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Review

Emergent gauge symmetries and particle physics

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ABSTRACT

Hadron properties and interactions are emergent from QCD. Atomic and condensed matter physics are emergent from QED. Could the local gauge symmetries of particle physics also be emergent? We give an introduction to this question and recent ideas connecting it to the (meta)stability of the Standard Model Higgs vacuum. With an emergent Standard Model the gauge symmetries would “dissolve” in the ultraviolet. This scenario differs from unification models which exhibit maximum symmetry in the extreme ultraviolet. With emergence, new global symmetry violations would appear in higher dimensional operators.

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1. Emergent particle physics

One of the big surprises from the LHC is that the Standard Model works so well! The Standard Model including QCD describes particle physics up to at least a few TeV as revealed in experiments at the Large Hadron Collider and in low-energy precision experiments such as electron electric dipole measurements and precision measurements of the fine structure constant α . Quantum Chromodynamics, QCD, gives us hadrons with their properties and interactions emergent from more fundamental quark and gluon degrees of freedom. The world of everyday experience (atoms, molecules, superconductors etc.) is emergent from Quantum Electrodynamics, QED. The sun and nuclear reactors are powered by radioactive β -decays through the weak interaction.

In high-energy particle physics the Higgs boson discovered at CERN in 2012 [1,2] completes the particle spectrum of the Standard Model. The discovered boson behaves very Standard Model like. It provides masses to the Standard Model particles. With the masses and couplings measured at the LHC the Standard Model works as a consistent theory up to the Planck scale with a Higgs vacuum very close to the border of stable and metastable, which may be a signal for new critical phenomena in the ultraviolet. So far there is no evidence for new particles and interactions either from collider experiments or from precision measurements. Perhaps the symmetries of the Standard Model are more special than previously anticipated. Where do the gauge symmetries come from? Do they unify in the ultraviolet or might the gauge symmetries of the Standard Model be emergent in the infrared, “dissolving” in the ultraviolet close to the Planck scale?

In this article we discuss possible emergent gauge symmetries in particle physics. With main focus on phenomenology, we emphasise signatures that might show up in future experiments.

Emergence in physics occurs when a many-body system exhibits collective behaviour in the infrared that is qualitatively different from that of its more primordial constituents as probed in the ultraviolet [3,4]. As an everyday example of emergent symmetry, consider a carpet which looks flat and translational invariant when looked at from a distance. Up close, e.g. as perceived by an ant crawling on it, the carpet has structure and this translational invariance is lost. The symmetry perceived in the infrared, e.g. by someone looking at it from a distance, “dissolves” in the ultraviolet when the carpet is observed close up.

New local gauge symmetries, where we make symmetry instead of breaking it, are emergent in many-body quantum systems beyond the underlying QED symmetry and atomic interactions [5–7]. Examples include high temperature superconductors [8,9], the Quantum Hall Effect [10] and the A-phase of low temperature ^3He [11]. Emergent Lorentz invariance is also observed in the infrared limit of many-body quantum systems starting from a non-relativistic Hamiltonian, though some fine tuning may be needed to ensure the same effective limiting velocity c for all species of (quasi-)particles [12].

To understand how emergent symmetry might work in particle physics, consider a statistical system near its critical point. The long range tail is a renormalisable Euclidean quantum field theory with properties described by the renormalisation group [13–16]. The Landau–Ginzburg criterion tells us that fluctuations become important for space–time dimensions of four or less. This coincides with the dimensionality of space–time. With four space–time dimensions one finds an interacting quantum field theory. With five or more space–time dimensions the physics reduces to a free field theory with long range modes decoupled. With analyticity the Wick rotation means that the Euclidean quantum field theory is mathematically equivalent to the theory in Minkowski space. Renormalisable theories with vector fields satisfy local gauge invariance [17]. If the physics contains massive gauge bosons, then renormalisability [18–20] and unitarity [21–24] require Yang–Mills structure for the gauge fields (when we go beyond massive QED) together with a Higgs boson.

For emergence, the key idea is that for a critical statistical system close to the Planck scale, the only long range correlations – light mass particles – that might exist in the infrared self-organise into multiplets just as they do in the Standard Model. The vector modes would be the gauge bosons of $U(1)$, $SU(2)$ and $SU(3)$. In the self-organisation process small gauge groups will most likely be preferred. Gauge invariance is then exact (modulo spontaneous symmetry breaking) in the energy domain of the infrared effective theory with gauge invariance determining the number of polarisation degrees of freedom for the vector fields. With parity violating chiral gauge interactions, chiral anomaly cancellation in the ultraviolet limit of the effective theory then groups the fermions into families. In this scenario the gauge theories of particle physics (and perhaps also General Relativity) would be effective theories with characteristic energy of order the Planck scale [25].

The physics of the critical system residing close to the Planck scale would be inaccessible to our experiments. Theoretically, the challenge would be to understand the universality class of systems which exhibit identical critical behaviour, e.g. Standard Model like long-range behaviour. By analogy, the physics of the extreme ultraviolet would be like probing the transition from quarks to hadrons as we go through the ultraviolet phase transition to (very possibly) completely different physics with different degrees of freedom. Whether the Standard Model is the unique stable low-energy limit of the critical Planck system is an interesting subject for conjecture.

With emergence the Standard Model becomes an effective theory. The usual Standard Model action is described by terms of mass dimension four or less. In addition, with emergence one also finds an infinite tower of higher mass dimensional interaction terms with contributions suppressed by powers of a large ultraviolet scale which characterises the limit of the effective theory. If we truncate the theory to include only operator terms with mass dimension at most four, then gauge invariant renormalisable interactions strongly constrain the global symmetries of the theory which are then inbuilt. For example, electric charge is conserved and there is no term which violates lepton or baryon number

conservation. The dimension-four action describes long distance particle interactions. Going beyond mass-dimension four one finds gauge invariant but non-renormalisable terms where global symmetries are more relaxed and which are suppressed by powers of the large ultraviolet scale associated with emergence. Possible lepton number violation, also associated with Majorana neutrino masses, can enter at mass-dimension five, suppressed by a single power of the large emergence scale [26]. Baryon number violation can enter at dimension six, suppressed by the large emergence scale squared [26,27]. The strong CP puzzle – the absence of CP violation induced by the non-perturbative glue which generates the large η' meson mass – might be connected to a possible new axion particle, which is a postulated new pseudoscalar with coupling that enters at mass-dimension five [28,29]. Dark matter [30] might also involve dimension five (or higher) interactions – that is, with non-gravitational interaction strength very much suppressed by power(s) of the large emergence scale. With the preference for small gauge groups, extra massive U(1) gauge bosons might also be possible at mass-dimension four.

Possible emergent gauge symmetries in particle physics were discussed in early work by Bjorken [31–33], Jegerlehner [15,34,35] and Nielsen and collaborators [36]. The key idea has enjoyed some recent renaissance, see the Perspectives article by Witten [37] as well as Refs. [38–42]. This article serves as an invitation to explore this physics and its phenomenology.

Gauge symmetries act on internal degrees of freedom whereas global symmetries act on the Hilbert space. That is, local gauge symmetries are properties of the description of a system and global symmetries are properties of the system itself.

In the emergence scenario global symmetries would be restored with increasing energy (with energy exceeding symmetry breaking mass terms) until we reach some very high energy where higher dimensional terms become important. Then the system becomes increasingly chaotic with possible lepton and baryon number violation and also possible Lorentz invariance violation in the extreme ultraviolet. Additional sources of CP violation might also be possible and important for understanding the matter–antimatter asymmetry generated in the early Universe. This scenario differs from the situation in unification models which exhibit maximum symmetry in the extreme ultraviolet and where symmetries are spontaneously broken in the infrared, e.g. through coupling to the Higgs and dynamical chiral symmetry breaking in QCD.

Lorentz invariance might also be emergent in the infrared along with gauge symmetry. Nielsen and collaborators considered the effect of adding a Lorentz violating term and found that it vanishes in the infrared through renormalisation group evolution, e.g. with Lorentz invariance emerging as an infrared fixed point [43,44]. In early work Bjorken suggested that the photon might be a Goldstone boson associated with spontaneous breaking of Lorentz invariance [33]. He also suggested that any violation of Lorentz invariance might be proportional to the ratio of the tiny cosmological constant scale, 0.002 eV, to the scale of emergence, that is a ratio of about 10^{-27} and much beyond the range of present experiments [31]. Lorentz invariance is very strongly constrained by experiments [45,46]. Being linked to Lorentz invariance, any violations of CPT symmetry would also be very tiny in this scenario.

The plan of the paper is as follows. In Section 2 we review the status of the successful phenomenology of the Standard Model. In Section 3 we discuss the role of gauge invariance in defining the interactions of QED, QCD and the electroweak Standard Model and the constraints on global symmetries in these theories. Following this introductory material, Section 4 discusses the issue of vacuum stability and the interplay of Standard Model parameters and the physics of the extreme ultraviolet. In Section 5 we compare the unification and emergence scenarios. Section 6 discusses the Standard Model as a low energy expansion and the physics of higher dimensional operators. Here we discuss the signatures of emergent gauge symmetry that might show up in future experiments. Section 7 summarises the discussion of emergence versus unification and the open puzzles in this approach.

