

Experimental determination of the Dalitz plot for positronium decay using the J-PET detection system

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Abstract

We present the first measurements of the Dalitz plot for ortho-positronium annihilation to three photons. Our measurements, accurate to about 3% statistical and 2-3% systematic uncertainty in angular representation over almost the entire available phase space, were performed using the Jagiellonian Positron Emission Tomograph (J-PET) based on organic scintillator strips. Until now, the Dalitz plot for the three-body positronium decay has been poorly explored. The new measurements presented here are consistent with the leading-order QED prediction for the Dalitz plot.

Keywords: positronium, QED, Dalitz plot

1. Introduction

Studies of positronium are focused on measurements of its decays and spectroscopy. For positronium in vacuum these observables are described by QED bound-state theory. The predictions are being tested up to and beyond next-to-leading order corrections in the fine structure constant α [1, 2, 3, 4]. Recent theoretical developments in the context of spin-zero para-positronium are reported in Refs. [5, 6]. In addition, positronium is being used to perform new tests of QED discrete symmetries [7, 8, 9, 10] which go beyond the tests in single-electron systems. Beyond measurements in vacuum, positronium decays in medium play an important role in bio-medical and material sciences applications [3, 11]. A highlight is the new technique of positronium imaging [12, 13, 14, 15, 16, 17, 18] that is being pioneered in medical diagnostics including the use of quantum entanglement of the final state photons resulting from the decay [19, 20]. For the three-photon decay of the longest-lived spin-one state ortho-positronium, an essential ingredient is the Dalitz plot which encodes information about the momentum dependence of the final state photons and connects to key observables like the energy spectrum and decay rate. The Dalitz plot so far has not been measured in previous experiments due to limited kinematical coverage. Here we report the first measure-

ments using the J-PET tomograph detector system in Cracow. Our result shows the potential of the experimental setup. We presently determine the Dalitz plot up to about 3% statistical uncertainty and 2-3% systematic uncertainty over almost the full phase space. Planned measurements with an upgraded detector system will allow about an order of magnitude improvement in precision.

This paper is structured as follows. Section 2 provides an overview of the Dalitz plot in the context of o-Ps decays. Section 3 introduces the Jagiellonian Positron Emission Tomograph (J-PET) detection system. The subsequent section outlines the experimental methodology, followed by Section 5, which presents the results. The final section presents the conclusions and perspectives for future measurements.

2. Dalitz Plot for $o\text{-Ps} \rightarrow 3\gamma$ decay

The positronium ground state occurs in two hyperfine configurations: para-positronium, $p\text{-Ps}$ (the singlet 1S_0 state with total spin zero, lifetime in vacuum of 0.125 ns and $J^{PC}=0^{-+}$), and ortho-positronium, $o\text{-Ps}$ (the triplet 3S_1 state with total spin one, lifetime of 142 ns and $J^{PC}=1^{--}$). Precision measurements of their annihilation decay rates yield $\Gamma_{p\text{-Ps}} = 7990.9 \pm$

$1.7 \mu\text{s}^{-1}$ [21], corresponding to a vacuum lifetime of $\tau_{p-Ps} = 125.14 \pm 0.03 \text{ ps}$, and $\Gamma_{o-Ps} = 7.0401 \pm 0.0007 \mu\text{s}^{-1}$ [22], corresponding to $\tau_{o-Ps} = 142.043 \pm 0.014 \text{ ns}$. Positronium is an eigenstate of parity (P), charge (C) and thus the CP operator which makes it ideal for studies of the discrete symmetries in quantum electrodynamics (QED) [23, 24]. Charge conjugation symmetry means that p - Ps and o - Ps decay into even and odd numbers of photons, respectively.

In contrast to the two-photon back-to-back para-positronium decay, the longer-lived ortho-positronium decays into three photons with a more complex phase space. In the three-photon decay process, energy-momentum conservation requires the total photon energy to be equal to the ortho-positronium mass. That is $E_1 + E_2 + E_3 = 2m_e$, where E_1, E_2, E_3 are the energies of the three emitted photons and m_e is the electron mass¹. The photon momenta lie in a plane $\vec{k}_1 + \vec{k}_2 + \vec{k}_3 = 0$ so that the four-momentum of any photon is completely determined by the momenta of the other two. The photon energies take values from 0 to 511 keV. The differential cross-section of the o - $Ps \rightarrow 3\gamma$ decay in the centre-of-mass frame is determined from the low energy limit of the cross section for the $e^+e^- \rightarrow 3\gamma$ process

$$d\sigma_{3\gamma} = \frac{(2\pi)^4 |M|^2}{4m_e^2 v} \delta\left(\sum_{i=1}^3 \vec{k}_i\right) \delta\left(\sum_{i=1}^3 E_i - 2m_e\right) \frac{1}{(2\pi)^9} \prod_{i=1}^3 \frac{d^3 k_i}{2E_i} \quad (1)$$

