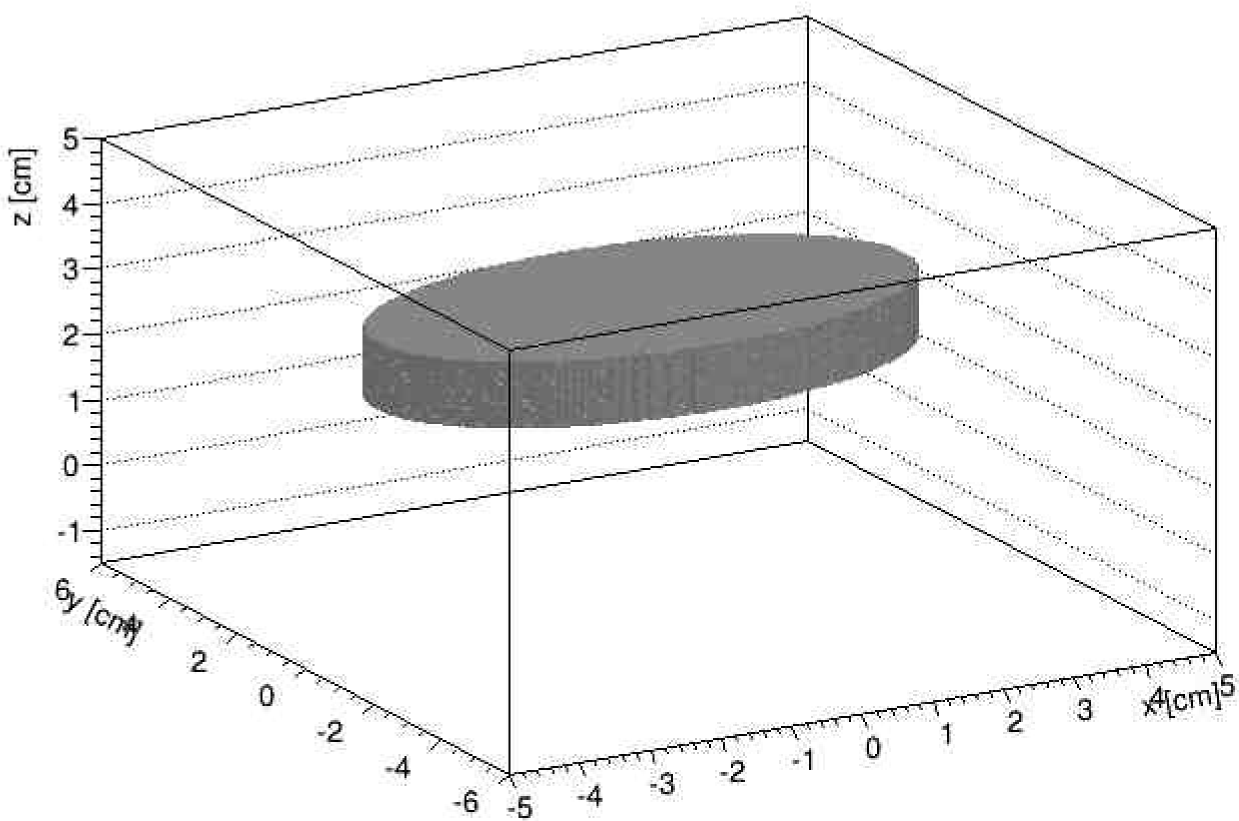
center [cm]	(1.0,2.0,3.0)
major axis [cm]	3.7
minor axis [cm]	2.5
inclination [rad]	1.3

Phantom: Sphere



center [cm]	(0.4,1.3,6.7)
radius [cm]	2.7

Phantom: Cylinder



center [cm]	(1.0,2.0,3.0)
major axis [cm]	3.7
minor axis [cm]	2.5
inclination [rad]	1.3
height [cm]	1.0