2. Particle physics today – the very successful Standard Model

The Standard Model is built on the gauge group of SU(3) colour, chiral SU(2) and U(1) which are associated with QCD, weak interactions and QED respectively. The ground state of QED is in the Coulomb phase, QCD in the confining phase and weak interactions in a Higgs phase. Whereas QED through massless photons has infinite range, quarks and gluons in QCD can propagate up to about 1 fm before strong confinement forces take over and the relevant degrees of freedom are colour singlet hadrons. Weak interactions operate over a distance scale of about 0.01 fm through massive W and Z boson exchanges.

The QCD and the SU(2) weak couplings are asymptotically free meaning that they decrease logarithmically in the ultraviolet. In contrast, the QED coupling or fine structure constant increases logarithmically in the ultraviolet with any divergence very much above the Planck scale, the scale where quantum gravity effects are believed to become important. Gluon–gluon interactions between gauge bosons in QCD and W–Z coupling with weak interactions give us asymptotic freedom. The large QCD coupling in the infrared leads to confinement and dynamical chiral symmetry breaking with pions and kaons as the corresponding Goldstone bosons.

In QED and QCD the photons and gluons interact with equal strength with left- and right-handed fermions. The weak SU(2) interaction breaks parity and acts just on left handed quarks and leptons grouped into lepton doublets consisting of a charged lepton and neutrino and quark doublets with an up-type quark (electric charge $+\frac{2}{3}$) and a down-type quark (electric charge $-\frac{1}{3}$).

The Standard Model has 18 parameters, or up to 27 if we also include neutrino mixing and tiny neutrino masses:

- 3 gauge couplings,
- 15 in Higgs sector (6 quark masses, 3 charged leptons, 4 quark mixing angles, W and Higgs mass),
- 9 neutrino parameters (3 masses plus 6 mixing angles with Majorana neutrinos) which might be connected to a dimension 5 operator.

The fermion multiplet structure is reproduced three times in families, also called generations. Precision measurements of Z^0 decays at the LEP experiments at CERN revealed the number of light neutrinos as 2.984 ± 0.008 [47]. The number of neutrino families is determined independently from the cosmic microwave background. One finds 3.13 ± 0.32 [48].

Photons and gluons are massless. The masses of W and Z bosons and of charged fermions (leptons and quarks) come from the Higgs sector. The W and Z bosons have mass 80 and 91 GeV and the Higgs boson has mass 125 GeV. The charged leptons and their masses are

$$m_e = 0.51 \text{ MeV}, \quad m_\mu = 105.66 \text{ MeV}, \quad m_\tau = 1776.86 \pm 0.12 \text{ MeV}. \quad (1)$$

Neutrinos come with tiny masses, see Section 6.1, as evidenced by neutrino oscillation data [49]. The down-type quarks have masses

$$m_d = 5 \text{ MeV}, \quad m_s = 93 \text{ MeV}, \quad m_b = 4.18 \text{ GeV} \quad (2)$$

whereas the up-type quarks have masses

$$m_u = 2 \text{ MeV}, \quad m_c = 1.27 \text{ GeV}, \quad m_t = 173.1 \pm 0.9 \text{ GeV}. \quad (3)$$

Strong interactions of QCD are an additional source of mass generation. Scalar confinement of quarks and gluons in the proton generates the large proton mass. About 99% of the mass of the hydrogen atom 938.8 MeV is associated with the confinement potential with the masses of the electron 0.5 MeV and the proton 938.3 MeV. Inside the proton the masses of the proton's constituent two up quarks and one down quark contribute about 9 MeV.

The number of degrees of freedom for the gauge bosons are dependent on the ground state, whether in the Coulomb, confining or Higgs phase. The massless photons carry two transverse polarisation states whereas the massive W and Z bosons have three polarisations with longitudinal polarisation also included. Massless gluons come with two transverse polarisations but are always virtual because of confinement meaning that longitudinal gluons can play a role. The charge-neutral scalar Higgs has just one degree of freedom.

Particle physics experiments measure interactions described by the Standard Model action with mass dimension at most four. Higher dimensional terms are suppressed by powers of some large ultraviolet scale which we are not yet sensitive to with the present energies and precision of our experiments. The Lagrangian restricted to terms with mass dimension at most four is renormalisable. Ultraviolet divergences from loop diagrams can be self-consistently absorbed in a re-definition of the parameters [50]. Renormalisability tells us that, with parity violating vector interactions, we have to worry about chiral anomaly cancellation in the ultraviolet. For example, the triangle Feynman diagrams with two photon or gluon vertices and a chiral Z^0 coupling cannot be gauge invariantly renormalised unless the relevant charges of the fermions propagating in the loop sum to zero. This in turn groups the fermions into families with combinations of charges perfectly able to cancel any local chiral anomalies, meaning that gauge invariance and renormalisability are preserved.

Three families are also needed for CP violation with the Cabibbo–Kobayashi–Maskawa (CKM) matrix. This is a unitary 3×3 quark mixing matrix with three real mixing parameters and one CP violating angle. The measured mixing parameters from the Cabibbo angle to heavy-quark mixing angles are consistent with a unitary matrix and no new CP violating physics in the energy range of present experiments, e.g. up to LHC energies. Precision studies of the electroweak Standard Model come from measurements at the Z^0 pole from LEP at CERN and SLD at SLAC – for a review see [51]. Key observables include the weak mixing angle $\sin^2 \theta_W$, forward–backward asymmetries and τ lepton polarisation.

Our everyday experience is described by just the first generation of light quarks and leptons (protons, neutrons, pions, electrons, plus neutrinos from the Sun). However, our existence is not insensitive to the physics of the highest scales. After radiative corrections the values of the top-quark and Higgs boson masses are essential for whether the electroweak particle physics vacuum is stable or not [52–59]. With the value of the top quark mass measured at the LHC, the Higgs mass is about the minimum necessary for the Standard Model vacuum to be stable, see Section 4. One finds a delicate balance between Standard Model parameters and the physics of the extreme ultraviolet.

Going beyond the Standard Model, there were hopes of the Standard Model couplings meeting in the ultraviolet, perhaps with help from some new particles or interactions. This would lead to unification of the three Standard Model forces, generalising the electroweak unification of electromagnetism and weak interactions to include QCD. With just Standard Model couplings, they nearly do meet but not exactly – see Section 4 below.

The observed matter–antimatter asymmetry in the Universe requires some extra source of CP violation beyond the quark mixing described by the CKM matrix in the electroweak Standard Model. In the neutrino sector recent measurements by the T2K Collaboration in Japan are consistent with CP violation at the level of two standard deviations [60,61]. In addition, low-energy precision experiments are used to look for possible new sources of CP violation. Key experiments involve the search for electric dipole moments [62–64] plus precision measurements of CP sensitive observables in positronium decays [65–67]. So far there is no evidence for new extra sources of CP violation from these low-energy precision experiments.

2.1. Precision QED tests

QED is the most accurately tested theory with remarkable precision achieved in different measurements of the fine structure constant α . The most accurate determinations of α come from precision measurements of the electron's anomalous magnetic moment [68] and atom interferometry measurements with Caesium, Cs [69]. The electron anomalous magnetic moment $a_e = (g - 2)/2$ is generated by radiative corrections, which have been evaluated to tenth-order in QED perturbation theory plus tiny QCD and weak contributions [70]. The electron a_e value gives a precision measurement of α (modulo any radiative corrections from new physics beyond the Standard Model). Atom interferometry experiments with Cs provide a more direct determination (less sensitive to details of radiative corrections) but also involve a combination of parameters measured in experiments: the Rydberg constant R_∞ , the ratio of the atom to electron mass m_{atom}/m_e and new precision measurements of the Cs mass from recoil of a Cs atom in an atomic lattice, viz. $\alpha^2 = (2R_\infty/c) (m_{\text{atom}}/m_e) (h/m_{\text{atom}})$. (Here c is the speed of light and h is Planck's constant.) Comparing these different determinations of α gives a precision test of QED as well as constraining possible new physics scenarios. Any "beyond the Standard Model" effects involving new particles active in radiative corrections will enter a_e but not the Cs measurements. The new most accurate Cs atomic physics measurement corresponds to

$$a_e^{\text{exp}} - a_e^{\text{th}}|_{\text{Cs}} = (-88 \pm 36) \times 10^{-14} \quad (4)$$

when we substitute the α value measured in these atomic physics experiments into the perturbative expansion of a_e to obtain the value $a_e^{\text{th}}|_{\text{Cs}}$. That is, one finds agreement to 1 part in 10^{12} .

2.2. QCD and emergent hadrons

QCD is fundamentally different because of confinement in the infrared. Quarks carry a colour charge and interact through coloured gluon exchange, like electrons interacting through photon exchange in QED. QCD differs from QED in that gluons also carry colour charge whereas photons are electrically neutral. This means that the Feynman diagrams for QCD include 3 gluon and 4 gluon vertices (as well as the quark gluon vertices) and that gluons self-interact. The three gluon vertex leads to gluon bremsstrahlung resulting in gluon induced jets of hadronic particles which were first discovered in high energy e^-e^+ collisions at DESY. The decay amplitude for $\pi^0 \rightarrow 2\gamma$ and the ratio of cross-sections for hadron to muon-pair production in high energy electron-positron collisions, $R_{e^-e^+}$, are each proportional to the number of dynamical colours N_c , giving an experimental confirmation of $N_c = 3$.

