Simulation studies of time resolution with multi-SIPM readout of the J-PET

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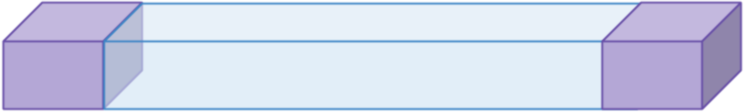
Model description



$$x_\gamma = x_\gamma(t^L - t^R) \quad (1)$$

$$t_\gamma = \frac{t^L + t^R}{2} - const \quad (2)$$

$$\sigma(t_\gamma) = \frac{1}{2}\sigma(t^L + t^R) = \frac{1}{2}\sigma(t^L - t^R) \quad (3)$$

Photon emission time distribution

$$f(t|\Theta) = K \int_{-\infty}^{+\infty} (e^{-\frac{t-\Theta}{t_d}} - e^{-\frac{t-\Theta}{t_r}}) G(2.5\sigma + \Theta, \sigma, \tau - t) d\tau, \quad (4)$$

where

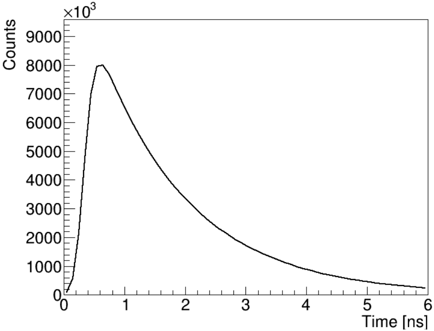
$$G(X, \sigma, x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-X)^2}{2\sigma^2}}$$

Used parameter values

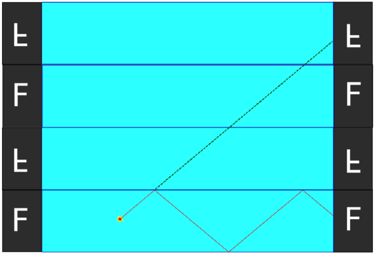
$t_r =$ 0.08 ns

$t_d =$ 1.50 ns

$\sigma =$ 0.11 ns



Geometry simplification

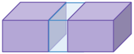


$$P_{reach} = P_{refl}(sin\alpha_x)^{n_x} P_{refl}(sin\alpha_y)^{n_y} \quad (5)$$

$$t_{reach} = t_e + \frac{\Delta L}{\frac{c}{n} cos\theta} \quad (6)$$

Statistical limit for time resolution

Ideal model:

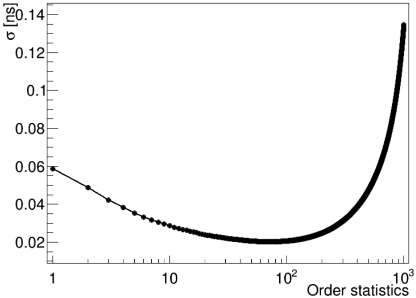


Photon absorption	no
Photomultiplier efficiency	100%
Photomultiplier TTS	0 *ns*
Scintillator size	infinitely small

We calculate $\sigma(t^L - t^R)$, where

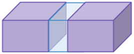
$$t^L - t^R = \frac{\sum_i \frac{t_i^L - t_i^R}{\sigma^2(t_i^L - t_i^R)}}{\sum_i \frac{1}{\sigma^2(t_i^L - t_i^R)}} \qquad (7)$$

