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COMPUTER SCIENCE**

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**Comparative analysis of image reconstruction
algorithms based on NEMA NU2 phantom
measurements for the J-PET scanner**

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1 Introduction

Positron Emission Tomography (PET) is one of the most rapidly developing diagnostic imaging techniques in modern nuclear medicine. An example of an innovative approach in this field is the J-PET scanner [1], developed at the Jagiellonian University, which uses plastic scintillator strips. The specific geometry and timing characteristics of this type of detector require a dedicated approach to the mathematical modelling of the data acquisition and reconstruction process. In particular, the new generation of the device – Modular J-PET – is based on independent detection modules equipped with arrays of silicon photomultipliers (SiPM). Such an architecture makes it possible to optimise detection efficiency and to configure the tomograph geometry flexibly, while at the same time introducing new challenges in the precise modelling of the detector response.

The aim of this thesis is to develop a stand-alone image reconstruction code for radioactive sources in the NEMA NU2 phantom [2] based on the MLEM (Maximum Likelihood Expectation Maximization) algorithm [3], and to analyse the data by comparing the images and line profiles with the reference results of the CASToR software [4].

The thesis consists of an introduction, four chapters, a summary and a bibliography. The first chapter presents the main theoretical foundations of positron emission tomography and the specifics of data recording in the list-mode format. A simple algorithm for determining annihilation points is presented, and the operation of the MLEM algorithm is described in its classical version (NON-TOF) and with the time of flight of photons taken into account (TOF). The second chapter contains a description of the geometry and experimental parameters of the phantom used, as well as the specifics of the experimental data used in the analysis. The third chapter gives a complete description of the implementation and evolution of the algorithm in the programming environment. The fourth chapter is devoted to the presentation of the tomographic images obtained at successive stages of the program's development and to their qualitative and quantitative evaluation by comparing the images with each other and with the reference images from the CASToR software.

2 Physical and mathematical foundations of PET tomography

2.1 Principle of operation of Positron Emission Tomography (PET)

Positron Emission Tomography (PET) is an advanced diagnostic imaging technique of nuclear medicine that enables non-invasive study of metabolic processes taking place in a living organism.

The physical principle of operation of a PET scanner is based on the phenomenon of electron-positron pair annihilation and consists of several main stages:

- Administration of a radiopharmaceutical: before the examination, the patient is given a safe chemical substance containing a radioactive isotope that undergoes β^+ decay. The most common choice is fluorodeoxyglucose (FDG, containing ^{18}F), which accumulates in regions of increased metabolism (e.g. cancer cells).
- Positron emission: the nucleus of the isotope emits a positron, which travels in tissue over a distance of a fraction of a millimetre [5] and loses its kinetic energy through Coulomb interactions in matter.
- Annihilation: once the positron has lost most of its energy, it interacts with one of the surrounding electrons and annihilation occurs, in which the e^+e^- particles are converted into two gamma quanta (γ) with an energy of 511 keV travelling in opposite directions (at an angle of about 180°) [6].

The patient is surrounded by a cylindrical arrangement of scintillation detectors (e.g. plastic strips in the J-PET tomograph), which registers the gamma quanta in coincidence.

2.2 Data recording in the list-mode format

As a result of the measurement, we obtain the hit positions of both annihilation quanta (photons) in the format (x_1, y_1, z_1) and (x_2, y_2, z_2) , together with their registration times t_1 and t_2 . The virtual line that begins at the point (x_1, y_1, z_1) and ends at the point (x_2, y_2, z_2) is called the Line of Response (LOR). The coordinate system used in the J-PET detector is shown in Fig. 2.1. The origin of the coordinate system (the zero point) is placed at the geometric centre of the scanner. Experimental data in the list-mode format are stored in a ROOT file, which is used for further analysis.

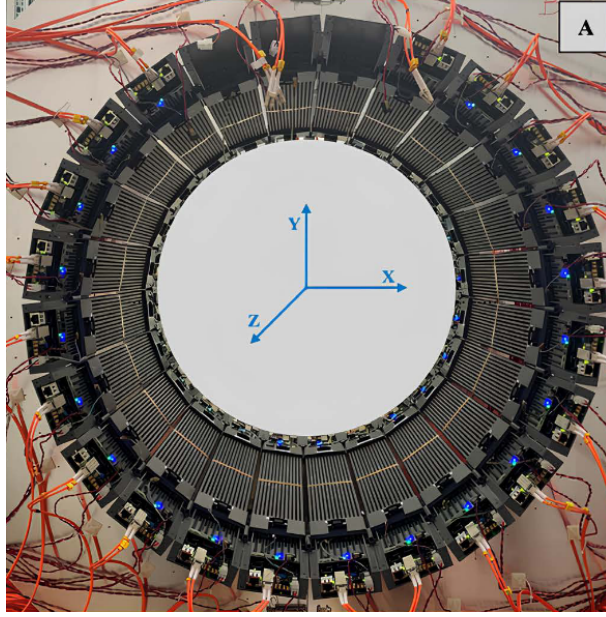


Figure 2.1: Coordinate axes system of the J-PET detector [7].

2.3 Method of determining annihilation points using the Time-of-Flight (TOF) technique

The simplest method of PET image reconstruction is the determination of annihilation points on the LOR using the difference in the photon time of flight (TOF).

The procedure for determining the annihilation point can be described as follows. Each registered annihilation event ($e^+e^- \rightarrow 2\gamma$) is characterised by the coordinates of the hit in the first and the second detector. A LOR is drawn between these two points, initially assuming the annihilation point to be at the middle of the LOR (x_{mid} in Fig. 2.2).

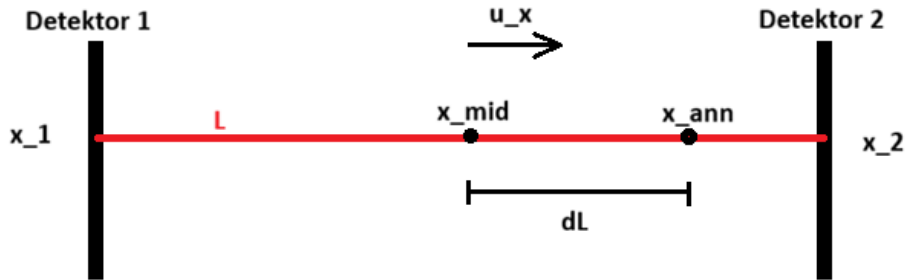


Figure 2.2: Illustration of determining the position of the annihilation point on the LOR, using the X coordinate as an example, based on the geometry and the photon time-of-flight (TOF) information.

The coordinate of the LOR midpoint on the X axis is determined as:

$$x_{mid} = \frac{x_1 + x_2}{2} \quad (2.1)$$

where:

x_1 – position of the interaction point of the γ quantum with the first scintillation detector (the first detector in a LOR pair is taken to be the one with the lower identification number (ID)),

x_2 – position of the interaction point of the γ quantum with the second scintillation detector.

The coordinates y_{mid} and z_{mid} are obtained analogously.

Next, the point $(x_{mid}, y_{mid}, z_{mid})$ must be shifted using the photon interaction time information t_1 and t_2 .

For this purpose, one can define a vector $\vec{u}$, which is the direction vector of the shift of the annihilation point along the LOR, with the normalisation factor L (the LOR length):

$$L = \sqrt{\Delta x^2 + \Delta y^2 + \Delta z^2}, \quad \vec{u} = \left(\frac{\Delta x}{L}, \frac{\Delta y}{L}, \frac{\Delta z}{L} \right), \quad (2.2)$$

where: $\Delta x = x_2 - x_1$, $\Delta y = y_2 - y_1$, and $\Delta z = z_2 - z_1$.

Figure 2.2 shows the direction of the vector $\vec{u}$ along which the point should be shifted from the LOR midpoint in order to obtain the position of the annihilation point on the x axis.

The X coordinate of the annihilation point can be calculated as:

$$x_{ann} = x_{mid} - u_x \cdot dL, \quad dL = \frac{dt \cdot c}{2} \quad (2.3)$$

where:

$dt = t_2 - t_1$ – difference in the photon time of flight,

dL – distance by which the point x_{mid} is shifted along the LOR,

c – speed of light in air ($299\,792\,458 \frac{m}{s}$) [8].

The factor of 2 in the denominator results from the fact that we are shifting the LOR midpoint. The coordinates y_{ann} and z_{ann} are obtained analogously.

Unfortunately, this method generates serious artefacts in the reconstructed image and is not sufficient for a full analysis of experimental data. Therefore, it is necessary to use more advanced reconstruction methods such as MLEM, which is described in the next section.

2.4 Mathematical foundations of the MLEM algorithm

The modern standard of image reconstruction in PET is the MLEM (Maximum Likelihood Expectation Maximization) algorithm [3]. It is an iterative method that aims to fit the activity of each cell of the tomographic image (voxel) to all the events (LORs). There are two versions of this algorithm: with TOF (TOF MLEM) and without TOF (NON-TOF MLEM). In this thesis, reconstruction was carried out using each of these methods.

2.4.1 NON-TOF MLEM

The algorithm written in Eq. (2.4) is fully equivalent to the classical matrix notation using the parameter λ_j^k described in the literature [3].

$$f_j^{n+1} = \frac{f_j^n}{\sum_i a_{ij}} \sum_{i \in \text{events}} a_{ij} \frac{1}{\sum_k a_{ik} f_k^n}, \quad (2.4)$$

where:

f_j^n – activity value of voxel j in iteration n ,

$\sum_i a_{ij}$ – sensitivity – the probability that a signal originating from voxel j will be registered by the system,

a_{ij} – element of the system matrix, which in the code represents the weight of voxel j for the registered event i (the geometric interpretation of determining this value is presented in Figs. 2.3 and 4.1 and in Section 4.3),

$\sum_k a_{ik} f_k^n$ – forward projection – the number of events expected after passing through one LOR, based on the currently estimated image (in the ideal case it equals one, because there is one LOR and one event is expected),

$\sum_{i \in \text{events}}$ – backprojection – the process of distributing the error (the ratio of the measured data to the forward projection) back onto the voxels in order to update the image.