In the infrared quark-gluon interactions become strong. Low energy QCD is characterised by confinement and dynamical chiral symmetry breaking. The physical degrees of freedom are emergent hadrons (protons, mesons etc.) as confined bound states of quarks and gluons. Spontaneous chiral symmetry breaking is associated with a non-vanishing chiral quark condensate. The light mass pions (and kaons) are the corresponding would-be Goldstone bosons with mass squared proportional to the light quark masses, $m_\pi^2 \sim m_q$. In the isosinglet channel non-perturbative gluon dynamics increase the masses of the η and η' mesons by about 300–400 MeV relative to the masses they would have if they were pure Goldstone states [71].

The proton's mass and spin are emergent from quark and gluon degrees of freedom.

High energy deep inelastic scattering experiments probe the deep structure of hadrons by scattering high energy electron or muon beams off hadronic targets. Deeply virtual photon exchange acts like a microscope which allows us to look deep inside the proton. These experiments reveal a proton built of nearly free fermion constituents, called partons. Quark and gluon partons play a vital role in high energy hadronic collisions, e.g., at the LHC [51]. Deep inelastic scattering experiments also tell us that about 50% of the proton's momentum perceived at high Q^2 is carried by gluons, consistent with the QCD prediction for the deepest structure of the proton. Polarised deep inelastic scattering experiments have taught us that just about 30% of the proton's spin of one half is carried by its quarks [72]. The rest is carried by gluons and by quark and gluon orbital angular momentum. Confinement generates a transverse momentum scale in the proton leading to finite quark and gluon orbital angular momentum contributions. Scalar confinement also induces dynamical chiral symmetry breaking, e.g., in the Bag model the Bag wall connects left and right handed quarks leading to quark-pion coupling and the pion cloud of the nucleon [73]. The pion cloud takes further orbital angular momentum through quark-pion coupling in the nucleon [74]. One finds a consistent picture where pion cloud dynamics, modest gluon polarisation (up to about 50% of the proton's spin at the scale of typical deep inelastic experiments) and perhaps non-local gluon topology describe the internal spin structure of the proton [72,75].

Hadron physics is our first example of emergence in particle physics with change to totally new degrees of freedom as one goes through the confinement transition from coloured quarks and gluons to colour-neutral hadrons.

3. Global and local gauge symmetries

We next focus on the symmetries in the particle physics Lagrangian, first with QED and then QCD and the electroweak Standard Model. Our aim in this Section is to show the role that local gauge invariance plays in constraining the interaction terms and global symmetries of the Standard Model. This discussion will lead into the exploration of global symmetry breaking terms with higher mass dimension in Sections 5 and 6.

Local gauge symmetries determine the dynamics. Poincare invariance is an important part of quantum field theory together with the associated discrete symmetries of P, C, T, CP (which can be broken) and fundamental CPT symmetry which is exact. The usual particle physics Lagrangian includes fields and interaction terms with mass dimension at most four. For example, the mass dimensions of fermion fields ψ , scalar bosons ϕ and vector fields A_μ and their interaction terms are

- $[\psi] = \frac{3}{2}$
- $[\phi] = 1$
- $[A_\mu] = 1$
- $[m] = 1$
- $[\partial_\mu] = 1$
- $[m\bar{\psi}\psi] = 4$
- $[\partial^\mu\phi\partial_\mu\phi] = 4$

Starting with the theoretical Lagrangian, Noether's theorem tells us that there are conserved currents associated with continuous global symmetries. For example, translational invariance is associated with momentum conservation. Rotational invariance is associated with angular momentum conservation. Electric charge conservation is associated with global U(1) invariance in QED and the conserved (and gauge invariant) vector current. Invariance under global axial rotations of the phase of the fermion fields leads to the Noether current $j_{\mu 5} = \bar{\psi}\gamma_\mu\gamma_5\psi$. Conservation of $j_{\mu 5}$ which also corresponds to fermion helicity conservation is softly broken by fermion mass terms (and also sensitive to anomalous terms in its divergence equation in the singlet channel where one couples through two gauge-boson intermediate states [76,77]).

Fermion masses which break chiral symmetry between left- and right-handed fermions represent a continuous deformation of the massless theory. Gauge boson masses which enter through the Higgs mechanism with spontaneous symmetry breaking change the degrees of freedom meaning we have a different theory (where longitudinal polarisation of the gauge bosons becomes physical).

If we truncate the theory to operators of mass dimension at most four, then the global symmetries are strongly constrained by the operators that are allowed by gauge invariance and renormalisability. Particle masses and global symmetry breaking becomes less important with increasing energy, especially when the energy is much greater than the particle masses, $E \gg m$. Global symmetries which are compelled to hold at dimension four can be broken in non-renormalisable higher dimensional operators which are suppressed by powers of some large ultraviolet scale and become active only in the extreme ultraviolet [15,37]. Examples include lepton and baryon number violation discussed in Section 6. If we allow for new higher dimensional terms in the action, then we find increasing restoration of global symmetries with increasing energy and resolution until we become sensitive to these higher dimensional terms, say at energies within about 0.1% of the large ultraviolet scale.

We next look in detail at gauge symmetry which is intrinsic to particle physics interactions.

3.1. Quantum Electrodynamics

Quantum Electrodynamics, QED, follows from requiring that the physics is invariant under local U(1) changes of the phase of charged particles, e.g. the electron, viz.

$$\psi(x) \rightarrow e^{i\alpha(x)}\psi(x) \quad (5)$$

where $\alpha(x)$ is a function of the space-time co-ordinates. Derivative terms ∂_μ acting on ψ will also act on the phase factor $\alpha(x)$ so that the phase factor does not flow through the combination $\partial_\mu\psi$. Instead, consider the gauge covariant derivative

$$\partial_\mu \mapsto D_\mu = \partial_\mu + ieA_\mu \quad (6)$$

where A_μ is the gauge field and e is the electric charge with the fine structure constant

$$\alpha = e^2/4\pi. \quad (7)$$

With A_μ transforming under the phase rotation in Eq. (5) as

$$A_\mu(x) \rightarrow A'_\mu(x) = A_\mu(x) - \frac{1}{e}\partial_\mu\alpha(x), \quad (8)$$

the combination $D_\mu\psi$ transforms as

$$D_\mu\psi \rightarrow e^{i\alpha(x)}D_\mu\psi \quad (9)$$

and the phase factor is pulled through the derivative term. The gauge field A_μ describes the photon after quantisation. The photon field tensor $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ is invariant under transformations of A_μ , Eq. (8).

The QED Lagrangian

$$\mathcal{L} = \bar{\psi}i\gamma^\mu D_\mu\psi - m\bar{\psi}\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} \quad (10)$$

is gauge invariant. After quantisation it describes the QED dynamics: the electron and photon propagators and the electron-photon interaction vertex. The first term includes the kinetic energy for the electron together with the electron photon interaction, $e\bar{\psi}\gamma^\mu A_\mu\psi$. The second term is the electron mass. The third term describes the photon kinetic energy.

- Real photons come with two transverse polarisations. The time and longitudinal components of A_μ (for real photons) are really not dynamical degrees of freedom. They can be set equal to zero by a suitable choice of gauge – the radiation or Coulomb gauge, $\nabla \cdot A = 0$. In general, under a Lorentz transformation A_μ does not transform as a four-vector but is supplemented by an additional gauge term which ensures that only gauge invariant Maxwell equations are Lorentz covariant [50].
- Conserved electric charge corresponds to global U(1) transformations.
- The electron mass term breaks the chiral symmetry between left- and right-handed electrons. Helicity is conserved for massless electrons. (Without the electron mass, left- and right-handed electron fields, $\psi_L = \frac{1}{2}(1 - \gamma_5)\psi$ and $\psi_R = \frac{1}{2}(1 + \gamma_5)\psi$, transform independently under chiral rotations.)
- Mass is important in quantum field theories and is needed even in QED; charged particles should also carry mass [78–80]. Starting from Eq. (10) one cannot perturbatively renormalise massless QED on-shell. If one renormalises the massive theory on-shell and then takes the mathematical limit that the electron mass goes to zero, then the Landau pole in the running coupling gets pulled towards the infrared,

$$\alpha(\lambda_{UV}^2) = \frac{\alpha(m^2)}{1 - \frac{\alpha(m^2)}{3\pi} \ln \frac{\lambda_{UV}^2}{m^2}}. \quad (11)$$

- If we treat QED as an effective theory, then one might add also the gauge invariant but non-renormalisable Pauli term

$$i\frac{e}{2M} \bar{\psi}(\gamma^\mu\gamma^\nu - \gamma^\nu\gamma^\mu)\psi F_{\mu\nu} \quad (12)$$

which is suppressed by power of some large mass scale M [81]. This is our first example of a non-renormalisable dimension 5 operator. It gives a contribution to the electron magnetic moment of $4e/M$. Experiment through Eq. (4) constrains M to be at least 3×10^{10} GeV.

The dimension 5 Pauli term with finite large M gives a finite value of α without Landau pole or triviality issues [82].