Here v is the electron positron relative velocity and the δ functions express energy and momentum conservation in the decay. The factor $|M|^2$ denotes the squared absolute value of the Lorentz invariant amplitude for the o - $Ps \rightarrow 3\gamma$ transition summed over the final state spins of the outgoing photons and averaged over the initial state polarization of the electron and positron. For the lowest-order (LO) decay process (involving 6 Feynman diagrams) $|M|^2$ is [25, 26]:

$$|M|^2 = 16(4\pi)^3 \alpha^3 \left[\left(\frac{m_e - E_1}{E_2 E_3} \right)^2 + \left(\frac{m_e - E_2}{E_1 E_3} \right)^2 + \left(\frac{m_e - E_3}{E_1 E_2} \right)^2 \right] \quad (2)$$

When the calculations are extended to include next-to-leading-order corrections at $O(\alpha)$, $|M|^2$ takes a much more complicated form; for details see Ref. [27]. Due to the energy-momentum conservation, the massless nature of the emitted photons and rotational symmetry of the decay, the kinematics of the three photons originating from o - Ps decay is determined entirely by specifying only two independent photon parameters, such as the energies or relative angles between photons. This leads to simplification of the differential annihilation cross-section formula, which in energy representation can be expressed as:

$$d\sigma_{3\gamma} = \frac{1}{6m_e^2 v} \frac{1}{16(2\pi)^3} |M|^2 dE_1 dE_2 \quad (3)$$

¹The sum of the three photon energies is slightly below $2m_e$ since positronium comes with a small binding energy $E_B \approx -6.8 \text{ eV}$ [2]

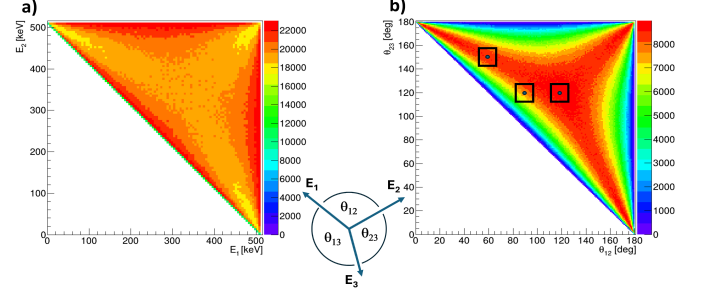


Figure 1: Energy (a) and corresponding angular (b) distribution for o - $Ps \rightarrow 3\gamma$ decay. The distribution was generated for 10^8 events based on the QED predictions for LO decay process [25, 26]. The black rectangles denote three angular configurations measured in Ref. [35].

Here, the factor of six in the denominator refers to $6=3!$ possible permutations of three identical photons. The o - $Ps \rightarrow 3\gamma$ differential decay rate is determined by averaging the cross-section over initial spin states ($\frac{4}{3}d\sigma_{3\gamma}$, three of the four possible positronium spin states undergo 3γ annihilation), multiplying by the flux density $v|\psi(0)|^2 = \frac{m^3 \alpha^3}{8\pi}$ where $|\psi(0)|^2$ is the wave function of the positronium evaluated at the origin, and finally taking the limit that $v \rightarrow 0$ [26].

The complete kinematics of the o - $Ps \rightarrow 3\gamma$ process can be visualised using a Dalitz plot, where the axes correspond to two independent observables of the emitted photons. This excellent tool introduced by Dalitz [28] has been widely and successfully applied in nuclear and particle physics for many years to study three-body decays, see e.g. [29, 30, 31, 32, 33]. Each point in the Dalitz plot corresponds to a specific kinematic configuration of the three photons and is weighted by the squared QED matrix element $|M|^2$ so that the event density directly reflects the differential decay rate. The total decay rate is recovered by integration over the full kinematically allowed region, taking into account the Bose symmetry of the photons and the associated phase-space constraints. As an example, the Dalitz plot of o - Ps decay into three photons in energy and angular representation generated based on LO QED calculations is presented in Fig. 1.