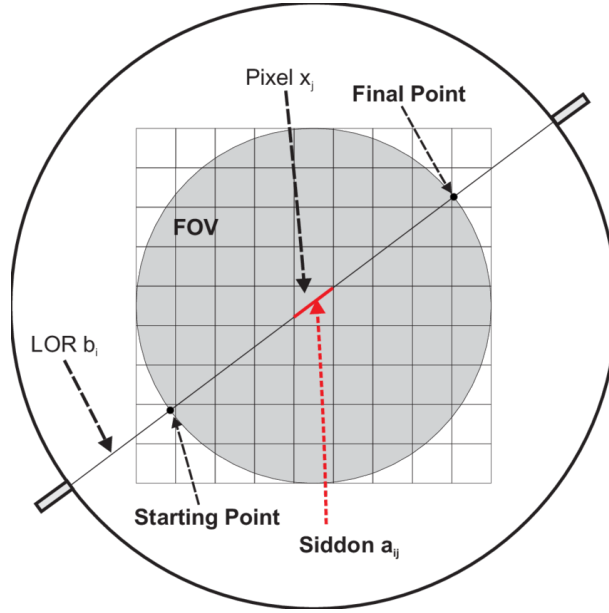


Figure 2.3: Diagram of the operation of the Siddon algorithm for the line of response LOR b_i . The determined element a_{ij} (marked in red) represents the length of the segment of line i passing through a given voxel x_j inside the field of view (FOV). The starting point is the position of the interaction point of the γ quantum with the first scintillation detector, and the final point is the position of the interaction point of the γ quantum with the second scintillation detector [9].

2.4.2 TOF MLEM

The MLEM algorithm with TOF taken into account can be presented as follows:

$$f_j^{n+1} = \frac{f_j^n}{\sum_i a_{ij}} \sum_{i \in \text{events}} a_{ij} \cdot w_{ij} \frac{1}{\sum_k a_{ik} \cdot w_{ik} \cdot f_k^n}, \quad (2.5)$$

where: w – weighting factor which assigns a weight, on a scale from 0 to 1, to the LOR segment currently being computed, depending on how far this segment is from the predicted annihilation point. The closer the segment is to the annihilation point, the greater its influence on the predicted number of events or on the correction of the activity.

The weighting factor for voxel k and LOR i is therefore expressed by the formula:

$$w_{ik} = \exp\left(-\frac{diff^2}{2\sigma^2}\right), \quad (2.6)$$

where:

$diff$ – difference between the distance of the end of the currently computed segment and the annihilation point,

σ – time resolution converted into distance (analogously to Eq. 2.3).

The total experimental time resolution of the J-PET scanner measurement system is 628 ps (FWHM) [10]. In practical numerical implementations this value is approximated by a reference time resolution of 600 ps. The standard deviation depends on the FWHM as follows [11]:

$$\sigma = \frac{FWHM}{2.35}. \quad (2.7)$$

3 Experimental data

The validation of the image reconstruction algorithms was carried out on the basis of data from measurements performed with the J-PET detector with a ProNM NEMA NU2 phantom [2] placed inside, as shown in Fig. 3.1.

3.1 NEMA IQ phantom

The NEMA IQ phantom contains six spheres with internal diameters of: 10 mm, 13 mm, 17 mm, 22 mm, 28 mm and 37 mm (left panel in Fig. 3.1). All the spheres were filled with an aqueous solution with a high concentration of the radiotracer fluorodeoxyglucose (^{18}F -FDG) [10]. The body of the phantom surrounding the spheres constituted the background, in which the radiotracer concentration was 10 times lower than inside the spheres. In addition, a lung insert model was located in the very centre of the phantom geometry, used to simulate a region of low density. The phantom was placed on a dedicated Plexiglas plate with wooden guide rails and introduced into the scanner.

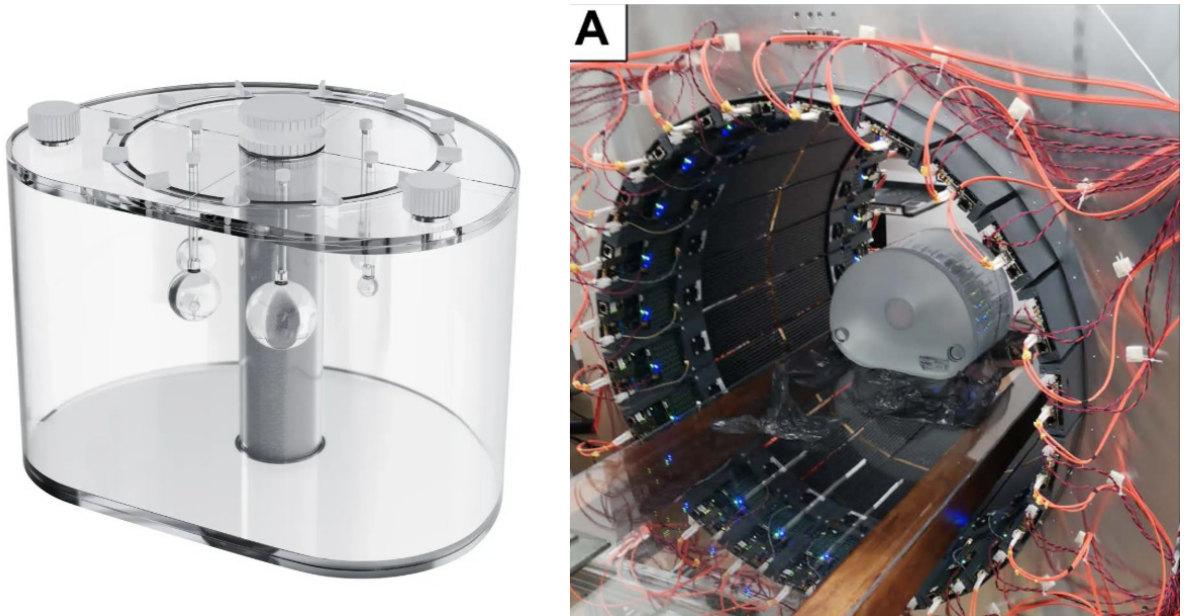


Figure 3.1: Left panel: NEMA IQ phantom used in the measurement. Right panel: NEMA phantom with spheres filled with FDG placed inside the Modular J-PET tomograph [10].

3.2 Description of the Modular J-PET experimental setup and data acquisition conditions

The study used data from the digital Modular J-PET scanner in a single-layer configuration [1]. The system consists of 24 independent detection modules arranged in a cylinder with an inner diameter of 739 mm. Each individual module contains 13 strips of EJ-230 plastic scintillator with dimensions of $500 \times 24 \times 6 \text{ mm}^3$.

The light signal from both ends of each scintillator strip is registered by arrays of silicon photomultipliers (SiPM) from Hamamatsu. The scanner uses a dedicated digital data readout system (Front-End Electronics — FEE), which allows threshold-free registration of the time and amplitude of the signals (triggerless readout). Such a design provides high time resolution, which is crucial for time-of-flight (TOF) reconstruction techniques.

The acquisition of the experimental data was carried out continuously in the triggerless mode and lasted 60 minutes. The exact signal detection times were recorded for the individual voltage comparison thresholds. The collected data stream was subjected to a time calibration procedure and to filtering using a coincidence time window (CTW = 3 ns), an energy window and an angular separation criterion of more than 60° in the transaxial plane. These steps allowed a preliminary selection of the registered signals for the further reconstruction and data correction process. As a result, 40.38 million valid coincidence events that met the assumed time, energy and geometric filtering criteria were collected for the final reconstruction [10]. In order to optimise the computation time and to test the reconstruction code prepared in this thesis, a sample constituting 50% of the total number of collected data was used for the final analyses.

4 Software implementation and evolution of the MLEM algorithm

4.1 Choice of programming environment: Python, NumPy and Numba acceleration

The entire source code developed, including the implementation of the MLEM algorithm, the procedures for loading list-mode data and the scripts used for qualitative analysis and visualisation of the results, has been deposited in a public GitHub repository [12]. The code has been made openly available, which allows full reproducibility of the reconstruction results obtained in this thesis.

The `Python` programming language was chosen for implementing the reconstruction code, owing to the number of libraries that can help in implementing the code and the author's level of familiarity with the language. The `NumPy` library was used for processing, managing lists and performing mathematical operations on vectors and matrices. `SciPy` was also used for advanced mathematical operations, `Matplotlib` for presenting projections of the reconstructed images, and `Uproot` [13] for extracting data from files intended for the ROOT framework [14].

The environment used for preparing the code is `Jupyter notebook`. At the early stages of developing the code, an attempt was made to carry out the reconstruction using the Google Colab service, but code execution on the author's own computer is 20 times faster.

All calculations were carried out on the processor of the author's own computer, which has 6 cores. To speed up program execution, to use all the cores of the computer and to compile the program into machine code, the `Numba` library [15] was used.

Preparing the code for computations on a graphics card would have been justified in order to shorten the program execution time, but it required knowledge of additional topics such as working with graphics card memory and the `CuPy` library [16], which has its own specifics.

4.2 Modelling the geometry and defining the reconstruction grid (Grids 100x100x57 and 200x200x200)

In order to carry out discrete reconstruction, the continuous space of the field of view (FOV) was divided into a grid of voxels of defined dimensions.

The first grid had dimensions of 100 voxels along the X and Y axes and 57 voxels along the Z axis, in the range from -20 to 20 cm for each axis, which gives a voxel size of 0.4 cm along the X and Y axes and 0.7 cm along the Z axis. The grid sizes were chosen as a compromise between image quality and program execution time. This grid was used for all images before the application of the sensitivity map from the professional CASToR software [4] (see Sections 5.3.2, 5.3.3 and 5.4).

The second grid, ranging from -25 to 25 cm along each axis with a voxel size of $(0.25, 0.25, 0.25) \text{ cm}^3$, is the standard for J-PET studies. It was used in the image reconstruction with the sensitivity map from CASToR (see Section 4.5.2) [10].

4.3 Application of the Ray Marching method in determining the LOR

In order to determine the LOR length in each voxel and to determine the system matrix element, the analytical Siddon-Jacobs method [17] is used, which makes it possible to determine exactly the LOR length in each voxel. At an early stage of software development, however, in order to avoid excessive implementation complexity and potential numerical problems related to handling edge cases (e.g. a ray passing exactly through the edges or vertices of voxels), it was decided to use an alternative approach in the form of the so-called Ray Marching method [18].

The Ray Marching algorithm discretely samples the space along the LOR with a constant numerical step (e.g. 0.2 cm), which ensures stable operation and excellent suitability for parallelisation of the computations using the `Numba` library. An illustration of the operation of this method is shown in Fig. 4.1.