3.2. QCD and non-abelian gauge theories

The same arguments that led to QED can be generalised to non-abelian groups, e.g. SU(2) and SU(3). For QCD we require the physics to be invariant under

$$\Psi(x) \rightarrow U(x)\Psi(x) \quad (13)$$

where

$$U(x) = e^{i\frac{1}{2}\vec{\lambda}\cdot\vec{\alpha}(x)} \quad (14)$$

and λ_a are the eight 3×3 Gell-Mann matrices. Here Ψ is a 3-dimensional spinor corresponding to quarks carrying the SU(3) colours red, green and blue. The QCD gauge covariant derivative is

$$D_\mu\Psi = \left[\partial_\mu + \frac{i}{2}g_3\vec{\lambda}\cdot\vec{A}_\mu \right] \Psi \quad (15)$$

where g_3 is the SU(3) colour charge and the gluon gauge fields $A_\mu = \frac{1}{2}\vec{\lambda}\cdot\vec{A}_\mu$ satisfy the transformation rule

$$A_\mu(x) \rightarrow A_\mu'(x) = UA_\mu U^{-1} + \frac{i}{g_3}(\partial_\mu U)U^{-1}. \quad (16)$$

Corresponding to SU(3) phase rotations (13) and (14), the QCD gauge covariant derivative transforms as

$$D_\mu \rightarrow UD_\mu U^\dagger. \quad (17)$$

There are eight gluon fields. The field tensor for these gauge fields is

$$G_{\mu\nu}^a = [D_\mu, D_\nu]_- = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a - g_3 f_{abc} A_\mu^b A_\nu^c \quad (18)$$

where the f_{abc} are the structure constants of SU(3), $[t^a, t^b] = if_{abc}t^c$ with $t^a = \frac{1}{2}\lambda^a$. Putting things together the QCD Lagrangian

$$\mathcal{L} = \bar{\Psi}i\gamma^\mu D_\mu\Psi - m\bar{\Psi}\Psi - \frac{1}{4}\text{Tr} G_{\mu\nu}G^{\mu\nu} \quad (19)$$

is invariant under the SU(3) gauge transformations in Eqs. (13) and (16).

- The QCD running coupling is asymptotically free. At leading order

$$\alpha_s(K^2) = \frac{g_3^2}{4\pi} = \frac{4\pi}{\beta_0 \ln(K^2/\Lambda_{\text{QCD}}^2)}. \quad (20)$$

Here K^2 is the four-momentum transfer squared, $\beta_0 = \frac{11}{3}N_c - \frac{2}{3}f$ where $N_c = 3$ is the number of colours and f is the number of active flavours; Λ_{QCD} is the renormalisation group invariant QCD infrared scale, about 200 MeV.

- Rising α_s in the infrared leads to QCD confinement of quarks and gluons. A scalar confinement potential leads to dynamical mass generation through the strong interactions. It also spontaneously breaks the chiral symmetry between left- and right-handed quarks with pions and kaons as the would-be Goldstone bosons with mass squared $m^2 \propto m_q$ where m_q is the light-quark mass. The light-quarks then acquire an effective large constituent quark mass of about 300 MeV in the infrared.
- In the isosinglet channel the η and η' mesons are too heavy by about 300–400 MeV to be pure Goldstone bosons. They receive extra mass from non-perturbative gluon processes in the flavour-singlet channel [71] connected to non-perturbative gluon topology and the QCD axial anomaly in the divergence of the flavour-singlet axial-vector current [76,77]. While the non-singlet axial-vector currents like $J_{\mu 5}^{(3)} = \bar{u}\gamma_\mu\gamma_5 u - \bar{d}\gamma_\mu\gamma_5 d$ are partially conserved (they have just mass terms in the divergence), the singlet current $J_{\mu 5} = \bar{u}\gamma_\mu\gamma_5 u + \bar{d}\gamma_\mu\gamma_5 d + \bar{s}\gamma_\mu\gamma_5 s$ satisfies the divergence equation

$$\partial^\mu J_{\mu 5} = 6Q_t + \sum_{k=1}^3 2im_k \bar{q}_k \gamma_5 q_k \quad (21)$$

where

$$Q_t = \frac{\alpha_s}{8\pi} G_{\mu\nu} \tilde{G}^{\mu\nu} \quad (22)$$

is called the topological charge density. Here $G_{\mu\nu}$ is the gluon field tensor and $\tilde{G}^{\mu\nu} = \frac{1}{2}\epsilon^{\mu\nu\alpha\beta} G_{\alpha\beta}$. Beyond perturbative QCD, $\int d^4x Q_t$ quantises with integer or fractional values [83]. Non-perturbative gluon processes act to connect left- and right-handed quarks whereas left- and right-handed massless quarks propagate independently in perturbative QCD with helicity conserved for massless quarks.

The non-perturbative gluon dynamics which generate the large η' mass also has the potential to induce strong CP violation in QCD. Provided the quark masses are non-zero, e.g. through the Yukawa couplings associated with the Higgs mechanism, one finds a new effective CP-odd term in the QCD Lagrangian at mass dimension-four [84]

$$\mathcal{L} = -\theta Q_t. \quad (23)$$

Experimentally, $\theta < 10^{-10}$ [63]. We return to this physics in Section 6.2. Finite θ values in Eq. (23) are induced by non-zero quark masses which enter at mass dimension four. The suppression of strong CP violation may involve a new axion particle (which enters with mass and coupling at mass dimension five) with delicate interplay of interactions at dimension four and dimension five.

3.3. The Higgs mechanism and the Standard Model

Before discovery of the W and Z bosons parity violating weak interactions were described using Fermi's four-fermion point interaction with coupling

$$\frac{1}{\sqrt{2}} G_F = \frac{g^2}{8m_W^2}. \quad (24)$$

The coupling here G_F comes with mass dimension minus-two with the four-fermion interaction violating unitarity and renormalisability signalling the need for new physics at a deeper level – the Standard Model.

We now know in the Standard Model that weak interactions are mediated by massive W and Z gauge boson exchange with left-handed fermion doublets and gauge group SU(2). Restricting to one family of fermions the lepton and quark doublets are

$$L = \begin{pmatrix} \nu \\ e \end{pmatrix}, \quad Q = \begin{pmatrix} u \\ d^* \end{pmatrix}_c \quad (25)$$

where u is the up quark and d the down quark. Going beyond the first fermion family, d^* in the lower component of the quark doublet includes mixing from the Cabibbo angle (including strange quarks) and the full Cabibbo–Kobayashi–Maskawa matrix taking into account the three families of fermions. The subscript c denotes the three colours of quarks (red, green and blue) associated with QCD.

QED and weak interactions unify through mixing between the charge neutral photon and Z boson with the gauge group $SU(2)_L \otimes U(1)$. We let B_μ denote the $U(1)$ gauge boson and W_μ^i denote the $SU(2)$ bosons. The $U(1)$ gauge bosons interact equally with left- and right-handed fermions. Fermions transform under the $SU(2)$ and $U(1)$ gauge transformations as

$$\Psi_L(x) \rightarrow e^{i\frac{1}{2}\vec{\tau}\cdot\vec{\alpha}(x)}\Psi_L(x) \quad \text{and} \quad \Psi(x) \rightarrow e^{i\frac{y}{2}\beta(x)}\Psi(x) \quad (26)$$

with gauge covariant derivative

$$\begin{aligned} D_\mu \Psi_L &= \left[\partial_\mu + \frac{1}{2}ig\vec{\tau}\cdot\vec{W}_\mu + \frac{1}{2}ig'yB_\mu \right] \Psi_L \\ D_\mu \Psi_R &= \left[\partial_\mu + \frac{1}{2}ig'yB_\mu \right] \Psi_R. \end{aligned} \quad (27)$$

Here τ denotes the $SU(2)$ Pauli matrices and g and g' are the $SU(2)$ and $U(1)$ couplings. The electric charge is $Q = t_3 + y/2$ where $t_3 = \tau_3/2$. The hypercharge y is $y = -1$ for left-handed leptons l_L , $y = -2$ for the right-handed leptons l_R , $y = \frac{1}{3}$ for the left-handed quarks, $y = -\frac{2}{3}$ for right-handed down-type quarks, and $y = \frac{4}{3}$ for right-handed up-type quarks. With these assignments the electron carries electric charge and the neutrino is electric charge neutral.

The electric charge neutral gauge bosons mix as

$$\begin{pmatrix} W_\mu^3 \\ B_\mu \end{pmatrix} = \begin{pmatrix} \cos \theta_W & \sin \theta_W \\ -\sin \theta_W & \cos \theta_W \end{pmatrix} \begin{pmatrix} Z_\mu \\ A_\mu \end{pmatrix} \quad (28)$$

where A_μ is the photon field, Z_μ is the Z boson field and θ_W is the Weinberg angle. The neutral current with left-handed fermions interaction is then

$$ig \sin \theta_W \bar{\Psi}_L \gamma^\mu \left[A_\mu (t_3 + \frac{1}{2}y) + Z_\mu (-\frac{1}{2}y \tan \theta_W I + \cot \theta_W t_3) \right] \Psi_L. \quad (29)$$

The photon is massless provided that

$$g \sin \theta_W = g' \cos \theta_W = e \quad (30)$$

or

$$\tan \theta_W = g'/g. \quad (31)$$

Mixing fixes the Weinberg angle θ_W

$$\cos \theta_W = \frac{g}{\sqrt{g^2 + g'^2}}, \quad \sin \theta_W = \frac{g'}{\sqrt{g^2 + g'^2}} \quad (32)$$

- that is,

$$W_\mu^\pm = \frac{1}{\sqrt{2}}(W_\mu^1 \mp W_\mu^2), \quad B_\mu = \frac{-g'Z_\mu + gA_\mu}{\sqrt{g^2 + g'^2}}, \quad W_\mu^3 = \frac{gZ_\mu + g'A_\mu}{\sqrt{g^2 + g'^2}}. \quad (33)$$

The W^\pm connect different members of the electroweak lepton and quark doublets whereas the photon and Z^0 are electric-charge neutral bosons.