It is worth emphasizing that, although the energy and angular representations describe the same three-body kinematics, their event densities are related by a non-trivial Jacobian. The event density in the angular Dalitz plot is therefore determined by both the squared QED matrix element $|M|^2$ and the Jacobian of the transformation arising from the kinematic relations between the photon energies and relative angles [34]. Consequently, even for a constant matrix element, the angular Dalitz distribution would not, in general, be uniform, since the Jacobian varies across the available phase space. In contrast, in the conventional Dalitz representation based on two independent photon energies, the phase-space factor is uniform, so that a constant matrix element would result in a flat distribution.

If we can measure the complete Dalitz plot for the three-photon o - Ps decay, this would provide a multidimensional view of the o - $Ps \rightarrow 3\gamma$ decay dynamics, enabling sensitive tests of differential QED amplitudes (differential decay rates) beyond traditional observables such as total decay rates. The Dalitz plot

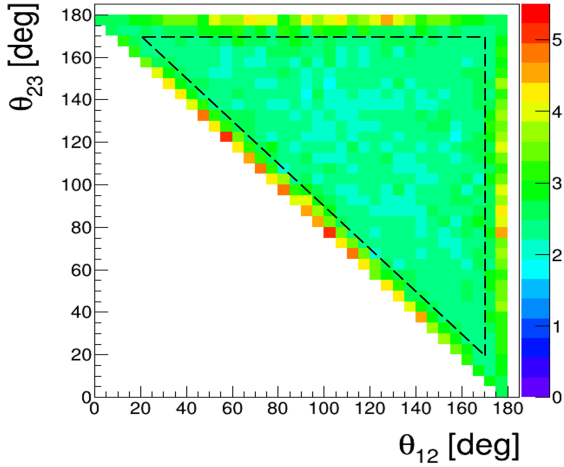


Figure 2: The relative percentage difference between the QED leading-order (LO) prediction and LO + $O(\alpha)$ result, normalized to the LO prediction, in the angular distribution for the $o\text{-Ps} \rightarrow 3\gamma$ decay ($\frac{N_{\text{LO}} - N_{\text{LO}+O(\alpha)}}{N_{\text{LO}}} \cdot 100\%$). The distribution was obtained based on simulations including $|M|^2$ for the LO term and $O(\alpha)$ correction, as described in [27]. The black lines indicate the experimental limits of J-PET (see Sec. 5). In the region of interest (marked with the dashed line) the difference between LO and LO+ $O(\alpha)$ correction varies between 1.5 - 2.5%, indicating only a weak dependence of the relative correction on the angular variables over most of the accessible region.

for ortho-positronium decays has never been measured in the entire available phase space. So far, only three possible angular configurations of annihilation photons (marked in Fig. 1b)) have been measured with limited statistics [35]. For each of these configurations, the measurement provided only the total number of counts within an angular range of $\pm 8.5^\circ$ (marked as black rectangles in Fig. 1b)) resulting from the geometry of the detector used, that is, using crystal diameters of 3.8 cm placed at a distance of 12.7 cm from the source. The first measurements of the Dalitz plot for $o\text{-Ps} \rightarrow 3\gamma$ annihilation process in almost entire available kinematic region are presented below in Section 4. This experiment was performed by the J-PET group in Kraków with a multiphoton detector system based on organic scintillators.

Ultimately, as a long-term goal, one would like to measure the Dalitz plot to sufficient precision that we become sensitive to higher-order QED corrections at $O(\alpha)$ and beyond. Experiments have verified the $O(\alpha)$ correction to the total decay rate as about 2-3% correction to the leading term [36, 37, 22]. Theoretical predictions are available to $O(\alpha^3)$, with testing the non-leading terms a challenge for future experiments. Also, $O(\alpha)$ corrections to the energy spectrum in the decay were found to be needed to understand the data in Ref. [38]. Positronium spectroscopy is also a topic of vigorous investigation [1]. Positronium energy levels have been calculated to order $m_e\alpha^6$ (plus some terms to higher order) with the leading term starting at $O(m_e\alpha^4)$. Recent transition frequency experiments are reported in Refs. [39, 40, 41, 43]. While these experiments are mostly in good agreement with QED theory, there is also a more than 4 standard deviations (or one part in 10^4) anomaly in the

$2^3S_1 \rightarrow 2^3P_0$ transition [40, 41].