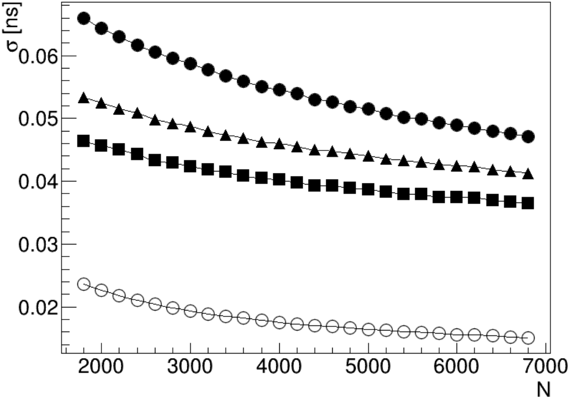
i — order statistics



The condition for index range in (7)

$$\sigma(t_i^L - t_i^R) \leq \sigma(t_1^L - t_1^R) \qquad (8)$$





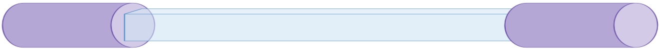
Ideal model:

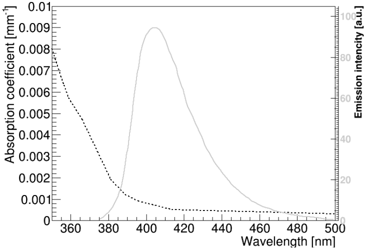
$\sigma(t_1^L - t_1^R)$
$\sigma(t_2^L - t_2^R)$
$\sigma(t_3^L - t_3^R)$

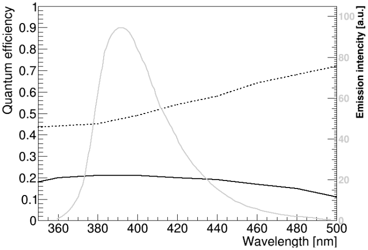
$\sigma(t^L - t^R)$

There's a possibility of increasing the time resolution if photons are registered separately.

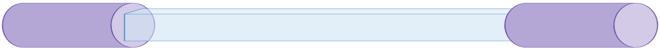
Realistic model of detector with photomultiplier tubes

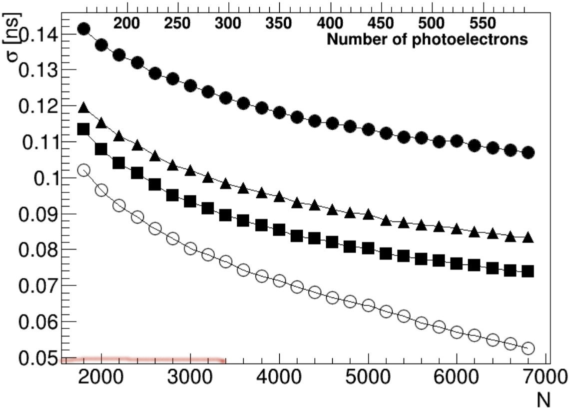






Photon absorption in scintillator	Left figure (dashed)
Photomultiplier efficiency	Right figure (solid)
Photomultiplier TTS	0.068 *ns*
Scintillator size	19*5*300 *mm*





Photomultiplier tubes:

$\sigma(t_1^L - t_1^R)$ (measured)

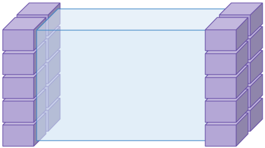
$\sigma(t_2^L - t_2^R)$
$\sigma(t_3^L - t_3^R)$

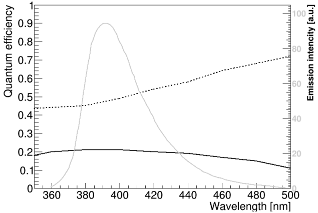
$\sigma(t^L - t^R)$

Using of realistic model does not change the previous conclusion.

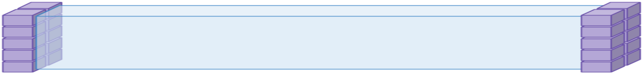
Some trick is needed for registering photons with small time difference separately.