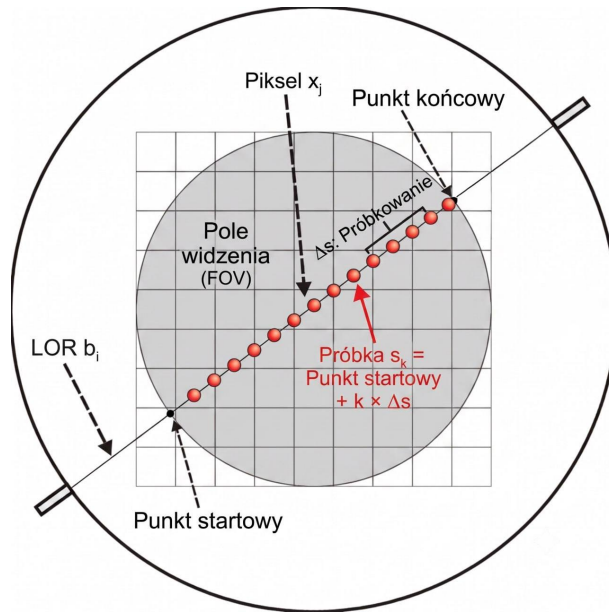


Figure 4.1: Diagram of sampling the space with the Ray Marching method along the LOR b_i . The sampling points s_k (marked as red circles) are the distances of the end of the currently computed LOR segment from the starting point. Δs determines the length of the LOR segment and is the difference between the distances of consecutive sampling points. k is the sampling point number.

Taking into account the experimental time resolution of the J-PET system of about 600 ps, the higher geometric accuracy of analytical methods would not noticeably improve the quality of the final image.

4.4 Masking of the field of view (FOV)

In the MLEM algorithm, cylindrical masking of the field of view (FOV) was implemented in order to improve numerical stability and to correct the visualisation of the results.

A mask with a radius of 18 cm was applied in order to select the region in which the phantom is located. There is no activity outside the phantom, but the MLEM algorithm tends to inflate noise during its operation. According to Eq. (2.4), in the “forward” projec-

tion, owing to the low activity of voxels outside the phantom, the value of the denominator is small and the algorithm does not expect an event, although it was registered, as a result of which it drastically inflates the activity in voxels outside the phantom. During a single iteration this effect is weak, but in subsequent iterations the algorithm increases the activity in the same voxels outside the phantom and the image quality deteriorates. Therefore, after each iteration, the activity of pixels outside the mask (at a radius > 18 cm) is set to zero.

4.5 Sensitivity map

A sensitivity map is needed to perform PET image reconstruction. In this thesis, three possible maps were used: a uniform map, a map generated using the CASToR framework, and a map created by the author.

The sensitivity map takes into account the density distribution of LOR lines in the field of view (FOV) of the scanner. In the very centre of the detector there is the maximum number of possible LORs, which may lead to an artificial inflation of activity, because the number of LORs registered at this point is greater than at the place of actual highest activity. The tomographic image must therefore be corrected using this map.

4.5.1 Uniform sensitivity map

A uniform sensitivity map (filled with ones) is the simplest solution, but not an ideal one, since it can generate artefacts in the image. However, it can be used to verify the correct implementation of the MLEM algorithm.

4.5.2 Sensitivity map from CASToR

The literature [10] describes the process of creating a sensitivity map for the modular J-PET detector using the CASToR software. In the first stage, a high-statistics simulation of a measurement of a uniform cylinder completely filling the field of view of the J-PET tomograph was performed using the GATE environment [19] (based on Monte Carlo methods). Such a measurement took into account the physical detection efficiency of the plastic scintillation strips. In the second stage, the **SENS** optimiser used backprojection of the registered LORs, creating a three-dimensional weight matrix.

4.5.3 Determining the author's own sensitivity map

The sensitivity map was created on the basis of information about the positions of all detectors (pseudo-crystals) in the format (x, y, z) , consistent with the axes marked in Fig. 2.1. The total number of detectors is 62400.

The author's own backprojection code was used, in which, instead of the photon interaction coordinates, the coordinates of each of the detectors were used. Each detector was connected to every other detector by exactly one LOR. The graphical connection of the detectors is shown in Fig. 4.2. At each step of the passage through a LOR, one is added to the sum.

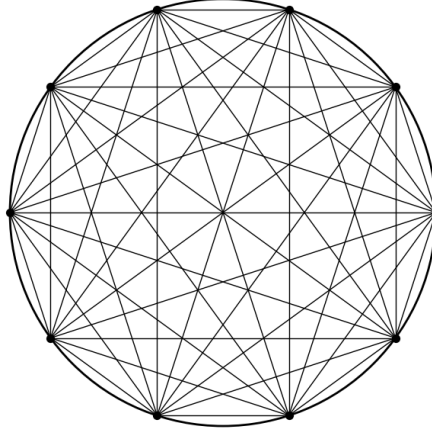


Figure 4.2: Illustration of connecting ten detectors on an every-with-every basis.

The resulting sensitivity map has the shape of a moiré effect, shown in Fig. 4.3. This result is a direct consequence of the discretisation of the detector positions (division into pseudo-crystals) and of the geometric discontinuity of the scanner ring, in which there are empty spaces between the detection modules.

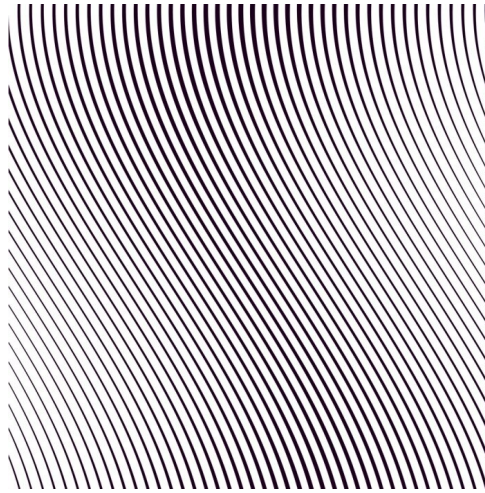


Figure 4.3: Optical moiré effect [20].

A possible solution to this problem is to add a Gaussian blur to the sensitivity map and to test the reconstruction for different values of the blur width σ .

4.6 Multi Ray Marching

The next stage in the development of the image reconstruction program is the application of the multi Ray Marching method. This method consists in tracing ten lines instead of a single LOR. The main LOR is located in the centre and the nine remaining ones are located around it at a distance of the voxel side (2.5 mm), forming not a LOR (Line of Response) but a TOR (Tube of Response). A graphical representation of this method in two dimensions is shown in Fig. 4.4.

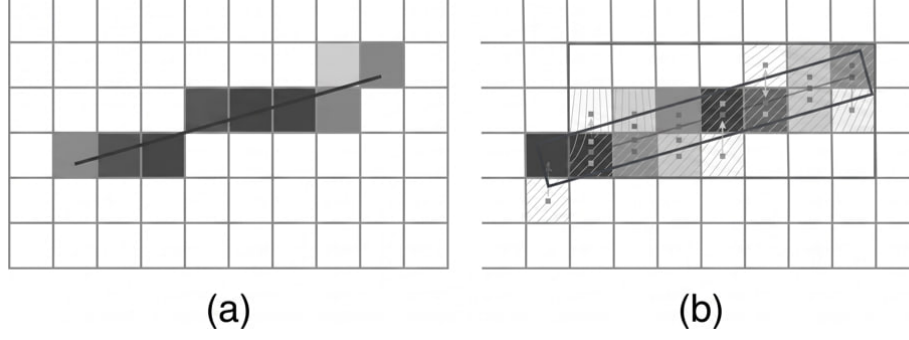


Figure 4.4: Graphical representation of the Ray Marching method for a) a single LOR line, b) 3 LOR lines. [21].

A single pseudo-crystal has the shape of a cuboid with dimensions of $24 \times 6 \times 2.5 \text{ mm}^3$ (width, thickness and axial length, respectively). It is not a physical element of the detector, but a virtual segment separated programmatically by dividing each of the scintillator strips into 200 equal parts along the Z axis [10]. The TOR model takes into account the finite geometric dimensions of the scintillation elements of the scanner, eliminating the unrealistic assumption of a point-like nature of signal registration. In addition, the model prevents the formation of so-called discretisation gaps. The problem of discretisation gaps results from the nature of sampling a discrete space (the voxel grid) with steps of constant length in the Ray Marching method. When a ray is projected at an oblique angle with respect to the grid axes, the standard step length of the algorithm may turn out to be too large. As a result, the next sampling point on the ray may “jump over” a neighbouring voxel, omitting it completely, even though the physical ray passes through it. This leads to the formation of artificial gaps in the data and to distortion of the reconstructed image. The introduced model prevents this phenomenon. This guarantees geometric continuity and drastically limits the formation of banding artefacts.

For this model to work correctly, the diameter ϕ of the tube must be slightly larger than the diagonal of the voxel (3.53 mm for the adopted voxel size of $2.5 \times 2.5 \times 2.5 \text{ mm}^3$). In my analysis I assume $2R_{tube} = 4.8 \text{ mm}$.

The mathematical implementation of this model is as follows:

- We look for a vector $\vec{n}$ perpendicular to the direction of the vector $\vec{u}$, which is directed along the LOR. The coordinates of the vector $\vec{u}$ are described by Eq. 2.2. The vectors $\vec{n}$ and $\vec{u}$ are perpendicular when they satisfy the condition $\vec{u} \cdot \vec{n} = 0$.
- The vector $\vec{u}$ is three-dimensional (left panel of Fig. 4.5). We assume that one of the coordinates of the vector $\vec{n}$ is equal to 0 (n_x or n_y). This allows the immediate determination of the remaining two coordinates by swapping their places and changing the sign of one of them.
- We distinguish two cases of the vector $\vec{n}$ (Eqs. 4.1 and 4.2), which is necessary in order to avoid division by zero. This guarantees that the value under the square root in the denominator will always be greater than zero, regardless of the direction in which the vector $\vec{u}$ points.

$$- |u_x| > |u_y|:$$

$$\vec{n} = \frac{[-u_z, 0, u_x]}{\sqrt{u_z^2 + u_x^2}}. \quad (4.1)$$

$$- |u_x| \leq |u_y|:$$

$$\vec{n} = \frac{[0, -u_z, u_y]}{\sqrt{u_y^2 + u_z^2}}. \quad (4.2)$$

- The third vector of the orthogonal basis is determined by the cross product of the vectors $\vec{u}$ and $\vec{n}$.

$$\vec{v}_{ort} = \vec{n} \times \vec{u} \quad (4.3)$$

- We determine the initial coordinates of the starting point for all (ten) LORs ($r \in (0,9)$) of the TOR:

$$x_{1,r} = x_1 + \Delta n_r \cdot n_x + \Delta v_r \cdot v_{ortx}, \quad (4.4)$$

where:

x_1 – coordinate of the first detector on the x axis,

Δn_r – value of the shift along the vector $\vec{n}$,

Δv_r – value of the shift along the vector $\vec{v}_{ort}$.