An interesting aspect is the radiative hyperfine transition $o\text{-Ps} \rightarrow p\text{-Ps} + \gamma$, which has been directly induced and observed using radiation at the hyperfine-splitting frequency [42]. Followed by $p\text{-Ps} \rightarrow 2\gamma$, this transition formally gives rise to the sequential three-photon process $o\text{-Ps} \rightarrow p\text{-Ps} + \gamma \rightarrow 3\gamma$. Its contribution is experimentally negligible (branching ratio of order 10^{-15}). However, in an ideal Dalitz plot, such a process would generate a structure located essentially at the kinematic boundary, corresponding to one nearly zero-energy photon and a 2γ invariant mass close to the $p\text{-Ps}$ mass.

Eventually, one would like to connect especially the decay rate and energy spectrum measurements with the Dalitz plots. So far, higher-order effects, in particular $O(\alpha)$ corrections in the angular distribution have not been investigated experimentally. Fig. 2 shows the relative difference between the leading order QED calculation and the calculation [27, 38] where $O(\alpha)$ corrections are included. If one could measure to this sensitivity, then this detailed characterization would allow to test higher-order effects, in particular $O(\alpha)$, provide a reference dataset for future theoretical studies, and put much tighter constraints on three-body annihilation amplitudes.

3. The J-PET experimental setup

The J-PET apparatus [44, 45], located at the Jagiellonian University in Kraków, Poland, is a unique and innovative detection system developed based on plastic scintillation material for simultaneous multi-photon measurements. This feature makes it an excellent tool for studying positronium atom decays, in both, fundamental research [8, 7] and medical imaging applications [13, 12, 15, 16, 17, 46, 47, 48].

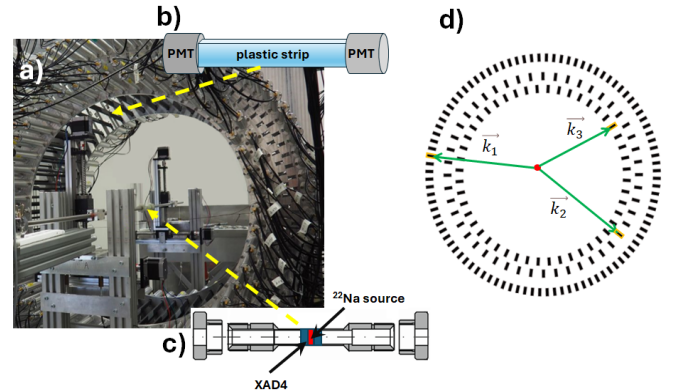


Figure 3: (a) Photograph of the J-PET detector built from 192 strips of EJ-230 scintillators, each read out by R9800 Hamamatsu PMTs [45] (b); with a small annihilation chamber containing a sodium ^{22}Na radioactive β^+ embedded in porous XAD-4 material, fixed in the center [49] (c). (d) Schematic of the J-PET detector showing photons (green arrows) associated with the $o\text{-Ps} \rightarrow 3\gamma$ annihilation occurring inside the chamber (red dot).

The analysis presented here relies on measurements conducted with the J-PET detector using a small annihilation chamber as a positronium target (see Fig. 3). The J-PET detector consists of 192 plastic scintillator strips, each 500 mm long, 7

mm thick, and 19 mm high, arranged in three concentric cylindrical layers (Fig. 3 (a)). Each of the two inner layers consists of 48 strips and is rotated by 3.75° with respect to the other. These layers are positioned at radii of 42.5 cm and 46.75 cm, respectively. The outer layer, located at a radius of 57.5 cm, is composed of 96 strips. Each scintillator strip is wrapped in two types of foils: an outer foil for optical isolation and an inner reflective foil to enhance light collection in R9800 Hamamatsu photomultipliers (PMTs) optically connected to both its ends (Fig. 3 (b)). The light is generated as a result of photons undergoing Compton scattering in the scintillator material and converted into electrical signals by PMTs. The J-PET data acquisition system (DAQ), based on Field Programmable Gate Arrays (FPGAs) operates in a trigger-less mode [51, 50]. The analog signals from the PMTs are probed at four amplitude thresholds: 30, 80, 190, and 300 mV, allowing for up to eight time-point measurements (four at the leading edge and four at the trailing edge), which are subsequently digitized by a Time-to-Digital Converter (TDC) and distributed to the Trigger Readout Boards (TRBs). To ensure consistent and precise timing measurements across the entire detector system, proper time synchronization is performed [52]. The longitudinal position of the photon interaction in the strip is reconstructed based on the difference in the arrival times of light signals at the two ends, while the interaction time is determined as the average of the two times [44]. Fast timing response of the plastic scintillators allows one to achieve an interaction time resolution of approximately 250 ps, while an angular resolution reaches about 1° [7].