Naively, gauge boson mass terms break gauge invariance: the mass term $m^2 W_\mu W^\mu$ is not invariant under gauge transformations of the W_μ^a . Gauge invariance is maintained through the Higgs mechanism [85–88]. One adds the scalar doublet Φ with potential

$$V(\Phi) = \frac{1}{2}\mu^2 \Phi^2 + \frac{1}{4}\lambda \Phi^4 \quad (34)$$

and transforming as

$$\Phi(x) \rightarrow e^{i\frac{1}{2}\vec{\tau}\cdot\vec{\alpha}(x)}\Phi(x) \quad \text{and} \quad \Phi(x) \rightarrow e^{i\frac{y}{2}\beta(x)}\Phi(x) \quad (35)$$

under the $SU(2)$ and $U(1)$ gauge transformations (26). In Eq. (34) $\lambda \geq 0$ to ensure that the potential has a finite minimum, as required for vacuum stability. The Higgs scalar comes with the covariant derivative coupling

$$D_\mu \Phi = \left[\partial_\mu + \frac{1}{2}ig\vec{\tau}\cdot\vec{W}_\mu + \frac{1}{2}ig'y_\phi B_\mu \right] \Phi. \quad (36)$$

Here the scalar hypercharge $y_\phi = +1$ for the Standard Model to ensure that the photon does not couple to the Higgs boson and ϕ^+ has the correct charge. For $\mu^2 > 0$ Eq. (34) describes the potential for a particle with mass μ . More interesting is the case $\mu^2 < 0$. In this case the potential has a minimum at

$$|\Phi| = \frac{v}{\sqrt{2}} \equiv \sqrt{-\frac{\mu^2}{2\lambda}} \quad (37)$$

where v is the vacuum expectation value, vev. One takes the vev to be real,

$$\langle \Phi \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}, \quad (38)$$

and expands the scalar field about this vev, viz.

$$\Phi = \begin{pmatrix} \phi^+ \\ \frac{1}{\sqrt{2}}(v + \rho + i\zeta) \end{pmatrix}. \quad (39)$$

The phase of Φ is then chosen using the gauge freedom to make Φ real. Three components of the Higgs doublet then “disappear” through the gauge choice – the ζ and ϕ^\pm have become “eaten” – to become the longitudinal components of the massive gauge bosons, which acquire a mass term

$$\mathcal{L}_\phi = \left(1 + \frac{h}{v}\right)^2 \left\{ m_W^2 W_\mu^\dagger W^\mu + \frac{1}{2} m_Z^2 Z_\mu^\dagger Z^\mu \right\}. \quad (40)$$

Here $h = \rho$ is the remaining scalar degree of freedom – the Higgs boson. The linear combination proportional to

$$g' W_\mu^3 + g B_\mu \quad (41)$$

remains massless,

$$m_W = m_Z \cos \theta_W = \frac{1}{2} g v \quad (42)$$

and

$$v = (\sqrt{2} G_F)^{-\frac{1}{2}} = 246 \text{ GeV} \quad (43)$$

$$\lambda = \frac{m_h^2}{2v^2} = 0.13. \quad (44)$$

The Higgs boson h comes with Lagrangian terms

$$\mathcal{L}_h = \frac{1}{2} \partial_\mu h \partial^\mu h - \frac{1}{2} m_h^2 h^2 - \frac{m_h^2}{2v} h^3 - \frac{m_h^2}{8v^2} h^4 + \frac{1}{8} m_h^2 v^2. \quad (45)$$

Fermion masses are constructed by contracting the Higgs doublet with the left-handed fermion doublet and then coupling to right-handed fermion singlets, viz.

$$\mathcal{L}_Y = -y_d \bar{Q}_L \Phi d_R - y_u \bar{Q}_L \bar{\Phi} u_R - y_l \bar{L}_L \Phi l_R + \text{h.c.} \quad (46)$$

which for the first generation gives

$$\mathcal{L}_Y = - \left(1 + \frac{h}{v}\right) \{m_d \bar{d} d + m_u \bar{u} u + m_l \bar{l} l\}. \quad (47)$$

Note that with parity violating couplings, the QED mass term is not possible and the Higgs doublet is required for fermion masses. This means also that without the Higgs there is no bare fermion mass term in the Standard Model [89].

Summarising, the fermion and gauge boson masses are then

$$m_f = y_f \frac{v}{\sqrt{2}}, \quad (f = \text{quarks and charged leptons}) \quad (48)$$

and

$$m_W^2 = \frac{1}{4} g^2 v^2, \quad m_Z^2 = \frac{1}{4} (g^2 + g'^2) v^2. \quad (49)$$

The electroweak interaction Lagrangian is

$$\begin{aligned} \mathcal{L} = & \bar{\Psi}_L i \gamma^\mu D_\mu \Psi_L + \bar{\Psi}_R i \gamma^\mu (\partial_\mu + i g B_\mu) \Psi_R - \frac{1}{4} \text{Tr} W_{\mu\nu} W^{\mu\nu} - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} \\ & + \mathcal{L}_\phi + \mathcal{L}_h + \mathcal{L}_Y. \end{aligned} \quad (50)$$

Note that gauge boson masses change the degrees of freedom (with longitudinal polarisation) whereas fermion masses are a continuous deformation of the massless theory. The mass dimension four Lagrangian (50) respects lepton and baryon number conservation although tiny effects can be induced by electroweak vacuum tunnelling processes associated with the axial anomaly [90].

The Higgs mechanism allows the gauge bosons to acquire mass while respecting gauge invariance, which becomes “hidden”. Beyond the tree-level Lagrangian, 't Hooft and Veltman showed that the Higgs mechanism is also needed to give renormalisable massive Yang–Mills [18–20]. Further, going beyond any massive U(1) gauge bosons, unitarity in high

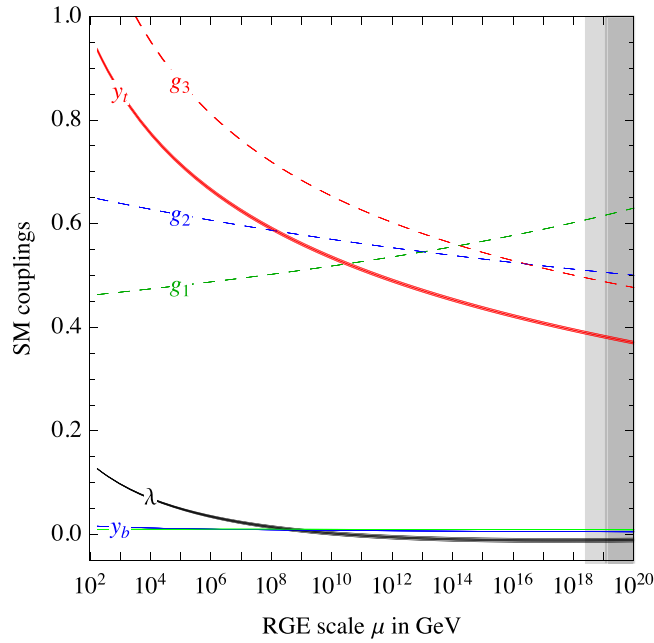


Fig. 1. Running couplings of the Standard Model gauge couplings g_1 , g_2 , g_3 , top quark Yukawa coupling y_t and Higgs self-coupling λ . Whether/where the Higgs self-coupling λ crosses zero determines the (meta)stability of the vacuum.
Source: The figure is from [52].

energy collisions involving massive spin-one particles requires that they satisfy Yang–Mills gauge invariance, also with fermion couplings, and that there is a Higgs providing the mass generation [21–24]. The Higgs propagation in intermediate states cancels otherwise unitarity violating terms from the longitudinal component of the massive Z^0 boson. The Higgs cannot be too heavy to do its job with maintaining unitarity. Indeed, if the Higgs had not been found at the LHC, some alternative mechanism would have been needed in the energy range of the experiments, e.g. involving strongly interacting WW scattering with the Higgs replaced by some broad resonance in the WW system [91].

4. Renormalisation group and vacuum stability

There are hints for possible critical phenomena in the ultraviolet if we can extrapolate LHC data up to close to the Planck scale.

The scale dependence of the running couplings α_3 for QCD and α_2 for the SU(2) weak interactions plus the U(1) α_1 are shown in Fig. 1. The non-abelian QCD and weak couplings are asymptotically free, decreasing logarithmically with increasing resolution. Their β functions

$$\beta(\alpha_i) = \mu^2 \frac{d}{d\mu^2} \alpha_i(\mu^2) \quad (51)$$

have zeros in the extreme ultraviolet at $\mu = \infty$. In contrast, the U(1) coupling α_1 increases logarithmically in the ultraviolet. Since the Planck scale is very much less than the scale of any ultraviolet Landau pole, α_1 is always finite. With the particle masses and couplings measured at the LHC, the Standard Model works as a consistent theory with finite couplings up to the Planck scale.