Shift values:

- for $r = 0$:

$$\Delta n_r = 0, \quad \Delta v_r = 0 \quad (4.5)$$

- for $r > 0$:

$$\Delta n_r = R_{tube} \cdot \cos\left(\frac{2\pi \cdot r}{9}\right), \quad \Delta v_r = R_{tube} \cdot \sin\left(\frac{2\pi \cdot r}{9}\right) \quad (4.6)$$

A graphical representation of this method is shown in Fig. 4.5. Next, the MLEM algorithm is carried out as presented in Eq. 2.4 or 2.5.

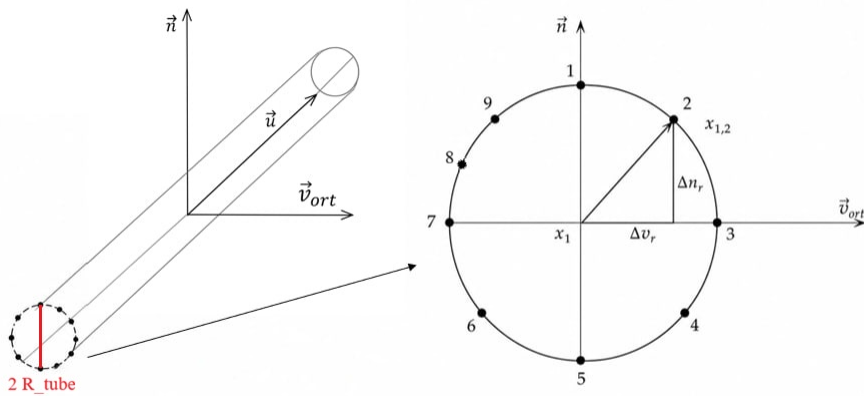


Figure 4.5: Graphical illustration of the orthogonal basis around the LOR and of the shift values Δv_r and Δn_r .

The purpose of applying this method is to bring the mathematical projection operator closer to the real physical conditions of the J-PET scanner and to improve the numerical stability of the algorithm. CASToR uses the Multi Siddon algorithm, which creates the TOR in the same way.

4.7 Attenuation correction

In the process of reconstructing a PET tomographic image, one must take into account the physical phenomena related to the interaction of gamma radiation with matter, which include: attenuation (absorption of gamma quanta in the object), intra-object scatter, random coincidences, as well as geometric and normalisation effects resulting from the discontinuity of the scanner ring structure and the varying efficiency of the detectors [10].

In order to take the attenuation effect into account in the image reconstruction, an attenuation map μ was used, which was generated on the basis of a computed tomography (CT) scan of the examined object and converted using an appropriate method described in the literature [22].

The implementation of the map in the MLEM algorithm is as follows:

$$\mu_{sum} = \sum_{s=0}^{steps-1} \mu(x_s, y_s, z_s) \cdot \Delta s, \quad A_n = e^{-\mu_{sum}} \quad (4.7)$$

where:

μ_{sum} – accumulated linear attenuation coefficient along the photon path (LOR),

s – index of the successive discrete step of sampling the space,

steps – total number of numerical steps for sampling a given LOR,

$\mu(x_s, y_s, z_s)$ – local value of the linear attenuation coefficient in a given medium (taken from the attenuation map),

Δs – constant length of a single numerical step (here 0.2 cm),

A_n – attenuation factor (the probability that both photons leave the patient's body without interacting with matter), determined according to the Beer-Lambert law [23].

The attenuation factor is taken into account for each LOR in the main formula of the algorithm (Eq. 2.4 or 2.5):

$$f_j^{n+1} = \frac{f_j^n}{\sum_i a_{ij}} \sum_{i \in \text{events}} a_{ij} \cdot w_{ij} \frac{\left(\frac{1}{A_i}\right)}{\sum_k a_{ik} \cdot w_{ik} \cdot f_k^n}, \quad (4.8)$$

where:

$\left(\frac{1}{A_i}\right)$ – attenuation correction weight for the i -th event, which compensates for the physical absorption of photons and restores their original intensity.

5 Data analysis results

5.1 CASToR reconstructions (reference result)

Figures 5.1 and 5.2 below present images reconstructed using the CASToR software, which were compared with my reconstructed PET tomographic images. The Amide program [24] was used for data analysis and for creating the dynamic line profile. It is a program adapted to the analysis of reconstruction images. A Gaussian blur of 2σ ($\sigma = 1$ voxel of the neighbourhood) was applied to each of the images. The reconstruction images are from the tenth iteration of the MLEM algorithm.

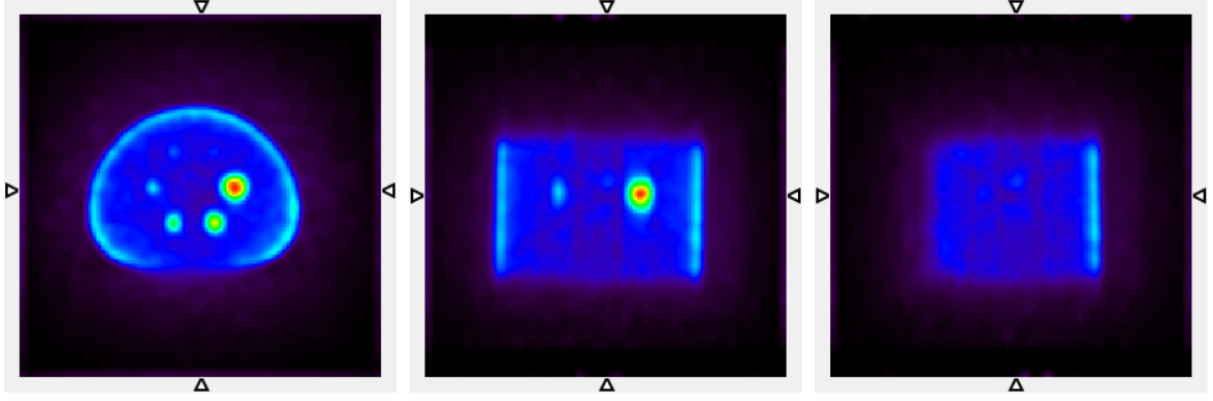


Figure 5.1: Reconstruction of the NEMA NU2 phantom using the CASToR software without taking attenuation into account, presented in three planes. Left panel: transverse (axial) section ($Z=0$), coronal section ($Y=0$) and sagittal section ($X=0$) [10].

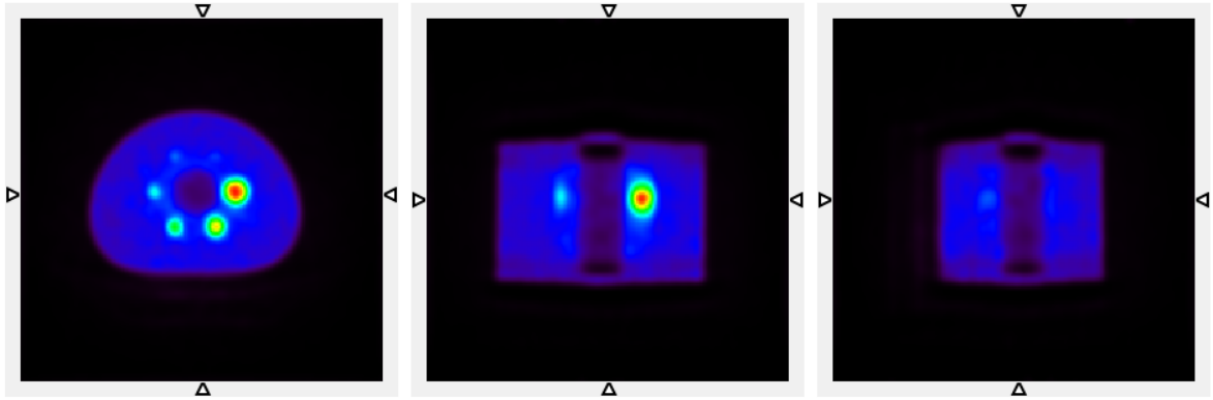


Figure 5.2: Reconstruction of the NEMA NU2 phantom using the CASToR software with attenuation taken into account, presented in three planes. Left panel: transverse (axial) section ($Z=0$), coronal section ($Y=0$) and sagittal section ($X=0$) [10].

5.2 Image of annihilation points

Based on the method described in Section 2.3, the annihilation points were determined. The distribution of annihilation points in the XY plane and the line profile (at $Y=0$ cm) are shown in Fig. 5.3a.

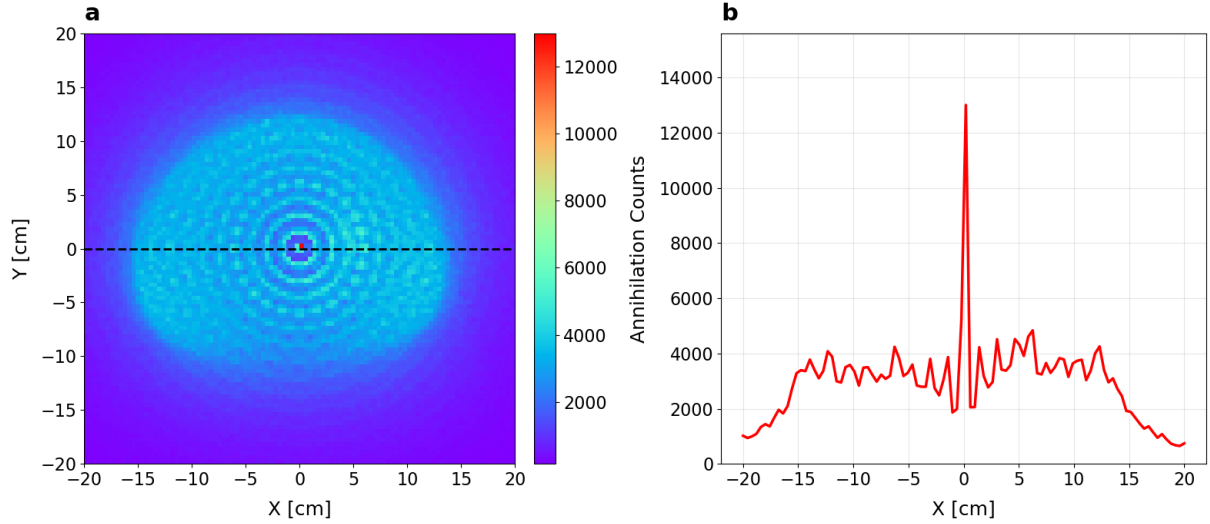


Figure 5.3: a) Annihilation points in the XY projection (the number of annihilations is summed along the Z axis), b) line profile at $Y=0$.