For efficient production of positronium atoms, a properly designed annihilation chamber was used and positioned at the centre of the J-PET detector (Fig. 3(a,c)) [53, 49]. The chamber consists of an internal pipe containing a ^{22}Na positron source with activity of 0.7 MBq, surrounded by a porous XAD-4 polymer, and placed inside a larger, external cylindrical chamber. The XAD-4 polymer is characterized by a high fraction of positron-electron 3γ annihilations (29%) and a high o - Ps formation probability [53, 49]. The external chamber, 1 mm thick and made of lightweight PA6 polyamide (1.14 g/cm³), attenuates annihilation photons by only 1%. The chamber is connected to vacuum pump system (two rotary pumps) by steel pipes, which also set the whole chamber in the axis of J-PET detector. The cylinder is closed from both ends with the plastic partitions that allows to reach the vacuum at the order of $1.5 \cdot 10^{-5}$ Pa. The measurements reported here are based on data from 218 effective days of measurements with the sodium source resulting in the identification of 33 million o - $Ps \rightarrow 3\gamma$ signal events. The volume of processed data amounts to approximately 950 TB.

4. Selection of o - $Ps \rightarrow 3\gamma$ annihilation event candidates

In the experiment, interactions of positrons produced in the decay of a sodium source ($^{22}\text{Na} \rightarrow ^{22}\text{Ne}^* e^+ \nu_e \rightarrow ^{22}\text{Ne} \gamma_d e^+ \nu_e$) with electrons in the porous material lead to annihilation processes to two- or three-photons. The annihilation may occur directly or via the formation of a positronium atom and is pre-

ceded by the emission of a prompt photon (γ_d) from the excited daughter nucleus ($^{22}\text{Ne}^*$), having energy of 1275 keV.

To extract three-photon o - Ps decays from the experimental data, appropriate selection criteria were applied. These were primarily based on Monte Carlo simulations. The dedicated simulations were performed using GEANT4 simulation package [54, 55, 56] with implemented geometries of J-PET tomograph and annihilation chamber, and material composition. The fraction of direct positron-electron annihilations, p - Ps and o - Ps formations known for the XAD-4 porous material were used. The simulated data, once convoluted with the detector response, were then analysed with dedicated framework [13] using the same procedure as for the experimental data.

In the first step of selection, the accepted events corresponded to the registration of three photon hits in the active regions of three different scintillator strips (hits located within a ± 23 cm around the strip's center), within a 2.5 ns coincidence time window (see Fig. 4(a)). To distinguish between photons from different origins, i.e., annihilation and de-excitation photons, the Time-over-Threshold (TOT) technique was applied [57]. TOT, which is related to the energy deposited by photons in plastic scintillators, was determined for each hit as the sum of signal widths at four predefined voltage thresholds [57, 52]. The TOT spectrum for three-photon events is shown in Fig. 4(b). For further analysis, a TOT range from 1 to 17 ns was selected (area between red dashed vertical lines), where a contribution from annihilation photons is expected.

To substantially reduce background originating from secondary scattering of γ quanta (see Fig. 5a,b), cosmic rays, and random coincidences, selection criteria were applied based on the distance d between the decay plane and the annihilation point (defined by the position of the annihilation chamber), as well as the emission time difference between the first and last hit in the cluster $\delta t_{13} = t_{\text{hit1}} - t_{\text{hit3}}$ (for details see [13, 19]). The corresponding distributions are shown in Fig. 6. For further analysis, events corresponding to $d < 5$ cm and $\delta t_{13} < 1.5$ ns were accepted.

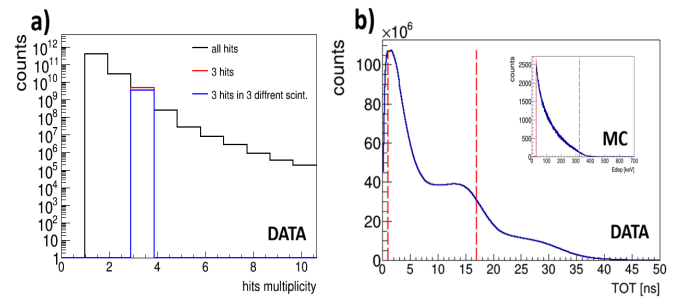


Figure 4: (a) Experimental hits multiplicity before and after applying the requirement of the strip's active region and the condition of three interactions in three different scintillators. (b) Experimental TOT distribution related to energy deposition in the scintillators. The red dashed line on the right indicates the Compton edge corresponding to 511 keV photons from 2γ annihilation, while the line on the left corresponds to the energy threshold set in the electronics. The corresponding energy deposition distribution obtained from MC simulations of annihilation into 3γ for the signal is shown in the inset plot.