Matrix of silicon photosensitive elements (12572-100P)





Size of sensitive area	3*3 *mm*
Insensitive edge	0.5 *mm*
Efficiency	dashed line on plot
TTS	0.128 *ns*
Signal registering time	1st photon (worst resolution)
Matrix size	5*2
Scintillator size	19*7*300 *mm*



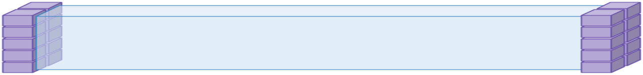
For each registered event,
times of signals from photosensitive elements

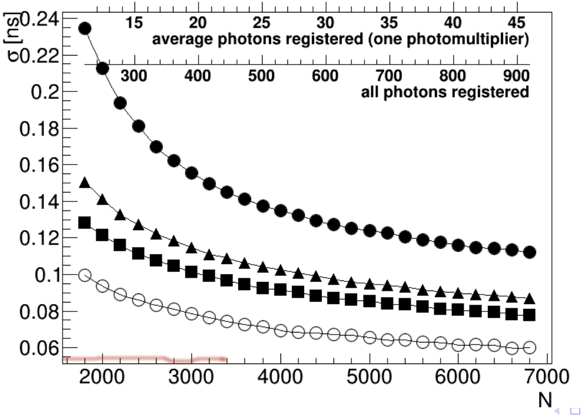
Left: $t^L_{x,y}$ Right: $t^R_{x,y}$

are sorted in ascending order

$t^L_{x^L_1,y^L_1} \leq \dots \leq t^L_{x^L_i,y^L_i} \leq \dots$ $t^R_{x^R_1,y^R_1} \leq \dots \leq t^R_{x^R_i,y^R_i} \leq \dots$

and the time resolution is calculated due to (7) and (8)





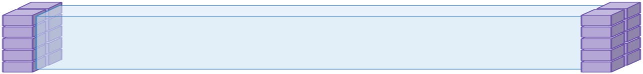
Size 5*2:

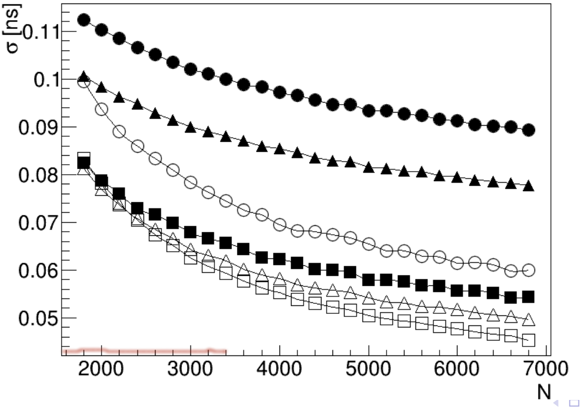
$\sigma(t^L_{x_1^L,y_1^L} - t^R_{x_1^R,y_1^R})$

$\sigma(t^L_{x_2^L,y_2^L} - t^R_{x_2^R,y_2^R})$

$\sigma(t^L_{x_3^L,y_3^L} - t^R_{x_3^R,y_3^R})$

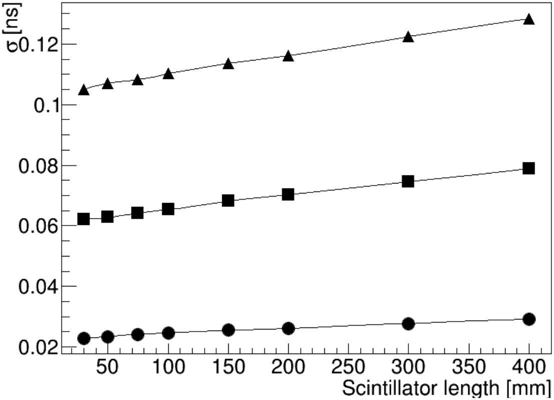
$\sigma(r^L - t^R)$





Size	edge width
2*1	0 mm
3*1	0 mm
5*2	0.5 mm
5*2	0 mm
7*2	0 mm
7*3	0 mm

Comparement of different detectors



Photomultiplier tubes

Silicon (5*2)

Ideal model

$N = 3400$