The absolute majority of annihilation points are located in the very centre of the phantom and, because of one bright voxel, the data from the line profile have a small amplitude compared to the centre. In addition, periodic artefact structures are visible in the uncorrected image. They are a direct effect of the spatial discretisation of the interaction positions inside the detector (division into a finite number of pseudo-crystals) and of the lack of a correction for geometric sensitivity and system normalisation.

In order to smooth the image, a Gaussian blur and masking of the centre by the average of the surrounding points were applied. This operation improves the image and makes it possible to notice structures of increased activity. The result is shown in Fig. 5.4, where in the line profile one can notice a peak corresponding to the activity from the sphere with $\phi = 37\text{mm}$. This procedure is, however, insufficient for correct image reconstruction.

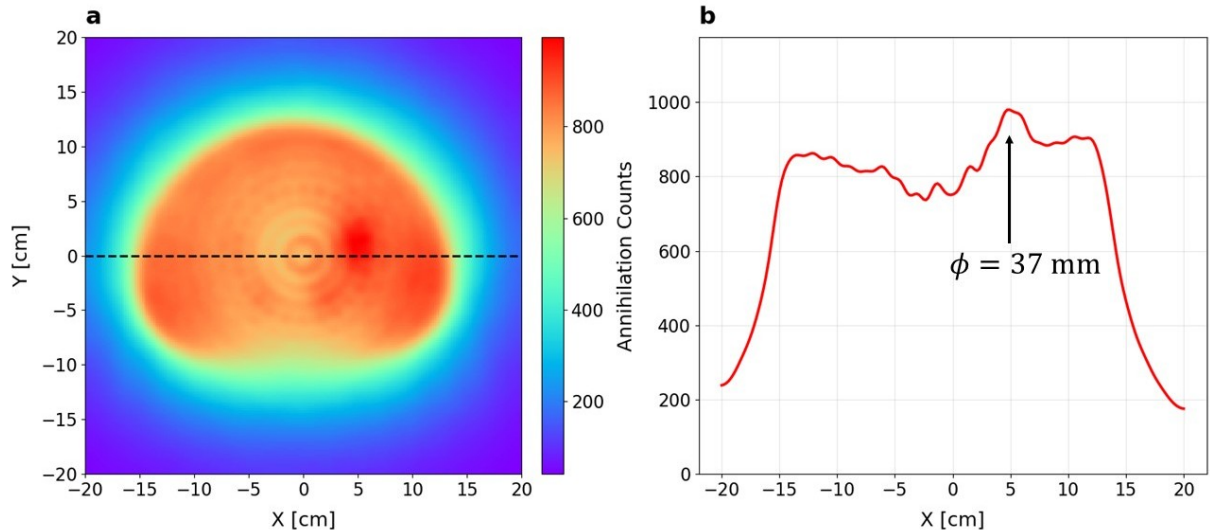


Figure 5.4: a) Annihilation points in the XY projection, b) line profile at $Y=0$ (with Gaussian blur 3σ and masking of the centre applied).

5.3 MLEM reconstruction

The results presented in this section were obtained using the MLEM algorithm described in Section 2.4, applying two versions of this algorithm (NON-TOF and TOF), different sensitivity maps and attenuation correction. The most optimal result (adopted as a compromise between spatial resolution and the noise level in the image) was obtained after a certain number of iterations of the MLEM algorithm. This number differs depending on the reconstruction method used. For the TOF method, the optimal result was achieved after 10 iterations, while for the NON-TOF method after 15 iterations.

5.3.1 Images for the uniform sensitivity map

The image reconstruction was performed using the uniform sensitivity map (Sec. 4.5.1) and a grid with dimensions of 100x100x57 (Sec. 4.2). In addition, FOV masking was applied (Sec. 4.4). The result for MLEM without TOF is shown in Fig. 5.5, and with TOF in Fig. 5.6. The value of the Gaussian blur is 0.8 voxel and is related to the grid size.

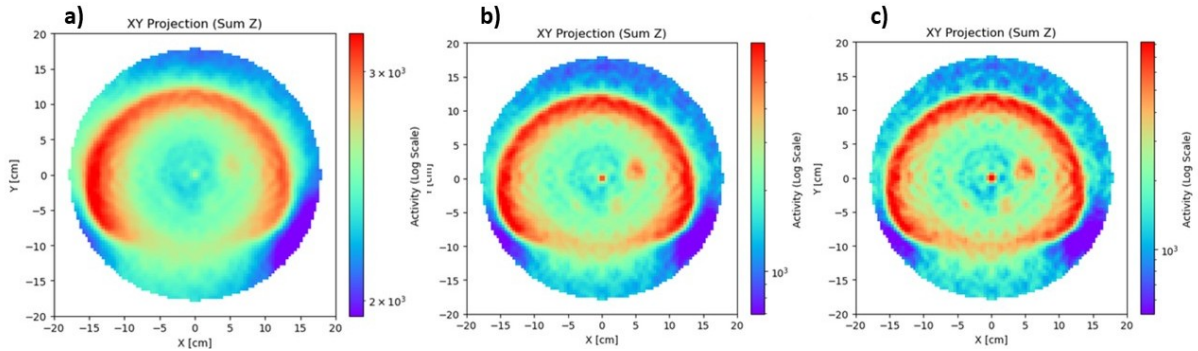


Figure 5.5: Image reconstruction using the MLEM algorithm in the version without TOF for: a) one iteration, b) five iterations, c) fifteen iterations.

In the first iteration (Fig. 5.5a) the algorithm starts to detect structures of increased activity, in the fifth (Fig. 5.5b) foci of the radiopharmaceutical are visible, and in the fifteenth (Fig. 5.5c) the centre of the phantom contains less noise and the focus from the sphere with $\phi = 17mm$ can be seen. The reconstruction reveals structures of higher activity, including those originating from the spheres filled with FDG, as well as a large enhancement in the very centre of the phantom.

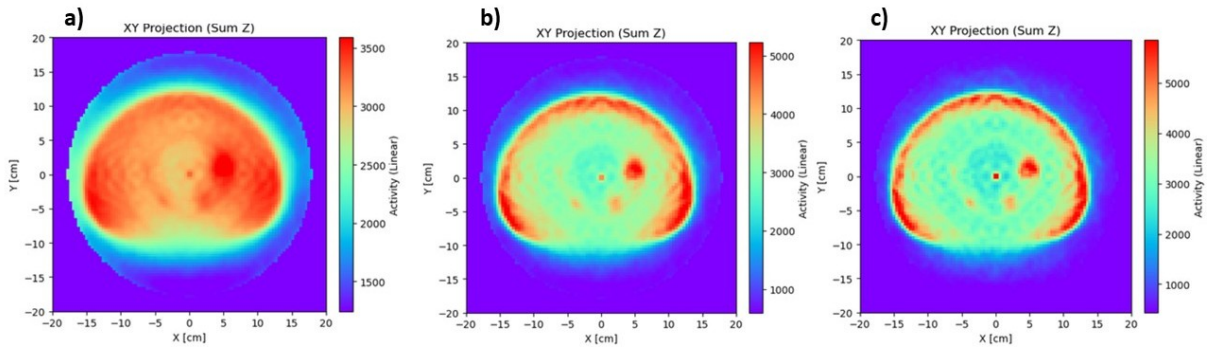


Figure 5.6: Reconstruction using the MLEM algorithm in the version with TOF for: a) one iteration, b) five iterations, c) ten iterations.

The use of the MLEM algorithm in the version with TOF (Fig. 5.6) gives a less noisy image compared with the result in Fig. 5.5. However, it does not eliminate the peak in the centre of the phantom.

5.3.2 Image using the sensitivity map from CASToR

The MLEM reconstruction was performed using the sensitivity map obtained from CASToR (Sec. 4.5.2), implemented by the authors of the article [10] and shown in Fig. 5.7.

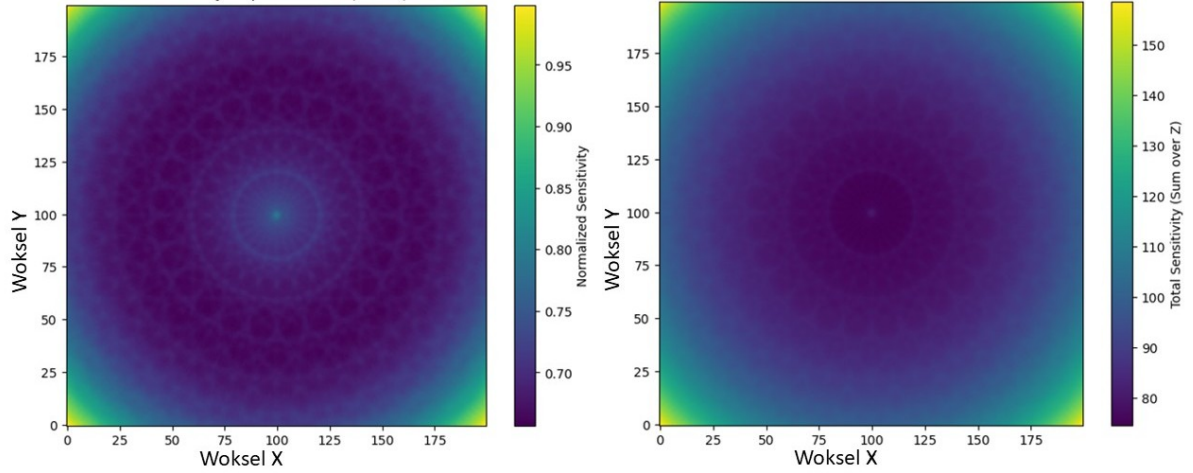


Figure 5.7: Sensitivity map generated using the CASToR software in the XY plane (as described in Sec. 4.5.2). The left panel shows the sensitivity map for $Z=0$, and the right panel the projection along the Z axis.