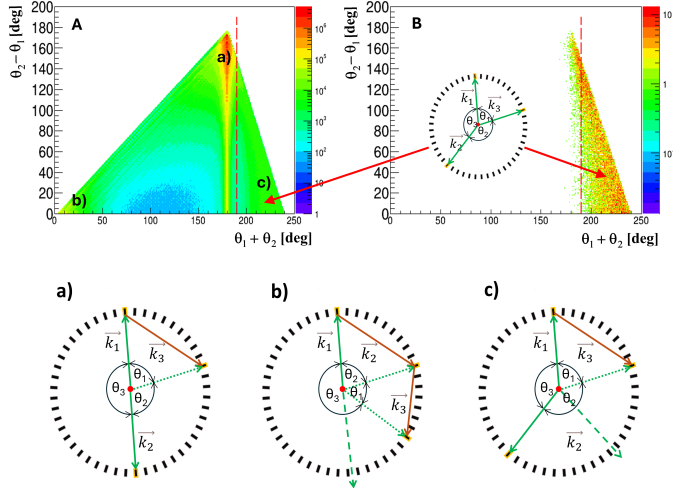


Figure 5: Spectrum of $\theta_2 - \theta_1$ vs. $\theta_2 + \theta_1$ for 3γ events being candidates for $o\text{-}Ps \rightarrow 3\gamma$ decays for experimental data (Fig. A) and obtained from simulations of the signal (Fig. B). Lower panel represents main background contributions corresponding to regions a), b) and c) shown in the upper panel. The background contributions are: a) 2γ annihilation with secondary Compton scattering, b) two back-to-back photons, where one photon is not registered and the other undergoes double scattering, c) misreconstructed $o\text{-}Ps \rightarrow 3\gamma$ decay. Registered annihilation photons are denoted by solid green arrows, not registered annihilation photons by the dashed green arrows, while wrongly assigned photons by the dotted green arrow.

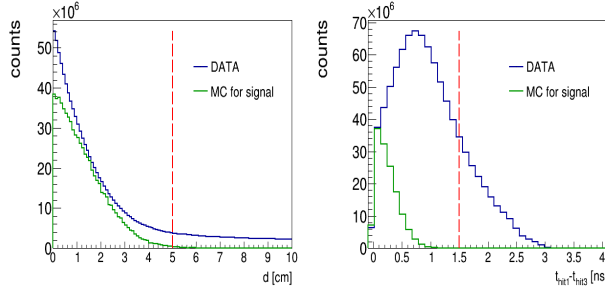


Figure 6: (left) Distance between annihilation origin and the decay plane. (right) Emission time difference between the first and the third hit in event. Applied cuts $d < 5\text{cm}$ and $t_{\text{hit}1} - t_{\text{hit}3} < 1.5\text{ns}$ are marked with red dashed vertical lines.

The remaining background associated with single- and double-scattered events (schematically shown in Fig. 5a and Fig. 5b) was suppressed using angular correlations between photon momenta. Fig. 5A and Fig. 5B present the difference between the two smallest relative angles ($\theta_2 - \theta_1$) as a function of their sum ($\theta_2 + \theta_1$) for experimental data and Monte Carlo simulations for the signal, respectively. The $o\text{-}Ps \rightarrow 3\gamma$ events correspond to the high $\theta_2 + \theta_1$ values with a peak around 240° (Fig. 5B). Two- γ annihilation followed by scattering appears as a vertical band around 180° , while small values of $\theta_2 + \theta_1$ (around 50°) correspond to situation where only one photon is detected and undergoes two scatterings. The signal-rich region was selected requiring $\theta_2 + \theta_1 > 190^\circ$.