Phantom: SheppLogan 2D

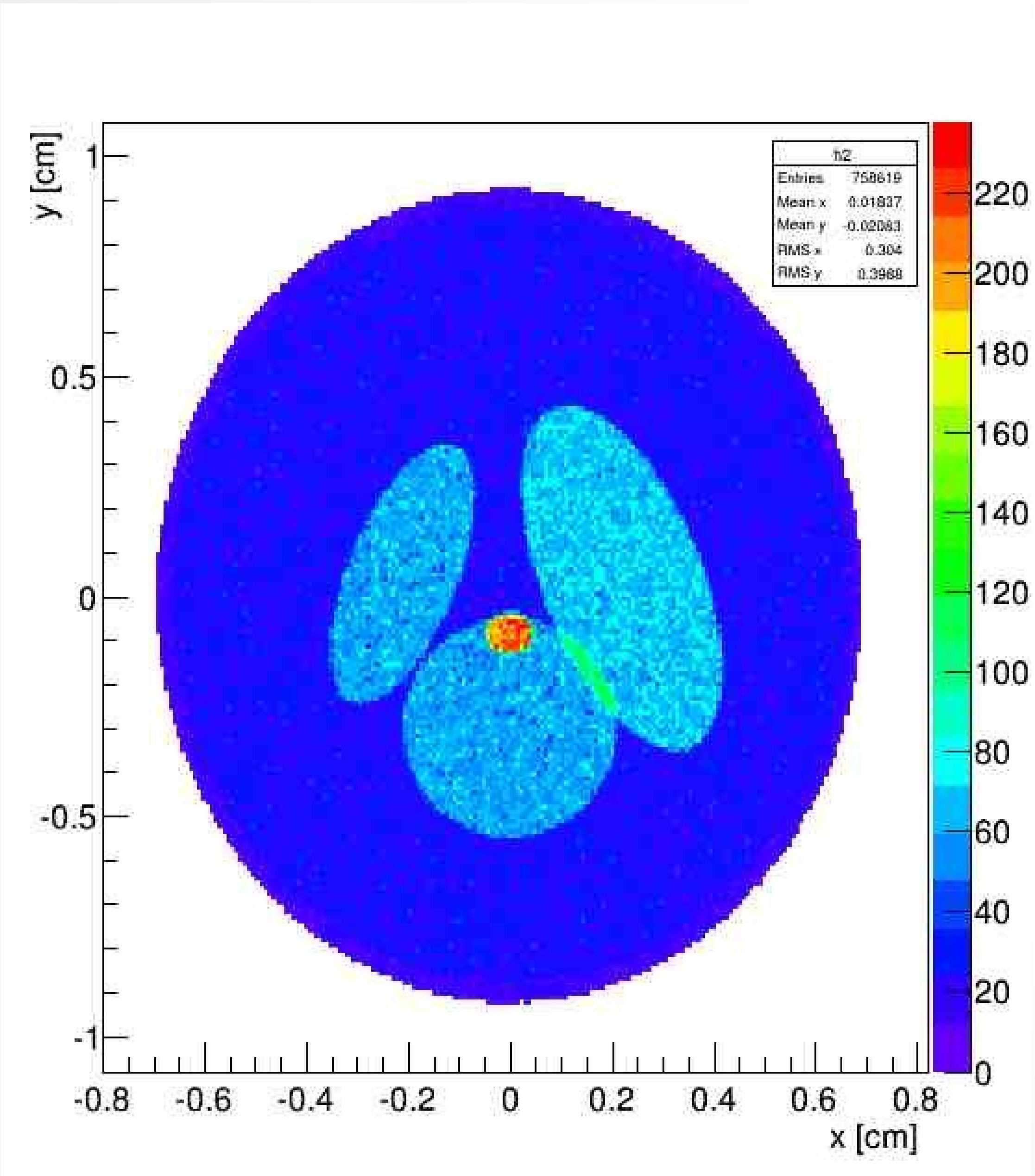
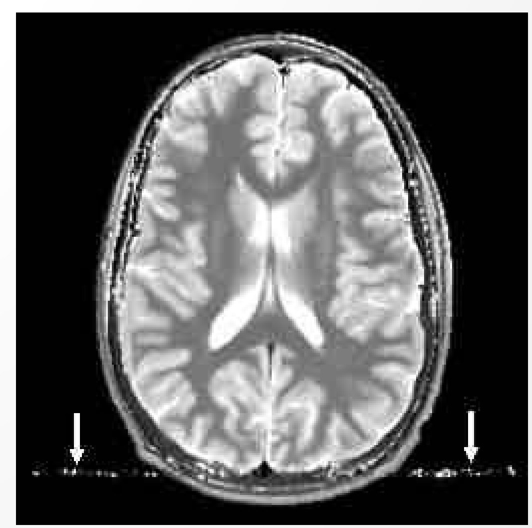


figure	relative probability
ellipse 1	0.6
ellipse 2	0.32
ellipse 3	0.18
ellipse 4	0.08
ellipse 5	0.12
ellipse 6	0.02



[3]

Reconstruction Methods

We use:

→ <u>**fast**</u> reconstruction methods

Those methods might be used for:

→ <u>**online**</u> monitoring in PET

Aim:

→ do <u>**fast**</u> reconstruction of images from PET

Why:

→ <u>**huge**</u> number of events from PET online

Backward Projection Algorithm

$$F^{(0)}(x, y) \rightarrow B^{(0)}(r, \theta) \qquad \theta \in (0, \pi)$$

$$\Delta^{(0)}(r, \theta) = B^{m} - B^{(0)}$$

$$\Delta^{(0)}(r, \theta) \rightarrow \Delta^{(0)}(x, y)$$

$$F^{(1)} = F^{(0)} + \Delta(x, y)$$

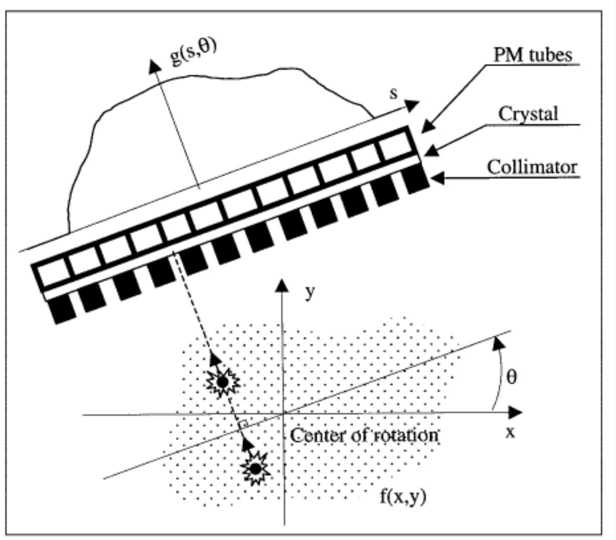


Fig.3 [1]

In our method we only use **one** step.