The Higgs self interaction coupling λ also runs under renormalisation group – see Fig. 1. Its β -function

$$\beta(\lambda) = \mu^2 \frac{d}{d\mu^2} \lambda(\mu^2) \quad (52)$$

is found to have a zero close to the Planck scale. The important issue for vacuum stability is that the β function for the Higgs four-boson self-coupling λ has a zero and when (if at all) this coupling λ crosses zero.

The sign of $\beta(\lambda)$ is dominated by the large negative top quark Yukawa coupling which yields a negative β -function at laboratory energies and remains negative up to (close to) the Planck scale. In the absence of the large top quark coupling (and also with the Yukawa sector switched off), the sign of $\beta(\lambda)$ would be driven by the positive contribution from λ (and the other bosons). It turns out that the running of λ is more sensitive to the value of y_t than on λ itself. It is a surprising property of the Standard Model and its specific parameter values, which leads to an asymptotically free behaviour of the Higgs boson self-coupling up to not far below the Planck scale. A similar flip of the renormalisation group behaviour

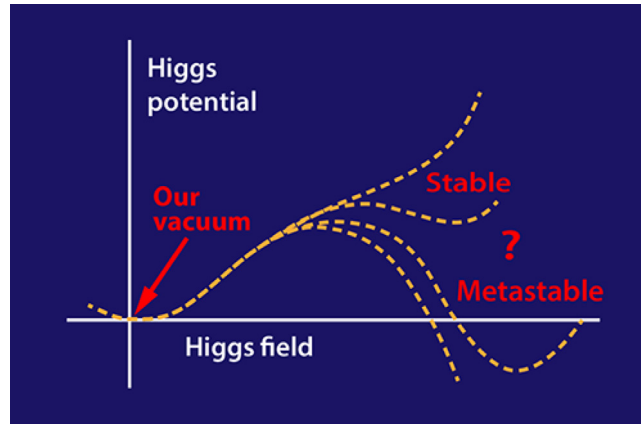


Fig. 2. Vacuum stability scenarios. The figure is from [94], APS/Alan Stonebraker. Here the curve is normalised with the minimum of “our vacuum” at $|\phi| = \frac{v}{\sqrt{2}}$ with $v = 246$ GeV, see Eqs. (37) and (45).

applies to the top quark Yukawa coupling. A pure Yukawa model would not behave as an asymptotically free theory. It is the interplay with QCD (with the top quark strongly interacting) that yields a negative β -function at low energies which remains negative up to the Planck scale. These properties cannot be accidental, because if the Standard Model parameters would change only somewhat this important behaviour would be lost, and the emerging low energy effective theory would vastly differ from the Standard Model.

Electroweak vacuum stability requires that λ remains positive. Otherwise, with λ negative definite the vacuum is unstable. The metastable case is that λ goes negative and comes back positive with half-life of the Universe much bigger than its present age – see Fig. 2. Vacuum metastability is a delicate issue and requires a more sophisticated form of the potential which only appears when radiative corrections are taken into account. The resulting effective potential then allows for a second minimum of the potential not far below the Planck scale. Globally, the vacuum remains unstable, viz. for large Higgs field values the potential takes the form $\propto \lambda_{\text{eff}}(\mu = \phi)\phi^4$ where $\lambda_{\text{eff}}(\mu = \phi)$ turns negative near the Planck scale and beyond. An unstable electroweak vacuum would require some new additional interaction at higher scales to stabilise it.

Taking the masses and couplings measured up to LHC energies, one finds that the electroweak vacuum resides very close to the border of stable and metastable [15,52–59] – see Fig. 3 – suggesting possible new critical phenomena in the ultraviolet. The Higgs vacuum sits within 1.3 standard deviations of being stable on relating the top quark Monte-Carlo and pole masses if we take just the Standard Model interactions with no coupling to undiscovered new particles and evolve using three-loop perturbative evolution and two-loop matching conditions up to the highest scales of order the Planck mass [59]. In these calculations electroweak vacuum stability is very sensitive to Higgs couplings, especially the values of the Higgs and top quark masses and to the technical details of higher-order radiative corrections. In calculations with a metastable vacuum λ typically crosses zero around 10^{10} GeV [52,53] whereas it remains positive definite with a stable vacuum [15]. For the measured value of m_t, m_h is very close to the smallest value to give a stable vacuum. The 1.3σ difference from a stable vacuum is reduced if one includes the difference, about 600 MeV, in the top quark Monte-Carlo and pole mass definitions discussed in [92]. One finds a delicate interplay of Standard Model masses and couplings and the physics of the deep ultraviolet. This opens the possibility that the Higgs scale might be set by physics close to the Planck scale with an implicit reduction in the number of fundamental parameters [15,59,93]. The Standard Model might behave as an effective theory with characteristic energy close to the Planck scale with no new scale between the electroweak scale and close to the Planck mass.

The Higgs vacuum sitting “close to the edge” of stable and metastable suggests possible new critical phenomena in the ultraviolet [15,52,53]. One interpretation is a statistical system in the ultraviolet close to the Planck scale close to its critical point. Criticality might be an attractor point in the dynamical evolution [52,53]. For interpretation in terms of multiverse ideas see [52,53]. The theoretical challenge is to identify the universality class of theories which have the Standard Model (plus any new particle interactions waiting to be discovered) as their long range asymptote [15].

A possible extra source of critical phenomena in the ultraviolet is connected to quantum corrections to the Higgs mass. The Standard Model Higgs mass squared comes with a quadratically divergent counterterm. The renormalised mass squared is related to the bare mass term by

$$m_{h \text{ bare}}^2 = m_{h \text{ ren}}^2 + \delta m_h^2 \quad (53)$$

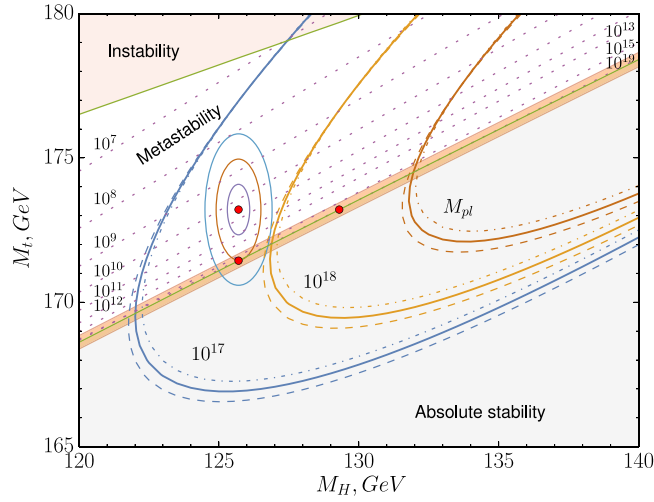


Fig. 3. Vacuum (meta)stability of the Standard Model showing the Monte-Carlo top quark mass and Higgs mass with one, two and three standard deviations ellipses. The figure is from [59].

where

$$\begin{aligned} \delta m_h^2 &= \frac{\Lambda^2}{16\pi^2} \left(12\lambda + \frac{3}{2}g_1^2 + \frac{9}{2}g_2^2 - 12y_t^2 \right) \\ &= \frac{\Lambda^2}{16\pi^2} \frac{6}{v^2} \left(m_h^2 + m_Z^2 + 2m_W^2 - 4m_t^2 \right). \end{aligned} \quad (54)$$

Here λ is the Higgs self coupling, g_1 and g_2 are the U(1) and SU(2) couplings and y_t is the Yukawa top quark coupling to the Higgs; v is the Higgs vacuum expectation value, about 246 GeV at the scale of the experiments. The particle masses and Higgs Yukawa couplings are related through Eqs. (48), (49). Here we take the same ultraviolet cut-off scale Λ for each particle and, for simplicity, include just the heaviest t , h , W , Z particles. The effect of including the two-loop order correction is moderate as discussed in [15,95].

The Higgs mass hierarchy puzzle enters when Λ is taken as a physical scale. Why is the physical Higgs mass so small compared to the cut-off? For the Standard Model, the quadratic divergence in the Higgs mass self energy would cancel if the coefficient of Λ^2 vanishes, viz.

$$2m_W^2 + m_Z^2 + m_h^2 = 4m_t^2. \quad (55)$$

This equation is the Veltman condition [96]. The Higgs mass hierarchy puzzle would be resolved if the Veltman condition would hold (at some scale) as a collective cancellation between fermion and boson contributions. The Veltman condition does not work with the PDG masses $m_t = 173$ GeV, $m_W = 80$ GeV and $m_Z = 91$ GeV, where one would need a Higgs mass about 314 GeV before renormalisation group evolution. With $m_h = 125$ GeV, δm_h^2 exceeds m_h^2 for Λ values bigger than about 600 GeV.

The terms in the coefficient of Λ^2 enter with different signs for boson and fermion contributions. Further, the running masses and couplings each have different renormalisation group scale dependence. This means there is a chance of crossing zero at some much higher scale. The scale of Veltman crossing is calculation dependent. Values reported are 10^{16} GeV with a stable vacuum [15], about 10^{20} GeV [97] and much above the Planck scale of 1.2×10^{19} GeV [52] with a metastable vacuum. With the Standard Model evolution code [98] crossing is found at the Planck scale with a Higgs mass about 150 GeV, and not below with the measured mass of 125 GeV [99]. These results are very sensitive to the value of the top quark Yukawa coupling, which presently is known only via the top quark mass and conceptually is difficult to measure, with connection to differences in the Monte-Carlo and pole masses.