The result of the reconstruction for MLEM without TOF and with TOF is shown in Figs. 5.8 and 5.9, respectively. A grid size of $200 \times 200 \times 200$ was used (matched to the dimensions of the sensitivity map). For both reconstructions the problem of the bright centre remained, and artefacts in the form of rays became visible. They result from the fact that the sensitivity map was created by an algorithm that uses the Siddon-Jacobs LOR projector, while the MLEM reconstruction uses the Ray Marching LOR projector.

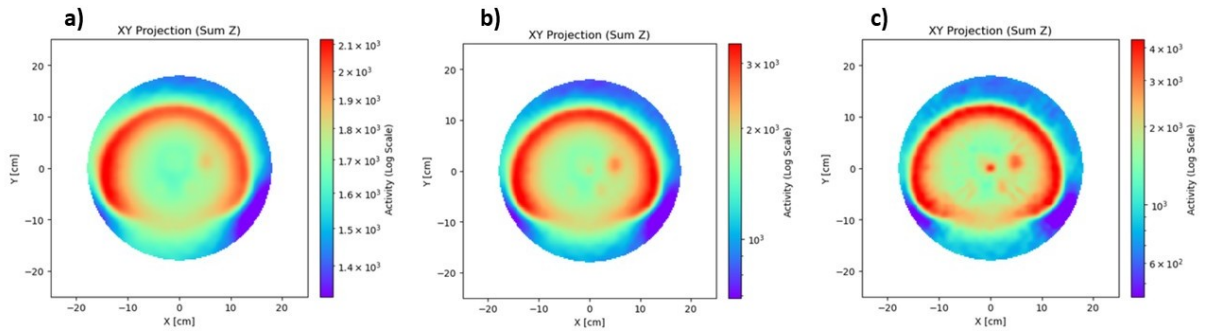


Figure 5.8: MLEM reconstruction with the sensitivity map generated using CASToR without TOF for: a) one iteration, b) five iterations, c) fifteen iterations.

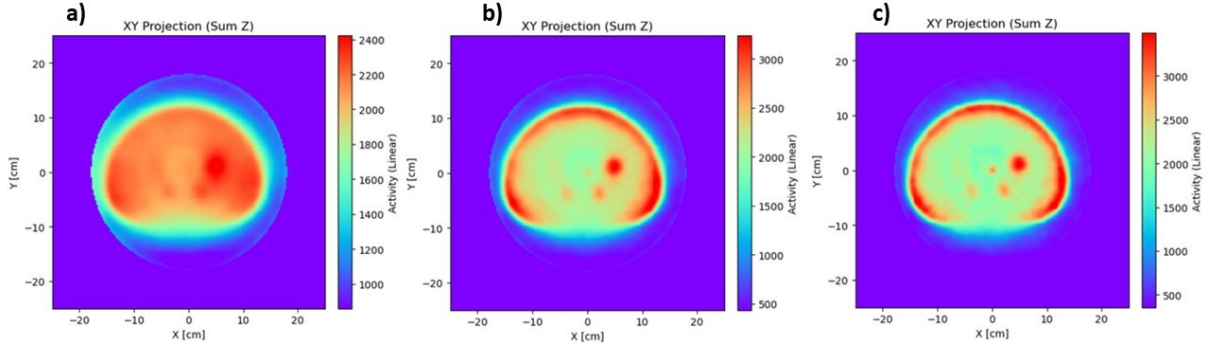


Figure 5.9: MLEM reconstruction with the sensitivity map created using CASToR with TOF for: a) one iteration, b) five iterations, c) ten iterations.

The algorithm with TOF gives a better result. Artefacts in the form of rays appear in the images starting from the tenth iteration, and the artefact in the very centre of the phantom is smaller.

5.3.3 Reconstruction with Multi Ray Marching taken into account

MLEM reconstructions with the sensitivity map generated using the CASToR software and with the Multi Ray Marching method described in Sec. 4.6 are presented in Figs. 5.10 and 5.11 for NON-TOF and TOF, respectively.

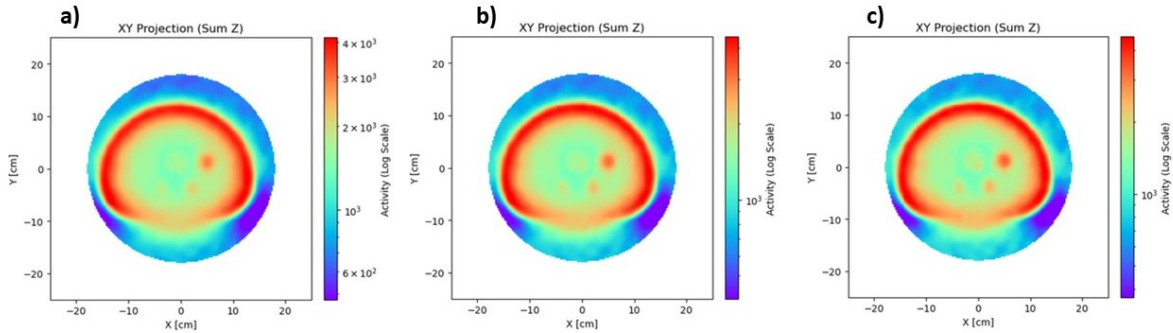


Figure 5.10: MLEM reconstruction with the sensitivity map generated using CASToR without taking TOF into account, with Multi Ray Marching, for: a) ten iterations, b) fifteen iterations, c) twenty iterations.

Compared with the images shown in Figs. 5.8 and 5.9, the resulting phantom images do not have artefacts in the form of rays, and the peak in the centre is considerably reduced. The positions of the radiopharmaceutical foci are clearly visible.

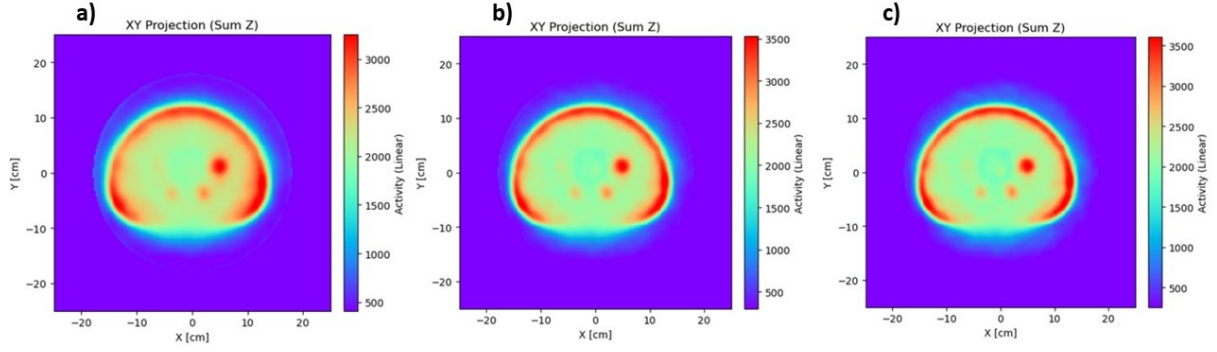


Figure 5.11: MLEM reconstruction with the sensitivity map created using CASToR with TOF, with Multi Ray Marching, for: a) five iterations, b) ten iterations, c) fifteen iterations.

The algorithm with TOF naturally gives a better image. The position of the sphere ($\phi = 17mm$) is visible, there are no ray artefacts, and the artefact in the centre of the phantom is almost invisible. A comparison of the line profile obtained for 10 iterations of the TOF MLEM algorithm, including Multi Ray Marching and the sensitivity map created using the CASToR software, with the line profile of the image reconstructed in the article [10] is shown in Fig. 5.12.

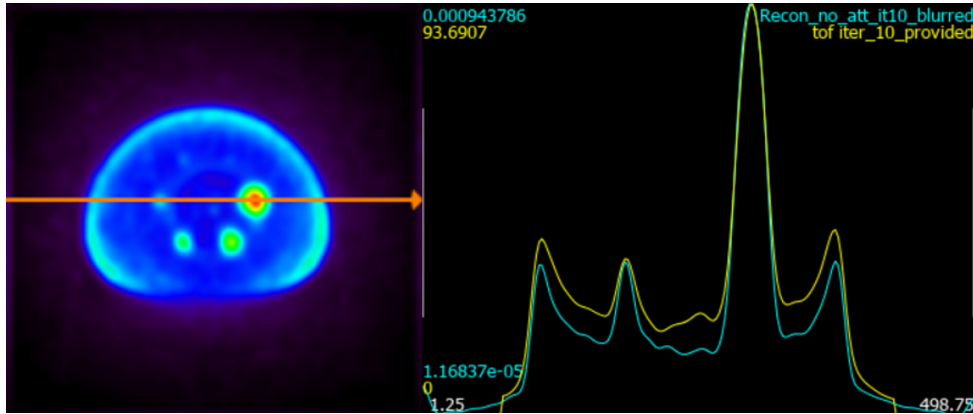


Figure 5.12: Comparison of the line profile of the reconstruction performed using the CASToR software and of the author's own reconstruction with TOF with the sensitivity map from CASToR and Multi Ray Marching for the tenth iteration. The blue colour marks the profile of the CASToR image 5.1, and the yellow one the profile of the image obtained with the algorithm developed in this thesis.

The line profiles show maxima at the same locations (the spheres with $\phi = 37mm$ and $\phi = 17mm$). The line profile of the image obtained with the algorithm developed in this thesis shows increased activity in the region between the spheres compared with the image obtained using the CASToR software.

5.3.4 Reconstruction with the author's own sensitivity map

The sensitivity map determined as described in Sec. 4.5.3, using the Multi Ray Marching method, is shown in Fig. 5.13.