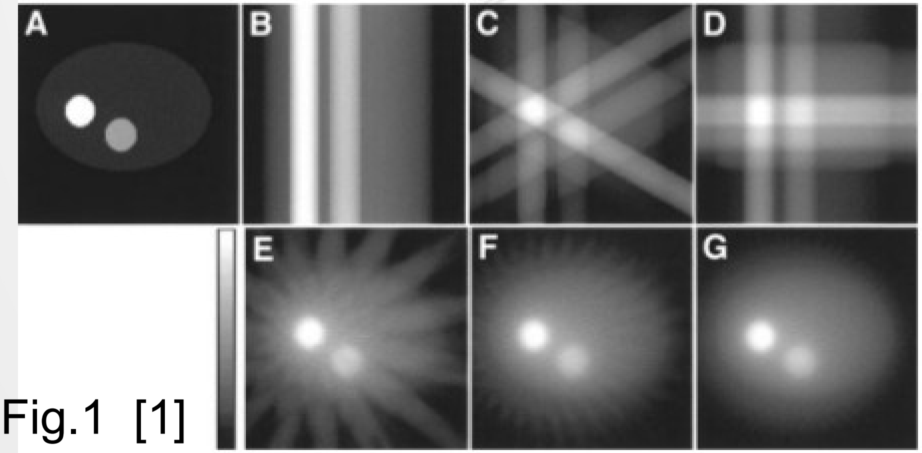
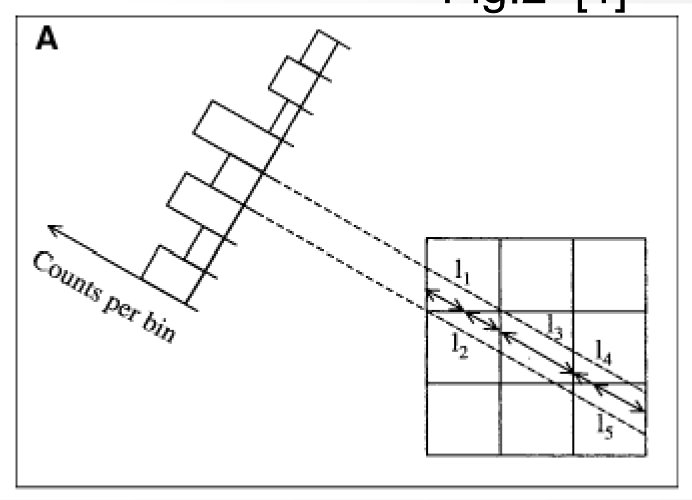
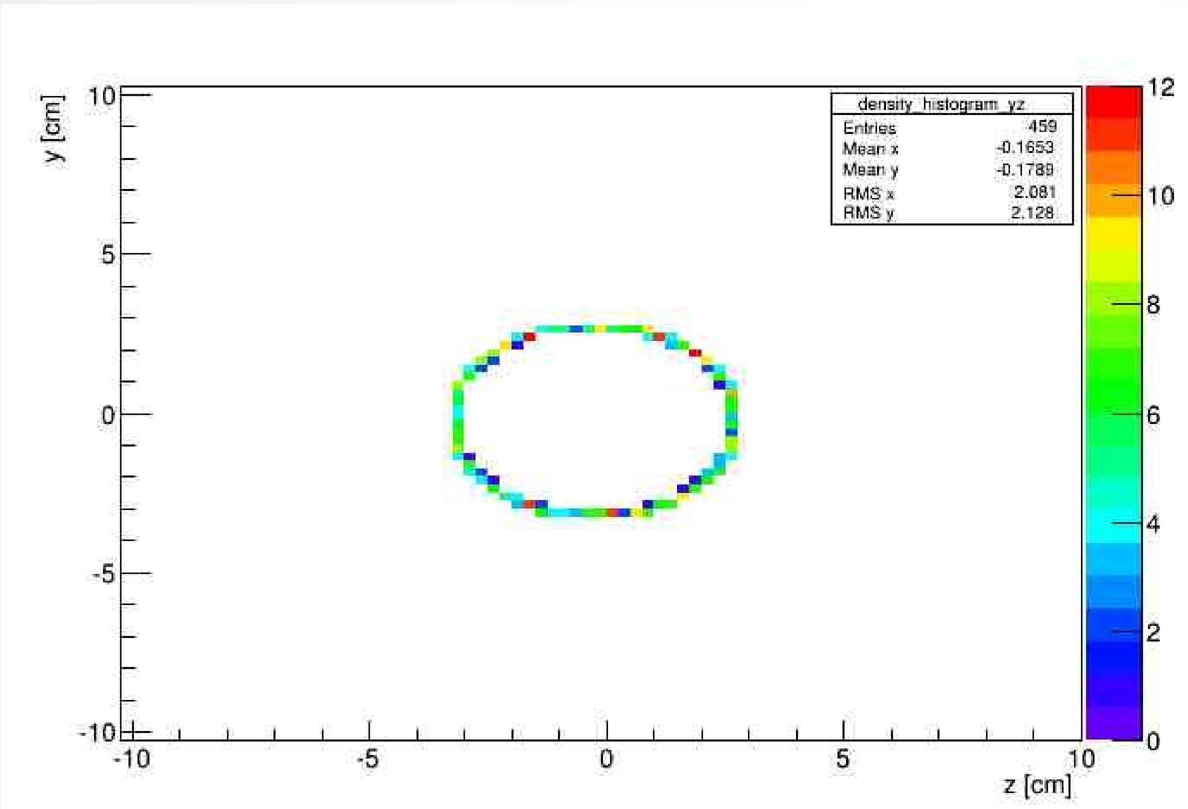