The last stage of the selection was aimed at subtracting events corresponding primarily to misreconstructed $o\text{-}Ps \rightarrow 3\gamma$ de-

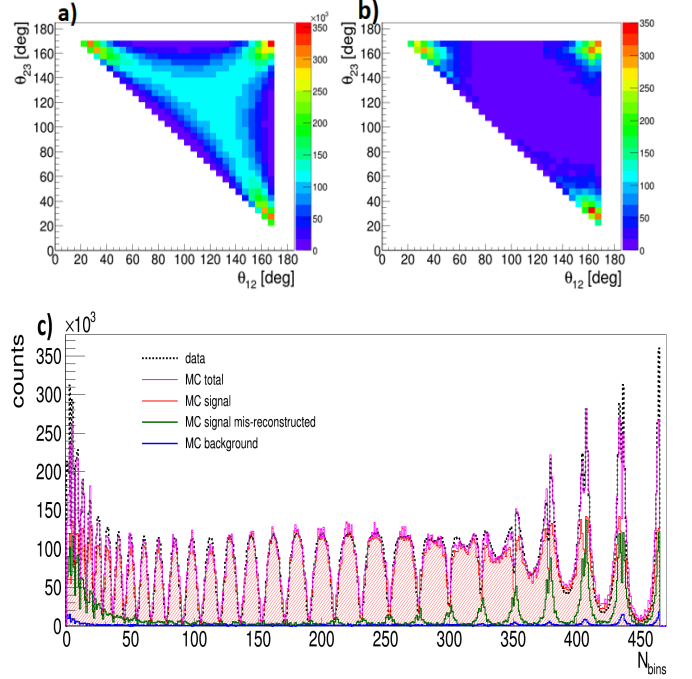


Figure 7: a) Experimental Dalitz Plot in angular representation. b) Dalitz Plot for misreconstructed signal events obtained from Monte Carlo simulations. c) The experimental Dalitz distribution (a) transferred to one-dimensional histogram, excluding empty bins. The distribution was fitted with three contributions: $o\text{-}Ps$ signal (red shaded area), misreconstructed signal (green solid line), the background (blue solid line), and overall fit (magenta solid line) as explained in the text.

cays. This occurs when one of the annihilation photons is not detected and instead a scattered photon is registered (or less likely when only one annihilation photon is detected along with two scattered photons originating from it) as shown in Fig. 5c.

To extract the misreconstructed $o\text{-}Ps$ events and the non-signal background from the sample, the experimental Dalitz distribution in the angular representation (Fig. 7a) was transferred to a one-dimensional histogram, N^{exp} , (Fig. 7c). The transformation was performed by looping over the bins of the two-dimensional distribution, first along the x-axis and then along the y-axis, and assigning each non-zero two-dimensional bin content to a consecutive bin of the one-dimensional histogram. The resulting distribution was fitted with three components: reconstructed N^{sig} and misreconstructed $N^{\text{mis-rec-sig}}$ signal, and the background N^{bkg} .

The fit proceeds through the minimization of a chi-square function (using the TMinuit library [58]):

$$\chi^2 = \sum_{i=1}^{N_{\text{bins}}^{1D}} \frac{(N_i^{\text{exp}} - A \cdot N_i^{\text{sig}} - A \cdot N_i^{\text{mis-rec-sig}} - B \cdot N_i^{\text{bkg}})^2}{N_i^{\text{exp}} + A^2(N_i^{\text{sig}} + N_i^{\text{mis-rec-sig}}) + B^2 \cdot N_i^{\text{bkg}}} \quad (4)$$

Here i represents the bin index, N_{bins}^{1D} is the number of bins in the one-dimensional histogram shown in Fig. 7c, and A and B are the fit parameters for the signal (reconstructed and misreconstructed) and the background, respectively.

As can be seen in Fig. 7b, the contribution from partially reconstructed signal is significant in the "corners" of the Dalitz