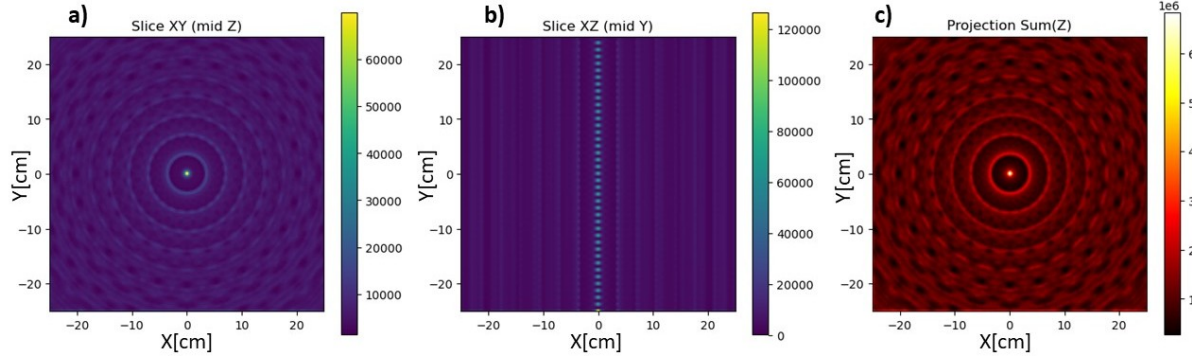


Figure 5.13: Sensitivity map in: a) the XY plane at $Z=0$, b) the XZ plane at $Y=0$, c) the XY projection.

The artefacts visible in it were smoothed using Gaussian blur. Fig. 5.14 shows sensitivity maps for different values of the blur (6σ , 7σ , 8σ). As the value of the Gaussian blur increases, the previously visible structures gradually disappear.

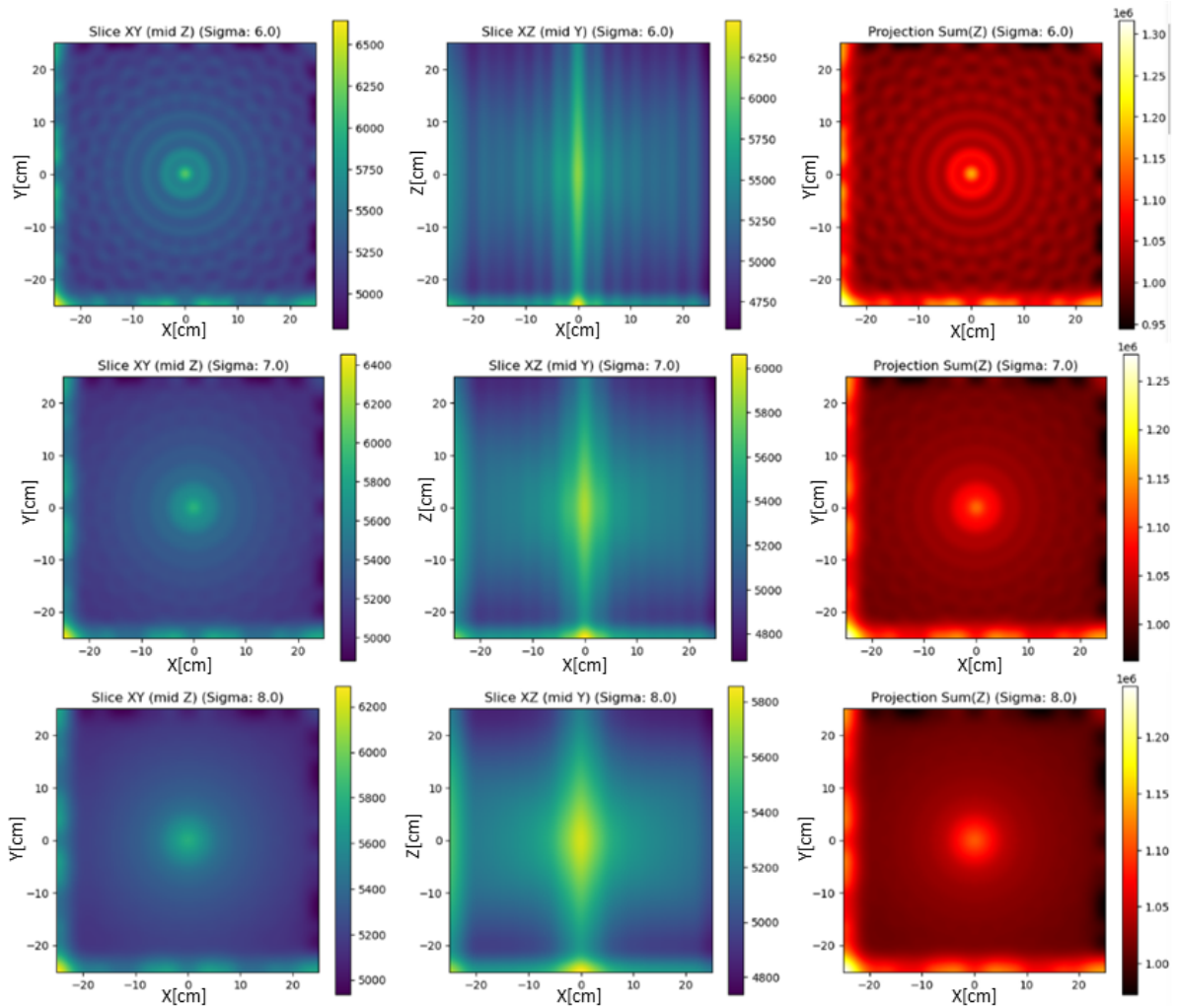


Figure 5.14: Sensitivity map obtained using the *backprojection* method. Left panel: XY profile ($Z=0$), middle panel: XZ profile ($Y=0$), right panel: XY projection. The top, middle and bottom panels show the results with blurs of 6σ , 7σ , 8σ , respectively.

The MLEM reconstruction with the author's own sensitivity map for different values of sigma is presented in Fig. 5.15.

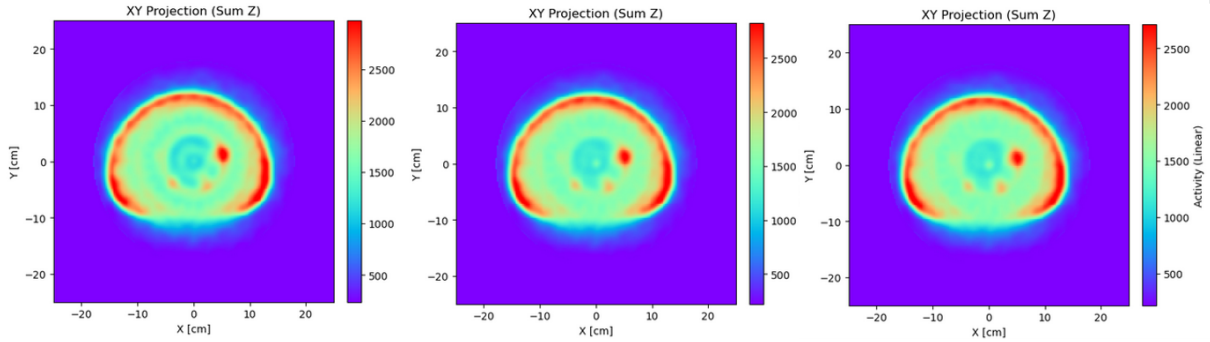


Figure 5.15: Results of the MLEM reconstruction with the author's own sensitivity maps with different blurs in the XY projection. Left panel: Gaussian blur 6σ , middle panel: 7σ , right panel: 8σ .

A comparison of the line profiles for these 3 images with the line profile of the image obtained using the CASToR software is shown in Fig. 5.16.

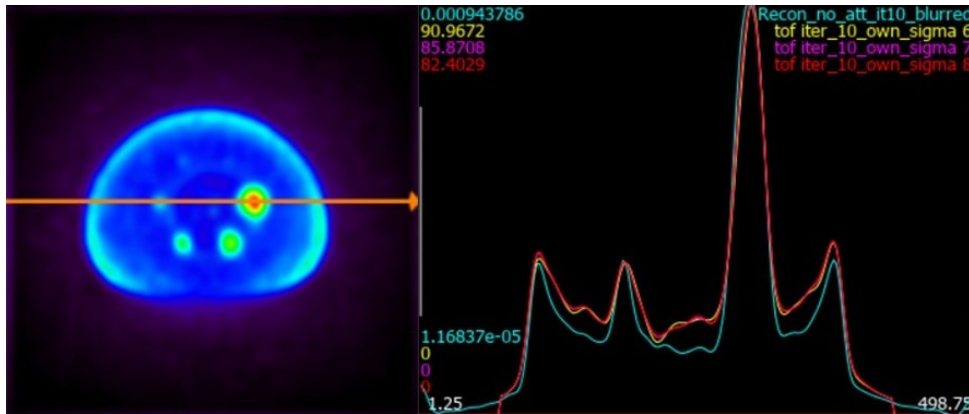


Figure 5.16: Comparison of the line profile of the reconstruction performed using the CASToR software (blue) and of the author's own reconstructions with TOF with the sensitivity map with different Gaussian blurs 6σ (yellow), 7σ (purple), 8σ (red).

The results of the reconstruction with the author's own sensitivity map are more consistent with the reconstruction with the map from the CASToR software, because they show a similar inflation of activity at the edges of the phantom and in its centre, as in Fig. 5.17.

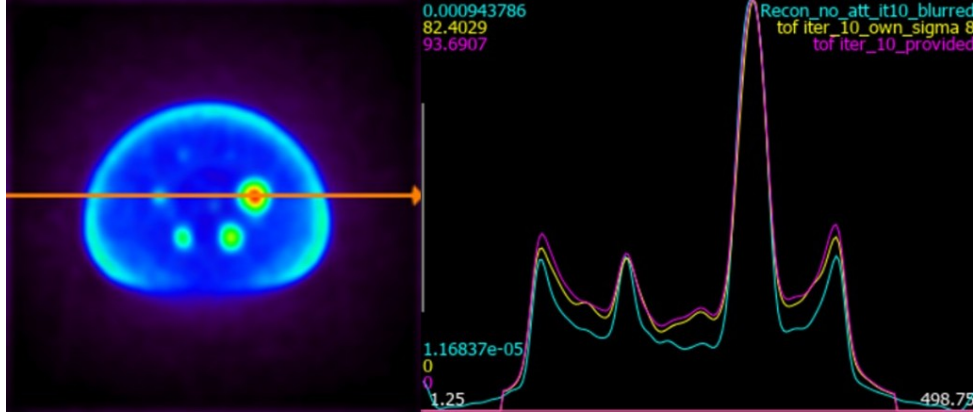


Figure 5.17: Comparison of the line profile of the reconstruction performed using the CASToR software (blue), of the author's own reconstruction with TOF with the author's own sensitivity map with Gaussian blur 8σ (yellow) and of the author's own reconstruction with the sensitivity map created using the CASToR software (purple).

The line profiles show maxima at the same locations (the spheres with $\phi = 37mm$ and $\phi = 17mm$) as well as the phantom edges. The profile of the image with the author's own sensitivity map shows an increase of activity at the edges of the phantom and in the centre, but smaller than the increase of activity in the profile of the image with the sensitivity map from CASToR.

