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Near threshold kaon-kaon interaction in the reactions $e^+e^- \to K^+K^-\gamma$ and $e^+e^- \to K^0\bar{K}^0\gamma$

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Abstract. Strong interactions between pairs of the K^+K^- and $K^0\bar{K}^0$ mesons can be studied in radiative decays of $\phi(1020)$ mesons. We present a theoretical model of the reactions $e^+e^- \to \phi \to K^+K^-\gamma$ and $e^+e^- \to \phi \to K^0\bar{K}^0\gamma$. The K^+K^- and $K^0\bar{K}^0$ effective mass dependence of the differential cross sections is derived. The total cross sections and the branching fractions for the two radiative ϕ decays are calculated.

1 Description of the theoretical model

The kaon-kaon strong interaction near threshold is largely unknown. Also the parameters of the scalar resonances $f_0(980)$ and $a_0(980)$ are still imprecise. The $\phi(1020)$ meson decays into $\pi^+\pi^-\gamma$, $\pi^0\pi^0\gamma$ and $\pi^0\eta\gamma$ have been measured, for the ϕ transition into $K^0\bar{K}^0\gamma$ only the upper limit of the branching fraction has been obtained in Ref. [1] but there are no data for the $\phi \to K^+K^-\gamma$ process.

In this paper we outline a general theoretical model of the e^+e^- reactions leading to final states with two pseudoscalar mesons and a photon. At the beginning we derive the amplitude $A(m)$ for the $e^+e^- \to K^+K^-\gamma$ process. It is a sum of the four amplitudes corresponding to diagrams (a), (b), (c) and (d) in Fig. 1:

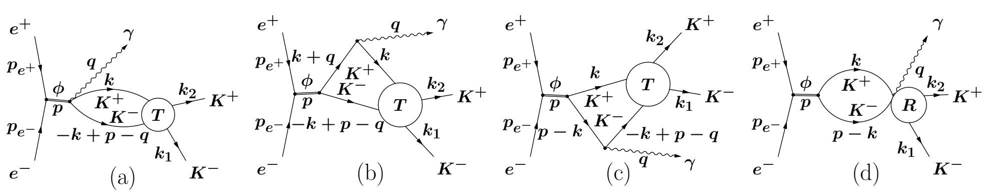

Figure 1. Diagrams for the reaction $e^+e^- \to K^+K^-\gamma$ with final-state K^+K^- interaction. The K^+K^- elastic amplitude is labelled by T and R denotes the difference of the K^+K^- amplitudes T in Eq. (4).

$$A_a = 2i \int \frac{d^4k}{(2\pi)^4} \frac{J_\nu \epsilon^{\nu *} T(k)}{D(k)D(-k+p-q)}, \tag{1}$$

$$A_b = -4i \int \frac{d^4k}{(2\pi)^4} \frac{J_\mu \epsilon^{\nu *} k_\nu (k_\mu + q_\mu) T(k)}{D(k+q)D(k)D(-k+p-q)}, \tag{2}$$

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$$A_c = -4i \int \frac{d^4k}{(2\pi)^4} \frac{J_\mu \epsilon^{\nu *} (k_\nu - p_\nu) k_\mu T(k)}{D(p-k)D(k)D(-k+p-q)}, \tag{3}$$

$$A_d = -2i \int \frac{d^4k}{(2\pi)^4} \frac{J \cdot k \, \epsilon^* \cdot \tilde{k}}{D(k)D(p-k)} \frac{[T(k-q) - T(k)]}{q \cdot \tilde{k}}. \tag{4}$$

One can show that the amplitude $A(m) = A_a + A_b + A_c + A_d$ is gauge invariant. In Eqs. (1-4) $D(k) = k^2 - m_{\rm K}^2 + i\delta$, $\delta \to +0$, is the inverse of the kaon propagator, $m_{\rm K}$ is the charged kaon mass, the four-vector $\tilde{k} = (0, \hat{\mathbf{k}})$ with the unit three-vector $\hat{\mathbf{k}} = \mathbf{k}/|\mathbf{k}|$. In the above expressions q is the photon four-momentum, $p = p_{e^+} + p_{e^-}$ is the ϕ meson four-momentum, ϵ^ν is the photon polarization four-vector and J_μ is defined as

$$J_\mu = \frac{e^3}{s} F_K(s) \bar{v}(p_{e^+}) \gamma_\mu u(p_{e^-}), \tag{5}$$

where e is the electron charge, $s = (p^2$ is the Mandelstam variable, v and u are the e^+ and e^- bispinors, respectively, γ_μ are the Dirac matrices and $F_K(s)$ is the kaon electromagnetic form factor. The K^+K^- elastic scattering amplitude is given by

$$T(k) = \langle K^-(k_1) K^+(k_2) | \tilde{T}(m) | K^-(-k+p-q) K^+(k) \rangle, \tag{6}$$