Fig.1 [1]

Fig.2 [1]

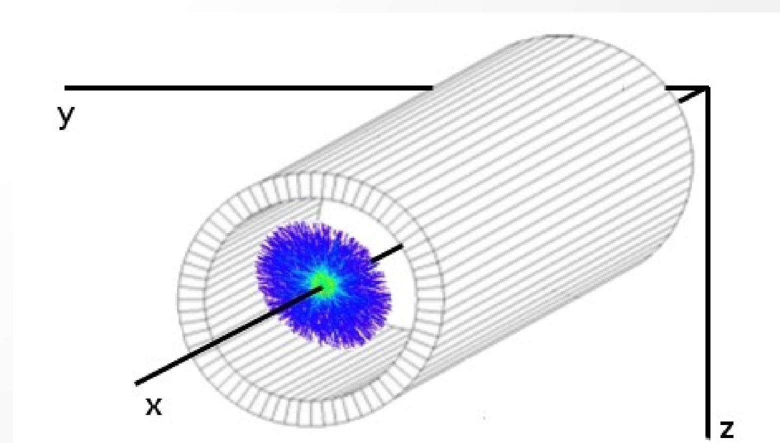


Original Object - Sphere

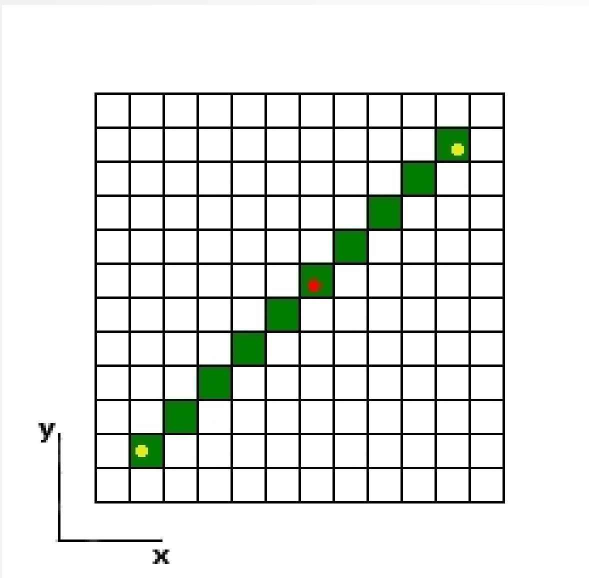


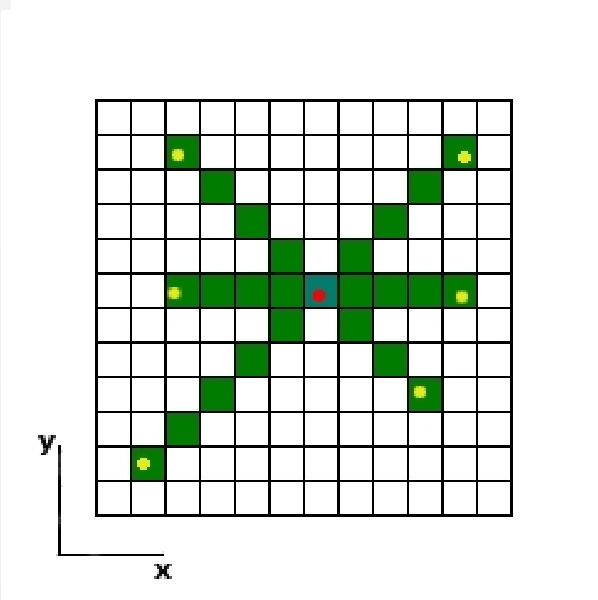
center [cm]	(0,0,0)
radius [cm]	3
points	20 000

detectors	240
decay	uniformly throughout the circle



Classical Method





Step.1 : Load X1, X2.