5.4 Reconstruction with attenuation correction

In order to correct the image for the attenuation of photons inside the phantom (visible as increased activity at the edges of the phantom), the attenuation correction described in Section 4.7 was applied.

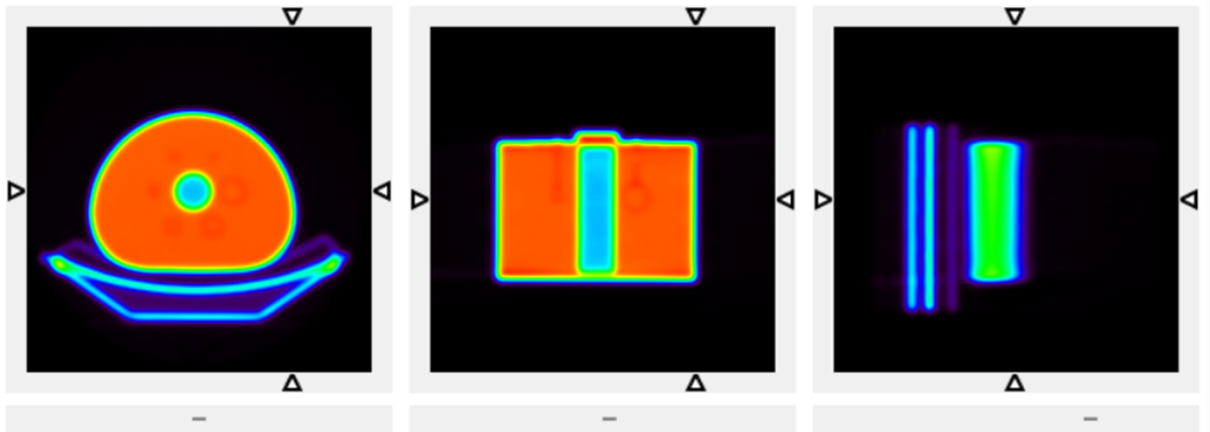


Figure 5.18: Map of the values of the matter attenuation coefficients μ (obtained using computed tomography, CT) [10].

The attenuation map presented in Fig. 5.18 was implemented in the MLEM algorithm as described in Section 4.7. The result of the PET tomographic image reconstruction with attenuation taken into account is shown in the right panel of Fig. 5.19.

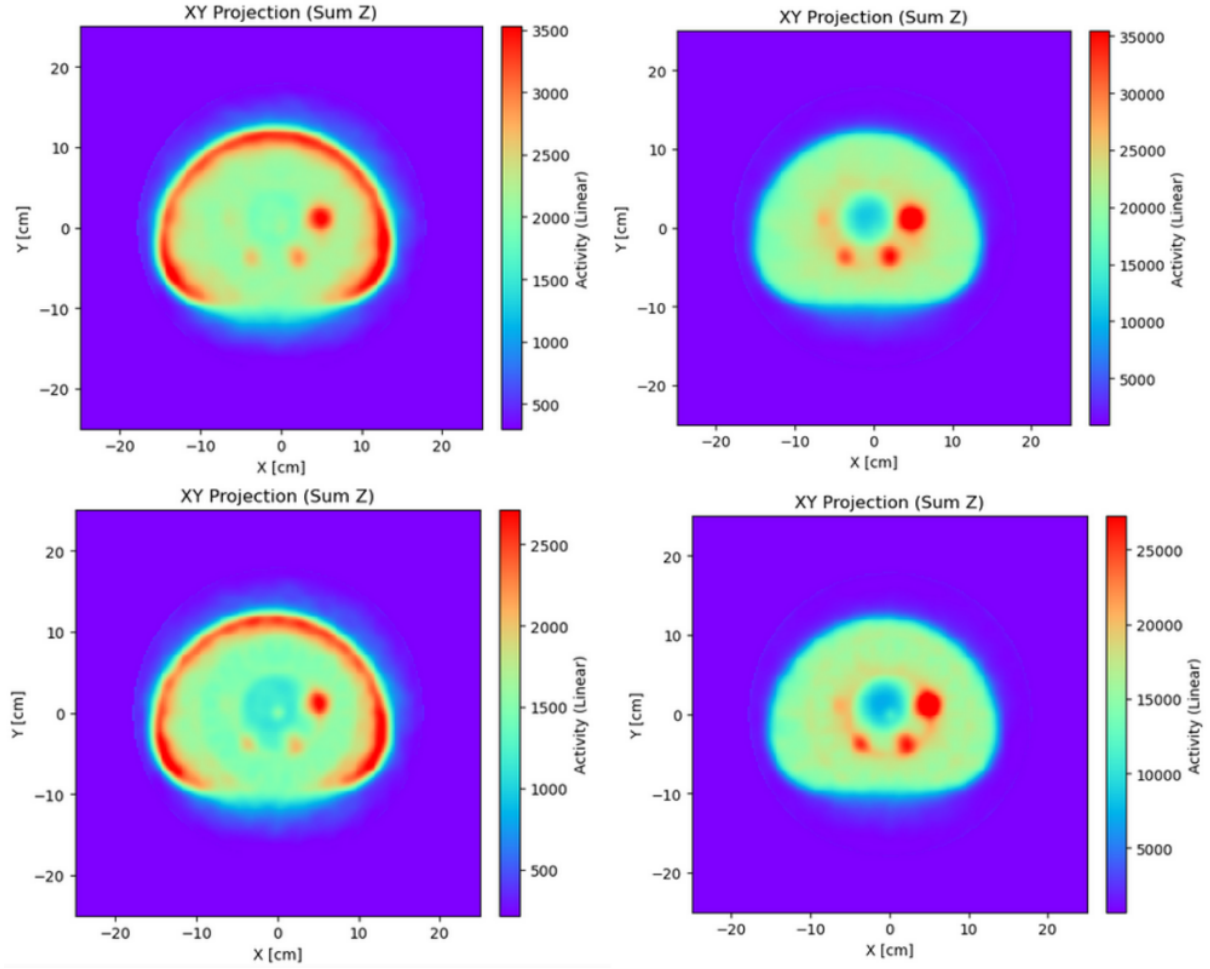


Figure 5.19: Results of the reconstruction using the MLEM algorithm without attenuation correction (left panel) and with attenuation correction taken into account (right panel). The top panel shows images made using the sensitivity map from CASToR with Multi Ray Marching, while the bottom panel shows images made using the author's own sensitivity map with a blur of 8σ .

Fig. 5.20 shows a comparison of the line profiles of the images obtained from three different reconstructions after applying the attenuation correction.

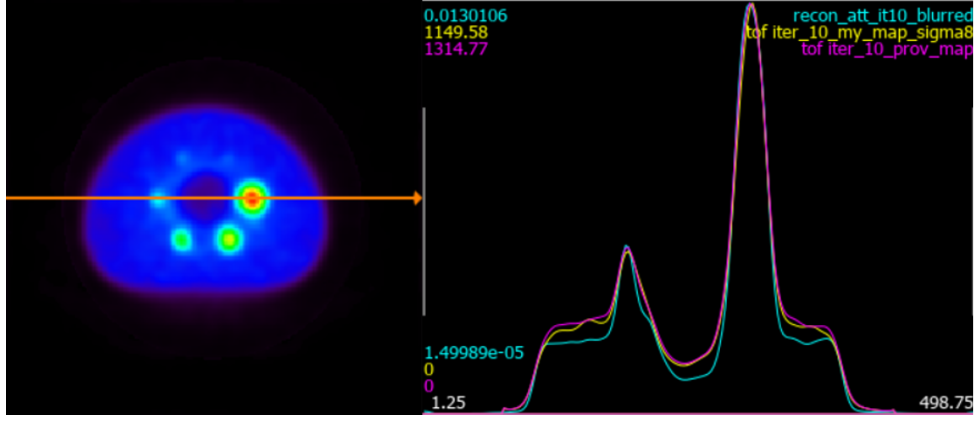


Figure 5.20: Comparison of the line profile with attenuation taken into account for the reconstruction performed using: the CASToR software (blue), the author's own reconstruction with TOF with the author's own sensitivity map with Gaussian blur 8σ (yellow), and the author's own reconstruction with the sensitivity map made using the CASToR software (purple).

After applying the attenuation correction, the activity at the edges of the phantom decreased clearly in all cases. Two main peaks are visible, corresponding to the activity in the spheres with $\phi = 37$ mm and $\phi = 17$ mm in the phantom.

For all reconstructions, the centres of the peaks are located at the same positions. We see a slight increase in the number of counts between the spheres and at the edges in the case of the reconstruction algorithm developed in this thesis.

6 Summary and conclusions

On the basis of the research carried out, the analysis of line profiles, the comparison of image reconstruction methods and the evolution of the code, the following final conclusions were formulated:

The simplest method of determining annihilation points from the photon time-of-flight (TOF) difference is characterised by low resolution and strong numerical artefacts in the centre of the field of view. The use of the iterative MLEM algorithm allowed a drastic improvement of contrast and made possible the correct identification and localisation of the smaller spheres of the phantom with diameters of 22 mm and 28 mm. Introducing TOF information into the reconstruction equation significantly accelerated the convergence of the algorithm compared with the classical method and markedly reduced the background noise outside the boundaries of the phantom.

The standard Ray Marching method generated strong artefacts in the form of rays, resulting from the discretisation of the grid. The implementation of the Multi Ray Marching method effectively eliminated the banding artefacts, ensuring geometric continuity of the projection and bringing the image quality closer to the CASToR standard.

The use of the sensitivity map generated by CASToR in combination with the author's own Ray Marching reconstruction code led to the appearance of artefacts owing to the mathematical mismatch of the projection operators. To eliminate them, the author's own sensitivity map was created on the basis of the given detector geometry. The author's own sensitivity map made it possible to obtain image line profiles with smaller artificial inflation of activity in the centre and at the edges.

The implementation of radiation attenuation correction based on the map of attenuation coefficients μ significantly improved the quantitative contrast of the reconstructed spheres, compensating for the physical absorption of photons inside the phantom.

The choice of the Python language together with the appropriate libraries turned out to be fully sufficient for accomplishing the task set.

The software created allows autonomous reconstruction of tomographic data for the J-PET scanner without strict dependence on external general-purpose open-source reconstruction packages. The results obtained are in good agreement with the results from the CASToR software.

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