where $m^2 = (k_1 + k_2)^2$ is the square of the K^+K^- effective mass and $\tilde{T}(m)$ is the $K\bar{K}$ scattering operator. The on-shell K^+K^- amplitude can be expressed as $T_{K^+K^-}(m) = \langle K^-(k_1) K^+(k_2) | \tilde{T}(m) | K^-(k_1) K^+(k_2) \rangle$. The four-momenta of kaons in the K^+K^- center-of-mass frame are: $k_1 = (m/2, -\mathbf{k_f})$ and $k_2 = (m/2, \mathbf{k_f})$, where $k_f = \sqrt{m^2/4 - m_{\rm K}^2}$ is the kaon momentum in the final-state. We can assume that $T(k)$ is related to $T_{K^+K^-}(m)$ as follows:

$$T(k) \approx g(k) T_{K^+K^-}(m), \tag{7}$$

where $g(k)$, as a real function of the modulus of the kaon three-momentum $k \equiv |\mathbf{k}|$, takes into account the off-shell character of $T(k)$. From Eqs. (6-7) one infers that $g(k_f) = 1$.

Under a dominance of the pole at $m = 2E_k \equiv 2\sqrt{\mathbf{k}^2 + m_{\rm K}^2}$, the amplitude $A(m)$ can be written in the following form:

$$A(m) = \vec{J} \cdot \vec{\epsilon}^{\,*} \, T_{K^+K^-}(m) \, [I(m) - I(m_\phi)], \tag{8}$$

where the integral $I(m)$ reads

$$I(m) = -2 \int \frac{d^3k}{(2\pi)^3} \frac{g(k)}{2E_k m(m - 2E_k)} \left[1 - 2 \, \frac{|\mathbf{k}|^2 - (\mathbf{k} \cdot \hat{\mathbf{q}})^2}{2p_0 E_k - m_\phi^2 + 2\mathbf{k} \cdot \mathbf{q})} \right]. \tag{9}$$

In Eq. (9) $p_0 = m + \omega$, where ω is the photon energy in the K^+K^- center-of-mass frame, m_ϕ is the ϕ meson mass and $\hat{\mathbf{q}} = \mathbf{q}/|\mathbf{q}|$ is the unit vector indicating the photon direction.

In the model of Close, Isgur and Kumano [2] the momentum distribution of the interacting kaons has been expressed by the function

$$\phi(k) = \frac{\mu^4}{(k^2 + \mu^2)^2}, \tag{10}$$

with the parameter $\mu = 141$ MeV. This function is normalized to unity at $k = 0$, however the function $g(k)$ in Eq. (7) has to be normalized to 1 at $k = k_f$, so the function $g(k)$ corresponding to $\phi(k)$ should be defined as

$$g(k) = \frac{(k_f^2 + \mu^2)^2}{(k^2 + \mu^2)^2}. \tag{11}$$

In Ref. [2] kaons are treated as extended objects forming a quasi-bound state. If the K^+K^- system is point-like, like in Refs. [3] or [4], then the function $g(k) \equiv 1$. Let us note that both models can be treated as special cases of our approach.

Separable kaon-kaon potentials can be used to calculate the kaon-kaon amplitudes needed in practical application of the present model. Then the function g(k) takes the following form:

$$g(k) = \frac{k_f^2 + \beta^2}{k^2 + \beta^2}, \tag{12}$$

where β is a range parameter. In Ref. [5] for the isospin zero $K\bar{K}$ amplitude the value of β close to 1.5 GeV has been obtained. In order to get an integral convergence at large k in Eq. (9) we use an additional cut-off parameter $k_{\max}$=1 GeV.

2 Numerical results

In Fig. 2 we show the K^+K^- effective mass distributions at the e^+e^- energy equal to m_ϕ. On the left panel one observes some dependence on the form of the function $g(k)$. A common feature is a presence of the maximum of the differential cross section situated only a few MeV above the K^+K^- threshold. On the right panel we see a comparison of our model (solid line) with two other models, named the "no-structure" model [6] and the "kaon-loop" model, and described in Refs. [3] and [4]. The parameters of the latter models have been taken by us from experimental analysis of the data of the reaction $\phi \to \pi^+\pi^-\gamma$ [7] and then used in calculation of the results shown as dashed and dotted lines.