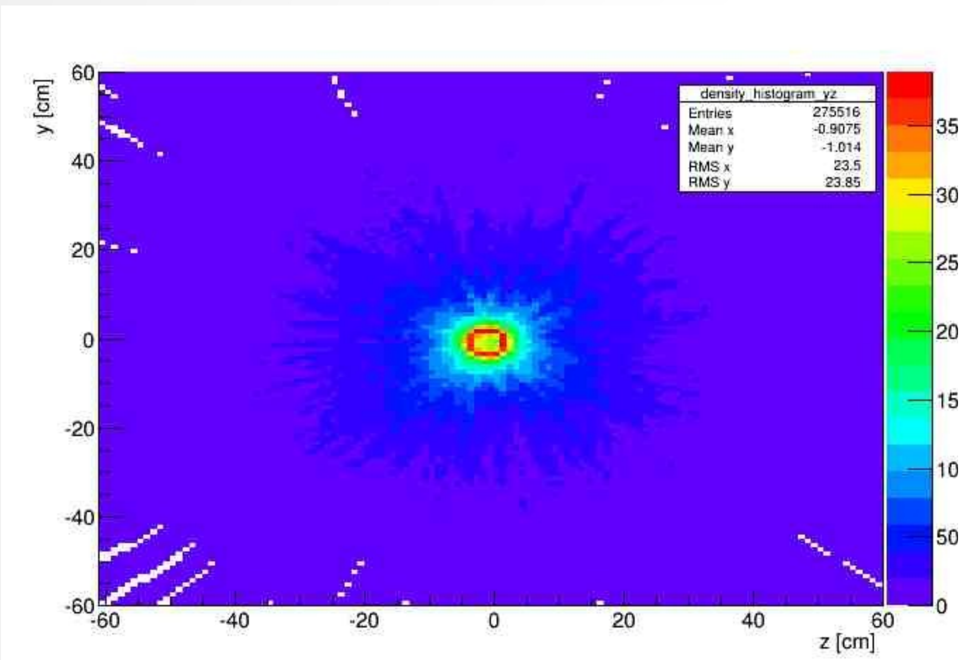
Step.2 : Follow these steps every (X2-X1)/N.

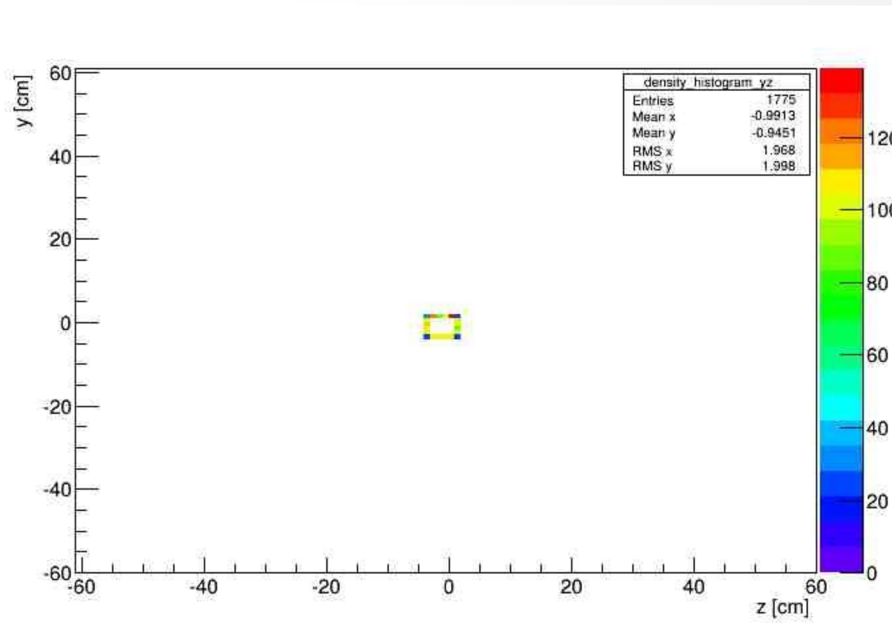
Step.3 : Start for X = X1.

Step.4 : If you are a new cuboid, then increase the value by 1.

Step.5 : Follow until X!=X2.

Classical Method





TOF PET

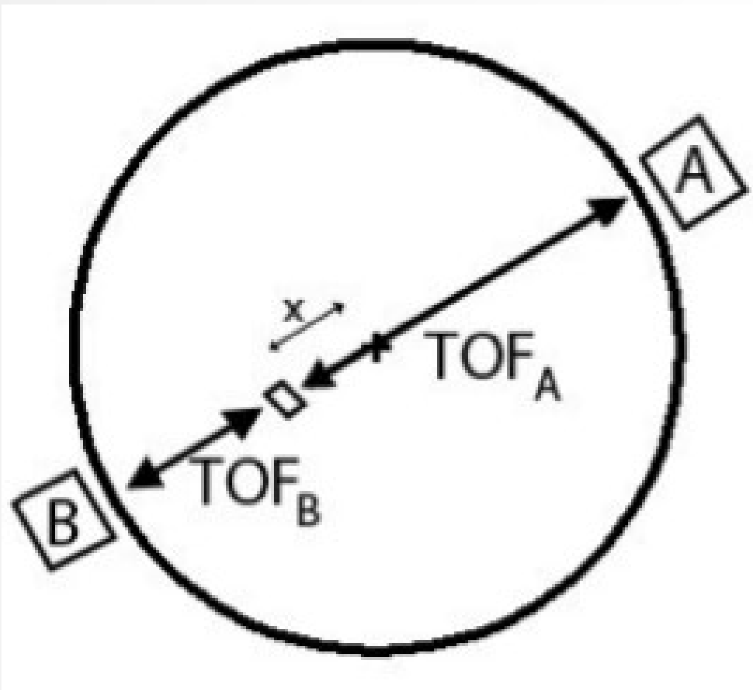


Fig.4 TOF PET geometrical model. [2]

“Time Of Flight Positron Emission Tomography (TOF PET) uses difference between the time of fight for gamma quanta from the same annihilation event.