Veltman crossing means that the renormalised and bare Higgs mass squared first coincide. If we can extrapolate above this scale, then the bare mass squared changes sign with first order phase transition to a symmetric phase with vanishing Higgs vev [15]. In this case the W and Z bosons and charged fermions become massless, and the Nambu–Goldstone modes eaten to become the longitudinal components of the W and Z are liberated to become massive spin-zero bosons. Jegerlehner argues that in this scenario the Higgs might act as the inflation at higher mass scales in a symmetric phase characterised by a very large bare mass term for the Higgs scalars and vacuum energy contribution, about 10^{15} GeV [100]. Alternatively, the Higgs might be emergent from some more primordial degrees of freedom at the scale of electroweak symmetry breaking together with the onset of mass generation.

Table 1

Typical operators in a low energy expansion. The large ultraviolet scale M is expected to be greater than about 10^{15} GeV and close to the Planck scale.

Source: Table adapted from [101].

	Dimension	Operator	Scaling behaviour
↑ no data 	·	∞-many irrelevant operators	
	·		
	·		
	$d = 6$	$(\square\phi)^2, (\bar{\psi}\psi)^2, \dots$	$(E/M)^2$
	$d = 5$	$\bar{\psi}\sigma^{\mu\nu}F_{\mu\nu}\psi, \dots$	(E/M)
 experimental data ↓	$d = 4$	$(\partial\phi)^2, \phi^4, (F_{\mu\nu})^2, \dots$	$\ln(E/M)$
	$d = 3$	$\phi^3, \bar{\psi}\psi$	(M/E)
	$d = 2$	$\phi^2, (A_\mu)^2$	$(M/E)^2$
	$d = 1$	ϕ	$(M/E)^3$

5. Possible emergent gauge symmetries in particle physics

At this point it is helpful to recollect where we are.

The tremendous success of the Standard Model at predicting and explaining the results of all our experiments, together with the curious result that the Standard Model Higgs vacuum sits close to the border of stable and metastable when LHC measured couplings are extrapolated up to the Planck scale raise interesting questions about its symmetries. Might the Standard Model gauge symmetries be more special than previously expected? Perhaps the Standard Model is an effective theory with characteristic energy close to the Planck scale. Might there be some new critical phenomena in the ultraviolet with Standard Model physics as the long range tail of a critical Planck-scale system?

With the Standard Model gauge symmetries taken as emergent and dynamically generated, the full theory is not the physics truncated to mass dimension four operators but also includes an infinite tower of higher-dimensional terms suppressed by powers of the large emergence scale — see Table 1. At low energies the physics is determined by a relatively small number of operators with mass dimension at most four. For these terms gauge invariance and renormalisability restrict the number of possible operator contributions and strongly constrain the global symmetries of the system. Extra symmetry breaking terms can occur in higher dimensional operators which enter the action suppressed by powers of the large scale of emergence, for example with lepton and baryon number violation discussed in Section 6. These higher-dimensional terms only become active in the particle dynamics when we are sensitive to mass and energy scales close to the large emergence scale. In the formal language of renormalisation group, the operators with mass dimension less than four are called relevant operators, dimension four operators are called marginal operators, and higher dimensional operators are called irrelevant operators.

The few relevant and marginal operators can be invariant under a wider range of field transformations than a generic irrelevant operator would be. The effects of irrelevant operators are strongly suppressed at low energies (suppressed by powers of the large emergence scale), making it appear that the theory has a larger symmetry group. Symmetry can be emergent in the low energy theory even if it is not present in the underlying microscopic theory, e.g. associated with an infrared fixed point in the language of the renormalisation group. This scenario differs from unification models which have maximum symmetry at the highest energies and where symmetry breaking is generally understood as originating from spontaneous symmetry breaking, e.g. mass terms induced by the Higgs mechanism and chiral symmetry breaking in QCD. In a low-energy expansion any unitarity violating terms are suppressed by powers of the large ultraviolet scale M .

6. The Standard Model in a low-energy expansion

We have seen examples of higher-dimensional operators with the Pauli operator in QED and Fermi four-fermion interaction with pre Standard Model weak interactions. Searches for evidence of higher mass dimension corrections in LHC data are so far consistent with zero meaning the ultraviolet cut-off scale is much above present LHC energies, for recent discussion see [102]. We next discuss key examples where higher dimensional operators may contribute to new global symmetry breaking beyond the Standard Model as truncated to mass-dimension four.

6.1. Neutrino masses and mixings

Tiny neutrino masses and neutrino flavour mixing are deduced from solar, atmospheric and reactor neutrino disappearance experiments as well as from accelerator based appearance and disappearance experiments. In addition, recent measurements by the T2K Collaboration in Japan are consistent with CP violation in the neutrino sector at the level of two standard deviations [60,61]. One finds differences between the lightest and second lightest neutrino masses

$$\delta m^2 = 7.37^{+0.17}_{-0.16} \times 10^{-5} \text{ eV}^2 \quad (56)$$

and between the second lightest and heaviest neutrinos

$$|\Delta m^2| = 2.525^{+0.042}_{-0.030} \times 10^{-3} \text{ eV}^2. \quad (57)$$

Mixing angles are

$$\sin^2 \theta_{12} = 0.297^{+0.017}_{-0.016}, \quad \sin^2 \theta_{13} = 0.0215^{+0.0007}_{-0.0007}, \quad \sin^2 \theta_{23} = 0.425^{+0.021}_{-0.015} \quad (58)$$

with measurement of the CP violating phase

$$\delta = 1.38^{+0.23}_{-0.20} \times \pi. \quad (59)$$

These numbers are quoted without bias on the mixing ordering, whether the neutrinos follow the normal ($\nu_e < \nu_\mu < \nu_\tau$) or inverted ($\nu_\tau < \nu_\mu < \nu_e$) ordering of masses [103].

Neutrinos have no electric charge and might be either Dirac or Majorana particles. Majorana neutrinos have the property that each mass eigenstate with a given helicity coincides with its own antiparticle with the same helicity. Dirac neutrinos would come with the usual mass terms $m_\nu \bar{\nu} \nu$ generated by Higgs-like Yukawa couplings. Dirac neutrinos would imply the existence of (possibly sterile) right-handed neutrinos. Majorana neutrinos come with mass generation through the Weinberg dimension-five operator [26]

$$O_5 = \frac{(\Phi L)_i^T \lambda_{ij} (\Phi L)_j}{M} \quad (60)$$

which naturally explains the tiny neutrino masses with large values of the ultraviolet scale M , about 10^{15} GeV. In Eq. (60) Φ is the Higgs doublet, L_i denotes the SU(2) left-handed lepton doublets defined in Eq. (25), and λ_{ij} is a matrix in flavour space. Neutrinos and antineutrinos can be identified if lepton number is not conserved and not a good quantum number. The Majorana mass term $\nu_L^T \nu_L$ violates lepton number conservation by two units (through the process $\nu \rightarrow \bar{\nu}$). With lepton number non-conservation, Majorana neutrinos come with the experimental signature that they trigger neutrino-less double β -decays, with a vigorous experimental programme dedicated to search for evidence of this process.

In general Majorana neutrinos come with two extra CP mixing angles. On the basis of present measurements one cannot say anything about the possible size of these extra angles.

6.2. Axion couplings

The large η' mass in QCD is induced by non-perturbative glue, the detailed dynamics of which are still discussed. Independent of the detailed QCD dynamics one can construct low-energy effective chiral Lagrangians which include the effect of the QCD axial anomaly and use these Lagrangians to study low-energy processes involving the η and η' [104–106].

Define $U = e^{i(\phi/F_\pi + \sqrt{2/3}\eta_0/F_0)}$ as the unitary meson matrix where $\phi = \sum \pi_a \lambda_a$ denotes the octet of would-be Goldstone bosons π_a associated with spontaneous chiral symmetry breaking with λ_a the Gell-Mann matrices, η_0 is the singlet boson and F_0 is the singlet decay constant (at leading order in the chiral expansion taken to be equal to $F_\pi = 92$ MeV). The gluonic mass contribution $\tilde{m}_{\eta_0}^2$ is introduced via a flavour-singlet potential involving the topological charge density Q_t in Eq. (22) which is constructed so that the Lagrangian also reproduces the axial anomaly. This potential reads

$$\frac{1}{2} i Q_t \text{Tr} \left[\log U - \log U^\dagger \right] + \frac{3}{\tilde{m}_{\eta_0}^2 F_0^2} Q_t^2 \mapsto -\frac{1}{2} \tilde{m}_{\eta_0}^2 \eta_0^2 \quad (61)$$

where Q_t is eliminated through its equation of motion to give the gluonic mass term for the η' . The Lagrangian contains no kinetic energy term for Q_t , meaning that the gluonic potential does not correspond to a physical state; Q_t is therefore distinct from mixing with a pseudoscalar glueball state. The $Q_t \eta_0$ coupling in Eq. (61) describes a picture of the η' as a mixture of chirality-two quark-antiquark and chirality-zero gluonic contributions [71].