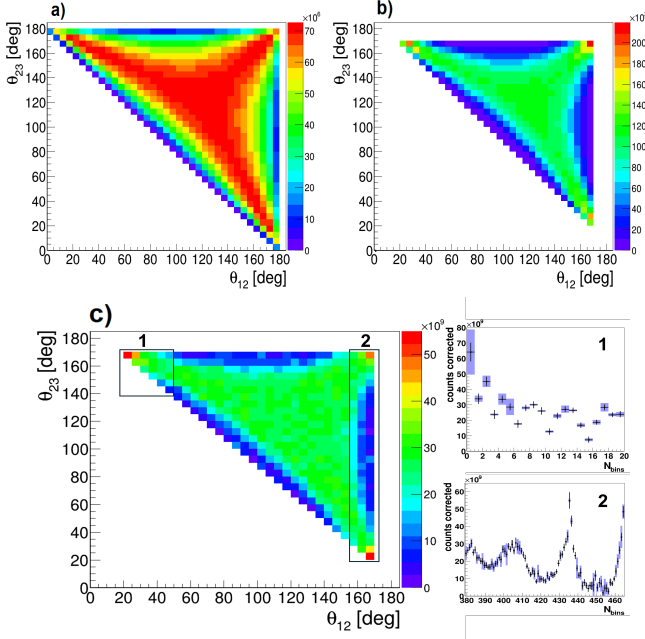


Figure 8: a) Generated Dalitz distribution based on the QED predictions for LO decay process with the same binning as that applied in experimental data analysis. b) Experimental Dalitz Plot in angular representation after background subtraction. c) The experimental Dalitz Plot (b) corrected with efficiency. The right subpanels show the systematic uncertainties for regions 1 and 2 of the Dalitz Plot.

distribution which corresponds to the first and last bins in Fig. 7c. A similar pattern is observed for the remaining background component. Based on the agreement between the reconstructed Monte Carlo and the experimental data, the purity and overall efficiency are estimated to be 84% and 0.03%, respectively.

5. Results

The Dalitz distribution after subtraction of $N^{\text{mis-rec-sig}}$ signal, and the background N^{bcg} contributions (shown in Fig. 7c) is presented in Fig. 8b and contains of 33.42×10^6 identified $o\text{-Ps} \rightarrow 3\gamma$ events. The distribution was corrected for overall detection and reconstruction efficiency map determined based on performed Monte Carlo simulation. The efficiency was calculated as the ratio of the number of reconstructed events to the number of generated events for each angular interval in the Dalitz spectrum. The detector response was simulated for the generated events, including the registration efficiency of the J-PET detector as a function of photon energy deposition [59]. The analysis was then performed using the same selection criteria as those applied to the experimental data.

The obtained Dalitz plot for three-photon $o\text{-Ps}$ decay is shown in Fig. 8c. This distribution was determined for the first time for almost the entire phase space, except for the region corresponding to low-energy photons correlated with relative angles $\theta_{ij} > 170^\circ$. This is shown in the experimental spectrum in Fig. 8c which compares with the QED Monte Carlo simulations in Fig. 8a. In most of the possible phase space, the green region of Fig. 8c, the systematic error is of order 2-3%

while the statistical uncertainty amounts to approximately 3% for a 15° angular binning. The large systematic errors, reaching up to 25%, are associated with bins in the "corners" of the plot. The systematic uncertainties were estimated by repeating the entire analysis procedure with independent variations of selection cuts. The total systematic uncertainties were then calculated by adding the respective deviations in quadrature. The systematic effect on the sample purity and efficiency is estimated to be about 7% and 4%, respectively.

6. Conclusion and perspectives

In this work we have presented the first-ever Dalitz distribution for ortho-positronium decay covering nearly the entire accessible phase space. The measurements were carried out using the three-layer J-PET detector, which has demonstrated its capability to register annihilation photons with energies above 30 keV. Although detection threshold limitations exclude very low-energy photons, this study confirms the feasibility to investigate the $o\text{-Ps} \rightarrow 3\gamma$ decay process with the J-PET detection system and establishes a foundation for future, more comprehensive measurements.

Within our experimental uncertainties, the obtained Dalitz distribution is in agreement with leading-order QED predictions (as shown in Fig. 2). The present statistical and systematic uncertainties are comparable to, or larger than, the expected $O(\alpha)$ modification of about 2%. Therefore, the current measurement does not yet provide sufficient sensitivity to discriminate experimentally between the LO and LO+ $O(\alpha)$ predictions. To go further, higher statistics and more precise measurements will be necessary to test higher-order QED corrections starting at $O(\alpha)$ [1, 27, 38] and to explore small-scale effects such as the search for any charge-conjugation symmetry breaking with high sensitivity. Such advancements are expected to be achievable with the high-acceptance modular J-PET detector [13, 46, 60] equipped with silicon photomultipliers, upgraded readout electronics [61], and a dedicated annihilation chamber. Performing measurements over the same time range as here with this new modular detector (assuming 20 times higher sensitivity for the signal reconstruction) equipped with a 5 MBq sodium source, the statistical uncertainty could be reduced by a factor of 10 over the whole available phase space. These combined improvements are expected to allow Dalitz plot reconstruction for $o\text{-Ps} \rightarrow 3\gamma$ decay across a wider kinematic range, including low-energy photon regions, and enable probing of higher-order effects.

Data availability

Data will be made available on request.

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