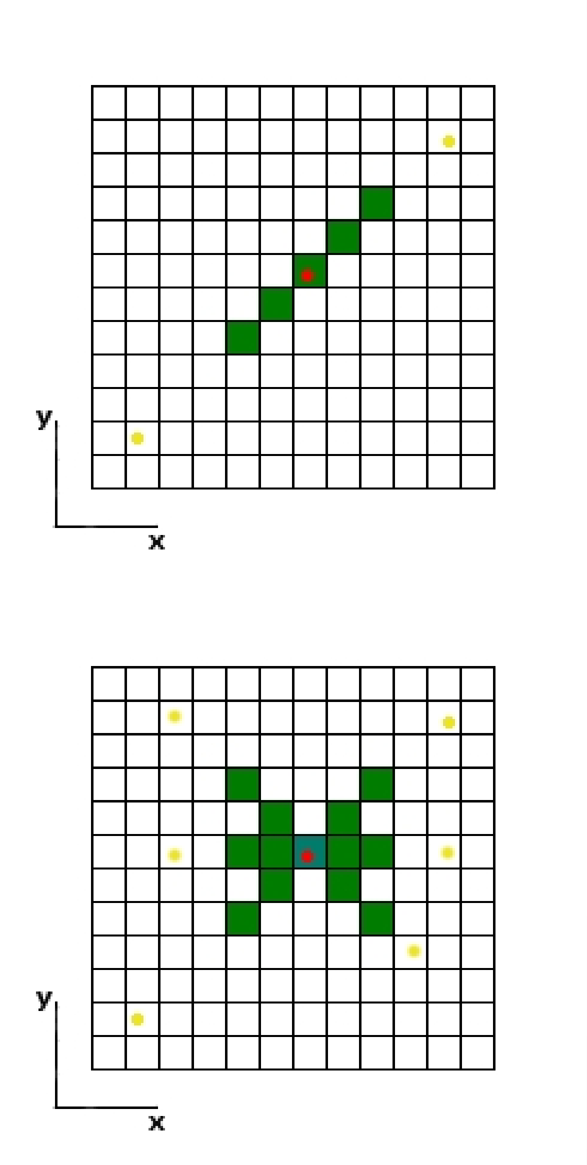
The time difference is related to the distance of annihilation point from detectors.” [2]

$$\sigma(r)=\frac{1}{2}c\,\sigma(t)$$

$$c=299\,792\,458\,m/s$$

σ(t) = 400 ps -under assumption that the error of TOF measurement can be estimated by the Gaussian distribution.

Gauss1 Method



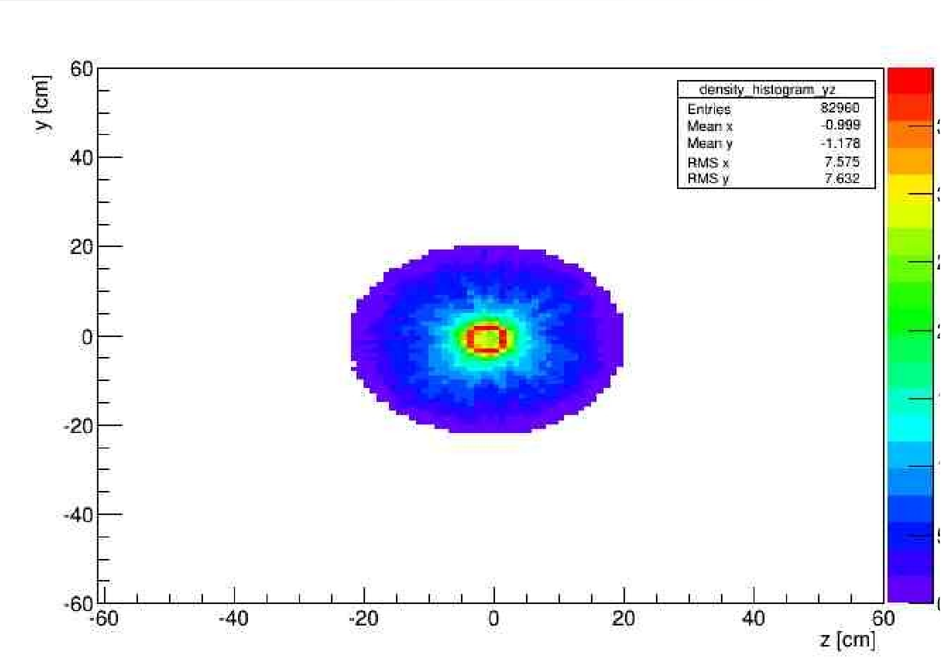
Step.1: Load X0,X1,X2.