The non-perturbative gluonic topology which generates the gluonic contribution to the η' mass also has the potential to induce strong CP violation in QCD. One finds an extra term, $-\theta_{\text{QCD}} Q_t$, in the effective Lagrangian for axial U(1) physics which ensures that the potential

$$\frac{1}{2} i Q_t \text{Tr} \left[\log U - \log U^\dagger \right] + \frac{3}{\tilde{m}_{\eta_0}^2 F_0^2} Q_t^2 - \theta_{\text{QCD}} Q_t \quad (62)$$

is invariant under global axial U(1) transformations with $U \rightarrow e^{-2i\alpha}U$ acting on the quark fields being compensated by $\theta_{\text{QCD}} \rightarrow \theta_{\text{QCD}} - 2\alpha N_f$.

The term $\theta_{\text{QCD}}Q_t$ is odd under CP symmetry. If it has non-zero value, θ_{QCD} induces a non zero neutron electric dipole moment [107]

$$d_n = 5.2 \times 10^{-16} \theta_{\text{QCD}} \text{ ecm.} \quad (63)$$

Experiments constrain $|d_n| < 3.0 \times 10^{-26} \text{ e.cm}$ at 90% confidence limit or $\theta_{\text{QCD}} < 10^{-10}$ [63].

Why is the strong CP violation parameter θ_{QCD} so small? QCD alone offers no answer to this question. QCD symmetries allow for a possible θ_{QCD} term but do not constrain its size. The value of θ_{QCD} is an external parameter in the theory just like the quark masses are.

Non-perturbative QCD arguments tell us that if the lightest quark had zero mass, then there would be no net CP violation connected to the θ_{QCD} term [84]. However, chiral dynamics tells us that the lightest up and down flavour quarks have small but finite masses. In the full Standard Model the parameter which determines the size of strong CP violation is $\Theta_{\text{QCD}} = \theta_{\text{QCD}} + \text{Arg det } \mathcal{M}_q$, where \mathcal{M}_q is the quark mass matrix. Possible strong CP violation then links QCD and the Higgs sector in the Standard Model that determines the quark masses.

A possible resolution of this strong CP puzzle is to postulate the existence of a new very-light mass pseudoscalar called the axion [28,29] which couples through the Lagrangian term

$$\mathcal{L}_a = -\frac{1}{2} \partial_\mu a \partial^\mu a + \left[\frac{a}{M} - \Theta_{\text{QCD}} \right] \frac{\alpha_s}{8\pi} G_{\mu\nu} \tilde{G}^{\mu\nu} - \frac{if_\psi}{M} \partial_\mu a \bar{\psi} \gamma_5 \gamma^\mu \psi - \dots \quad (64)$$

Here the term in ψ denotes possible fermion couplings to the axion a with $f_\psi \sim \mathcal{O}(1)$. The mass scale M plays the role of the axion decay constant and sets the scale for this new physics. The axion transforms under a new global U(1) symmetry, called Peccei–Quinn symmetry [108], to cancel the Θ_{QCD} term, with strong CP violation replaced by the axion coupling to gluons and photons. The axion here develops a vacuum expectation value with the potential minimised at $\langle \text{vac} | a | \text{vac} \rangle / M = \Theta_{\text{QCD}}$. The mass of the QCD axion is given by [84]

$$m_a^2 = \frac{F_\pi^2}{M^2} \frac{m_u m_d}{(m_u + m_d)^2} m_\pi^2. \quad (65)$$

Note that the axion coupling and mass enter at mass dimension five with the $1/M$ suppression factor.

Axions are possible dark matter candidates. Constraints from experiments tell us that M must be very large. Laboratory based experiments based on the two-photon anomalous couplings of the axion [109], ultracold neutron experiments to probe axion to gluon couplings [110], together with astrophysics and cosmology constraints suggest a favoured QCD axion mass between 1 μeV and 3 meV [30,111], which is the sensitivity range of the ADMX experiment in Seattle [112], corresponding to M between about 6×10^9 and 6×10^{12} GeV. The small axion interaction strength, $\sim 1/M$, means that the small axion mass corresponds to a long lifetime and stable dark matter candidate, e.g., lifetime longer than about the present age of the Universe. If the axions were too heavy they would carry too much energy out of supernova explosions, thereby observably shortening the neutrino arrival pulse length recorded on Earth in contradiction to Sn 1987a data [111].

6.3. Baryon number violation

In addition to lepton number violation, there is also the possibility of baryon number violation which can arise through the four-fermion operator [26,27]

$$O_6 = \frac{1}{M^2} QQQ L \quad (66)$$

which enters with $1/M^2$ suppression. Here L and Q are the lepton and quark doublets defined in Eq. (25). This leads to two body decays like $p \rightarrow e^+ \pi^0$ with rate $\Gamma \propto m_p^5 / M^4$ where m_p is the proton mass. Present experimental sensitivity to such decays is about 10^{34} years [113] from the SuperKamiokande experiment, corresponding to an ultraviolet scale $M \sim 10^{15}$ GeV.

The dimension-six operator in Eq. (66) becomes active at very high energies, e.g. close to the start of the Universe, and might play an important role in understanding the baryon asymmetry that we see. Left over from the early Universe, the number of baryons compared to antibaryons is finite and exceeds the number of photons in the Universe by a factor $\eta_B = (n_B - n_{\bar{B}}) / n_\gamma \sim 6 \times 10^{-10}$ [47]. Three conditions identified by Sakharov [114] to explain the baryon asymmetry are baryon number non-conservation, CP violation and C violation so that processes occur at different rates for particles and antiparticles in the early Universe, and departure from thermal equilibrium. Otherwise if the Universe starts with zero baryon number it will stay with zero baryon number. Further higher dimensional operators, with mass dimension 8, that violate charge conjugation and time reversal invariance while conserving parity are discussed in [115].

7. Conclusions and outlook: unification or emergence?

Emergent gauge symmetry involves making symmetry instead of breaking it.

Given the great success of the Standard Model at LHC collider energies and in precision experiments it is worthwhile to re-evaluate our ideas about fundamental symmetries and their origin in particle physics. The Standard Model provides an excellent description of nature up to the TeV scale with no evidence (so far) for new particles or interactions in the energy range of our experiments. Further, the Standard Model works as a consistent theory up to the Planck scale with vacuum that sits close to the border of stable and metastable. Might the gauge symmetries that determine particle interactions be emergent from some new critical system which exists close to the Planck scale? Emergent gauge symmetries are observed in quantum condensed matter systems.

With emergent gauge symmetries, the Standard Model is an effective theory with action containing an infinite series of higher-dimensional operators whose contributions are suppressed by powers of the large scale of emergence. In this scenario, the leading term (operators up to mass dimension four) contributions are renormalisable operators with greatest global symmetry. Experimental constraints on the size of the Pauli term, tiny neutrino masses and constraints on axion masses and proton decay suggest an ultraviolet scale M greater than about 10^{10} GeV and perhaps between 10^{15} GeV and the Planck scale of 1.2×10^{19} GeV. It is interesting that considerations of electroweak vacuum stability suggest either a stable vacuum or metastable vacuum with the Higgs self-coupling crossing zero in the same range of energy scales. The M -scale suppressed higher dimensional terms only start to dominate the physics when we become sensitive to scales close to M , e.g. sensitive to physics processes which happened close to the start of the Universe. That is, at the very highest energies the system becomes increasingly chaotic with maximum symmetry breaking in contrast to unification models which exhibit the maximum symmetry in the extreme ultraviolet.

Experimental signatures of this approach include lepton-number violating interactions at dimension-five with Majorana neutrino masses and neutrino-less double β -decays and, at dimension-six, baryon number violation which might also induce proton decays in future precision experiments. Small gauge groups are most likely preferred.

An emergent Standard Model gauge may help with explaining open puzzles in particle physics. The lepton and baryon number violating interactions in Eqs. (60) and (66) become active at very high energies, e.g. typical of processes in the very early Universe, and might play an important role in understanding the matter–antimatter asymmetry in the Universe. Cosmology observations point to an energy budget of the Universe where just 5% is built from Standard Model particles, 26% involves dark matter (possibly made of new elementary particles) and 69% is dark energy [48]. Dark energy – within present experimental errors the cosmological constant in Einstein's theory of General Relativity – is commonly interpreted as the energy density of the vacuum [116–118]. On distance scales much larger than the galaxy the Universe exhibits a large distance flat geometry. Dark matter clumps together under normal gravitational attraction whereas the cosmological constant is the same at all points in space–time and drives the accelerating expansion of the Universe. New axion like particles with masses and couplings suppressed by powers of the large emergence scale might be a vital ingredient in understanding the dark matter. Possible consequences of emergent symmetries for cosmology will be discussed elsewhere [99].

A key theoretical issue is the scale of emergence. If the scale of Veltman crossing happens below the scale of emergence, then the Higgs might play an essential role in inflation. If electroweak symmetry breaking and emergence were to happen at the same scale, then the physics of inflation would involve totally new physics with different unknown degrees of freedom. Electroweak physics comes with parity violating couplings of the gauge bosons and possible Majorana neutrinos. This prompts the question whether chirality (and neutrinos) might play a special role in any ultraviolet critical phenomena leading to emergent gauge symmetry in the infrared. Here it is interesting that the two-space-dimensional Ising model at its critical point is equivalent to an effective theory of Majorana fermions. One might speculate whether this result holds also in four space–time dimensions. The Ising model without external magnetic field has the same equation of state as the quantum vacuum [119].

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