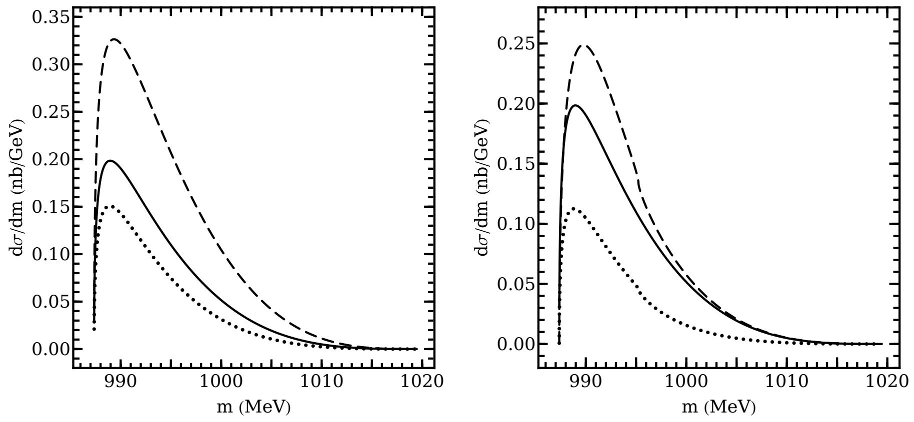

Figure 2. Dependence of the differential cross-section for the reaction $e^+e^- \to K^+K^-\gamma$ on the K^+K^- effective mass m. Left panel: the solid line corresponds to the case of the function $g(k)$ (Eq. 12) with the parameter $\beta \approx 1.5$ GeV and the cut-off $k_{\max} = 1$ GeV, the dotted line - to $g(k) \equiv \phi(k)$ given by Eq. (10) and the dashed curve - to $g(k)$ from Eq. (11) with $\mu = 141$ MeV; right panel: the dashed line is calculated for the no-structure model (Ref. [6]), the dotted one for the kaon-loop model of Ref. [4] with parameters obtained in Ref. [7] and the solid line is the same as in the left panel but with a different vertical scale.

The $K^0\bar{K}^0$ differential cross sections are presented in the left panel of Fig. 3. These cross sections are considerably lower than the K^+K^- cross sections seen in Fig. 2. This is due to a smaller $K^0\bar{K}^0$ phase space and to smaller absolute values of the transition amplitude $T(K^+K^- \to K^0\bar{K}^0)$ which replaces in this case the elastic K^+K^- amplitude in Eq. (8).

By integration of the K^+K^- and $K^0\bar{K}^0$ effective mass distributions, shown as solid lines in left panels of Figs. 2 and 3, one can calculate the total cross sections which are equal to 1.85 pb and 0.17 pb, respectively. The corresponding branching fractions are $4.5{\cdot}10^{-7}$ and $4.0{\cdot}10^{-8}$. In the right panel of Fig. 3 we have plotted the contours of the branching fraction for the $\phi \to K^0\bar{K}^0\gamma$ decay as a function of the $a_0(980)$ resonance position. We see that it is possible to generate lower values of the branching fraction by a moderate change of not well known resonance mass and width.

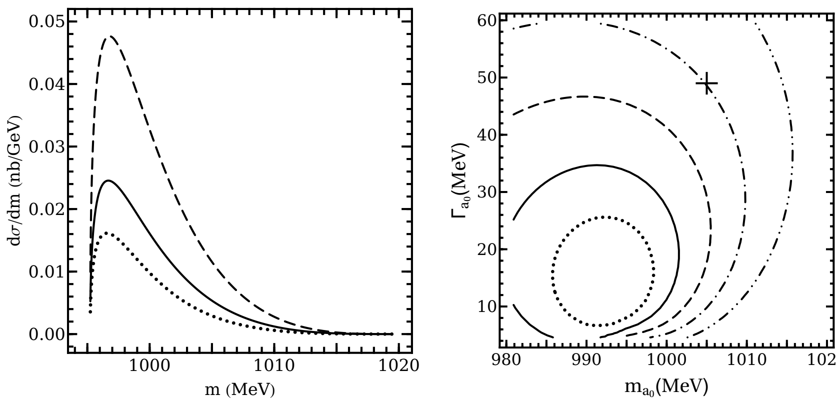

Figure 3. Left panel: differential cross-section for the reaction $e^+e^- \to K^0\bar{K}^0\gamma$ as a function of the $K^0\bar{K}^0$ effective mass. The curves are labelled as in the left panel of Fig. 2. Right panel: contours of the branching fraction Br for the decay $\phi \to K^0\bar{K}^0\gamma$ in the complex plane of the $a_0(980)$ pole position: $m_{a_0(980)}$ is the resonance mass and $\Gamma_{a_0(980)}$ is its width. The solid curve corresponds to the KLOE upper limit $Br = 1.9 \cdot 10^{-8}$, the dotted one to $Br = 1.0 \cdot 10^{-8}$, the dashed curve to $Br = 3.0 \cdot 10^{-8}$, the dashed-dotted one to $Br = 4.0 \cdot 10^{-8}$, and the dashed- double dotted one to $Br = 5.0 \cdot 10^{-8}$. The cross indicates the $a_0(980)$ resonance position on sheet $(-+)$ found in Ref. [8].

3 Conclusions

The above theoretical results for the reactions with charged and neutral kaon pairs indicate that the measurements of the $e^+e^- \to K^+K^-\gamma$ process can provide a valuable information about the pole positions of the $a_0(980)$ and $f_0(980)$ resonances. A coupled channel analysis of the radiative ϕ transitions into different pairs of mesons in the final state is possible after a relevant generalization of the present model.

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