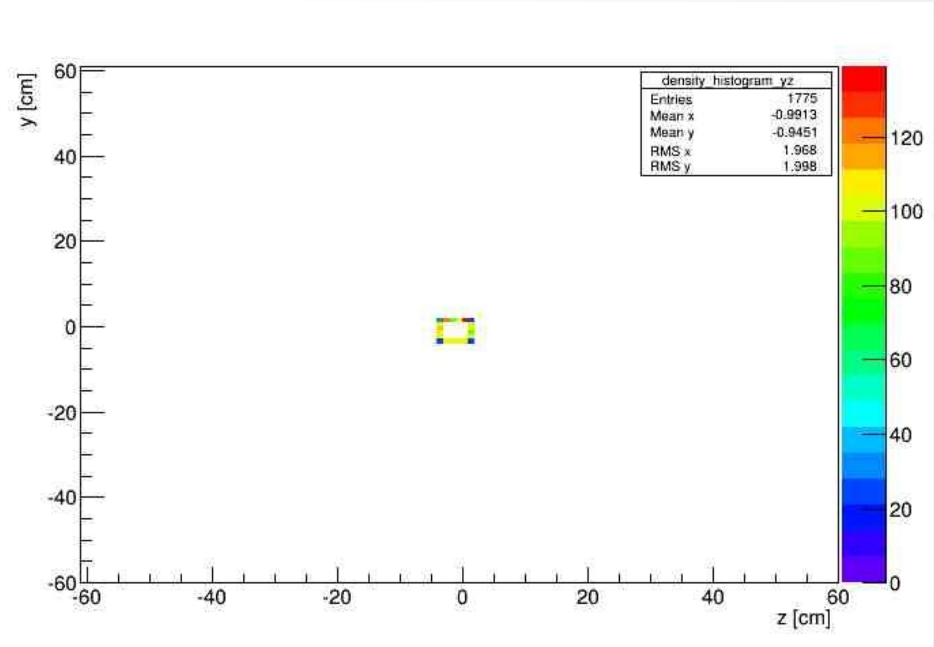
Step.2: Load σ(r).

Step.3: Apply the classical method to:

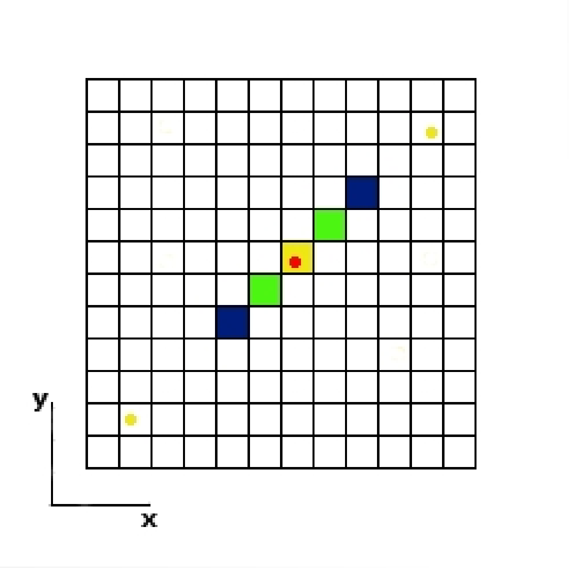
$$-3\sigma(r)+X_0<X_0<3\sigma+X_0$$

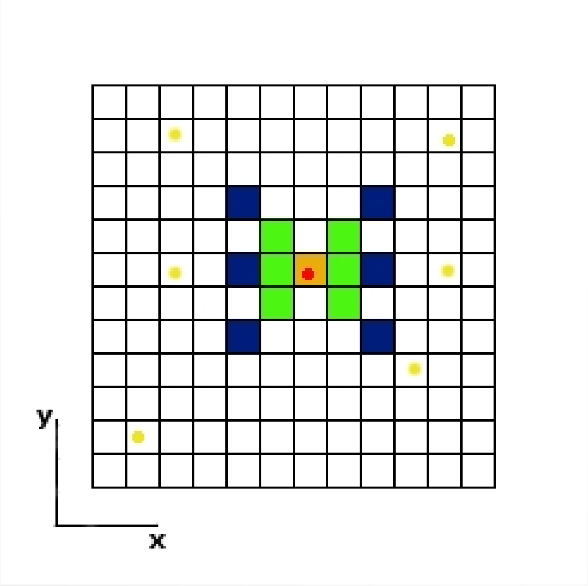
Gauss1 Method





Gauss2 Method





Step.1: Load X0,X1,X2

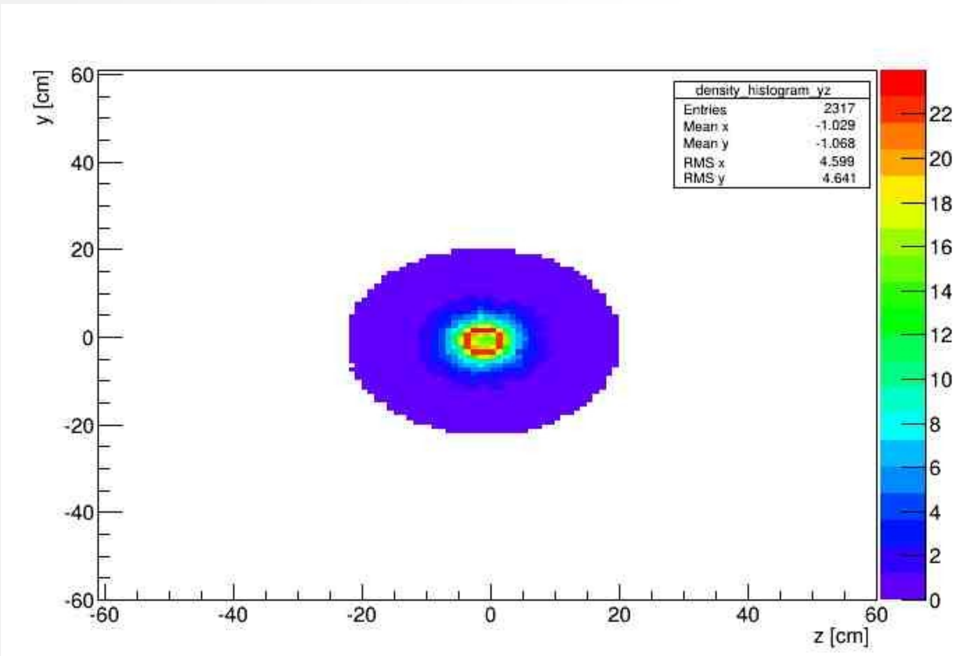
Step.2: Load σ(r)

Step.3: Apply method similar to that in Gauss1 assigning squareness values according to the formula:

$$G(r)=\frac{1}{\sigma(r)\sqrt{2\pi}}e^{-2\sigma^2(r)r^2}$$

$$r=\sqrt{(x-x_0)^2+(y-y_0)^2+(z-z_0)^2}$$

Gauss2 Method



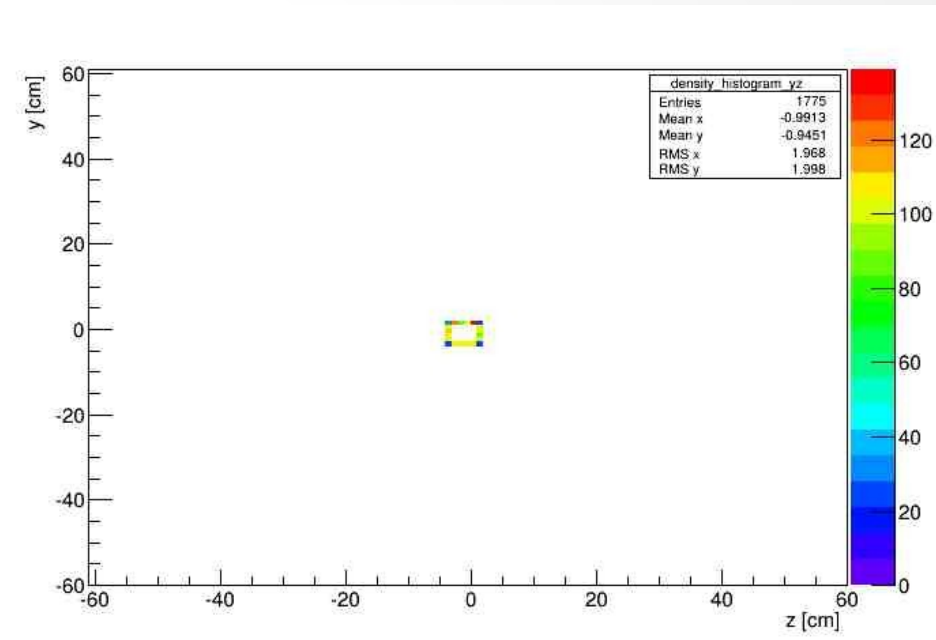


Image metrics

The quality of the reconstruction can depend in general on many things like :

- the reconstruction algorithm used,
- number of reconstruction iterations,
- input data,
- etc.

→ We need image quality metric to study the influence of those parameters on reconstructed images.

Image metrics

MSE (Mean Squared Error)

$$MSE(X,Y)=\frac{1}{N}\sum_{k=1}^{N}(x_k-y_k)^2$$

NRMSE 1 (Normalized Mean Squared Error v.1)

$$NRMSE1(X,Y)=\frac{\sum_{k=1}^{N}(x_k-y_k)^2}{\sum_{k=1}^{N}(x_k^2)}$$

NRMSE 2

$$NRMSE2(X,Y)=\frac{\sum_{k=1}^{N}(x_k-\alpha y_k)^2}{\sum_{k=1}^{N}(x_k^2)}$$

$$\alpha=\frac{\sum_{k=1}^{N}x_k\cdot y_k}{\sum_{k=1}^{N}y_k^2}$$

PSNR (Peak Signal-To-Noise Ratio)

$$PSNR(X,Y)=10\log_{10}\left(\frac{L^2(Y)}{MSE(X,Y)}\right)$$

$$L(Y)=\max(Y)-\min(Y)$$

Image metrics - Results

	Classical M.	Gauss1 M.	Gauss2 M.
MSE	5.70	4.15	0.05
NRMSE1	75.42	54.94	0.62
NRMSE2	0.78	0.70	0.54
PSNR	35.30	36.68	56.14

Gauss2 Method:
→ the best MSE
→ the best NRMSE1
→ the best PSNR

Summary

- we can produce a lot of different phantoms,
- we use phantoms to test image reconstruction methods,
- we use image metrics to quantify the quality of the reconstructed images and compare it.
- Gauss2 is the best fast reconstruction method

Bibliography

[1] Analytic and Iterative Reconstruction Algorithms in SPECT, Philippe P., The J. of Nuc. Med., vol. 43 (10,1343 - 1358), 10.2002

[2] Studies of detection of γ radiation with use of organic scintillator detectors in view of positron emission tomography,Szymon Niedźwiecki ,Cracow 2011